CORRESPONDENCE

May I make four comments on R. H. Daw's historical paper on J. H. Lambert in *J.I.A.* **107,** 345.

1. As there is no text in English on the history of actuarial techniques one could make a long list of authors "whose work has largely been overlooked" in Great Britain and North America. Lambert's name is at least mentioned in connection with the "law of mortality" in Westergaard (1932, p. 129) and a Swiss actuary just before the war (Linder, 1936) claimed that in the field of population statistics Lambert "is worth studying even in our days".
2. Linder was writing partly to draw attention to Lambert's priority with his Lebenskraft (force of vitality), namely μ_x^{-1}, and this priority was mentioned in *J.S.S.*, **6,** 197.
3. Lambert's use of a graph to estimate l_{45}, l'_{45} and l'_{90} besides observed decennial values of l_{60}, l_{70} and l_{90} to calculate a quintic polynomial for l_x is indeed a forerunner of osculatory interpolation. Because Lambert did not use the result in his further work this achievement has tended to be passed over in the history of graduation (e.g. Saar, 1917, p. 24) and emphasis laid on the poor agreement of his l_x-formula (Daw's § 2.6) with the observations on which it was based (Moser, 1839, p. 276). In the non-actuarial world osculatory interpolation is called 'Hermite interpolation' the word 'osculating' sometimes being inserted (e.g. Isaacson & Keller, 1966; Hildebrand, 1974). This is because Hermite (1878) generalized the well-known Lagrange interpolation formula based on n ordinates to one based on $m<n$ abscissa at some of which first derivatives were given, at others two orders of derivatives and so on, the aggregate data numbering n. The article anticipated Sprague's main exposition of his less general method but not his first use of it. The writer considers the ingenuity of Sprague's method to lie in his calculation of the required first and second derivatives from the observations themselves rather than, like Hermite, to assume them given. In any case Lambert must be given credit for the first departure from Lagrange.
4. The translation of the 'smallpox' portion of Lambert's text should make it easy to verify whether Seal (1977) was right when he said that Lambert did not add anything to Bernoulli's theory.

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H. L. SEAL

I would like to thank Mr Seal for his comments on my Lambert paper (*J.I.A.* **107,** 345), in particular for an interesting glimpse into the origin of mathematical (as distinct from practical) osculatory interpolation.

When writing the paper I was aware that there are occasional mentions of parts of Lambert's work, like those referred to by Mr Seal, but did not think it necessary to quote any of them. Had I wished to give an example I would have come nearer home and referred to the review in *J.I.A.* **59,** 104, by G. W. Richmond which briefly outlined Lambert's work as described in Loewy (1927).

https://doi.org/10.1017/S0020268100040713 Published online by Cambridge University Press

In item 3 Mr Seal says that Moser (1839, p. 276) emphasizes the poor agreement of Lambert's exponential formula (my § 2.6) "with the observations on which it is based". However, the scope for such criticism is small, since the largest difference which Lambert shows between formula and observations is 44 for l_x's of four figures. I interpret Moser as saying, principally, that the formula value of l_1 (for which no observed value is shown) i.e., 8146, shows too low a rate of infant mortality, as compared with other experiences. Moser does not notice that Lambert's formula value of l_2 is substantially incorrect (see my § 2.6.1) and this has some effect on his argument. So far as I know this error (and another of 3099 instead of 3071 for l_{40}) was not noticed until Eisenring (1948, p. 121).

Had Lambert calculated the correct value of l_2 by his formula, I suspect that he might have been able to improve the fit of his formula by adjusting the numerical constants. It would be an interesting exercise to try this.

I am puzzled by the last sentence of Mr Seal's item 3 in which he says that Lambert (1765) "must be given credit for the first departure from Lagrange". I find (e.g. Whittaker and Robinson, 1944, p. 29 and Karl Pearson, 1978, p. 620) that the so called 'Lagrange' interpolation formula was first given by Waring (1779), that is 14 years after Lambert wrote!

Details of any references in this letter not listed in my paper or Mr Seal's letter are set out below.

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