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Repeated Vibrational Motion Using an Inertial Drive

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Abstract

The primary purpose of this paper is to demystify the vibrational motion caused by a typical inertial (Dean) drive when is attached to a heavier object (vehicle). A Dean drive consists of two symmetric eccentric masses which are rigidly connected to rotating rods and driven by (say electric) motors. Therefore, the force components along the line *l* which joins them are cancelled and thus the two rotating masses operate together as a single equivalent driving mass which vibrates along the perpendicular midline to *l* passing through the object. Hence the system "vehicle + drive" reduces to a "two-body" problem. The contra-rotation of the two eccentrics is shown to cause a unidirectional displacement of the connected object for which closed-form analytical equations of motion are obtained in terms of the conservation of linear momentum. A first novel feature of the paper is that it discusses alternative design configurations. Within this context, the Dean-drive itself is shown to be almost equivalent to a spring-mass system attached above the vehicle whereas a similar vehicle's motion is obtained when a single spring is put below the vehicle. In the hitherto published classical version of the air-operated mechanism, the vehicle performs a reduced travel distance until it reaches the upper point of its trajectory. A second novel feature of this work is to demonstrate that repeated (endless) unidirectional motion of the vehicle is ensured when external support forces are exerted periodically during specific short time intervals at the end of each cycle of the travel.

Keywords: *Vibrational motion; Dean drive; Inertial propulsion; Support force; Transportation.*

Nomenclature

CM	center of mass
E	energy (Nm)
DOF	degree of freedom
F	force (N)
k_s	spring stiffness (m^2)
m	eccentric mass (kg)
M	vehicle's mass (kg)
P	linear momentum (kg m/s)
r	eccentricity (m)
t	time (s)
T	period of rotation (s)
*T**	period of travel cycle (s)
V	velocity (m/s)*Z* height (m)

Greek symbols

θ	angle (rad)
λ	constant (m)
ω	angular velocity (rad/s)

Subscripts

1	rotating mass No. 1
2	rotating mass No. 2

1. Introduction

For a long time, vibration was considered mostly as a harmful side effect which accompanied the operation of machines and apparatus. Only in the last several decades has vibration been applied to obtaining useful effects, and have the so-called vibratory machines and devices appeared (Blekhman [1]). Typical examples are vibro-compactors used in modern hydrotechnical

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construction, apparatus for concrete mixtures, vibro-machines and devices such as sizing screens, conveyors, feeders, mills, breakers, flotation machines, shakers and many other devices widely used in various industries (Blekhman [1]).

Among the abovementioned mechanisms, a “Dean drive” is a patented device having two eccentric masses that rotate in opposite direction about parallel linear axes which are rigidly connected and driven by (say electric) motors (Dean [2]). When attached to an object (vehicle), under special circumstances it has the ability to drive it. And since this motion is due to (internal) inertial forces, we usually refer to “inertial propulsion”. One additional class of industrial applications of such a device are the micro-robot driving platforms (see Ju et al. [3], and 21 papers therein), as well as larger mobile systems (see Lukanov et al. [4], and papers therein).

For the sake of completeness, it is worthy to mention that despite the fame of the abovementioned Dean drive (Norman L. Dean was an American citizen who granted his USA patent in 1959), the idea of “inertial propulsion” is very old and first appeared in Europe. Thus ancient Greeks used to hold and move the so-called “halteres” to elongate their long jump (Minetti and Ardigo [5]). Also, in 1933, the Italian professor Marco Todeschini granted a patent for a centrifugal water powered propulsion motor with dissociated gradual automatic speed variation (Todeschini [6]). A mechanical analysis of the latter idea and detailed reasoning for considering eccentrics which follow eight-shaped paths, instead of circular ones, was later reported (see Provatidis [7] and papers therein).

Although there are many out-of-stream older claims that a Dean drive is a supposed reactionless device producing motion without the exhaust of a propellant (for example, see Wikipedia [8], Dempewolff [9], Stine [10], and many YouTube videos accessible in Internet such as [11-19]), Provatidis [20] has shown that the secret is that the involved eccentrics must offer an *initial velocity* to the center of the mass of the system “object + drive” when it is left free to move. As a result, the object may perform a motion in the air, like an oscillating projectile, of course with decreasing average velocity at the center of mass due to the gravity. In other words, the system practically operates like a catapult which launches a projectile. For water [21] or ground [22] applications, the reduced travel distance is due to the dominating friction. Repeated endless motion on the ground may be achieved using the stick-slip phenomenon (see, Provatidis [23]). An extension of these ideas in the field of electromagnetic was reported by Provatidis and Gamble [24]; the latter study was carried out jointly by university and industry.

Despite the above findings, the mechanical model of the Dean drive and its categorization in dynamics is still unclear, thus this paper continues the research on the following issues:

1) To avoid any doubt, equations of motion will be derived in terms of the conservation of linear momentum.
2) It will be shown that, with adequate accuracy, a Dean-drive unit can be substituted by an undamped spring-mass system, or even by a single spring below the vehicle.
3) A specific procedure will be proposed to extend the reduced travel length and maintain motion in the air toward a certain (say vertical upward) direction.

2. Equations of Motion

Here the model restricts to an object (vehicle) of mass M lying on the horizontal ground. Two point masses (No. 1 and No. 2) also called “eccentrics”, each of mass m are connected to corresponding rigid rods of length (radius) r which are further attached to the aforementioned vehicle at the points O_1 and O_2, respectively, and contra rotate at an angular velocity ω (period $T = 2\pi/\omega$), as shown in Figure 1. The initial polar angle of the rigid bars at time $t = 0$ is denoted by θ_0, whereas the initial center of mass of the entire system [mass total $(2m+M)$] by CM_0 (note, in capital letters). The axis origin O has been chosen to be fixed in the middle of the initial segment O_1O_2, at a constant distance H_O from the ground, where the inertial observer sits. At the arbitrary time instance t, the midpoint O' of the segment O_1O_2 of the vehicle and the position of the masses are denoted by Z_M and Z_m, respectively (Figure 1).

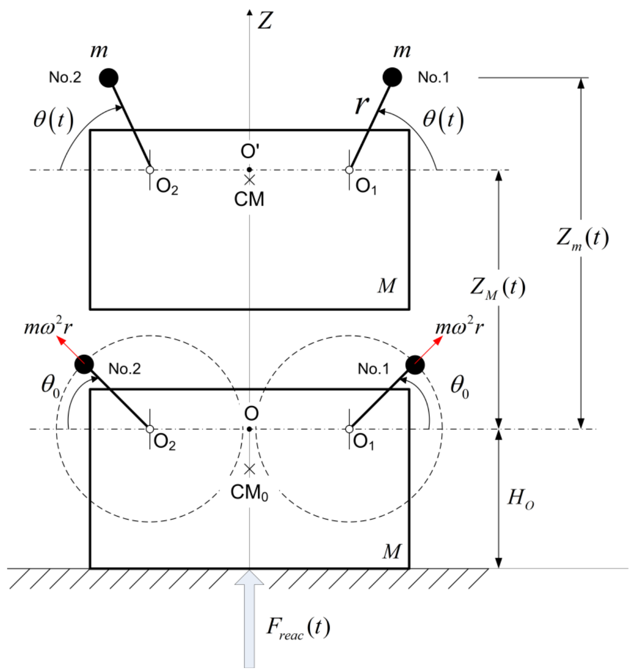

Figure 1. Initial and arbitrary position of the mechanical system.

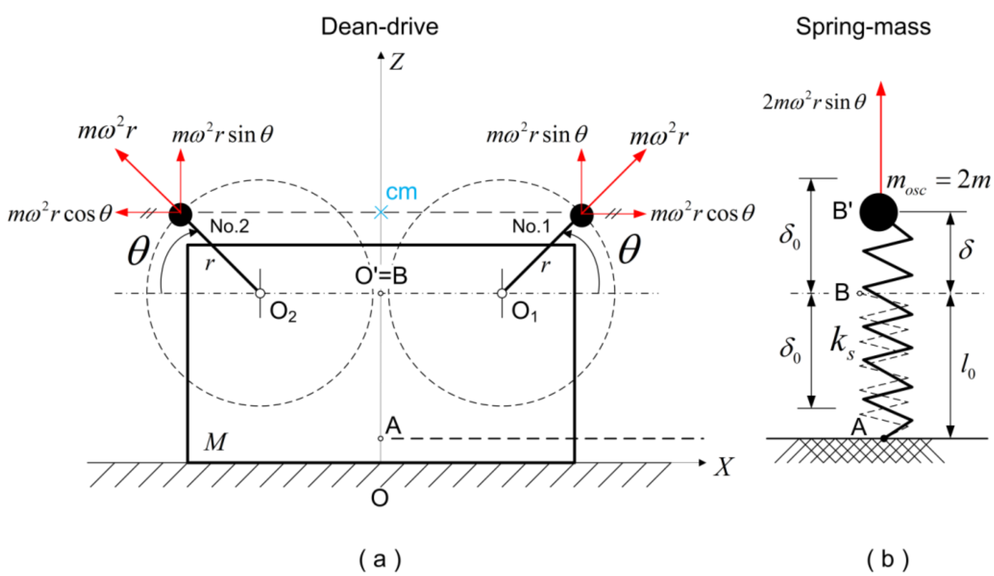

Figure 2. Equivalence of (a) Dean drive with (b) Spring-mass system.

Let us now focus on the action of the two rotating masses, of which the corresponding connecting rods form a polar angle $\theta = \theta(t)$. The inertial observer understands two centripetal forces, F_{cp1} and F_{cp2}, each of magnitude $m\omega^2 r$, which are further analyzed in horizontal and vertical components, as shown in Figure2a. Due to the symmetry in the polar angles, the horizontal components are cancelled thus the total vertical component becomes

$$F_{cp,vert} = 2m\omega^2 r \sin\theta \tag{1}$$

Following previous literature (Provatidis [7, 20]), the following parameter (in units of length) is introduced

$$\lambda = \frac{2mr}{(2m+M)} \tag{2}$$

Based on the above configuration, the equations of motion may be derived choosing one of the following procedures:

1) Newton's Second Law applied to each of the two point masses, i.e. $2m$ and M, and elimination of the interaction force (due to Newton's third law).
2) Newton's Second Law applied to the center of mass "CM" (of total mass $2m + M$).
3) Lagrange equations.
4) Conservation of linear momentum.

For the first time, we follow the procedure for the *conservation of linear moment* in order to derive the analytical expressions for the velocity $V_M(t)$ and the position $Z_M(t)$ of the vehicle.

Assuming that initially the vehicle is at rest ($V_{M,0} = 0$), we distinguish the initial (at time $t = 0$) and the arbitrary (at time $t > 0$) values of the system's linear momentum, as follows:

Initial: $$P_0 = M \cdot 0 + 2m \cdot \omega r \cos\theta_0 \tag{3}$$

Arbitrary: $$P_t = M \cdot V_M + 2m \cdot (V_M + \omega r \cos\theta) \tag{4}$$

The change $\Delta P = P_t - P_0$ of the above quantities equals to the impulse of the external forces, i.e. the time-dependent integral $\int_0^t [-(2m+M)g]\,\mathrm{d}\tau \equiv -(2m+M)gt$, which gives the vehicle's velocity:

$$V_M(t) = -gt - \lambda\omega(\cos\theta - \cos\theta_0)\ , \tag{5}$$

As the velocity is the first derivative of the position with respect to the time t, i.e.

$$dZ_M(t)/dt = V_M(t)\,, \tag{6}$$

the time integration of Eq. (6) leads to

$$Z_M(t) = -\tfrac{1}{2} gt^2 + (\lambda\omega\cos\theta_0)t - \lambda(\sin\theta - \sin\theta_0) \tag{7}$$

The reader may recognize the classical terms $(-gt)$ and $(-1/2\,gt^2)$ in Eq. (5) and (7), respectively, well known from college physics (e.g. Halliday and Resnick [25, p. 59]). One can also see the major importance of the initial polar angle θ_0, and particularly the product $(\lambda\omega\cos\theta_0)$, in the second (linear) term at the right-hand side of Eq. (7).

Equation (5) depicts that due to the oscillatory motion of the vehicle there are two time instances within each rotation of the eccentric rods at which its velocity vanishes (i.e., $V_M(t) = 0$).

Therefore it is safer to work with the velocity of the center of mass which is given by

$$V_{CM} = \frac{2mV_m + MV_M}{2m + M} \tag{8}$$

Since the vertical component of the velocity at each rotating mass is the sum of the absolute and relative motion:

$$V_m = V_M + \omega r \cos\theta \,, \tag{9}$$

by virtue of Eq. (2), at any time *t*, Eq. (8) finally becomes

$$V_{CM}(t) = V_M(t) + \lambda\omega\cos\theta(t) \tag{10}$$

Therefore, assuming a vehicle initially at rest ($V_{M,0} = 0$ at t = 0), according to Eq. (10) the initial vertical velocity component of the center of mass in the mechanical system will be

$$V_{CM,0} = V_{M,0} + \lambda\omega\cos\theta_0 \tag{11}$$

Elementary mechanics (e.g. [25, pp. 61-64]) referring to upward projectile motion, suggests that the upper point which the center of mass can reach at time $t_{\max} = V_{CM,0}/g$ is $Z_{\max}^{CM} = (V_{CM,0})^2/(2g)$, thus

$$Z_{\max}^{CM} = \frac{(V_{M,0} + \lambda\omega\cos\theta_0)^2}{2g} \tag{12}$$

and the time span from take-off to the upper point will be:

$$t_{\max} = \frac{(V_{M,0} + \lambda\omega\cos\theta_0)}{g} \tag{13}$$

When the upper point of the CM is reached, the position of the vehicle will be:

$$Z_M(t_{\max}) = -\tfrac{1}{2} g t_{\max}^2 + (\lambda\omega\cos\theta_0)t_{\max} - \lambda(\sin\theta(t_{\max}) - \sin\theta_0) \,, \tag{14}$$

with the corresponding velocity

$$V_M(t_{\max}) = -g t_{\max} - \lambda\omega(\cos\theta(t_{\max}) - \cos\theta_0) \,, \tag{15}$$

At the same time, $t_{\max}$, the position of each rotating mass will be:

$$Z_m(t_{\max}) = Z_M(t_{\max}) + r\sin\theta(t_{\max}) \,, \tag{16}$$

with the corresponding vertical velocity component given by

$$V_m(t_{\max}) = V_M(t_{\max}) + \omega r\cos\theta(t_{\max}) \,, \tag{17}$$

Clearly, despite the fact that at the upper point the velocity of the CM vanishes, it is not for sure that both the vehicle and the rotating masses will be at rest as well, unless $\cos\theta(t_{\max}) = 0$ [see Eq. (10)]. Otherwise, they are calculated using Eq. (15) and Eq. (17), respectively [see also Eq. (26a, b and c), for a numerical example].

3. Support Forces

If the angular velocity ω is assumed to be constant, the inertial (d' Alembert) forces induced by the two rotating masses m are directed outward in the radial direction and are finally transferred to the vehicle. As previously was mentioned, the contra rotation cancels the horizontal components of the inertial forces thus a vertical component of magnitude $F_{inert} = 2m\omega^2 r\sin\theta$ is exerted on the vehicle on excess of the dead weight $2mg$ of the rotating masses. Therefore, considering the total weight of the system, $(2m + M)g$, while the vehicle is on the ground, the reaction force varies into the interval $(2m + M)g \pm F_{inert}$, given by

$$F_{reac}(t) = (2m + M)g - 2m\omega^2 r \sin\theta \quad \text{with} \quad \theta = \omega t + \theta_0 \,. \tag{18}$$

Equation (18) is valid while the bottom of the vehicle is generally *bonded* to the horizontal ground. When the rods are in the horizontal position ($\theta = 0$ or π) the reaction force equals merely to the total weight $(2m + M)g$. Moreover, the maximum value of the reaction force appears for $\theta = 3\pi/2$ or $\theta = -\pi/2$, which corresponds to vertical downwards rods. Finally, when the rods become vertical upwards ($\theta = \pi/2$) the reaction force takes its minimum value, either positive (compression) or negative (tension). In the latter case, the future situation depends on the condition between the bottom of the vehicle and the ground surface. Thus a tensile reaction force is possible only when the vehicle is bonded (e.g. glued or welded) to the ground.

In contrast, when the vehicle is not bonded to the ground and the quantity $2m\omega^2 r$ is greater than the total weight, $(2m + M)g$, it is expected that loss of contact between the vehicle and the ground will occur at a certain value of the polar angle θ when the two rods are in the first and the second quadrants of the circle swept by each rod, respectively. In order to simplify things, we consider the easy-to-handle case in which the half-amplitude $2m\omega^2 r$ is much greater than the total weight. Also we assume that when the rods become horizontal ($\theta = 2k\pi$, where k is an integer) tending to move upward, the ground (say the cover of a deep well below) suddenly opens and the vehicle is left free to move. In this case, the support (ground) force suddenly reduces from $(2m + M)g$ (see Eq. (13)) to zero and remains to this value until probably the vehicle strikes to the ground and eventually on the bottom of the aforementioned well. The reason of resorting to the aforementioned meaning of the deep *well* is due to the avoidance of nonlinear contact problems, a matter which will be discussed in Section 6.

For the sake of brevity, the concept to apply periodical support forces when the vehicle is thrown into the air is discussed only in Section 6.

4. Alternative Configurations

4.1. Equivalent Spring-Mass Oscillator

The Dean drive is characterized by the following physical and geometric quantities:

i) Angular velocity: ω, thus frequency: $f = \omega/(2\pi)$.

ii) Eccentricity (radius): r.

iii) Total rotating mass: $m_{rot} = 2m$.

Exactly the same action is achieved by replacing the Dean drive with a 1D spring-mass oscillator having the following characteristics:

i) Spring stiffness: $k_s = 2m\omega^2$.

ii) Half-amplitude of displacement: $\delta_0 = r$.

iii) Oscillating mass: $m_{osc} = 2m$.

The abovementioned equivalency is valid only in case that an undamped oscillation is ensured in the spring-mass model. Ideally this equivalency holds only when the vehicle is *fixed* to the ground because only in that case we can ensure the *constraint* that the difference in the vertical position of the two masses (M and $2m$) is perfectly harmonic given by $u_2 - u_1 \equiv Z_M - Z_m = r \sin\omega t$. Otherwise u_1 and u_2 are independent one another as later will be shown in Eq. (21).

In more details, the kinetic energy of the two rotating masses (eccentrics) in the original Dean drive is

$$E_{kin}^{Dean} = 2 \cdot \left[\tfrac{1}{2} m(\omega r)^2 \right] = m\omega^2 r^2 \tag{19}$$

On the other hand, elementary dynamics suggests that in the spring-mass oscillator the maximum velocity will be equal to $v_0 = \omega\delta_0 = \omega r$. Therefore, at each arbitrary position of the oscillating mass m_{osc} (i.e., at phase angle $\theta = \theta_0 + \omega t$) the length change of the spring will be $\delta = \delta_0 \sin\theta$ thus, eventually, $\delta = r\sin\theta$. Also, the velocity of the point mass m_{osc} will be $v = v_0 \cos\theta$ thus, eventually, $v = \omega r \cos\theta$. Therefore, the sum of the kinetic and elastic strain energy of the 1D oscillator will be

$$\begin{aligned} E_{tot}^{m} &= E_{kin}^{m} + E_{strain}^{m} = \tfrac{1}{2} m_{osc} \cdot v^2 + \tfrac{1}{2} k_s \cdot \delta^2 \\ &= \tfrac{1}{2} 2m \cdot (\omega r \cos\theta)^2 + \tfrac{1}{2}(2m\omega^2) \cdot (r\sin\theta)^2 \\ &= m\omega^2 r^2 \end{aligned} \tag{20}$$

Comparing Eq. (19) with the final equality of Eq. (20), the coincidence in the kinetic energy between the Dean-drive and its spring-mass equivalent is obvious. Obviously, both systems exert the same maximum inertial force (i.e., $k_s\delta_0 = 2m\omega^2 r$; upward at $\theta = \pi/2$ and downward at $\theta = 3\pi/2$) and always have the same phase angle θ.

The advantage of the Dean drive is that it can *per se* ensure an undamped oscillation, whereas the advantage of the spring-mass model is that it can inspire the designer in order to choose a proper spring (k_s, δ_0) with a corresponding attached mass $m_{osc} = 2m$. It is worthy to mention that if the selected spring-mass system has an angular velocity $\bar{\omega}$ and a maximum length change $\bar{\delta}_0$ different than those of the Dean drive (i.e., ω and δ_0, respectively) so as $\bar{\omega}^2\bar{\delta}_0 \neq \omega^2\delta_0$, if this difference is bridged by modifying the attached mass so as $\bar{m}_{osc}\bar{\omega}^2\bar{\delta}_0 \equiv 2m\omega^2\delta_0$, then the same maximum inertial force will be derived and the same travel distance (i.e. maximum upper point) will be reached but the profile of the vehicle's velocity will change (see section 5).

It is worthy to mention that while the equivalence between Dean-drive and the spring-mass model is perfect when the vehicle is still at rest, however at later time instances the equation of undamped motion is obviously according to the general 2-DOF finite element formulation:

$$\begin{bmatrix} M & 0 \\ 0 & 2m \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + k_s \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = -\begin{Bmatrix} Mg \\ 2mg \end{Bmatrix} \tag{21}$$

In Eq. (21) the displacement variables $u_1 \equiv Z_M$ (vehicle) and $u_2 \equiv Z_m$ (equivalent rotating mass) are not interrelated through the previously mentioned constraint, thus they are independent and generally we have $u_2 - u_1 \neq r\sin\theta$. The solution of the system formed by the two differential equations in Eq. (21) can be solved using well known numerical analysis schemes such as Runge-Kutta (e.g. ode45 using MATLAB®) or even implementing analytical eigenvalue analysis, for the following initial conditions:

$$u_1(0) = 0,\ \dot{u}_1(0) = 0 \quad \text{and} \quad u_2(0) = r\sin\theta_0,\ \dot{u}_2(0) = \omega r\cos\theta_0 \tag{22}$$

4.2. Spring Below the Vehicle

In principle, the action of the rotating masses could be substituted by fixing a spring of stiffness $k_{s,below}$ on the bottom of the vehicle, i.e. between the vehicle and the ground. Although the primary aim is to move the vehicle of mass *M*, for the sake of comparison we continue to deal with the total mass (2*m* + *M*) assuming the mass 2*m* fixed to the mass *M*. Initially, the spring below the vehicle is compressed by $\delta_{compress}$ and when is left free to upspring the maximum velocity V_0 of the body (2*m* + *M*) obeys the following condition (energy conservation):

$$\tfrac{1}{2} k_{s,below} \delta_{compress}^2 = \tfrac{1}{2} (2m + M) V_0^2 \tag{23}$$

Setting the above initial velocity V_0 of the bonded masses to be exactly equal to the corresponding initial velocity of the center of mass in the system (2*m*, *M*) of the Dean-drive, which is given by Eq. (10), the stiffness $k_{s,below}$ can be arbitrary chosen. Among several choices, equal spring stiffnesses, i.e. $k_{s,below} = k_s = 2m\omega^2$, means that $\delta_{compress} = r$.

5. Application and Results

The condition, $2m\omega^2 r \gg (2m + M)g$, which was mentioned in Section 3, may be easily achieved, for example, considering the following data:

- Rotating mass at the end of every rigid rod: m = 1·0 kg
- Mass of the object B: M = 5·0 kg
- Radius of rigid rod: r = 0·10 m
- Angular velocity: ω = 314·16 s^{-1} (3000 rpm)
- Initial polar angle of the rods: $\theta_0 = 0$ deg
- Acceleration of gravity: g = 9·81m/s^2

Then the inertial force is 287 times the total weight.

In the text below, the subscripts "M" and "m" refer to the vehicle and the rotating masses, respectively.

In this particular position (initial angle $\theta_0 = 0$), the vehicle is at rest (initial velocity $V_{M,0} = 0$) thus the total linear impulse of the system equals to that of only the eccentrics, i.e. $V_{m,0} = 2m\omega r$ and is directed upward (Figure 3, bottom). But after the eccentrics rotate by 90 degrees ($\theta = \pi/2$ in Figure 3, top), the velocity vectors of the masses become horizontal thus the relative vertical component of the momentum with respect to the vehicle vanishes while the absolute value (with respect to the ground origin O) equals to $2mV_M$ (V_M is vehicle's vertical velocity which is transmitted through the pin joints O_1 and O_2 to the rigid bars and the attached rotating masses. In the absence of gravitational acceleration $(g = 0)$, the linear momentum is preserved, thus the kinematic state of the vehicle would change from calm to an upward velocity given by $V_M = 2m\omega r/(2m + M)$, or shortly by $V_M = \lambda\omega$ [cf. Eq. (5)] with $\lambda = 2mr/(2m + M)$. For the above specific data we have $\lambda \cong 0.0286$, which leads to a vehicle's velocity $V_M \cong 8.98\, m/s$ at the end of the first quarter of the first rotation of the rods. One may observe in Figure 4 that at the end of the first rotation the vehicle returns to the state of instantaneous calmness having elevated by about 0.18m, of course by absorbing energy from the motor. It is also worthy to observe that the difference between gravity and no-gravity paths is very small. It is left to the reader to prove that the formula which determines the aforementioned height is merely $2\lambda\pi$ ($\cong 0.18$). In the hypothetical case of absence of gravity the above procedure would repeat for ever thus an endless motion would occur. From a different point of view, when $g = 0$ (no external force) the existence of an initial velocity $V_{cm,0} = 2m\omega r/(2m+M)$ of the center of mass would lead to an endless upward motion (first Newton's Law).

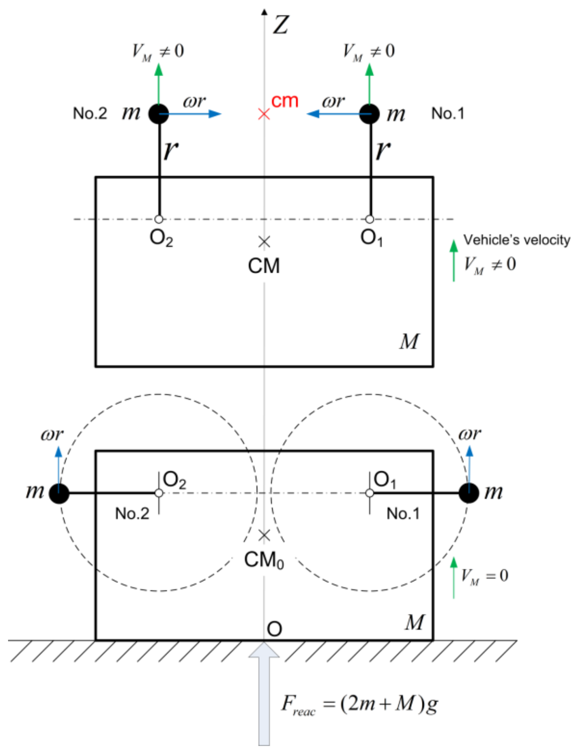

Figure 3. Velocities.

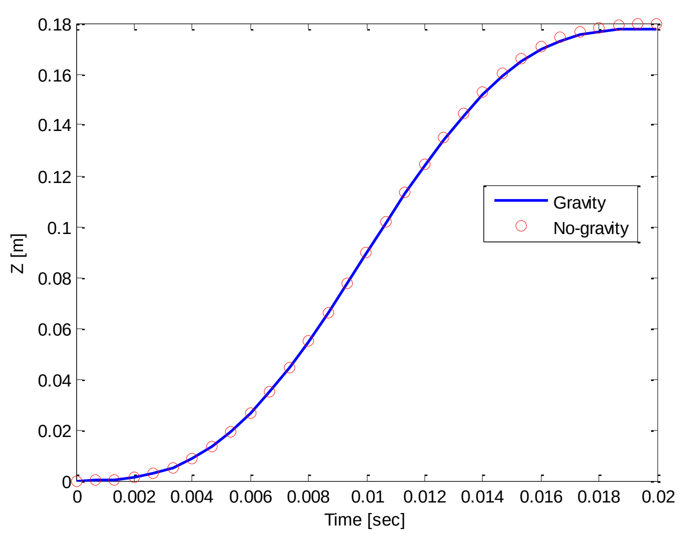

Figure 4. Vertical displacement of the vehicle for the first rotation.

But if gravity is taken into consideration as usual (i.e. $g \neq 0$), according to Eq. (5) the vehicle's velocity becomes:

$$\bar{V}_M\Big|_{(\theta=\pi/4)} = \frac{2m\omega r}{(2m+M)} - \frac{\pi g}{2\omega} \tag{24}$$

For the data assumed, the two terms in Eq. (24) equal to 8.9761 m/s (as previously for $g = 0$) and 0.04905 m/s, respectively.

To start the discussion, the key point is to determine the *initial velocity* of the system which equals to

$$V_{cm,0} = \frac{M \cdot 0 + 2m\omega r}{M+2m} = \frac{2mr}{M+2m}\omega \tag{25}$$

Again, the abovementioned factor $\lambda = 2mr/(2m+M)$ [see Eq. (2)] appears in Eq. (25) as well. Classical mechanics dictate that the highest point of the trajectory followed by system's center of mass (CM) is given by $h_{\max} = V_{cm,0}^2/(2g)$ and it appears at time $t_{\max} = V_{cm,0}/g$. At this time, the angular position of the rods is determined by the angle $\theta = \omega t_{\max}$, whereas the vehicle and the concentrated masses will have the following velocities:

Vehicle: $$V_M(t_{\max}) = -\lambda\omega\cos\theta \tag{26a}$$

Eccentrics: $$V_m(t_{\max}) = \frac{\omega M}{M+2m}\cos\theta \tag{26b}$$

Center of mass: $$V_{cm}(t_{\max}) = 0 \tag{26c}$$

Obviously, the highest point of the trajectory is reached after $(t_{\max}/T)$ rotations, which for the current date equals to 45.75, i.e. 45 full rotations and three-quarters of the circle (leading to polar angle of the rods at $\theta = 3\pi/2$). It is worthy to clarify that, generally, at the highest point of the trajectory the position and the velocity of the rods as well as those of the vehicle is not a standard but depend on the values of the variables (M, m, r, ω). To facilitate the discussion, we have chosen the latter data so as at the time $t = t_{\max}$ the rods are vertical downward (i.e. $\theta = 3\pi/2$ $\therefore$ $\cos\theta = 0$) thus both the vehicle and the eccentrics are instantaneously still (according to Eq. (26a) and Eq. (26b), respectively).

It is worthy to mention that at a very short time after the highest point is reached (i.e. $t = t_{\max} + \varepsilon$ with ε infinitesimal small), if no external force is applied to the system, the vehicle will accelerate moving downwards in a *free fall*, as shown in Figure 5. One may observe the slightly curly shape of the graph in both cases, the former being the absence of gravity (in red) and the latter the realistic case of $g = 9.81 m/s^2$ (in blue).

In contrast, if an external force (of variable magnitude) could be exerted on the bottom of the vehicle to preserve the vehicle *at rest* during the proceeding one-fourth of the period T [i.e. for a time interval $\Delta t_{reac} = T/4 = \pi/(2\omega)$] until the eccentrics become again *horizontal* (moving upward), the above cycle will be *repeated*, and so on. The aforementioned external force operates like an "invisible hand" which offers support, similar to the ground force shown in Figure 1 (bottom); the only difference is that it appears at a higher altitude Z_M.

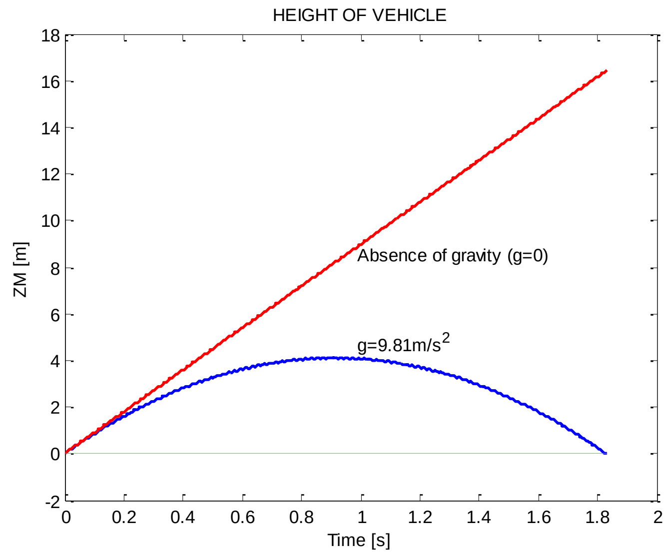

Figure 5. Vertical displacement $Z_M(t)$ of the vehicle for the first 92 rotations ($0 \le t \le 2t_{\max}$).

In Figure 6 the first two cycles are presented. Each cycle consists of 45.75 rotations of the masses completed in a time interval equal to $t_{\max} \cong 0.915\,\text{s}$ (unloaded vehicle) plus $T_{propulsion} \cong 0.005\,\text{s}$ (loaded vehicle), i.e. of period $T^* = t_{\max} + T_{propulsion} \cong 0.920\,\text{s}$ (46 full rotations). In Figure 6 one may distinguish the initial take-off time at which the cover of the well suddenly opens, as well as two successive peaks, P1 and P2, at the end of the first and second cycle. Such an external force could be implemented on the vehicle, for example, by releasing a small quantity of highly pressurized air in a vessel inside the vehicle. Although this means a certain mass loss in the vehicle ($M' < M$), if this fact is ignored (for the sake of brevity) the monotonically increasing in the average sense red line of Figure 5 (corresponding to zero gravity: $g = 0$) will be again obtained.

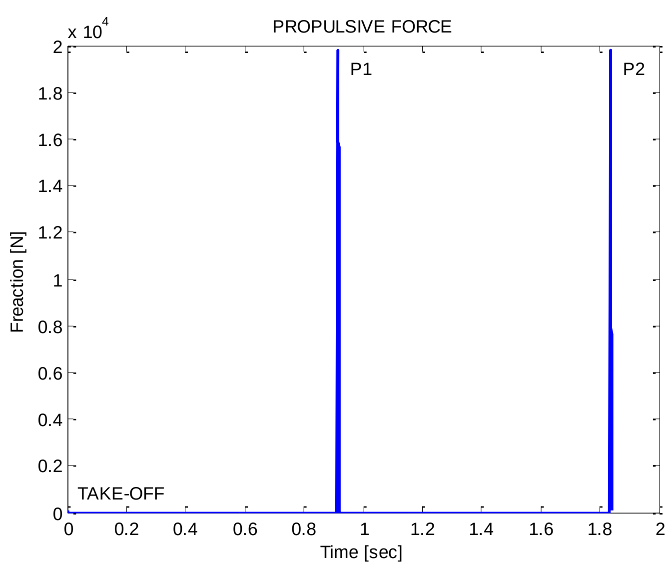

Figure 6. Support force exerted on the bottom of the vehicle.

In more details, from the beginning of the launch ($t=0$) until the highest point, i.e. for the time interval $T_f = t_{\max}$, the external force vanishes whereas immediately later it should suddenly take the maximum value $[(2m+M)g+2m\omega^2 r]$ and then it should decrease so as to withstand the inertial forces according to the expression (note that here, ignoring the integer multiples of 2π, we have ($3\pi/2 \le \theta \le 2\pi$):

$$F_{reaction} = \begin{cases} 0 & , 0 < t < t_{\max} \\ (2m+M)g + 2m\omega^2 r & , \ t = t_{\max} \quad (\theta = \frac{3\pi}{2}) \\ (2m+M)g - 2m\omega^2 r \sin\theta, & t_{\max} \le t \le T^* \end{cases} \tag{27}$$

If these conditions are continuously repeated into a gravitational field of acceleration g, then a continuous motion of the vehicle is ensured. Of course, the ejection of the pressurized air (or any other physical means of which the reader may think) probably cause some secondary effects which are open to be studied. Note that when M decreases the factor $\lambda = 2mr/(2m+M)$ increases as well, thus a greater height is obtained. Nevertheless, the general idea is that with the assistance of an external force of small duration (here, one-quarter of the full rotation), continuous vibrational projectile motion is theoretically possible.

6. Discussion

One may observe in Figure 7 that while minor difference exist in the vibration of the vehicle, all the three variations, i.e. the classical Dean-drive, the spring-mass model (section 4.1) and the spring below the vehicle (section 4.2) lead to the same upper point which depends only on the initial velocity of the center of mass. While in the pure Dean-drive and its approximate equivalent "spring-mass" system the vehicle oscillates, in the case of the spring below the vehicle (where no oscillating driving mass exists) no oscillation occurs.

Regarding the spring stiffness, for the particular data of Section 5, we have $k_s = 2m\omega^2 = 2\times 1.0 \times (314.16)^2 = 197,393 (N/m)$, which is within the range of realistic values.

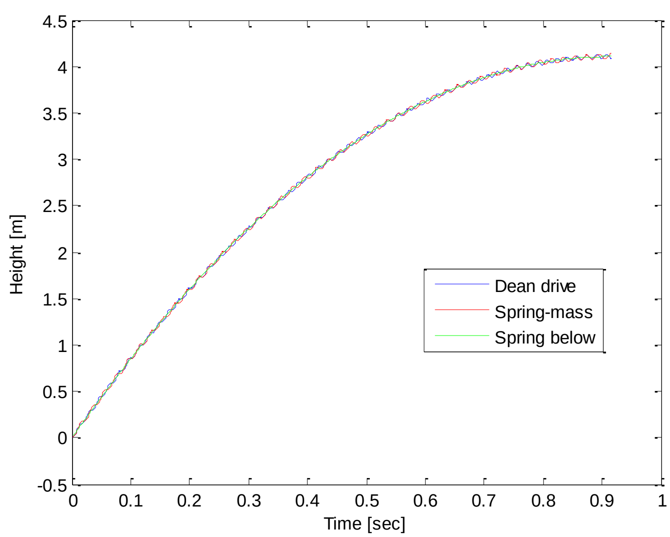

Figure 7. Paths followed by the center of mass.

Let us now resolve some "paradoxes" and see some limits. Previous discussion can justify the fact that if a Dean drive is put (say) inside a closed box (vehicle) and the latter is left free to move at the time that the eccentrics are horizontal but moving upwards, it is not strange that the box will perform an upward projectile motion. Again, this is because the centre of mass has an initial velocity and therefore it will move toward its direction. In Eq. (12), it was shown that the highest point $Z_{\max}^{CM}$ which may be reached by CM depends on the characteristic magnitude $\lambda = 2mr/(2m+M)$, where r is the eccentricity of the rotating masses (each of them m), the angular velocity ω of the eccentrics and the cosine of the initial polar angle θ_0. Depending on the choice of the masses m and M of the eccentric and the object, respectively, and the quantities (ω, r, θ_0), the ultimate height $Z_{\max}^{CM}$ may be small or large. For example, choosing the aforementioned masses to be equal to those in the old Rutherford-Bohr model of hydrogen atom and the radius equal to the smallest possible quantized value (called the *Bohr radius*), an isolated atom or a properly synchronized hydrogen molecule (if could occur) could jump up to 72 km above the surface of the earth (see details in [20, pp. 63-64]).

Therefore, the sudden jump of a closed box (when released to move) is neither magic nor should be attributed to new physics as claimed in the past [9, 10] but is simple Newtonian physics (i.e. the two-body problem) where a "hidden" initial velocity of the center of mass occurs. Newton's Third Law is perfectly fulfilled thus the action equals to the reaction force; in other words, the internal forces between eccentrics and the vehicle (at points O_1 and O_2) are cancelled and the motion is controlled by the center of mass which may be considered as a point mass of magnitude $(2m+M)$. Therefore, despite many out-of-stream claims which may be found in Internet under the keyword "Inertial Propulsion", Dean-drive is not a case study where "perpetual" motion could be produced. Although the reactionless propulsion is *not* possible because it violates the Newton's Third Law and the Conservation of Momentum, demonstrations in water (see [11-17]) and ground (see [18-19]) have shown such a motion, leaving the audience to form misguided conclusions about what they saw with only the information in the video. The discussion of this paper fully resolves the physical phenomenon.

No doubt that the conservation of laws of total energy and of linear momentum and angular momentum are fundamental to physics, being valid in all older and modern theories. Nevertheless, the possibility of an anomalous behavior due to the weak form of Newton's third law has attracted the interest of many investigators. Acoustic waves in elastic rotating bars were found *unsuccessful* in order to rectify the *alternating* support forces and this happens due to the appearance of Coriolis forces [24, p. 319]. And since Coriolis forces do *not* appear in electromagnetic phenomena, an analogue electromagnetic drive was developed but nevertheless the detailed analysis revealed *null* net thrust in average sense [24, p. 329]. Generally, the secret is that *the center of force cannot be at the same location as the center of mass*; it must be offset (polarized). Particularly, in the current case of the mechanical system a proper $\cos\theta_0$ should be used in both eccentrics; however it is noted that more pairs may be used and a possible setup may be found in [24, p. 323].

Although the spring stiffness which was introduced in this work for the first time was not involved in calculations, however it is a useful quantity to assist the design process. The designer has to look about the possible values the stiffness it may take and look for particular constructive solution that may replace the classical eccentrics by a different elastic structural component.

The reader may perhaps wonder why we have assumed that the cover of the well on which the vehicle stands is suddenly drawn when the system is left free to move. This was so because otherwise nonlinear contact effects could occur, thus modeling would become considerably more difficult without getting essential profit from this complexity. In fact, Figure 8 shows details for the actual path followed by the vehicle in the beginning of its motion. Clearly, despite the averaged upward motion a minor downward one (i.e. $Z_M < 0$) would occur when the initial situation refers to horizontal rods ($\theta_0 \cong 0$).

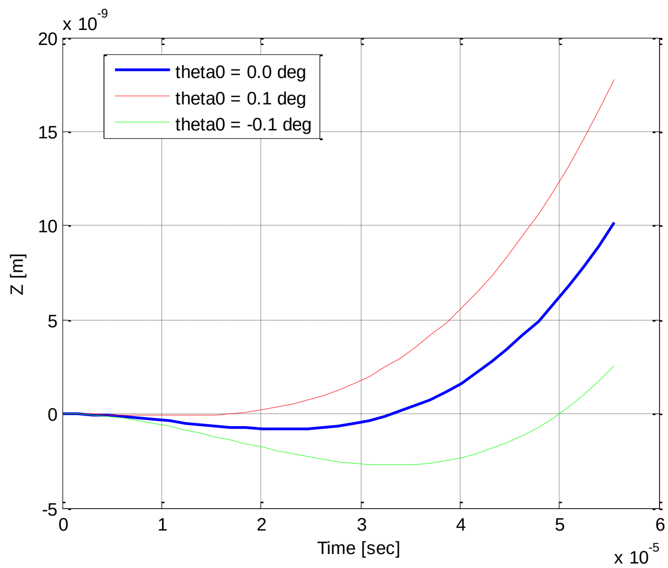

Figure 8. Path followed by the vehicle immediately after its motion.

Another issue is the imposition of external force for small time intervals at the end of each motion cycle (of which duration was about 46 rotations). In more detail, the motion could be continuous if we had ensured a way to activate sliding supports (S_1, S_2, S_3, etc) located at a given pitch Z_{max} along a vertical column parallel to the motion of the vehicle as shown in Figure 9. In more detail, the concept is that when the velocity of the vehicle vanishes nearby the upper point it can reach, the sliding support S_1 (initially being short) suddenly extends moving to the right (dashed line in Figure 9). After one-fourth of rotation (for the particular data of this problem) the same support (S_1) should be suddenly return back to the left (in order to avoid possible nonlinear contact phenomena as those shown in Figure 8) and thus leave the vehicle free to continue with a second travel cycle of almost another set of 46 rotations where the support S_2 will be now activated in the same way as S_1 did, and so on. Although this concept is obviously far away from the point to build an elevator to conquer the space, however it may be useful in the design of novel transport systems of goods.

7. Conclusions

It was shown that undamped vibrations caused by a Dean drive are almost equivalent to those induced by an auxiliary oscillating mass attached to a vehicle through a spring of proper stiffness. A third alternative is to use a spring below the vehicle and perform a certain compression. Under certain conditions, all the three aforementioned mechanisms may cause motion to the connected vehicle which will perform a travel of reduced length, say Z_{max}. In order to repeat the same travel length Z_{max}, in this paper two ways were proposed. The first is to apply an external force at nearby the time the maximum travel distance is reached and the vehicle is temporarily at rest. This can be achieved using a micro-jet or even by mass expulsion such as that of a gas carried on the vehicle. For the particular case of vertical upward motion, the proposed second way is to use a fixed column to provide a number of supports (S_1, S_2, etc.) in order to offer instantaneous support to develop the proper support force (initial velocity of the center of mass thus initial impulse) which will ensure the completion of a second elevation cycle, and so on. The initial concepts developed in this work require considerable design and manufacturing effort until such a system is ready to work in an industrial environment.

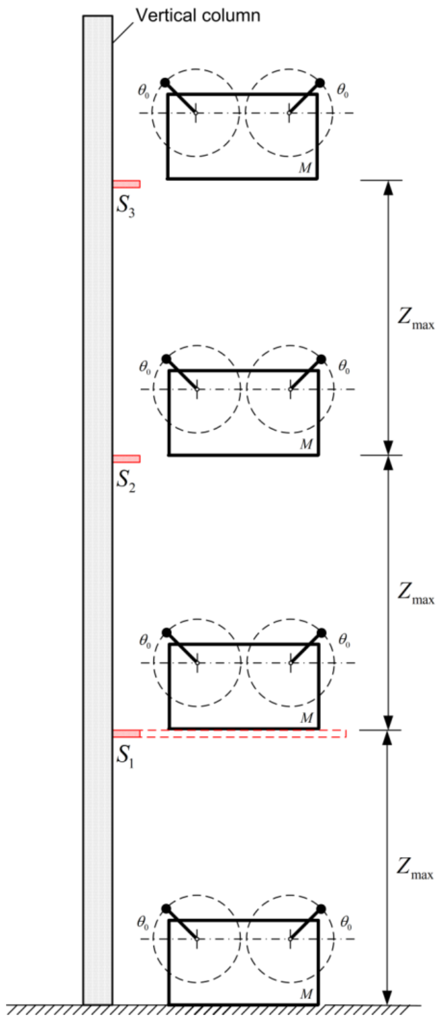

Figure 9. A vertical column with columns to assist continuous motion for the vehicle.

References

[1] I. I. Blekhman, Synchronization in science and technology, ASME Press, New York, 1988.

[2] N. L. Dean, System for converting rotary motion into unidirectional motion, USA Patent No. 2,886,976, submitted in July 1956 and granted on May 9, 1959. See also: https://en.wikipedia.org/wiki/Dean_drive.

[3] J. Ju, Q. Wang and K. Zhang, Design and analysis of a novel micro-robot driving platform, Proc IMechE Part C: J Mechanical Engineering Science vol 233(11), pp. 3849-3857, 2019.

[4] I. A. Lukanov, V. G. Vitliemov and I. V. Ivanov, Dynamics of a Mobile Mechanical System with Vibration Propulsion (VibroBot), International Journal of Research in Engineering and Science (IJRES), vol 4(6), pp. 44-51, 2016.

[5] A. E. Minetti and L. P. Ardigo, Halteres used in ancient Olympic long jump, Nature, vol 420, pp. 141-142, 2002.

[6] M. Todeschini, Motore a forza propulsive centrifuga alimentato ad acqua dissociate con variazione di velocita graduale automatic, Regno d'Italia, Ministero delle Corporazioni, Brevetto Industriale No. 312496, vol 17 November 1933; see the first two pages of the patent at the website https://www.circolotodeschini.com/brevetto-provatidis/

[7] C. G. Provatidis, Unidirectional motion using rotating masses along figure-eight-shaped trajectories, Journal of the Brazilian Society of Mechanical Sciences and Engineering, vol 37(1), pp. 397-409, 2015.

[8] https://en.wikipedia.org/wiki/Reactionless_drive.

[9] R. F. Dempewolff, Engine With Built-in Wings, Popular Mechanics, Sep 1961, pp. 131-134 and 264-266, 1961.

[10] G. H. Stine, Detesters, Phasers and Dean Drives, Analog, vol 96 (6), pp. 60-80, June 1976.

[11] Anonymous: http://www.youtube.com/watch?v=FjV5PaqDpR4.

[12] Anonymous: http://www.youtube.com/watch?v=vfQKGdADevA.

[13] Anonymous: http://www.youtube.com/watch?v=PaiYgB4q1gs&noredirect=1.

[14] Anonymous: https://www.americanantigravity.com/roy-thornsons-inertial-propulsion-demonstration.

[15] Anonymous: http://www.ntv.ru/novosti/84720/video.

[16] Anonymous: http://www.youtube.com/watch?v=1VW1y6isl18.

[17] Anonymous: https://www.youtube.com/watch?v=IeWk5vIEIbo.

[18] Anonymous: http://www.inertialpropulsion.com/.

[19] Anonymous: http://www.youtube.com/watch?v=68SiteyZgEo.

[20] C. G. Provatidis, An Overview of the Mechanics of Oscillating Mechanisms, American Journal of Mechanical Engineering, vol 1(3), pp. 58-65, 2013.

[21] C. G. Provatidis, On the inertial propulsion of floating objects using contra-rotating masses, Mechanics Research Communications, vol 62, pp. 117-122, 2014.

[22] Provatidis C. G., Mechanics of Dean Drive on Frictional Ground, Journal of Mechanical Design and Vibration, 2014, vol 1, No. 1, pp.10-19, 2014. [DOI:10.12691/jmdv-1-1-3]

[23] C. G. Provatidis, Design of A Propulsion Cycle for Endless Sliding on Frictional Ground Using Rotating Masses, Universal Journal of Mechanical Engineering, vol 2(2), pp. 35-43, 2014.

[24] C. G. Provatidis and M. A. Gamble, Support forces in a synchronized rotating spring-mass system and its electromagnetic equivalent, International Journal of Applied Electromagnetics and Mechanics, vol 41, pp. 313-334, 2013.

[25] D. Halliday, and R. Resnick, Physics, Parts I and II, John Wiley and Sons, Inc., New York, 1966.

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Christopher G. Provatidis was born in Athens, Greece, in November 3, 1956. He has a five-years Diploma in Mechanical-Electrical Engineering from the National Technical University of Athens, Greece (NTUA), 1979 and a PhD also in Mechanical Engineering from the NTUA, 1987. The author's major field of study is applied mechanics, statics and dynamics.

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Prof. Provatidis is a member of ASME, AIAA, European Society of Biomechanics (ESB), Greek Society of Biomechanics (Vice-president, 2008-2010), and Greek Association of Computational Mechanics (General Secretary, 2007-2009). In March 2011 he was elected as an active member (Class VI: Technical and Environmental Sciences) of the European Academy of Sciences and Arts (Salzburg, Austria).