Snow Accumulation and Ablation Model – SNOW-17

Eric Anderson, January 2006

The SNOW-17 snow accumulation and ablation model was first described by Anderson [1973] as a component of the National Weather Service River Forecast System (NWSRFS). SNOW-17 evolved from two earlier snow models [*Anderson and Crawford* (1964) and *Anderson* (1968)]. A few minor changes and the addition of snow depth computations have been made to the SNOW-17 model since 1973. This document describes the current version of SNOW-17. Besides just describing the model, this document also provides some insights into the reasoning and logic used by the author in developing the model. Hopefully this information will be helpful to future users.

I. Background

SNOW-17 is a conceptual model. Most of the important physical processes that take place within a snow cover are explicitly included in the model, but only in a simplified form. A few physical processes are not explicitly included, but are implicitly integrated into other process representations. These cases will be discussed as the components of the model are described.

SNOW-17 is an index model using air temperature as the sole index to determine the energy exchange across the snow-air interface. In addition to temperature, the only other input variable needed to run the model is precipitation. Many studies have shown air temperature to be a good indicator of snowmelt. Degree day factors have been used to estimate snowmelt for many years. Air temperature is an easy variable to measure. Air temperature data are readily available from climatological and real time networks. It is also reasonably easy to estimate the spatial variation of air temperature in most cases as compared to other meteorological variables that affect the snow energy balance. Though air temperature varies somewhat with forest cover and slope/aspect, the factor that explains most the variability over an area is elevation. The ability to reasonably extrapolate air temperature data to higher elevations is critically important for snow modeling since in many mountainous regions most of the snow runoff comes from areas that are higher than any measurement site. In addition, it is easier for meteorologists to predict air temperature for some period into the future than other energy exchange variables.

Even if someone is using a conceptual, index snow model it is beneficial to understand the basic physics of a snow cover. Appendix A contains a brief description of the physics of snow energy exchange for those not familiar with the processes involved. Appendix B contains a list of the symbols used in this document including Appendix A.

SNOW-17 was primarily designed for use in river forecasting. This means that the model needs to use data that are readily available everywhere, both historical climatological data for calibration and real time data for operational applications. Though the model has successfully been applied at point locations to simulate just the accumulation and melting of the snow cover, for river forecasting SNOW-17 is typically applied on an areal basis to estimate the outflow from the snow cover to a rainfall/runoff model, as well as the amount of snow. For river forecasting, large river basins are divided into headwaters and local areas generally based on where river observations are available, both historically and in real time. In flat terrain SNOW-17 is typically applied to a headwater drainage or local area though in some cases large drainages may be divided into several sub-areas. In mountainous regions due to the significant variation in the amount of snow and the timing of melt with elevation, watersheds are typically divided into 2 or 3 elevation zones when using SNOW-17. Since the model was not designed to calculate how melt rates might vary with various physiographic factors SNOW-17 is not generally used for applications such as predicting the effect of land use changes.

When generating operational river forecasts, simulated river conditions frequently don't match observations when the models are first run for a new period due to data and model errors. The forecaster generally has to make various updates to model states and computations and redo the simulations until there is a satisfactory agreement between computed values and observations. Some of these updates are objective, but many are subjectively applied and require an interactive trial and error process to determine the proper adjustments. Modifications can also be applied into the future in order to adjust melt estimates when abnormal weather conditions are predicted. Once it is felt that the models adequately represent the current state of the river system, then forecasts are generated based on predictions of the driving variables and any future melt adjustments. A single set of predictions can be used to get a deterministic forecast or an ensemble of possible future scenarios can be applied in order to generate a probabilistic prediction. Forecasts can be generated for a few days into the future or can extend out for many months. Besides describing the model, this document describes what takes place internally within SNOW-17 when various operational modifications are applied.

While some guidelines have been developed relating SNOW-17 model parameters to physiographic factors, the model needs to be calibrated in order to produce quality simulation results. Recommendations for determining initial parameter values and for calibrating the SNOW-17 model are included in a comprehensive guide for historical data analysis and model calibration for river forecasting applications [*Anderson,* 2002]. In order to get the best results from SNOW-17 for river forecasting applications three things must occur:

1. The model must be properly calibrated,
2. The input data (precipitation and temperature) used operationally must be unbiased compared to that used for calibration, and
3. Well devised, ideally objective, updating schemes must be used to remove bias and to minimize random errors to the maximum extent possible.

The values of the model parameters, as determined through calibration, represent normal conditions over a river basin (i.e. the typical spatial variation of precipitation and temperature, the prevailing storm directions and wind conditions that affect the measurement and distribution of snow, the typical climatological conditions during periods of melt, etc.). While many of the errors during calibration are random, there are certain biases that can't be overcome when using a temperature index snow model (e.g. very high melt rates, often associated with major runoff events, are generally the result of abnormal meteorological conditions, such as high winds and dew-points, resulting in a tendency to under estimate snowmelt during such events). When using the SNOW-17 model in an operational mode, good updating techniques are required in order to adjust the model states or internal computations to remove random errors and any bias to the maximum extent possible in order to reduce the uncertainty in the forecast.

When SNOW-17 was first developed, air temperature was the only meteorological variable to estimate snow energy exchange that was readily available most everywhere and in real time. Variables such as dew-point and wind speed were available at a limited number of synoptic sites and radiation measurements were generally only made at research locations. Today it is possible to obtain estimates of most of the variables involved in snow cover energy exchange over large regions based on the spatial analyses incorporated in mesoscale meteorological models. However, in order to use these data to compute snow cover energy exchange, the variables need to be downscaled to the surface and the effect of vegetative and terrain features need to be accounted for. Within the last few years the first attempt at running an energy budget snow model over the entire conterminous United States in real time has been attempted [*Carroll et al,* 2001]. This procedure uses a distributed, energy balance model that assimilates meteorological and snow cover data from satellite, airborne, and surface sources and analytical data from mesoscale atmospheric models. While such a procedure could be used to directly provide values of the outflow from the snow cover to use as input to rainfall/runoff models for river forecasting, this is believed not to be currently practical given the uncertainty of some of the data estimates, the complexity of the modeling procedures, difficulties in calibrating such a model, the need for multiple model runs with possible forecaster interaction for whatever model is used at a forecast office, the length and number of possible forecast ensembles, and the inability to predict many of the meteorological variables used by an energy balance model for more than a few days into the future. It seems more realistic to apply an index model for operational forecasting, but to use the information provided by an energy balance based procedure to adjust model states and internal computations when appropriate.

II. Model Description

The approach used when developing SNOW-17 was to first try to represent the physical processes that occur in a column of snow. Then features were added so that the model could be applied to an area. This is a similar approach as that later used when the Sacramento Soil Moisture model was developed. The main processes included in the model for a column of snow are:

- Form of precipitation,
- Accumulation of the snow cover,
- Energy exchange at the snow-air interface,

- Internal state of the snow cover,
- Transmission of water through the snow cover, and
- Heat transfer at the soil-snow interface.

In order to apply the model to an area, the areal extent of the snow cover is computed and used to determine the fraction of the area from which melt and outflow from the snow cover can occur. When the model is applied at a point location, the algorithm used to compute the areal extent of the snow cover is not used. When applied to an area, SNOW-17 keeps track of mean areal values of variables such as water equivalent and depth. In order to get the average over the snow covered area one must divide these mean areal values by the areal extent of the snow cover. Figure 1 shows a basic flowchart of the SNOW-17 model.

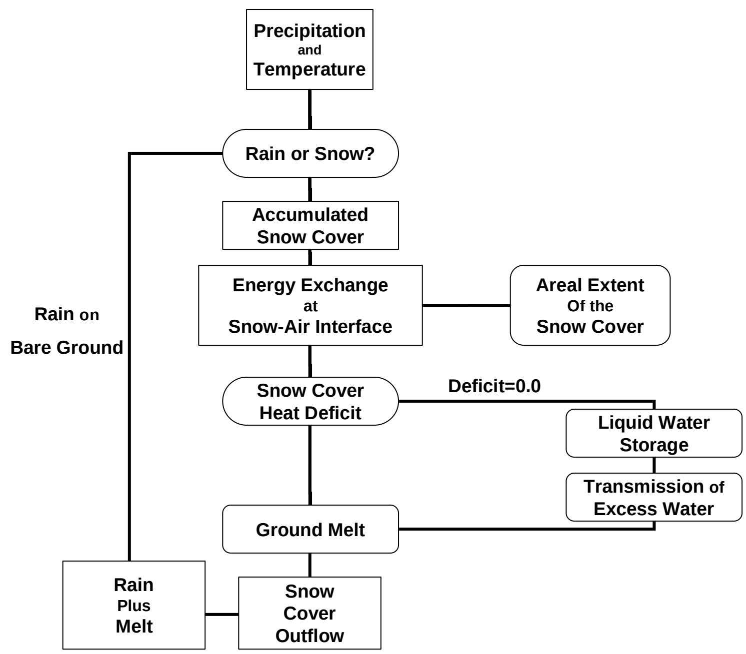

Figure 1. Flowchart of the SNOW-17 model

SNOW-17 is coded in NWSRFS to be run at computational time intervals of 1, 2, 3, 4, 6, 8, 12, and 24 hours, i.e. any hour that evenly divides into 24. The computational time interval, i.e. the minimum period for which the model can be run, is the time interval associated with the temperature data. For river forecasting the model has most frequently been applied at a 6 hour interval. Many of the model parameters are defined for that time period and are then adjusted if computations are done at a different interval.

The input data for SNOW-17 is precipitation and temperature. The precipitation is the total over a specified time interval. When doing areal computations, the precipitation is normally the mean amount over the area though the code does include a multiplying factor, PXADJ, that can be applied to all precipitation values entering the model (PXADJ is almost always equal to 1.0). The NWSRFS code does allow precipitation input to be provided more frequently than temperature data. This was done to allow the model to account for changes in the intensity of rain-on-snow at a short interval, e.g. hourly, even though air temperature data needed to compute melt are only available for a longer time period, e.g. 6 hours. The time interval of precipitation data must divide evenly into the period used for temperature data. Internally SNOW-17 loops through the model computations at the time interval associated with the precipitation data. If the time interval of the precipitation data is less than the interval for temperature values, energy exchange is calculated based on the average temperature over its time interval, but decisions concerning the effect of precipitation on melt and internal snowpack computations are made for each precipitation data period. This means that different melt equations can be used within a computational time interval depending on which periods rain occurs.

The temperature values are the time interval mean. For areal computations the temperature data normally represent the value at the mean elevation of the area. The NWSRFS code allows the user to input the mean elevation of the area and the elevation associated with the temperature data. Generally these values are the same. If they are different, the model will apply a lapse rate to the input temperature values in order to adjust them to the mean elevation of the area. This is sometimes done for elevation zones that are seldom fully covered by snow. In that case the typical average elevation of the snow covered portion of the area is supplied as the areal mean while the temperature data still correspond to the mean elevation. When the temperature data are adjusted for differences in the elevation, two lapse rates are input (units of ˚C/100m). These are the typical lapse rate at the time of the day when the maximum temperature generally occurs (assumed to be 3 pm local time) and the typical lapse rate when the minimum temperature most frequently occurs (assumed to be 6 am local time). The lapse rate at any given time of the day is determined by linearly interpolating between the two input lapse rates. No seasonal variation can be specified for these lapse rates.

III. Modeling a Column of Snow

Form of Precipitation

Background

Ground level temperature is a good, but not perfect, indicator as to whether precipitation is falling as rain or snow. Figure 2 illustrates how the form of precipitation can vary with surface level temperature. The variation of the form of the precipitation at a given location shown in Figure 2 is based on data from the Snow Investigations [*Snow Hydrology,* 1956]. It shows that rain can occur at temperatures below 30˚F and snow can occur when the air temperature is 40˚F. From that study the typical temperature separating rain from snow is around 34-35˚F (about 1.5˚C).

In mountainous regions it can be raining at lower elevations and snowing higher up in the mountains. This situation is prevalent in the United States during most storms in the winter along the West Coast, the Gulf of Alaska, and the far Southwest (such as the Mogollan Rim area of Arizona). In this case one needs to know the elevation dividing rain from snow, referred to as the rain-snow elevation. The surface temperature at the rain-snow elevation can vary, but is typically around the average value that separates rain from snow. Estimates of the rain-snow line can be calculated based on where the average temperature that separates rain from snow occurs. Input can be freezing level or temperature values at a specified elevation. When computing the rain-snow line, the saturated adiabatic lapse rate is normally used.

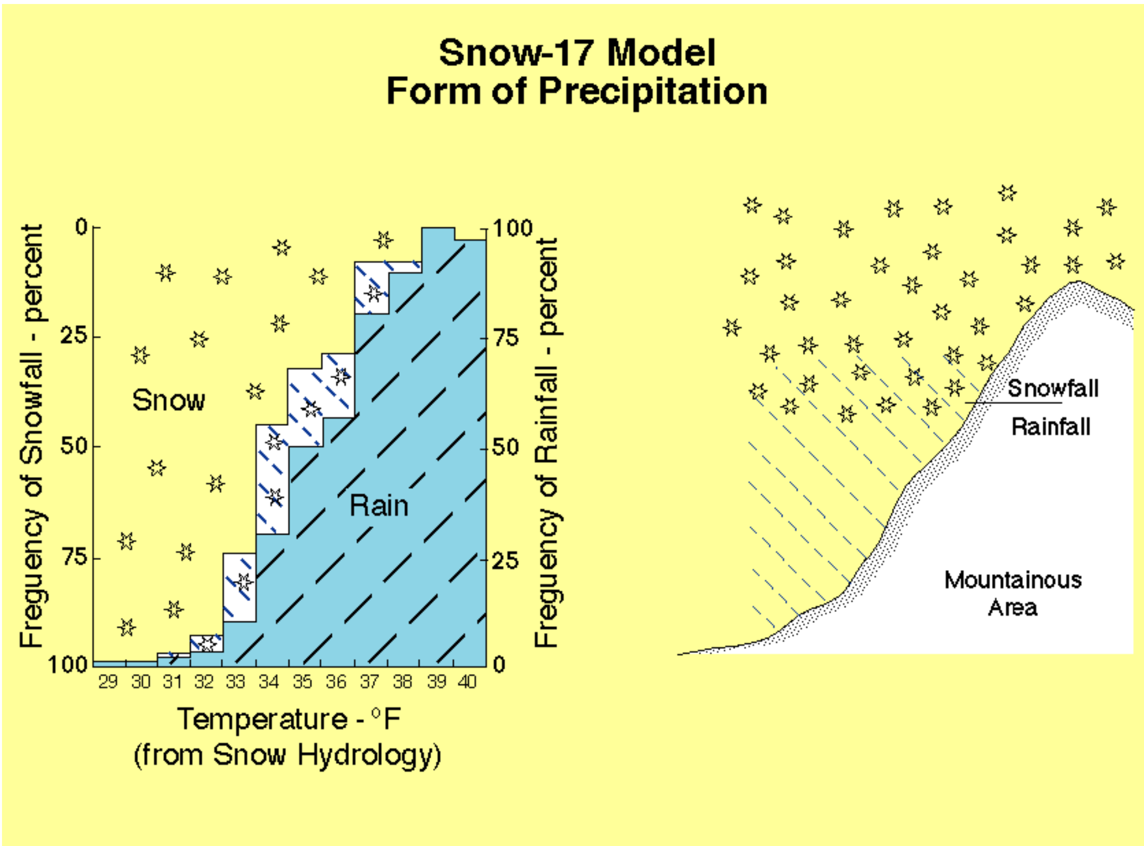

Figure 2. Variation of form of precipitation with temperature and elevation.

Model Options

SNOW-17 has 3 options for determining the form of precipitation values for a given computational time interval.

1. A single threshold temperature is specified as a model parameter, referred to as **PXTEMP** (units of ˚C). When the air temperature for the time interval is less than or equal to **PXTEMP**, all the precipitation is classified as snow. When the air temperature is greater than **PXTEMP**, all the precipitation is assumed to be rain.

2. A rain-snow elevation time series is input to the model at the computational time interval. When this option is selected, a definition of the area-elevation curve for the areal must also be provided (defined by the minimum and maximum

elevations and several elevations in between with the decimal fraction of the area below each elevation – in NWSRFS the user has the option to input elevations in feet or meters). If the rain-snow elevation is somewhere between the minimum and maximum for the area, then the model computes the fraction of the area where rain is occurring and the fraction where it is snowing. Of course if the rain-snow elevation is below the lowest point in the area, all the precipitation is snow and if above the highest point, it is all rain. With this option some of the precipitation for a given time interval can be treated as rain and the remainder treated as new snowfall.

3. A time series is supplied that indicates the decimal fraction of the precipitation that is in the form of snow for each precipitation data time interval. If the time series value is missing for a given time period, then option #2 is used if an area-elevation curve has been input and if not, option #1 is used. This option allows the user to override the model calculations of the form of precipitation for selected time intervals. This option would be used if direct observations of the form of precipitation were available (e.g. at a research site) or an external procedure, which could make use of additional meteorological data, was being applied. For this option the form of precipitation can vary within a computational interval as with option#2.

Accumulation of the Snow Cover

Background

Wind has a much greater effect on the ability of a precipitation gage to catch snow than rain. A precipitation gage located out in the open where it is exposed to the wind would have a very significant loss in catch during windy periods when it is snowing. Most precipitation gages are located in places where there is a reasonable amount of shelter from surrounding trees and structures. If a sheltered location can't be found for a precipitation gage in a region where snow is significant, then frequently a wind shield is used. Even with a good natural exposure or a wind shield, the gage catch during periods of snow is still typically considerably greater than when it is raining. Figure 3 shows some typical relationships between gage catch and wind speed.

In addition to being affected by deficiencies in the catch of precipitation gages, the accounting of the magnitude of the snow cover during an accumulation period is influenced by other factors. Some of the snow can be lost by sublimation. Sublimation can occur from the snow cover itself, from snow that is intercepted and temporarily held in trees, or from blowing snow. Sublimation occurs when the vapor pressure of the air is less than the vapor pressure at the snow surface. Sublimation can be computed using the latent heat transfer term of the energy balance equation. To calculate sublimation one needs to know the vapor pressure of the air (dew-point), the wind speed, and the snow surface temperature. Blowing snow can also result in a gain or loss from the snow cover, especially at a point location, due to redistribution. Over a fairly large area the movement of snow across area divides is typically negligible.

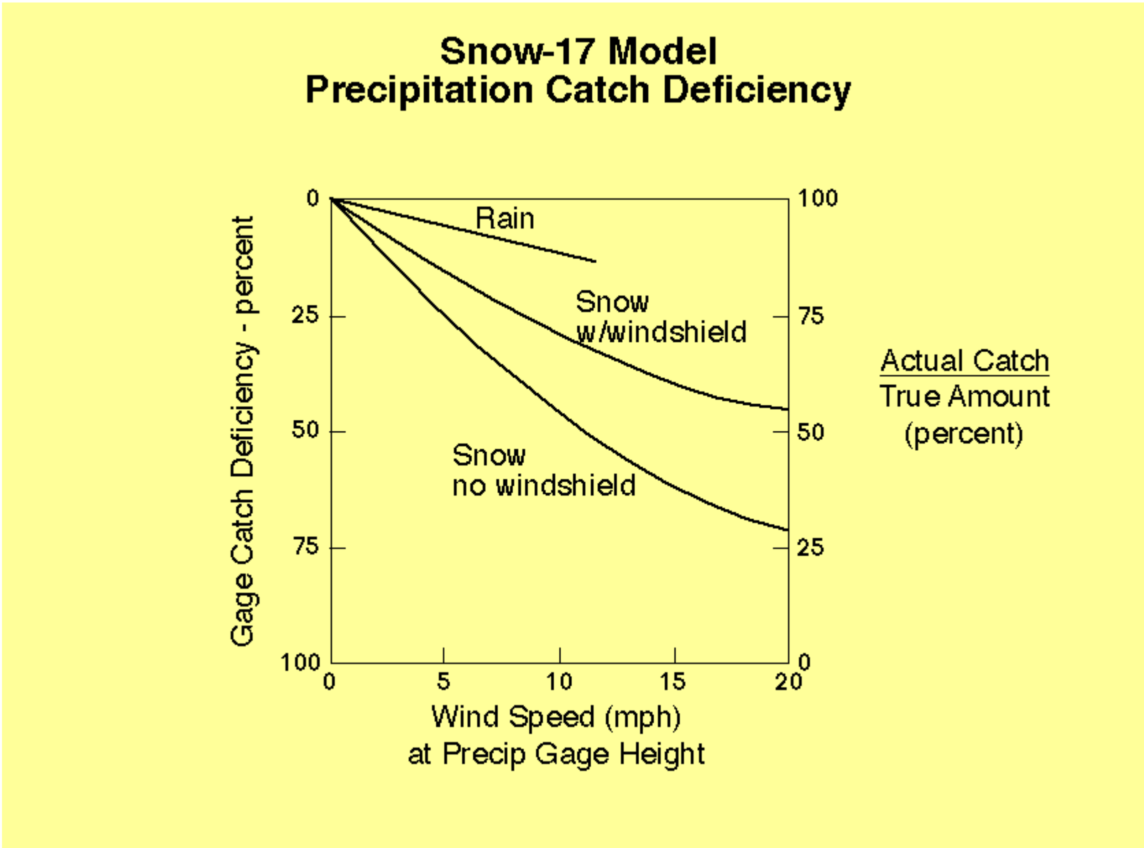

Figure 3. Effect of wind on precipitation catch.

The density of new snow varies based on meteorological conditions. Typically the colder and drier the air mass, the lower the density of new snow. While the frequently used value of 0.1 (one inch of water will produce 10 inches of snow) is a realistic average density of new snowfall in many regions, the average will be lower in regions with very cold, dry climates.

The temperature of precipitation, whether rain or snow, can probably best be approximated by the wet bulb temperature. When precipitation is occurring the relative humidity is generally quite high and thus under these conditions the wet bulb temperature is close to the air temperature. When snow falls at temperatures below freezing, it must eventually be warmed to 0˚C before melting.

Accumulation in the Model

SNOW-17 uses a multiplying factor, parameter **SCF**, to adjust all new snow amounts before they are added to the existing snow cover. Thus, the amount of new snow for each precipitation time interval is:

$$P_n = P \cdot f_s \cdot \mathbf{SCF} \tag{1}$$

where: P_n = water equivalent of new snowfall (mm),
P = total precipitation input to the model (mm), and
f_s = fraction of precipitation in the form of snow.

SCF was primarily included to account for the gage catch deficiencies that occur during periods of snow. However, **SCF** implicitly includes losses that occur during accumulation periods due to sublimation and redistribution caused by blowing snow. **SCF** is an average value over all the accumulation periods used to calibrate the model. The value of **SCF** is typically chosen to give the best estimate of the amount of water in the snow cover at the beginning of the melt season. In regions with large amounts of snow and many snowfall events, variations in the catch deficiencies from event to event tend to cancel out. When melt periods are preceded by only a couple snow storms, the errors can be much greater.

The density of new snow, ρ_n ($gm \cdot cm^3$), is computed based on the air temperature [*Anderson,* 1976]. When the air temperature is less than or equal to -15˚C:

$$\rho_n = 0.05 \qquad (2a)$$

When the air temperature, T_a (˚C), is greater than -15˚C:

$$\rho_n = 0.05 + 0.0017 \cdot (T_a)^{1.5} \qquad (2b)$$

This results in a density of new snow of about 0.15 when the air temperature is 0˚C. The depth of new snow is thus:

$$H_n = (0.1 \cdot P_n)/\rho_n \qquad (3)$$

where: H_n = depth of new snowfall (cm).

The temperature of the new snow is assumed to be equal to the air temperature or 0˚C, whichever is less. When the temperature of the new snow is less than 0˚C, the heat deficit of the snow cover is increased by:

$$\Delta D_p = -\frac{(T_n \cdot P_n)}{(L_f / c_i)} \qquad (4)$$

where: ΔD_p = change in the heat deficit due to snowfall (mm),
T_n = temperature of the new snow (˚C),
L_f = latent heat of fusion (80 $cal \cdot gm^{-1}$)
c_i = specific heat of ice (0.5 $cal \cdot gm^{-1} \cdot {}^\circ C^{-1}$)

This is the amount of heat that must be added to the new snowfall in order to bring it up to a temperature of 0˚C.

Energy Exchange at the Snow-Air Interface

Background

Energy exchange between the air and a snow cover is the result of net radiation, latent and sensible heat transfer, and heat associated with precipitation. Net radiation involves both solar (shortwave) radiation and atmospheric (longwave) radiation. The amount of incoming solar radiation is dependent on the time of the year and the time during the day. Incoming solar radiation is also affected by cloud and vegetation cover, as well as the slope and aspect of the surface. Snow has a high reflectivity or albedo in the shortwave portion of the spectrum, thus much of the incoming solar radiation is reflected back into the atmosphere. Fresh snow typically has an albedo of around 90%, while even well aged snow will reflect about 40% of the incoming solar energy. The net amount of solar radiation adsorbed by snow is typically greatest during the middle of the day in the late spring on a south facing slope when the snow cover is well aged. The net solar adsorption is minimal during the middle of the winter, especially soon after a new snowfall, and, of course, is zero during the night.

Longwave radiation is generated by objects in the atmosphere and on the surface of the earth. The amount of longwave radiation given off by an object is a function of its surface temperature and emissivity (efficiency of producing longwave radiation). Incoming longwave radiation is produced by the water vapor, CO_2, and other particles present in the atmosphere. Longwave radiation is also generated by trees and other objects on the earth's surface. Snow is a very efficient producer of longwave radiation (emissivity of around 0.99, where 1.0 indicates 100% efficient). Net longwave exchange with the snow cover is positive when the air is warm and either the sky is quite overcast or there is a dense conifer forest. The longwave exchange is negative at an open location with a clear sky and generally cool temperatures.

Latent and sensible heat transfers are turbulent exchange processes, i.e. the magnitude of the exchange is dependent on the wind speed. The amount of latent heat exchange is a function of the vapor pressure gradient between the air and the snow surface. The vapor pressure of the air can be computed from the dew-point temperature. The vapor pressure at the snow surface is assumed equal to the saturation vapor pressure at the snow surface temperature. When the vapor pressure of the air is greater than that at the snow surface, vapor is transferred from the air to the snow. Upon reaching the snow surface the vapor condenses and releases heat. When the vapor pressure of the air is lower than that at the snow surface, water vapor is transfer from the snow to the atmosphere. When sublimation occurs, the ice at the snow surface must be converted from a solid to a vapor which requires heat. Sublimation is more common during accumulation periods while condensation typically dominates during the melt season. In general, latent heat transfer produces a small net overall sublimation loss from the snow cover in most regions. The net sublimation loss is greater when low humidity and windy conditions persist like above the tree line in the Intermountain West. The amount of sensible heat exchange is a function of the temperature gradient between the air and the snow surface. When the air is warmer than the snow surface, the exchange is positive and when the air is colder, the exchange is negative. Sensible heat transfer is always positive when the air temperature is above freezing. The greater the wind speed, the greater the amount of latent and sensible heat transfer.

When rain occurs, heat is transferred to the snow based on the temperature of the rain water. As the rain enters the snow cover heat is released until the water temperature comes into equilibrium with the temperature of the snow. The amount of heat transferred from the rain water to the snow is a function of the temperature of the rain drops and the amount of rain.

When the net heat exchange between the air and the snow is positive and the snow surface is at 0˚C, the excess heat causes the snow to melt since the temperature of the snow can't rise any further. This generally occurs when the air temperature is above freezing, though depending on the meteorological situation and physiographic factors at a given location, melt can occur when the air temperature is below freezing and a negative heat exchange can occur when the air temperature is above freezing.

The heat exchange across the air-snow interface can be either positive or negative when the snow surface temperature drops below 0˚C. The direction of heat flow in this situation is dependent on the temperature gradient in the upper layers of the snow cover. Since snow is an excellent insulator, the rate of heat transfer within the snow cover is quite low. The snow surface temperature adjusts to reach a balance between the net heat exchange with the atmosphere and the transfer from within the snow cover. In general, the snow surface temperature drops below 0˚C when the air temperature is below freezing, though under certain conditions the snow surface temperature can be quite different than the air temperature. For example, when the air temperature is below freezing at an open site with clear skies, at night the snow surface temperature can be significantly colder than the air temperature while during the middle of the day the snow surface temperature could be quite a bit warmer than the air temperature.

The rate of heat exchange within the snow cover is largely a function of the effective thermal conductivity of the snow. The thermal conductivity is primarily a function of the density of the snow. The overall density of the snow cover is generally lowest during the mid winter period when new snow is frequent and temperatures are cold and then increases as the snow cover ages and becomes ripe as the melt season begins. The snow cover generally continues to increase in density as melt progresses.

A more complete description of the physics of snow energy exchange and heat transfer within a snow cover is in Appendix A.

Air Temperature has frequently been used to estimate snow melt. Early approaches used a simple degree day factor to compute the amount of melt water that would be produced by a snow cover on a daily basis. Such approaches didn't include an explicit representation of the ripening process or when applied to a watershed, the areal extent of the snow cover. Instead the degree day factor was changed over time as a function of accumulated degree days or fraction of seasonal runoff that had occurred or some other quantity in an attempt to include the effect of these processes. More recent models use air temperature to compute surface snowmelt and include various representations of the internal ripening process and the areal coverage. A comparison between a temperature index model and a detailed energy balance model at a point research location with high quality data [*Anderson* (1976)] showed that overall the results from the calibrated temperature index model compared quite well to output from the energy balance model. That study identified certain meteorological situations when the relationship between melt and air temperature deviated from the calibrated average relationship. The 3 cases identified were:

1. Periods with warm temperatures, high humidity, and strong winds – in these cases there were large amounts of sensible and latent heat transfer and the temperature index model clearly under computed the amount of melt (this was the situation during the snowmelt floods in the Northeastern U.S. in January 1996 [*Office of Hydrology* (1998)],
2. Clear sky periods with an aged snow surface (i.e. fairly low albedo) and cold temperatures – in these cases the amount of melt generated by solar radiation exceeded that estimated by the temperature index model, and
3. Periods with much above normal air temperatures but calm conditions – in these cases the temperature index model clearly over computed the amount of melt.

While an energy balance model, without much calibration, should produce better estimates of melt than a temperature index model at a research location with high quality data, when the models are applied with operationally available data across large river basins the result is not clear. It is much more difficult to estimate the spatial variation in the input for an energy balance model, especially in areas with considerable variations in terrain and vegetation, than to extrapolate air temperature over a watershed. It has yet to be shown that an energy balance model will improve overall results for applications like river forecasting. However, even if a temperature index model is used, energy balance computations may be able to be used to identify periods when the relationship between air temperature and melt deviates from normal and provide a reasonable magnitude of the corrections that need to be applied to melt computations in these situations.

<u>Model Surface Energy Exchange Computations</u>

The SNOW-17 model calculates surface melt in different ways depending on whether rain is occurring or not. Melt during rain-on-snow periods is computed differently than melt during non-rain periods because:

- the magnitude of the various energy transfer components tend to be quite different between the 2 situations,
- the dominant energy transfer components during rain-on-snow periods are known, and
- the seasonal variation in melt rates is generally quite different between non-rain and rain periods.

The model also keeps track of the heat deficit within the snow cover that develops when the temperature drops below 0˚C. SNOW-17 expresses energy exchange in terms of mm, where an mm of energy is the amount of heat required to melt or freeze 1 mm of ice or water, respectively, at 0˚C – approximately 8 cal/cm^2. This makes it easy to compare the heat deficit to the amount of melt or rain water required to overcome the deficit.

Surface Melt Computations

Rain-on-Snow Melt

When sufficient rain occurs, the model uses the energy balance to compute surface melt by making several assumptions about meteorological conditions:

- incoming solar radiation is negligible because overcast conditions generally prevail,
- incoming longwave radiation is equal to black body radiation (emissivity of 1.0) at the temperature of the cloud layer which should be reasonably close to the air temperature, and
- relative humidity is quite high (90% is assumed).

It is also reasonably assumed that the snow surface temperature is equal to 0˚C (273˚K) when rain occurs. With a 90% relative humidity the wet bulb temperature, the assumed temperature of the rain drops, is essentially equal to the air temperature. By making these assumptions, the energy budget equation for melt given in Appendix A (Equation A-16) can be used to compute snowmelt during periods when it is raining. The only variable that is unknown is the wind speed. SNOW-17 uses a parameter, **UADJ**, to indicate the average wind function during rain-on-snow events. By substituting into Equation A-16, the equation for melt during rain-on-snow periods is:

$$\begin{aligned} M_r = \sigma \cdot \Delta t_p \cdot \left[(T_a + 273.)^4 - 273.^4 \right] + 0.0125 \cdot P \cdot f_r \cdot T_r + \\ 8.5 \cdot \mathbf{UADJ} \cdot (\Delta t_p / 6) \cdot \left[(0.9 \cdot e_{sat} - 6.11) + 0.00057 \cdot P_a \cdot T_a \right] \end{aligned} \tag{5}$$

where: M_r = melt during rain-on-snow time intervals (mm),
σ = Stefan-Boltzman constant – $6.12 \cdot 10^{-10}$ mm/˚K/hr,
Δt_p = time interval of precipitation data (hours),
T_a = air temperature (˚C),
273. = 0˚C on the Kelvin scale,
f_r = fraction of precipitation in the form of rain,
T_r = temperature of rain (˚C) – (=T_a or 0˚C, whichever greater),
UADJ = average wind function (mm/mb/6 hr),
e_{sat} = saturated vapor pressure at T_a (mb) – computed from:

$$e_{sat} = 2.7489 \cdot 10^8 \cdot e^{(-4278.63/(T_a + 242.792))}$$, and

P_a = atmospheric pressure (mb) – computed using the 'standard atmosphere' altitude versus pressure relationship:

$$P_a = 33.86 \cdot \left(29.9 - 0.335 \cdot H_e + 0.00022 \cdot H_e^{2.4}\right)$$

where: H_e = elevation (meters).

The rain-on-snow melt equation is used whenever the amount of rain during a given precipitation data time interval is greater than 0.25 mm per hour (1.5 mm per 6 hours).

Equation 5 is independent of the time of the year, thus during rain-on-snow periods, the same quantity of melt is computed for a given air temperature and precipitation amount no matter whether it is the middle of the winter or late spring. This would not be the case in nature only if there was a definite seasonal variation in the wind speed during rain events.

Non-Rain Melt

When there is no rain or very light rainfall amounts (less than or equal to 0.25 mm/hr) during a precipitation data time interval and the air temperature is above a base value, SNOW-17 uses a melt factor to estimate the amount of surface snowmelt. If light rain exists, the small amount of melt due to the heat content of the rainwater is included. Thus the non-rain surface melt equation is:

$$M_{nr} = M_f \cdot \left(T_a - \mathbf{MBASE}\right) \cdot \frac{\left(\Delta t_p\right)}{\left(\Delta t_t\right)} + 0.0125 \cdot P \cdot f_r \cdot T_r \qquad (6)$$

where: M_{nr} = melt during non-rain periods (mm),
M_f = melt factor (mm/˚C/Δt_t),
Δt_t = time interval of temperature data (hours), and
MBASE = base temperature (˚C).

MBASE is a model parameter that allows the user to vary the temperature above which melt typically occurs. A value of **MBASE**=0˚C is normally used, though other values have been used in special situations, primarily when modeling a snow cover at a point location where the temperature measurements don't indicate when melt begins due to physiographic conditions at the site.

During non-rain melt periods solar radiation can have a significant effect on the energy balance. The amount of incoming solar radiation varies with the time of the year. In the northern hemisphere the maximum amount of incoming solar radiation occurs around June 21st while the minimum amount occurs around December 21st (at least south of the Artic circle). The albedo of the snow also tends to have a general seasonal variation because new fresh snow is more common in the mid winter and well aged snow is generally prevalent as the melt season progresses. This produces an even greater seasonal variation in the amount of net solar radiation than for incoming solar. This suggests that there is a strong possibility that the melt factor should vary seasonally. The amount of seasonal variation will depend on the relative magnitude of net solar radiation as compared to the other energy balance components when melt is occurring.

The seasonal melt factor variation used in SNOW-17 is based on energy balance computations. The energy balance equation was used to compute the amount of surface melt during non-rain periods and then, knowing the air temperature and assuming a base temperature of 0˚C, the melt factor that would produce an equivalent amount of melt was computed for each time period when the air temperature was above freezing [*Anderson,* 1968]. This was done using 6 hourly data from the Central Sierra Snow Laboratory near Donner Pass, California [*Snow Hydrology,* 1956]. Figure 4 shows the variation in computed melt factors and the seasonal variation relationship adopted for use in SNOW-17. Most of the points in Figure 4 that deviate considerably from the average relationship are during periods when the air temperature was only slightly above freezing or above freezing nighttime periods when very little melt was computed using the energy balance. This seasonal variation is used for the conterminous United States.

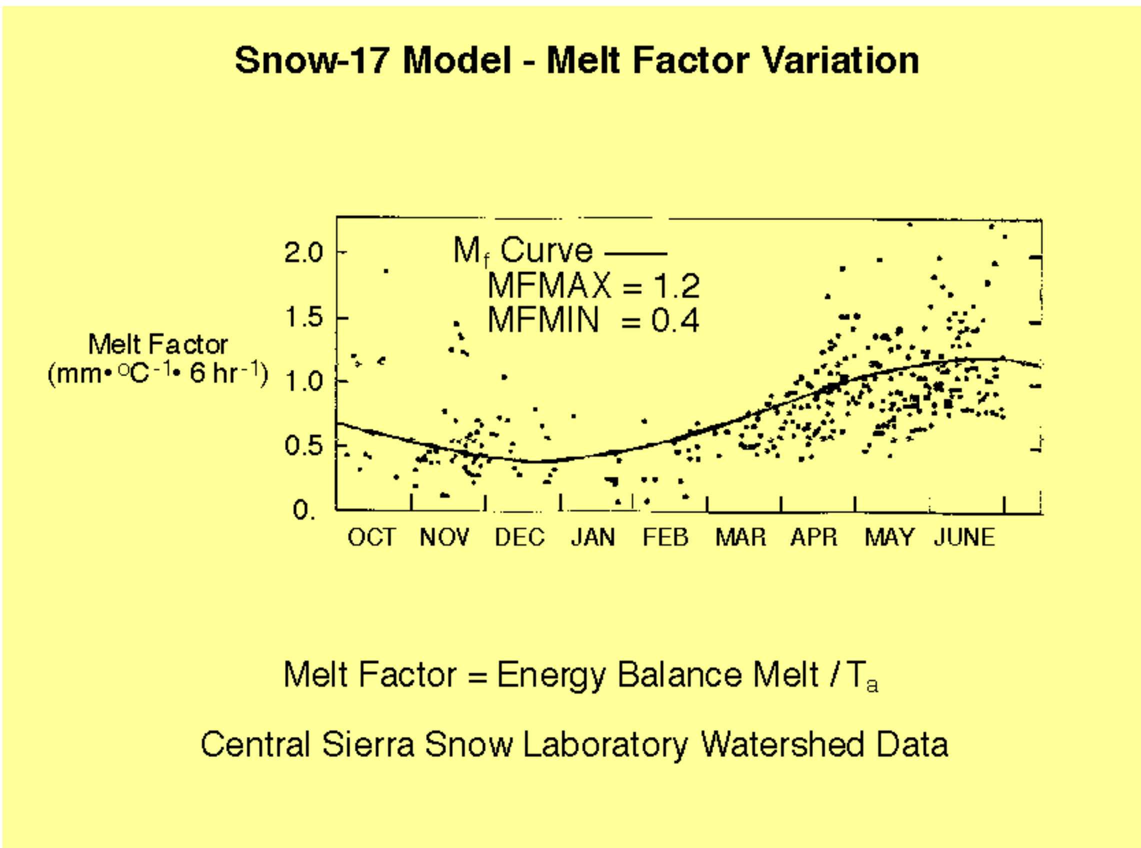

Figure 4. Seasonal melt factor variation for the conterminous United States.

When applying SNOW-17 in interior Alaska (near Fairbanks) it was clear that a different seasonal melt factor variation was needed due to the minimal sunlight that persists during much of the winter. Melt factors were again computed from the energy balance using synoptic data from Fairbanks to determine the values of the input variables. A limited number of melt factors could be computed for the winter months since temperatures above freezing occurred infrequently. In this case the minimum value of the melt factor persists throughout the fall and winter and then there is a rapid increase in the melt rate in the early spring (rapid decrease in the late summer) as the length of the days increase rapidly. The relationship developed using data from Fairbanks is used at all northern latitudes.

The seasonal variation in the non-rain melt factor used in SNOW-17 is expressed as:

$$M_f = \Delta t_t / 6 \cdot \{S_v \cdot A_v \cdot (\mathbf{MFMAX} - \mathbf{MFMIN}) + \mathbf{MFMIN}\}$$
$$S_v = 0.5 \cdot \sin\left(\frac{(N \cdot 2 \cdot \pi)}{366.}\right) + 0.5 \qquad (7)$$

where: N = day number since March 21st,
MFMAX = maximum melt factor – June 21st (mm/˚C/6 hrs),
MFMIN = minimum melt factor – Dec. 21st (mm/˚C/6 hrs), and
A_v = seasonal variation adjustment:
When latitude < 54˚ North, A_v = 1.0, and
When latitude ≥ 54˚ North:
A_v = 0.0 from September 24 to March 18,
A_v = 1.0 from April 27 to August 15, and
A_v varies linearly between 0.0 and 1.0 from 3/19-4/26 and between 1.0 and 0.0 from 8/16-9/23.

MFMAX and **MFMIN** are model parameters. Figure 5 shows how the melt factor varies seasonally between **MFMIN** and **MFMAX**.

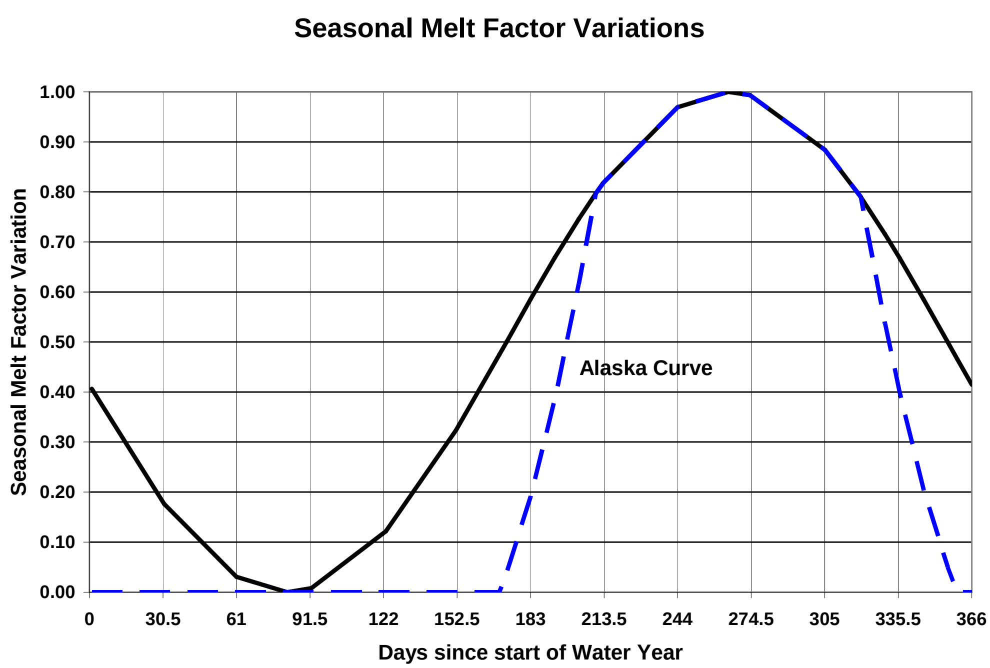

Figure 5. Seasonal melt factor variations used by SNOW-17.

In NWSRFS, SNOW-17 also contains an option for the user to specify the seasonal non-rain melt factor variation. This is done by specifying the decimal fraction that the melt factor is between **MFMIN** and **MFMAX** at the middle of each month. Linear interpolation is then used to get the melt factor on intermediate days. This option was added as a crude way of dealing with melt from glaciers where there is a significant delay in surface melt water flowing out of the glacier. Since then a procedure that accounts for the storage and transmission of water through a glacier has been added to NWSRFS. This new procedure offers a more explicit method of dealing with the timing of glacier melt.

Energy Exchange when No Surface Melt

SNOW-17 uses a heat deficit to keep track of the net heat loss from the snow cover. The heat deficit will change when the air temperature is below freezing due to energy exchange across the snow-air interface. When the air temperature is below freezing, a snow cover can be losing or gaining heat depending on the thermal gradient in the upper layers of the pack. SNOW-17 attempts to estimate this gradient by first assuming that the snow surface temperature, T_{sur} (˚C), is equal to the air temperature or 0˚C, whichever is less. The model estimates the temperature at some distance within the pack by computing an antecedent temperature index, ATI, by weighing the most recent air temperatures by decreasing amounts as one goes further back in time. When there is sufficient new snowfall (greater than 1.5 mm/hr water equivalent), ATI becomes equal to the temperature of the new snow. In SNOW-17 the ATI is computed as:

$$ATI_2 = ATI_1 + TIPM_{\Delta t_t} \cdot \left(T_a - ATI_1\right) \tag{8}$$

where: ATI = antecedent temperature index (˚C) where:
if ATI > 0˚C, ATI = 0˚C,
if $P_n > 1.5 \cdot \Delta t_p$, $ATI = T_n$,
$TIPM_{\Delta t_t} = 1.0 - \left(1.0 - \mathbf{TIPM}\right)^{\Delta t_t / 6}$, and
TIPM = model parameter (>0.0 and <1.0).

The gradient in the upper layers of the snow cover is then estimated as the difference between T_{sur} and ATI. When T_{sur} is less than ATI, the heat deficit is increasing and when T_{sur} is greater than ATI the heat deficit is decreasing. The rate of the increase or decrease is based on a negative melt factor. The negative melt factor is assumed to vary seasonally since typically the density of the snow cover tends to increase from the accumulation period to the melt season and the thermal conductivity of the snow is closely related to the density. Since the rate of heat gain or loss when the air temperature is below freezing is significantly less than when surface melt is occurring due to the insulating properties of snow and since the model uses a rough approximation to the temperature gradient in the upper layers of the snow, a unique seasonal variation is not used. Instead the seasonal variation in the negative melt factor is assumed to be the same as for the non-rain melt factor. Thus, the change in the heat deficit due to a temperature gradient in the surface layers of the snow cover for a precipitation data time interval is:

$$\Delta D_t = NM_f \cdot (ATI - T_{sur}) = \mathbf{NMF} \cdot (\Delta t_p/6) \cdot \frac{M_f}{\mathbf{MFMAX}} \cdot (ATI - T_{sur}) \tag{9}$$

where: ΔD_t = change in heat deficit due to a temperature gradient (mm),
NM_f = negative melt factor (mm/˚C/Δt_p), and
NMF = maximum negative melt factor (mm/˚C/6 hr).

NMF is a model parameter. The negative melt factor equals **NMF** on June 21st when M_f is equal to **MFMAX**.

Internal State of the Snow Cover

Background

A snow cover can contain water in both the solid and liquid form. While a snow cover mainly consists of ice granules, liquid water can adhere to the snow crystals just like water can be attached to soil particles. The amount of liquid water that can be held by well aged snow varies in the literature from about 2-10% by weight (i.e. 100 mm of ice can retain from 2 to 10 mm of liquid water). New fresh snow reportedly can hold more liquid water than aged snow; however, the addition of liquid water to fresh snow greatly speeds up the metamorphism process. Within a relatively short time the intricate snow flakes which can hold considerable water are converted to rounded, isometric grains as in an aged snow cover. Because of the instability of fresh snow there is no real data in the literature relating liquid water holding capacity to density. Sometimes a slush layer will form at the bottom of a snowpack. If the pack is deep, the slush layer has a minimal effect on the overall liquid water content, but if there is a shallow snow cover, the effect of the slush layer is to cause the overall percent liquid water to become greater than the values quoted in the literature.

When a snow cover first begins to lose heat, the temperature of the ice near the surface will drop below 0˚C and if any liquid water is present in the surface layer, it will refreeze. As cold temperatures persist and the snow continues to lose more heat, the below freezing temperatures will penetrate further into the snow cover and if present, more liquid water will refreeze. The movement of heat within a snow cover is fairly slow because the thermal conductivity of snow is quite low. Thermal conductivity varies primarily with density. Low density snow has a very low thermal conductivity and is thus an extremely good insulator. Besides conduction, heat is also transferred within a snow cover when phase changes occur. The main phase change process takes place when melt or rain occurs at the surface and then the resulting liquid water refreezes somewhere within the pack where the temperature is below freezing. Heat can also be transferred due to vapor movement produced by a temperature gradient. Sublimation occurs from a warmer snow crystal and then the vapor condenses on a nearby colder grain. The heat and vapor move in the direction of the colder temperature.

The density profile of a snow cover is also constantly changing causing the depth to change even if the water equivalent remains constant. The density changes due to new snowfall, compaction, and destructive, constructive, and melt metamorphism. Compaction increases the density due to the weight of the overlying snow. Compaction is primarily of importance in regions that experience a deep snow cover (generally over about 300 mm of water equivalent for the peak accumulation). Destructive metamorphism is the process that causes the intricate fresh snow flakes to change into rounded, isometric ice grains, thus increasing the density. The rate of destructive metamorphism decreases as the temperature of the snow is lowered. Destructive metamorphism is primarily of importance for fresh, low density snow. The rate of destructive metamorphism drops off significantly after a threshold density is reached (around 0.15 $gm \cdot cm^{-3}$). Constructive metamorphism changes the density profile within the snow cover as vapor is transferred from one grain to another whenever a temperature gradient exists though the overall density of the snowpack remains about the same. When there is a prolonged period of very cold surface temperatures and much warmer temperatures at the bottom of the pack, constructive metamorphism leads to the formation of depth hoar. Melt metamorphism occurs when snow at the surface is melted and then refreezes somewhere within the snow cover where the temperatures are below freezing. This produces an increase in the overall density of the snow cover. The presence of liquid water produced by melting or rain causes destructive metamorphism to proceed at a faster rate. This is another component of melt metamorphism.

Accounting for Internal Changes within the Model

SNOW-17 treats the snow cover as a single lumped entity. The model doesn't try to calculate the temperature, liquid water, or density profile within the pack; it only deals with the overall state of the snow cover. SNOW-17 accounts for the overall ripeness of the snow cover by keeping track of a heat deficit and the liquid water storage. The model also computes changes to the density of the snow in order to calculate the depth.

Ripeness of the Snow Cover

A snow cover is considered to be ripe when any additional melt or rain water cannot be held within the snow but will move through the pack and become outflow. This occurs when the snow cover is isothermal at 0˚C and the liquid water storage capacity is full. In SNOW-17 the snow cover is ripe when both the heat deficit is zero and the amount of liquid water held in the pack equals the holding capacity. The liquid water holding capacity in SNOW-17 is determined by:

$$W_{qx} = \mathbf{PLWHC} \cdot W_i \tag{10}$$

where: W_{qx} = liquid water capacity (mm),
PLWHC = percent liquid water holding capacity (decimal fraction), and
W_i = water equivalent of the ice portion of the snow cover (mm).

PLWHC is a model parameter. It is intended to represent the overall liquid water holding capacity of a well aged snow cover. The maximum allowed value of **PLWHC** is 0.4 (the maximum allowed density is 0.6 and the total of ice and liquid water can't exceed 1.0).

Before describing the internal snowpack accounting process in SNOW-17 it is important to understand the concept behind the heat deficit (the heat deficit is also referred to as negative heat storage – NEGHS). The heat deficit in SNOW-17 begins to accumulate when the air temperature drops below 0˚C. As the heat deficit increases it indicates that the snow cover is losing heat, but doesn't differentiate between whether the temperature of the ice particles are dropping below 0˚C or whether liquid water is refreezing or both are occurring since the model treats the snowpack as a lump. When the heat deficit returns to zero, it indicates that the snow cover is in the same relative state of ripeness as when the deficit began to accumulate. The actual physical amount of liquid water and the temperature of the ice within a real snowpack may be different from the time when the model heat deficit begins to accumulate to when it returns to zero, though the same relative state of ripeness will exist at both times. To help in understanding the relationship between the model heat deficit computations and what is occurring in an actual snow cover an example might be helpful.

> Assume that both the actual and model snow covers begin with isothermal conditions, 300 mm of ice, and 3 mm of liquid water. The liquid water holding capacity is 5%. Assume all of the liquid water within the actual snow cover is at the top of the pack, i.e. the top 20% in terms of the water equivalent of the ice. The heat deficit in the model is 0.0. Now examine what occurs as the snow cover goes through a cycle of heat loss and heat gain, with the heat deficit in the model returning to 0.0 at the end of the cycle.
>
> > Step 1 – A period of cold weather (air temperature remains below freezing) occurs resulting in a net heat loss of 20 mm of energy. For the actual snow cover the 3 mm of liquid water near the surface would all refreeze during this period causing the amount of ice in the pack to rise to 303 mm. The remaining 17 mm of heat loss would go into cooling the ice. The average snow cover temperature would drop to -8.98˚C (see Equation 4). For the model the heat deficit would become 20 mm while the amount of ice and liquid water would remain at their initial values, i.e. 300 mm and 3 mm.
> >
> > Step 2 – The weather turns milder but the air temperature remains below freezing resulting in a net heat gain of 7 mm of energy. The amount of ice and liquid water in the actual and model snow covers would remain the same as after step 1. For the actual snowpack the gain in heat would raise the average snow cover temperature to -5.28˚C. For the model the heat deficit would be reduced by 7 mm to 13 mm.
> >
> > Step 3 – The air temperature rises above freezing and remains there resulting in a net heat gain of 13 mm of which 12 mm produces melt at the surface (the other 1 mm of gain is from conduction due to the temperature gradient that will exist in the upper layers of the snow cover for at least a portion of the time period). For the actual snow cover the conduction and refreezing of some of the melt water will warm the upper part of the snowpack to 0˚C. The remaining melt water will

be held within that portion of the snow cover that is now at 0˚C. The ice in the lower layers of the pack will remain below freezing. A reasonable accounting of the melt water is that about 5 mm would refreeze, which along with the conduction of heat would cause the snow to be at 0˚C nearly half way down into the pack in terms of water equivalent (depth doesn't need to be considered in this example). The remaining melt water, 7 mm, would be held as liquid water within this portion of the pack. The amount of ice now in the actual snow cover would be 303 mm (step 2 value), minus 12 mm (melt), plus 5 mm (refreeze), or 296 mm. The average temperature of the snow cover would now be -2.16˚C with the average being -4.1˚C in the portion of the pack that is below freezing. This is equivalent to 4 mm of energy. For the model the heat deficit would be reduced by the 13 mm of heat gain and would now be back to 0.0. The 12 mm of surface melt would have all refrozen within the snow cover. The ice portion of the snowpack would still be 300 mm [300 – 12 (melt) + 12 (refreeze)] and the amount of liquid water remains at 3 mm.

When ripe a snow cover with 303 mm of total water equivalent and 5% liquid water holding capacity would contain 288.6 mm of ice and 14.4 mm of liquid water. At the end of the above cycle the model would require an additional 11.4 mm of surface melt to become ripe (11.4 mm of melt would reduce the ice content to 288.6 mm and raise the liquid water content to 14.4 mm with the heat deficit already zero). At the end of the cycle the actual snow cover would also require 11.4 mm of melt to become ripe [of the 11.4 mm of melt 4 mm would refreeze in order to make the entire pack isothermal at 0˚C and the remaining 7.4 mm would be held as liquid water – thus, the total amount of liquid water when ripe would be 7 mm (step 3 amount) plus the 7.4 mm or a total of 14.4 mm and the amount of ice in the ripe pack would be 296 mm (step 3), minus 11.4 mm (melt), plus 4 mm (refreeze), or 288.6 mm]. When ripe the total amount of water that refroze within the snow cover would be 12 mm in both the model and the actual snow cover (all occurs in step 3 for the model while for the actual snow cover 3 mm occurs in step 1, 5 mm in step 3, and 4 mm in order to become ripe).

Even though the temperature and liquid water profiles differ in the actual snow cover from the beginning to the end of a model heat deficit cycle, the overall ripeness of the snow cover would be the same at both times, i.e. it would take the exact same amount of additional melt or rain water to fill the liquid water storage and raise the temperature of the entire snow cover to 0˚C. The use of the heat deficit allows the model to reasonably represent the ripening process without having to make assumptions as to whether heat losses are refreezing liquid water or lowering the temperature of the snow cover or both. Measurements of the liquid water and temperature profiles for a real snowpack could be used to determine the relative ripeness, but not the heat deficit directly. Such measurements could be used to adjust the combination of the model liquid water storage and heat deficit.

SNOW-17 goes through the accounting process for the heat deficit and liquid water storage for each precipitation data time interval. The sequence is as follows:

1. First the amount of liquid water available at the surface of the snow cover due to melt and rain is calculated and the heat deficit is adjusted due to the temperature

of new snowfall and heat transfer caused by a temperature gradient in the upper layers of the snow cover:

$$Q_w = M_r + M_{nr} + P \cdot f_r \tag{11a}$$

$$D_2 = D_1 + \Delta D_p + \Delta D_t \tag{11b}$$

where: Q_w = liquid water available at the snow surface (mm), and
D = heat deficit (mm).

Of course both M_r and M_{nr} can’t be greater than zero during a given interval since either Equation 5 or 6 would be used to obtain surface melt depending on whether rain was occurring during that period.

2. Then if there is sufficient water available at the surface to overcome the heat deficit and exceed the liquid water storage capacity, the snow cover becomes ripe and the excess water will be available to move through the pack and become outflow. The amount of excess water in this case is:

$$E = Q_w + W_q - W_{qx} - D - (\mathbf{PLWHC} \cdot D) \tag{12}$$

where: E = excess liquid water (mm), and
W_q = liquid water held by the snow (mm).

At the end of the time interval the amount of liquid water held by the snow, W_q, is equal to the liquid water storage capacity, W_{qx}; the amount of ice in the snow, W_i, is increased by the heat deficit, D, since that much water ‘refroze’ in order to raise the temperature of the pack to 0˚C; and the heat deficit becomes zero.

3. If there is only sufficient water available at the surface to overcome the heat deficit, but not enough to fill the liquid water holding capacity of the snow cover, then the new amount of liquid water is computed as:

$$W_q = W_q + Q_w - D \tag{13}$$

In this case the amount of ice in the snow, W_i, is again increased by the heat deficit due to liquid water ‘refreezing’ in the pack and heat deficit becomes zero and there is no excess water available. The snowpack is not yet ripe.

4. If there is not enough surface water to overcome the heat deficit, then the heat deficit, D, is reduced by the amount of available water, Q_w; the amount of ice in the pack, W_i, is increased by the amount of water that ‘refroze’, Q_w; the amount of liquid water held in the pack, W_q, remains the same; and there is no excess water available. Again the snow cover is not yet ripe.

In items 2-4 the 'refreezing' of liquid water within an actual snow cover doesn't all occur during the same time interval as in the model due to the concept and logic behind the heat deficit. This could be seen in the preceding heat deficit example. Overall the net amount of surface water that refreezes within the actual snow cover during a ripening period is believed to be generally close to that computed by the model, though there can be differences depending on initial snowpack conditions and how fast the ripening occurs. Besides the heat deficit example, several other cases might be helpful in understanding how the timing of when water refreezes and the how the amount of water that refreezes can vary between the model and an actual snow cover.

1. At night during the active melt season a snow cover losses heat when the air temperature drops below freezing. In the case of the actual snow cover the liquid water in the surface layer refreezes overnight as the heat is lost and the temperature of the surface layer decreases. When melt begins the next day, the liquid water is replenished and the temperature of the ice returns to 0˚C in the surface layer. In the model the heat deficit increases overnight. When the temperature rises above freezing the next day, the heat deficit is first reduced somewhat by a positive change in the heat deficit due to the temperature gradient in the surface layer and then melt water is refrozen within the pack to return the heat deficit to zero. While the timing of the refreezing differs slightly between the model and the actual snow cover, the amount of water that refreezes should be very similar in this case.

2. A snow cover is ripe at some point during the accumulation season and then a long cold spell occurs that allows below freezing temperatures to penetrate to the bottom of the pack. In the case of the actual snow cover, during the cold spell all the liquid water in the snow would refreeze and the temperature of the snow would drop well below 0˚C. In the model during this period a large heat deficit would accumulate but no refreezing of liquid water would occur during the cold weather period. By the time when the snow again becomes ripe, the temperature of the actual snow cover would return to 0˚C and the liquid water would be replenished. In the model the amount of water that refroze would depend on the temperatures during the ripening period. If a sudden warm period which produced surface melt occurred, the heat deficit in the model would primarily be reduced by the refreezing of the melt water, thus the amount of refrozen water computed by the model would likely be greater than what occurred in nature. If a slow warm up occurred with temperatures staying near freezing, the heat deficit would first be reduced due to the temperature gradient in the surface snow layer though the rate of reduction in the heat deficit due to the warmer snow surface temperature would dissipate as the antecedent temperature index approached the surface temperature. This would likely occur before the heat deficit returned to zero, thus the remainder of the heat deficit would have to be removed by the refreezing of surface melt or rain water. The net result in this scenario could be fairly similar to what would occur in nature.

3. A dry snow cover accumulates during the winter as the temperature remains below freezing. The actual snow cover would contain no liquid water and its average temperature would be less than 0˚C. The model snow cover would also have no liquid water and a heat deficit would accumulate. If a sudden melt or rain-on-snow event then occurred, the increase in the temperature of the actual snow cover and the reduction of the heat deficit in the model would be primarily due to the refreezing of surface water as it moved down through the pack. If the temperatures warmed

slowly, the actual snow cover temperature would increase and the model heat deficit would decrease primarily due to the temperature gradient in the surface layers of the pack. In this case the liquid water storage would fill both in nature and in the model when melt or rain subsequently occurred.

Density and Depth Computations

The original SNOW-17 model [*Anderson,* 1973] didn't contain depth computations, but only included water equivalent. This feature was added in recent years so that model simulated depth could be compared to observations and to enable more physically based heat flow computations through a snow cover to estimate frozen soil conditions. In many regions the only readily available snow cover observations are of depth of new snowfall and depth of snow on the ground.

In order to compute the depth of the snow cover the model separates new snowfall from the snow that existed at the start of the computational interval. The change in density of the existing snow is calculated using a simplified version of the procedure developed by Anderson [1976]. This change in density is due to compaction, destructive metamorphism, and the component of melt metamorphism resulting from the presence of liquid water. Constructive metamorphism is not included since it only changes the density profile of a snow cover and SNOW-17 treats the entire snow cover as a single entity. The increase in density from these factors is computed using an extension of the analytical solution develop by Koren et. al. [1999]:

$$\rho_{x_2} = \rho_{x_1} \cdot \left(\frac{e^{B \cdot 0.1 \cdot W_{ix}} - 1}{B \cdot 0.1 \cdot W_{ix}} \cdot e^{A} \right) \tag{14}$$

where: $B = c_1 \cdot \Delta t_t \cdot e^{0.08 \cdot T_s - c_2 \cdot \rho_x}$

$A = c_3 \cdot c_5 \cdot \Delta t_t \cdot e^{c_4 \cdot T_s - c_x \cdot \beta \cdot (\rho_x - \rho_d)}$

ρ_x = density of the ice portion of the existing snow cover (gm·cm^{-3}),
T_s = average snow cover temperature (˚C),
$\beta = 0.0$ if $\rho_x \le \rho_d$ and $=1.0$ if $\rho_x > \rho_d$,
W_{ix} = remaining ice portion of the snow cover that existed at the start of the period (mm), and
c_1, c_2, c_3, c_4, c_5, c_x and ρ_d are constants defined in Anderson [1976]:

c_1 = fractional increase in density – 0.026 cm^{-1}·hr^{-1}
c_2 = constant estimated by Kojima [1967] – 21 cm^{3}·gm^{-1}
c_3 = fractional settling rate at 0˚C for $\rho_x < \rho_d$ – 0.005 hr^{-1}
c_4 = constant – 0.10 ˚C^{-1}
c_5 = increase in fractional settling rate when liquid water exists
= 0 when $W_{qt} = 0.0$, and
= 2.0 when $W_{qt} > 0.0$.
W_{qt} = total liquid water in snow (mm) - (see section on Transmission of Water through the Snow Cover)
ρ_d = threshold density above which destructive metamorphism decreases – 0.15 gm·cm^{3}

c_x = destructive metamorphism decay factor when $\rho_x > \rho_d - 23$.

In the Anderson (1976) report the values used for the compaction and destructive metamorphism constants were $c_1 = 0.01$, $c_2 = 21.0$, $c_3 = 0.01$, $c_4 = 0.04$, $\rho_d = 0.15$, and $c_x = 46.0$. These values were based on information in the literature and calibration using one year of data from the NOAA-ARS Snow Research station [*Anderson et al* (1977)]. When snow depth computations were first added to the SNOW-17 model, the parameter values were tested using 9 years of data from the same research station. Only the value of ρ_d was changed from 0.15 to 0.20 to improve the comparison of simulated and observed depth. The current values of the snow depth constants are based on comparisons of simulated and observed depth from a number of sites: the NOAA-ARS Snow Research station, the sites used for the SnowMIP project [*Etchevers et al* (2002)], 3 sites in interior Alaska, and Stampede Pass, Washington. This comparison showed that the original parameters over estimated snow depth for locations with a deep snow cover and under estimated snow depth for locations with very cold climates. The current parameter values produce the best overall results for the variety of snow depths and climates tested.

At the end of the period the depth of the snow that existed at the start of the computational time interval can be computed as:

$$H_x = \frac{0.1 \cdot W_{ix}}{\rho_x} \tag{15}$$

where: H_x = depth of the snow that existed at the start of a computational interval (cm).

Next the snow that was present at the start of the time interval is combined with any new snowfall during the period to get the average density of the total snowpack (If no snow existed at the beginning of the interval, then the overall density is the density of the new snowfall). Then the increase in density due to the component of melt metamorphism resulting from melt-freeze cycles is computed based on the amount of surface melt or rain water that refroze within the snow cover using (water equivalent is increased in this case with no change in depth):

$$\rho_2 = \rho_1 \cdot \frac{W_i}{(W_i - Q_f)} \tag{16}$$

where: ρ = average density of the ice portion of the total snow cover (maximum allowed value is 0.6 since the maximum value of **PLWHC** is 0.4), and Q_f = total water that refroze within the snowpack over Δt_t (mm).

The depth of the total snow cover can be then be computed as:

$$H = \frac{0.1 \cdot W_i}{\rho} \tag{17}$$

where: H = depth of the total snow cover (cm).

The average snow cover temperature, T_s, for use in Equation 14 is calculated as a weighted average of the temperature of the existing snow cover, T_x, and the new snowfall, T_n. The temperature of the existing snow cover is computed from an approximate solution of the heat transfer equation using the change in the air temperature since the last time interval. The average temperature of the existing snow cover is computed as:

$$T_{x,t+\Delta t_t} = T_{x,t} + \Delta T_a \cdot \frac{1.0 - e^{-\alpha \cdot 0.01 \cdot H_x}}{\alpha \cdot 0.01 \cdot H_x} \tag{18}$$

where: $\Delta T_a = T_{a,t} - T_{a,t-\Delta t}$

if $T_{a,t-\Delta t} > 0.$ and $T_{a,t} > 0.$; $\Delta T_a = \text{abs}(\Delta T_a)$

if $T_{a,t-\Delta t} > 0.$ and $T_{a,t} < 0.$; $\Delta T_a = T_a$

$$\alpha = \sqrt{\frac{\pi \cdot c}{\lambda \cdot 2 \cdot 3600. \cdot \Delta t_t}}$$

λ = thermal conductivity of snow (watts·m^{-1}·°C^{-1}) estimated from Djachkova's formula [*Koren,* 1991]:

$$\lambda = 0.0442 \cdot e^{5.181 \cdot \rho_x}$$

c = effective specific volumetric heat capacity of snow (watts·sec·m^{-3}·°C^{-1})

$$c = c_c \cdot \rho_x + c_a \cdot (1.0 - \rho_x - \theta_q) + c_q \cdot \theta_q$$

where: c_c = volumetric heat capacity of ice ($2.1 \cdot 10^6$),

c_a = volumetric heat capacity of air ($1.0 \cdot 10^3$),

c_q = volumetric heat capacity of water ($4.2 \cdot 10^6$), and

θ_q = fraction of liquid water in snow – $W_{qt}/(W_i + W_{qt})$, and

T_x = average temperature of the existing snow cover (˚C).

When there is new snowfall during a computational period, Equation 18 is modified to take into account the insulating effect of the new snow. In this case:

$$T_{x,t+\Delta t} = T_{x,t} + \Delta T_a \cdot \left[\frac{e^{-\alpha \cdot 0.01 \cdot H_n} - e^{-\alpha \cdot 0.01 \cdot H_x}}{\alpha \cdot 0.01 \cdot (H_x - H_n)} \right] \tag{19}$$

The weighted average snow cover temperature is computed as:

$$T_s = \frac{(T_x \cdot H_x) + (T_n \cdot H_n)}{H_x + H_n} \tag{20}$$

Transmission of Water through the Snow Cover

Background

Excess liquid water that is available at the snow surface, either from melt or rain, must move through the snow before becoming outflow from the pack. The time delay and the amount of dampening are related to the magnitude and condition of the snow cover and the amount of excess water. The larger the snowpack, the greater is the delay and the more dampening that occurs. The larger the amount of excess water, the quicker the water will move through the snow and the less dampening that will take place. Excess water will move most rapidly through well aged, i.e. ripe, snow that consists of spherical grains. The overall delay and attenuation of excess water through new fresh snow is considerably greater. As mentioned earlier, new fresh snow will hold more liquid water than ripe snow, but the presence of the water causes the transition from intricate snow flakes to spherical crystals to proceed more rapidly. The greater the amount of excess water, the faster the transition from fresh to ripe snow. Thus the net effect when rain or melt occurs on new fresh snow is to increase the lag and attenuation of the excess water through the pack. Very little detailed data are available on this phenomenon thus it is difficult to quantify the process.

Model Representation

SNOW-17 uses empirically derived equations to calculate the lag and attenuation of water through a ripe snow cover. The equations were derived from data collected with a lysimeter at the Central Sierra Snow Laboratory (CSSL) [*Snow Investigations*, 1955]. The same equations are used no matter what is the density of the snowpack. The amount of lag is computed as:

$$L = 5.33 \cdot \left[1.0 - e^{\left(\left(-0.03 \cdot (\Delta t_p / 6) \cdot W_i\right)/E\right)}\right] \tag{21}$$

where: L = lag time for excess water (hours).

Since this is a variable lag based on the W_i/E ratio, the solution used in the model breaks the excess water up into a number of increments based on the magnitude of E and computes the lag for each increment.

The attenuation portion of the excess water transmission process uses a withdrawal rate which is the portion of the lagged excess liquid water which drains from storage during a given time interval. The withdrawal rate for a one hour interval derived from the CSSL lysimeter data is:

$$R_1 = \frac{1.0}{1.0 + 5.0 \cdot e^{\left((-500. \cdot E_{ls})/W_{is}^{1.3}\right)}} \tag{22}$$

where: R_1 = one hour withdrawal rate (hr^{-1}),

W_{is} = mean water equivalent of the ice portion of the snow over the snow covered area (inches) - $W_{is} = W_i / (25.4 \cdot A_s)$,
E_{ls} = average hourly lagged excess liquid water available for Δt_p over the snow covered area (inches) - $E_{ls} = E_l / (25.4 \cdot A_s)$,
A_s = portion of the area covered by snow (decimal fraction), and
E_l = average hourly lagged excess water available for Δt_p (inches).

Since the water transmission equations were derived for a point location and the model can be applied over an area, the water equivalent and lagged excess water values in Equation 22 must to converted to represent the average for the snow covered portion of the area. No conversion is needed in Equation 21 since a ratio is used and thus the areal snow cover cancels out. The functional forms of Equations 21 and 22 were developed by plotting the experimental data. Final coefficient values were determined by minimizing the squared error between simulated and observed snow cover outflow from the lysimeter. English units are used for W_{is} and E_{ls} in Equation 22 because the coefficients were determined using English units and there is no simple conversion. Also since Equation 22 was derived using hourly data and again since there is no simple way to adjust the equation to other time intervals, the model always performs the attenuation and outflow computations on an hourly basis.

Figure 6 shows a comparison of the simulated versus observed snow cover outflow for 3 days at the beginning, middle, and near the end of the snowmelt period for the CSSL lysimeter. This comparison uses the same data that were used to derive Equations 21 and 22 and the coefficients used in the equations. The figure illustrates how the lag and attenuation change as the magnitude of the snowpack changes and show that the algorithms used in SNOW-17 reasonably mimic this variable response.

After lagging the excess surface water and computing the appropriate withdrawal rate, the amount of snow cover outflow due to surface melt or rain on an hourly basis is calculated as:

$$O_{mr_1} = (S_1 + E_1) \cdot R_1 \tag{23}$$

where: O_{mr_1} = hourly snow cover outflow from melt or rain-on-snow (mm), and
S_1 = amount of lagged excess liquid water in storage at the beginning of the hour (mm).

The amount of lagged excess liquid water in storage at the end of the hour, S_2, is then:

$$S_2 = S_1 + E_1 - O_{mr_1} \tag{24}$$

The outflow due to surface melt or rain for the time interval associated with the precipitation data, O_{mr} (mm), is the sum of the hourly outflow values for that period. The total amount of liquid water within the snow cover, W_{qt} (mm) is the sum of the liquid water held within the pack, W_q; the lagged excess surface water that hasn't yet entered storage; and the amount of lagged excess liquid water in storage, S. Thus, the total water equivalent of the snow cover, W_t (mm), is the sum of the ice and liquid portions:

$$W_t = W_i + W_{qt} \tag{25}$$

This is amount of water equivalent that would be measured by a snow tube or pressure sensing device.

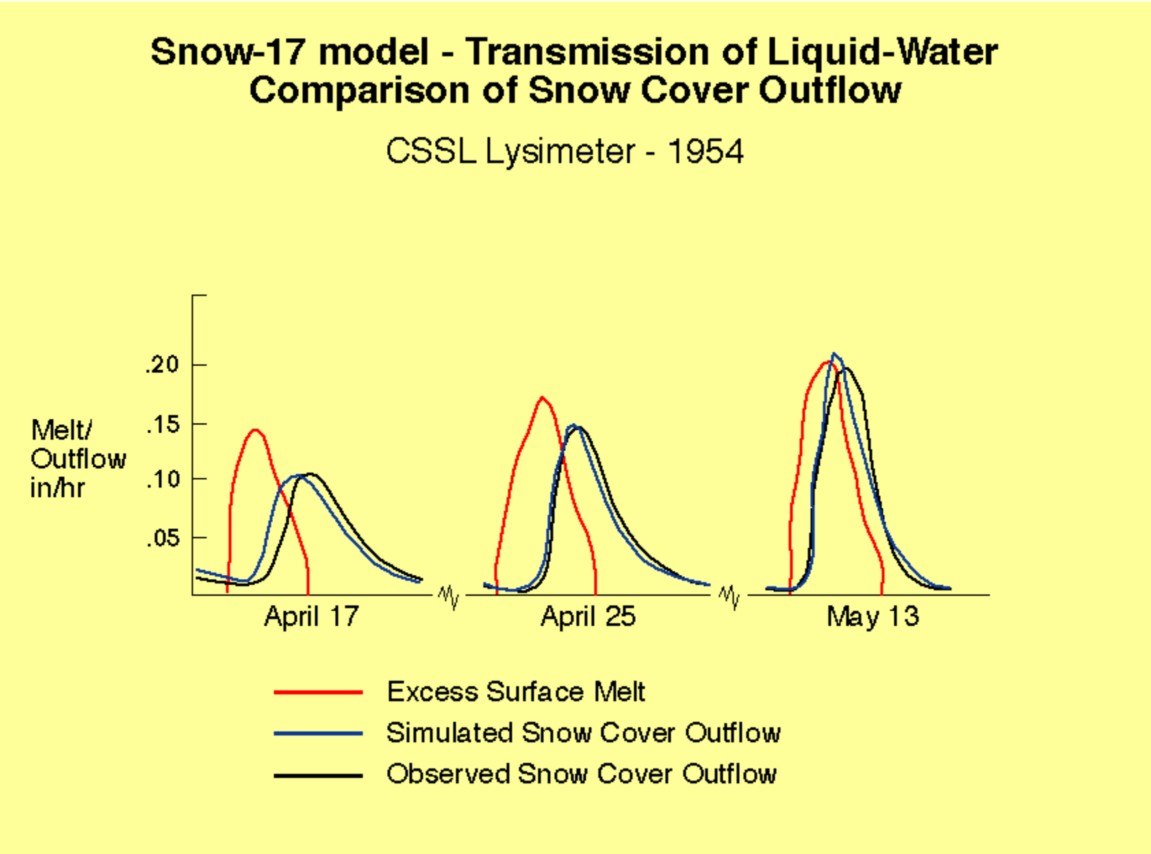

Figure 6. Verification of liquid water transmission algorithms.

Heat Transfer at the Snow-Soil Interface

<u>Background</u>

Heat is transferred from the soil to the snow or vice versa depending on the temperature gradient in the upper layers of the soil. When temperatures are below freezing for some period of time and there is little or no snow on the ground, frost will develop in the soil. When a sizeable snowpack exists, either an existing frost layer will cease to expand or frozen ground conditions will not develop due to the insulating properties of snow. When the ground is warmer than the snow at the bottom of the pack, some melt will occur at the interface. Typically since the soil temperatures are generally warmer when the snow cover begins to accumulate than later in the season, the amount of melt is greater at the beginning of the accumulation period than later in the winter and into the spring. The amount of melt is generally quite small compared to that which occurs at the snow surface unless one is dealing with a very shallow snow cover in a temperate region. However, when melt does occur at the snow-soil interface, it will increase the surface soil moisture over time and if the soil is already saturated, it will percolate into baseflow storages resulting in a baseflow recession that is flatter than during snow free periods of the year (baseflow levels may even increase in some situations due to melt at the soil interface).

Model Representation

SNOW-17 includes a parameter that specifies the daily amount of melt that occurs on the average at the snow-soil interface. This parameter is **DAYGM** with units of mm/day. The daily ground melt is assumed to be constant throughout the period when snow is on the ground. Even though this is not correct, it is adequate in most situations since the amount of ground melt is small compared to the total snow cover. The amount of melt at the snow-soil interface for the time interval of the precipitation data is:

$$M_g = \mathbf{DAYGM} \cdot \left(\Delta t_p / 24.\right) \qquad (26)$$

where: M_g = amount of ground melt during each precipitation data interval (mm) - equal to W_i if $M_g > W_i$.

The amount of outflow generated at the snow-soil interface for each precipitation data time interval is then the amount of ground melt plus any liquid water that is released due to a decrease in the amount of ice in the snowpack:

$$O_g = M_g + \left(M_g / W_i\right) \cdot W_q \qquad (27)$$

where: O_g = outflow due to ground melt for each precipitation data interval (mm).

The total outflow from the snow cover is then the sum of the outflow from surface melt or rain and the outflow from ground melt:

$$O_s = O_{mr} + O_g \qquad (28)$$

where: O_s = total snow cover outflow for each precipitation data interval (mm).

IV. Accounting for the Areal Extent of the Snow Cover

In order to apply SNOW-17 to an area, the model must calculate the areal extent of the snow cover and use it when producing areal output. The model keeps track of average areal values of state variables, energy exchange, and water balance quantities, thus when an equation is based on 100% cover, the result must be adjusted by the areal extent of the snow cover before it is used to adjust mean areal values. Estimates of surface melt (Equations 5 or 6) and the change in heat storage (Equation 9) are based on a complete snow cover and thus need to be multiplied by the areal snow cover prior to being used in snow cover accounting and outflow computations. The amount of ground melt also needs to be reduced by the areal cover prior to computing the areal average outflow from the snow cover. Any rainfall is divided into the portion that falls on the snow covered area and the portion that falls on bare ground (the rate of rainfall is assumed to be the same throughout the area). Rain that falls on the snow covered portion of the area is added to any melt before being used in the snow cover accounting computations. Rain that falls on bare ground is added to any outflow from the snow covered area to determine the total rain plus melt for each precipitation data time interval. Thus, the total amount of water exiting SNOW-17 for any precipitation data time interval is:

$$O = (O_s \cdot A_s) + [(1.0 - A_s) \cdot (P \cdot f_r)] \tag{29}$$

where: O = total outflow from SNOW-17 – snow cover outflow plus rain-on-bare-ground (mm), and
A_s = areal extent of the snow cover (decimal fraction).

This value is typically referred to as rain+melt. This is the amount of water available as input to a subsequent rainfall/runoff model.

Background

The areal extent of the snow cover over any given area is a function of various factors that influence both the accumulation and melting of the snowpack. Accumulation patterns are determined by factors such as storm type, storm direction, topography, wind both during and after a storm, and vegetation. These factors all play a role in determining the distribution of the snowfall from a storm over a given area and the possible redistribution of some of that snow afterward. During melt periods the rate of melt at any given point is determined by factors such as meteorological conditions (i.e. temperature, dew-point, wind, cloud cover, etc.), slope, aspect, and vegetation cover. Certain areas tend to melt faster than others (e.g. south slopes faster than north slopes and open areas faster than forested) though the difference depends on the current meteorological situation. The areal extent of the snow cover at any given time is a composite of how these factors vary from storm to storm and throughout ablation periods. In addition, the total amount of snow that builds up over the accumulation period is also a factor influencing the areal extent of the snow cover.

Depletion curves are frequently used to indicate how the areal extent of the snow cover changes as a melt season progresses. The most common depletion curve is a plot of the areal extent of the snow cover versus the mean areal water equivalent. Depletion curves that relate areal cover to the fraction of the seasonal runoff that has occurred or even to the time of the year have also been used. Because many of the factors that affect the areal snow cover are fairly consistent from year to year for a given area, the general shape of the depletion curve is also generally similar from year to year. A given area typically has a fixed topographic and vegetation pattern (unless large fires or timber cutting takes place), similar seasonal variation in storm types and storm direction, and similar average meteorological conditions during melt periods and between storms. This results in the variation in the relative amount of snow that accumulates from one point to another to be fairly consistent from one year to another. For example, drifts tend to develop in the same places, bare ground expansion patterns look alike, and the last remaining patches of snow are generally in the same locations from one snow season to another. Figure 7 shows how the areal depletion curves for a given area might vary from

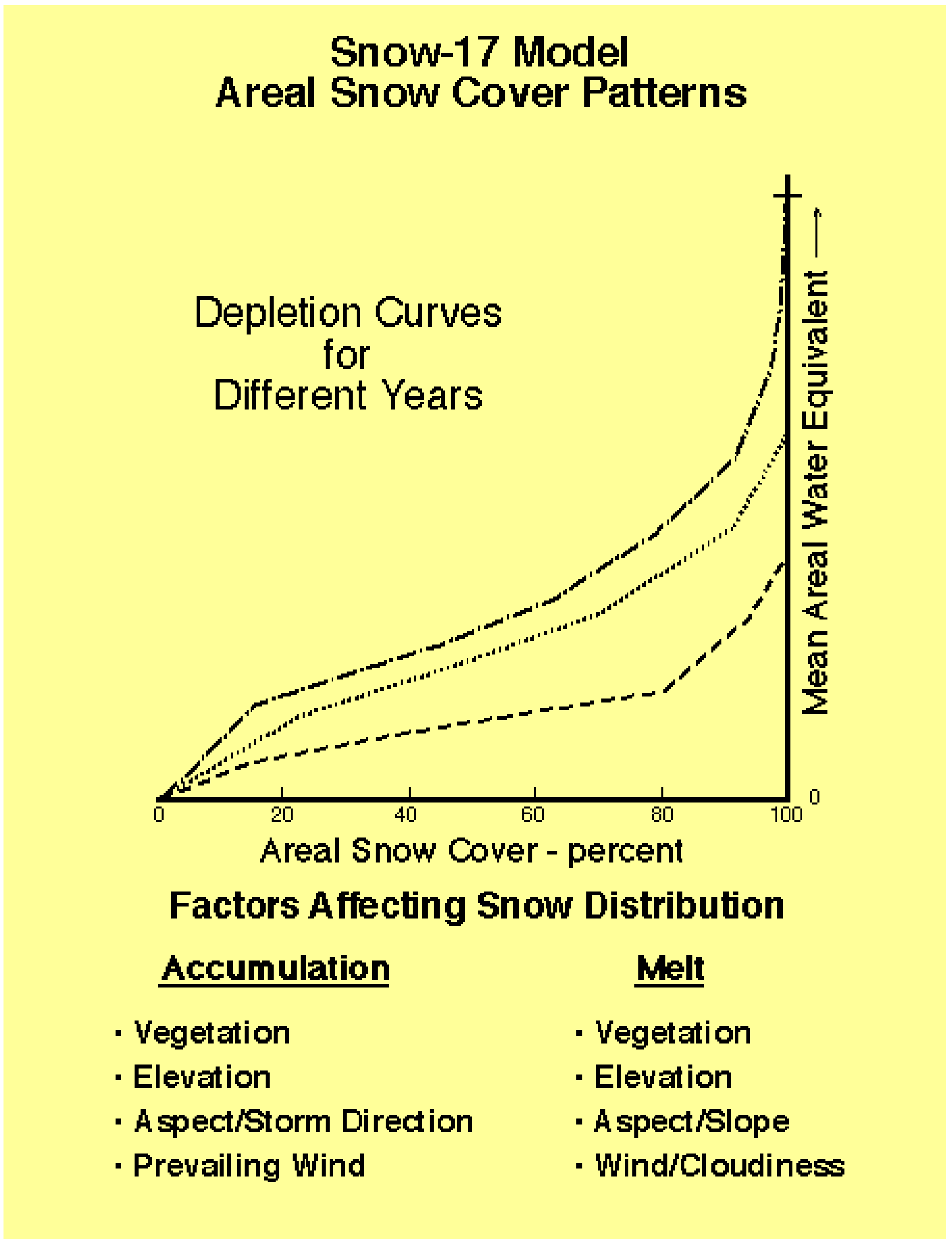

Figure 7. Variation in Depletion Curves from year to year.

year to year. As can be seen the shape is similar once the amount of bare ground starts to grow though the magnitude of the amount of snow that accumulates certainly varies from year to year. During many years, especially in mountainous regions, the bare ground portion of the area will begin to expand as soon as melt starts, however, during large snow accumulation years the area may remain at or near 100% cover for some period before the amount of bare ground begins to really expand.

It should also be noted that over most heterogeneous areas during periods of melt the average melt rate over the snow covered portion will varying with the amount of areal coverage. When there is complete cover the average melt rate is a weighted combination of portions of the area that melt quickly such as open south slopes and sections that melt much more slowly such as conifer forested north slopes. As the melt season progresses and the area extent of the snow cover decreases, the average melt rate also generally decreases because typically the portions of the area that still have snow are those with the slowest melt rates.

Computation of the Areal Extent of the Snow Cover in the model

In order to normalize the depletion curves from one year to another and to account for the situation where the area can remain at or near completely covered by snow during years with large accumulations SNOW-17 uses a depletion curve that relates the areal extent of the snow cover to the mean areal water equivalent, W (mm), divided by an areal index. The mean areal water equivalent used to compute the areal snow cover is the sum of the ice portion of the snowpack and the liquid water held against gravity drainage (water moving through the snow cover is not included), i.e.:

$$W = W_i + W_q \qquad (30)$$

The areal index is noted by A_i (units of mm of water equivalent). In SNOW-17 A_i is equal to the smaller of:

- W_{max} (mm), the maximum amount of water equivalent that existed during the accumulation period, or
- **SI** (mm) – a model parameter that specifies the mean areal water equivalent above which 100% snow cover always exists.

Figure 8 illustrates the depletion curve used by SNOW-17.

Since depletion curves can have a wide variety of shapes depending on the physiographic factors in a given area, SNOW-17 uses a table lookup to define the curve. The user specifies the areal extent of the snow cover for W/A_i ratios of 0.1 to 0.9 (in increments of 0.1). When W/A_i equals 1.0, then A_s equals 1.0 and when W/A_i equals 0.0, A_s equals 0.05. A value of $A_s = 0.05$ is used for a $W/A_i = 0.0$ ratio so that small amounts of snow don't continue to exist well past the time when all the snow is gone in nature. Generally when only small patches of snow remain, the advected energy from nearby bare ground quickly melts the remaining snow. In some mountainous regions small patches of dense snow remain throughout the summer, but these are hydrologically insignificant.

As seen in Figure 8 the model also attempts to represent what happens when new snowfall occurs on a partially bare area. In this case the areal extent of the snow cover reverts back to 100% temporarily. When subsequent melt occurs, the area cover stays at 100% until 25% of the new snowfall has melted and then proceeds linearly back to the point on the depletion where it was when the new snowfall occurred. The 25% figure seems like a reasonable value and is not based on any experimental data. The point on the depletion curve when the new snowfall begins is remembered by the model (W_{ns} is the water equivalent at that time and A_{ns} is the areal extent of snow cover). The amount of water equivalent when the areal extent first drops below 100% as the new snow melts is identified as W_{100}. To avoid increasing the areal cover to 100% for very small amounts of new snow, the new snowfall must exceed a specified value (currently 0.2 mm/hr water equivalent) before the model leaves the depletion curve.

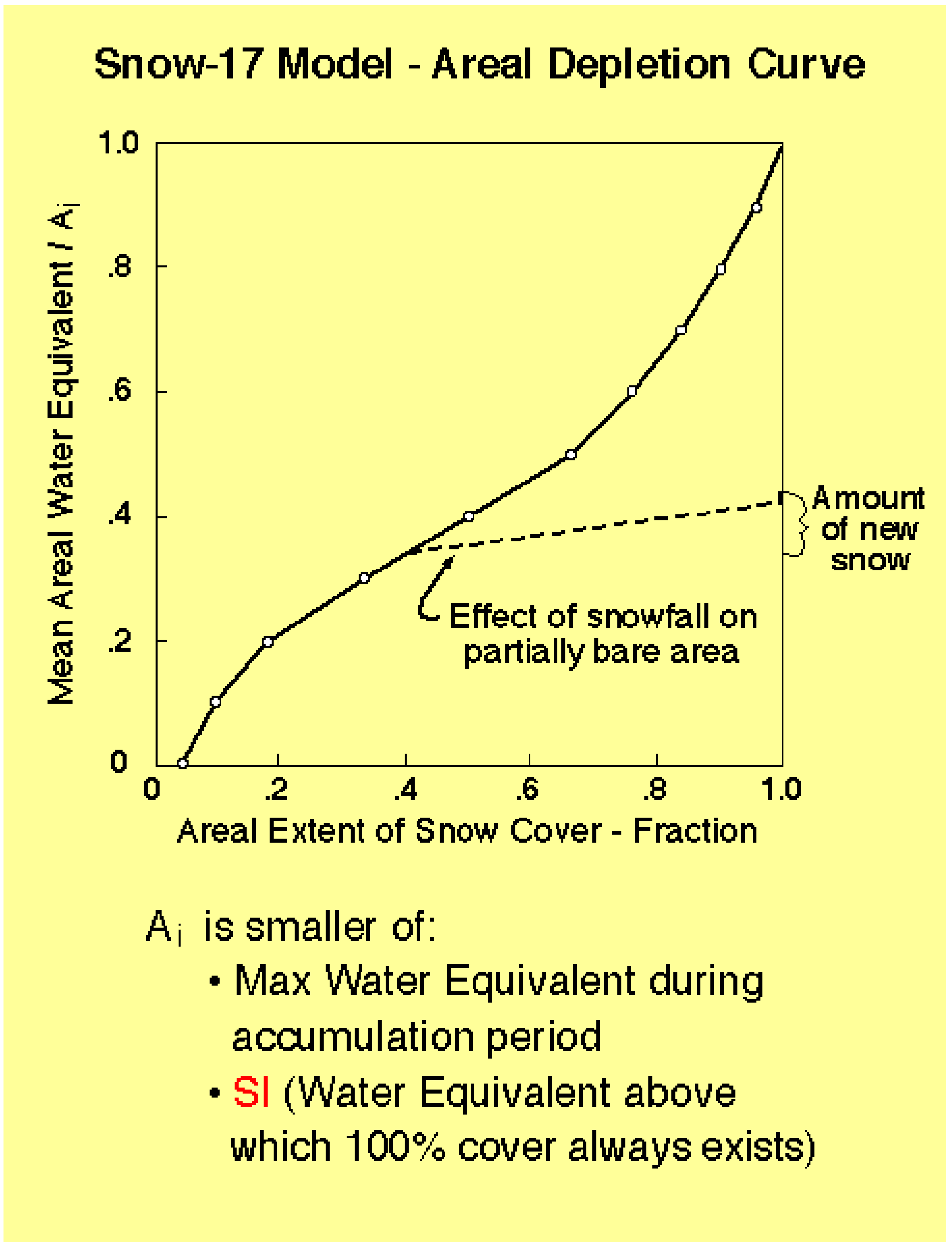

Figure 8. SNOW-17 areal depletion curve.

If sufficient new snowfall occurs on a partially bare area, the model assumes that it is a new accumulation period. If the mean areal water equivalent grows to 3 times the amount when the model was last on the depletion curve, then W_{max} is reset to the current value of W. This prevents problems that initially occurred when the snow that accumulated during a very large snow year didn't all melt off during the summer and thus, the W_{max} value from the large year was still being used for the next accumulation season. This feature is also enacted in regions with periodic accumulation and melt periods throughout the winter when the snow from one series of storms doesn't all melt prior to the next accumulation period.

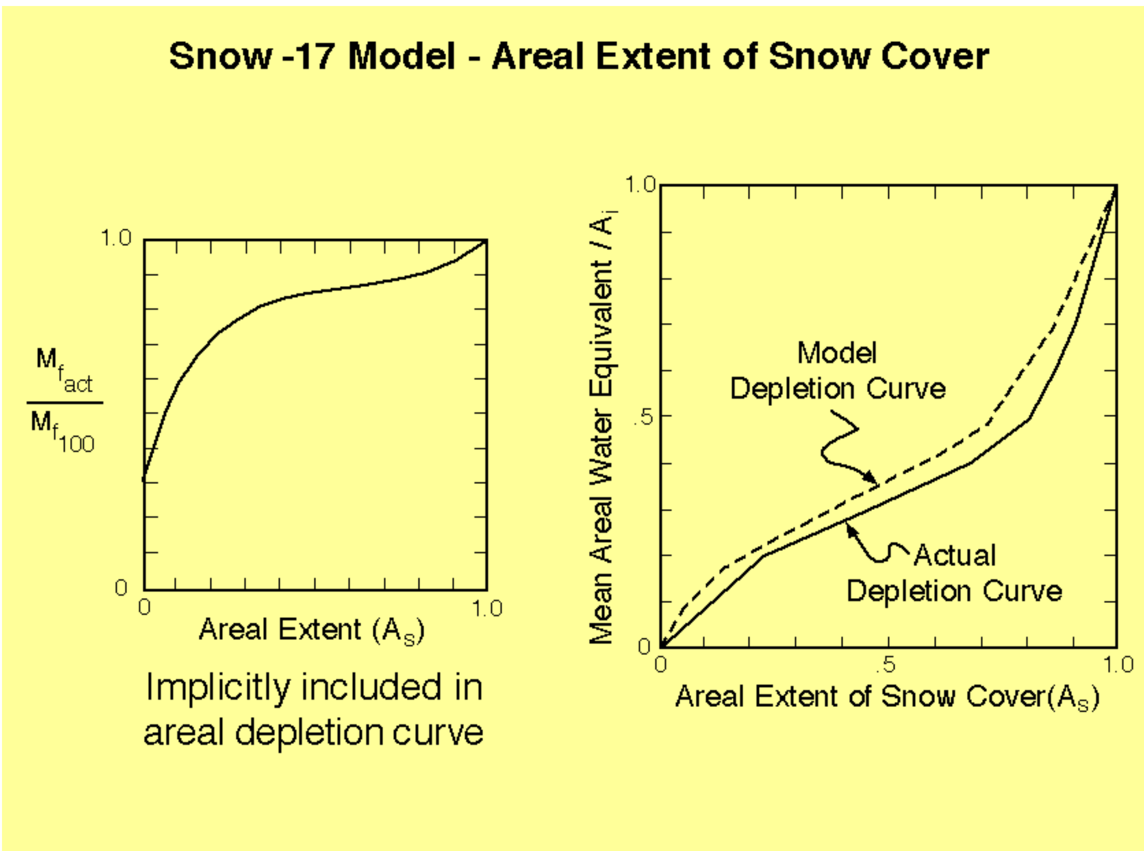

Figure 9. Effect of changing melt rates on the areal depletion curve.

The areal extent of snow cover calculated by SNOW-17 should probably be referred to as the 'effective' areal extent of snow cover. This is because the areal cover value needed by SNOW-17 likely includes items that are not explicitly included in the model. The primary factor not included in the model is the decrease that generally occurs in the melt rate over the snow covered area as the areal coverage decreases. The model parameters represent the melt rate when the area is 100% snow covered. As the snow cover depletes typically portions of the area with higher melt rates become bare first (e.g. open south facing exposed slopes), while those portions with slower melt rates (e.g. conifer covered north facing protected slopes) retain snow until near the end of the melt season. This general decrease in the average melt rate over the snow covered portion of the area will end up being included in the depletion curve that is derived during the model calibration process. Thus, the areal extent of snow cover needed by SNOW-17 is not the value that would be determined by a direct observation of the fraction of the area that is covered by snow. This situation is illustrated in Figure 9. On the left this figure shows an assumed variation in the ratio of the melt rate over the snow covered portion of an area to the melt rate when the area is 100% covered by snow as a function of the areal extent of the snow cover. On the right the variation in melt rates has been used to adjust the actual depletion curve for the area (i.e. the one that would be determined from direct observation) to get the 'effective' depletion curve needed by the model.

As noted earlier SNOW-17 keeps track of the mean areal value of state variables such as the water equivalent of the snow cover. It was indicated that in order to get the average value of a variable over the snow covered portion of an area one would need to divide by the areal extent of the snow cover. However, since the model actually uses an 'effective' areal cover, dividing a model variable by the computed areal extent doesn't really give a value that could be directly compared to any measured quantity. The comparison gets worse as the areal extent of the snow cover diminishes.

V. Operational Considerations

When SNOW-17 is applied operationally for river forecasting it is important to understand what takes place internally within the model when adjustments are applied to the model computations and state variables (these are referred to as 'Run Time Modifications' or 'Mods' in NWSRFS). Also in some cases the model is applied operationally at a different time and/or space scale than is used for calibration. The forecaster needs to understand how changing the scale will affect model computations.

Modifications to Model Computations and State Variables

As mentioned in the background section a forecaster can make updates or adjustments to model computations and states during real time operations in order to make the model results more closely match observations or to attempt to compensate for model limitations based on observed or predicted meteorological information. When making such updates it is important to understand what takes place inside the model, i.e. exactly which computations and/or states are modified. The internal changes that take place when the modifications that currently exist in the NWSRFS operational forecast system are applied to the SNOW-17 model are described below. It should be noted that when the water equivalent is adjusted, the areal extent of the snow cover is modified (unless it remains at 100%) while when the areal cover is changed, the water equivalent is not modified. Variations in the depletion curve from year to year, the fact that the model uses an effective areal cover that may not correspond to observations, and the reality that the depletion curve is generally determined by calibration and not measurements makes it unwise to modify the water equivalent when adjusting the areal cover.

<u>Change total water equivalent</u> – When the total water equivalent, W_t, is changed, adjustments are made to many of the state variables of the model.

1. The amount of lagged excess liquid water in storage, S, and the average hourly lagged excess water for each precipitation time interval, E_l, are not changed.
2. The percent liquid water held by the snow remains the same, i.e. the W_q/W_i ratio is unchanged. Thus the new W_i and W_q values are calculated as:
$$W_{i_{new}} = \left[W_{t_{new}} - \left(W_{qt_{old}} - W_{q_{old}}\right)\right] / \left[1.0 + \left(W_{q_{old}} / W_{i_{old}}\right)\right] \tag{31a}$$
$$W_{q_{new}} = W_{q_{old}} \cdot \left(W_{i_{new}} / W_{i_{old}}\right) \tag{31b}$$
3. The heat deficit, D, is not modified.
4. The maximum amount of water equivalent that existed during the accumulation period, W_{max}, will be changed in 3 situations – 1) if the new water equivalent, W, exceeds W_{max}, then W_{max} is set equal to the new W; 2) if the old W is within 80% of W_{max}, then W_{max} is adjusted by the ratio of the new water equivalent to the old water equivalent (this assumes that any changes to the water equivalent when it is near its maximum value for the accumulation period indicate that the amount of accumulated snow was likely also not correct); and 3) if the new water equivalent, W, is greater than or equal to 3 times the current value of W_{ns}, then the model assumes that a new accumulation period begins and W_{max} is set equal to the new W.
5. The depletion curve areal extent adjustment, A_{adj}, is set to zero (i.e. removed) when any of the following occur:
   a. The water equivalent is set to zero (i.e. there is no longer a snow cover),
   b. A new accumulation period is initiated (see item 3 under #4 above),
   c. A_{adj} is greater than or equal to the old A_i and the new W is greater than A_{adj}, or
   d. A_{adj}, is less than the old A_i and the new W is greater than the old A_i**.**
6. The variables that control the areal extent computations when new snowfall occurs on a partially bare area are adjusted as follows:
   a. If the new W is greater than the new A_i, then W_{ns} and W_{100} are set equal to the new W value,
   b. If the model was somewhere on the depletion curve and the new W is less than the new A_i, then W_{ns} and W_{100} are set equal to the new W value and a new areal extent, A_{ns}, is computed based on the new W/A_i ratio (the change in the areal extent of the snow cover when the water equivalent is adjusted is illustrated in Figure 10), and
   c. If the model was off the depletion curve due to new snowfall on a partially bare area, then W_{ns} and W_{100} are adjusted by the ratio of

the new W to the old W and A_{ns}, is recomputed from the depletion curve using the new W_{ns}/A_i ratio.

7. The snow depth is adjusted by the ratio of the new W_i to the old W_i.
8. The average snow cover temperature, T_s, and the air temperature for the previous time interval are unchanged.

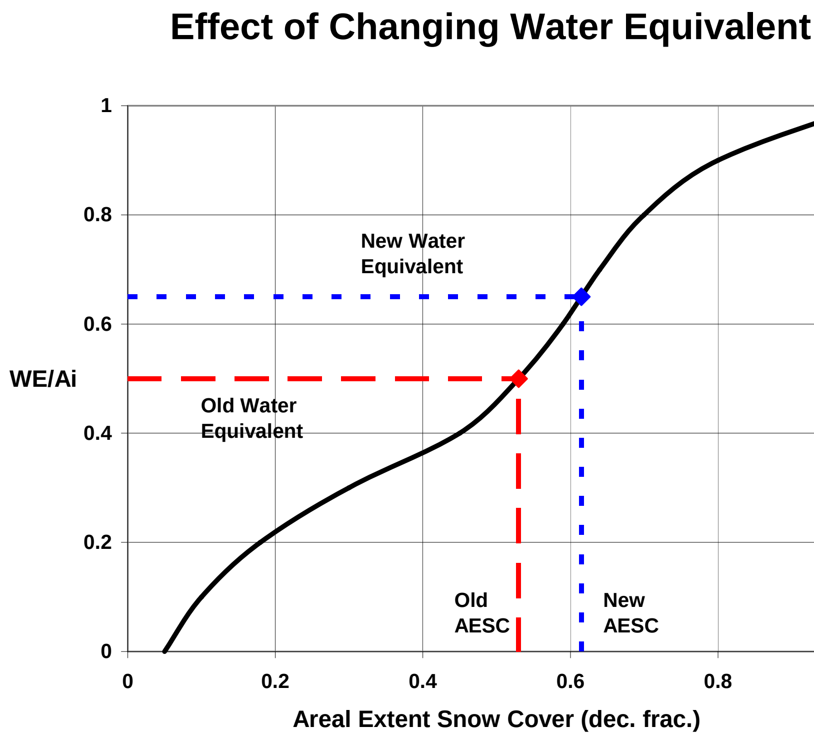

Figure 10. Effect of changing water equivalent on the areal extent of snow cover.

<u>Change areal extent of snow cover</u> – When the areal extent of the snow cover is changed, only the state variables involved in the areal extent of snow cover computations, i.e. A_{adj}, W_{ns}, W_{100}, and A_{ns} are altered. There is no change to the amount of water equivalent or other state variables.

1. If the model was on the depletion curve or should be based on the new areal cover (old cover 100% and new cover less than 100%, or new cover less than old A_{ns}, or new cover less than depletion curve value based on the current water equivalent), then the W/A_i ratio corresponding to the new areal cover is calculated from the depletion curve. The value of A_{adj} is then computed by dividing the water equivalent, W, by this W/A_i ratio. This value of A_{adj} is then used in place of A_i in computing the areal snow cover. Figure 11 illustrates this situation. A_{ns} is reset to the new areal cover. W_{ns} and W_{100} are unchanged. This value of A_{adj} is used until it is altered due to a subsequent change to the areal extent of the snow cover or it is set to zero. A_{adj} is set to zero when any of the situations listed under item #5 in the 'Change total water equivalent' section are encountered or when those situations are encountered due to normal model computations.

2. If the model is computing areal cover based on new snowfall on a partially bare area and should remain in that mode (new cover greater than depletion curve value based on the current water equivalent), then WE_{100} is recalculated as:

$$W_{100} = \{[(1.0 - A_{ns})/(A_s - A_{ns})] \cdot (W - W_{ns})\} + W_{ns} \qquad (32)$$

A_{adj}, W_{ns}, and A_{ns} are unchanged.

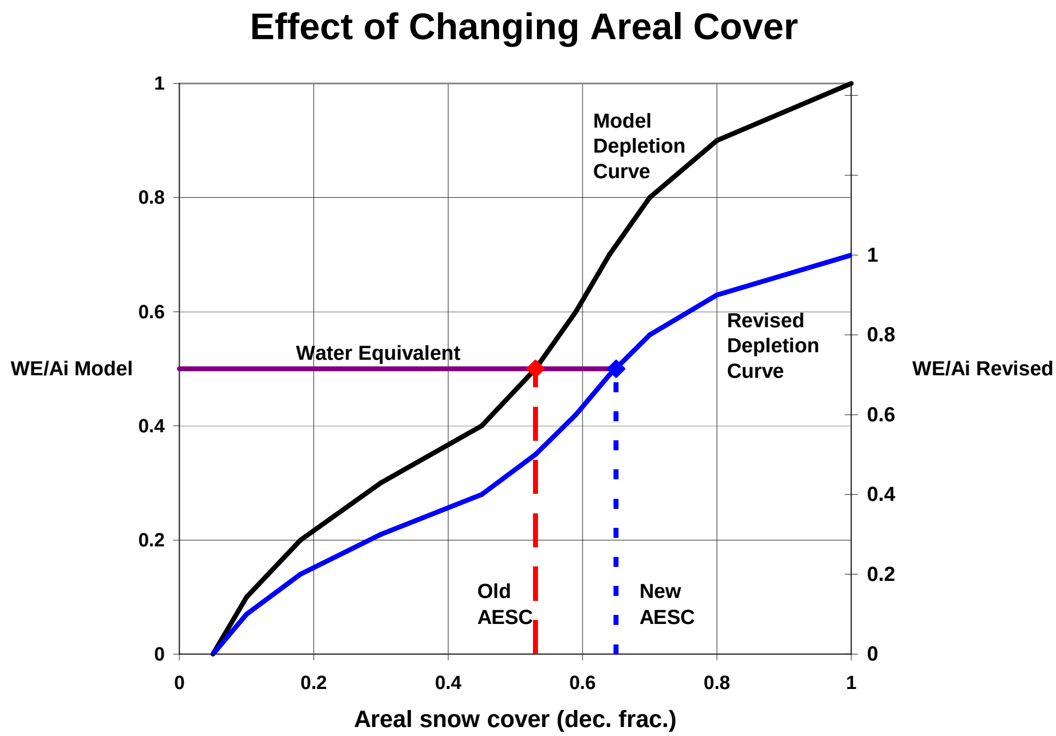

Figure 11. Effect of changing areal snow cover on the depletion curve.

Change non-rain melt computations – The forecaster can input a melt correction factor. The non-rain melt factor, M_f, in Equation 6 is then multiplied by this correction factor, thus the total melt during non-rain intervals during a specified time period is basically multiplied by the correction factor.

Change rain-on-snow melt computations – The forecaster can also input a wind function adjustment factor that is applied to any time intervals during a specified period that use the rain-on-snow melt equation. This correction factor is applied to the **UADJ** value in Equation 5, thus only the turbulent transfer portion of the rain-on-snow melt equation is modified.

Changing time or space scales

As new data sources and analysis procedures are implemented it is possible to apply models operationally at finer time and space scales than were realistic during calibration based on the available historical data. Non linear models, such as the Sacramento soil moisture accounting model, can be very scale dependent (i.e. the results can change significantly as the time interval or the spatial scale are altered). SNOW-17 is basically a linear model. To test the effect of using different computational time intervals on model results SNOW-17 was run at 6 and 1 hour intervals at several locations where hourly data were available. Differences in results occurred when:

1. The average 6 hour temperature was the same or close to the value of **MBASE** during a given period while hourly temperatures varied. In that case little or no melt was computed using 6 hourly data while some melt was generated for the hours when the temperature was above the **MBASE** value.

2. Hourly temperatures vary above and below the value of **PXTEMP** when precipitation was occurring. In that case the average 6 hour temperature was either above or below **PXTEMP** and all the precipitation was typed as either rain or snow while when using hourly data rain was occurring during some intervals and snow during others.

3. Areal snow cover was below 100%. When on the depletion curve, the areal cover used when adjusting the melt computations is based on the conditions at the start of a time interval plus the effect of any new snow. When running at an hourly interval, the model steps down the depletion curve during each 6 hour period while when running at a 6 hour time step, the model applies the initial values to the entire 6 hour interval.

In spite of the differences that occurred when these situations existed, the overall results from the comparisons showed essentially no difference in the values of water equivalent and depth generated over time. Thus it appears that SNOW-17 can be calibrated using 6 hour data and then applied operationally at an hourly computational interval with no significant effect on the results. Of course the data used operationally must be unbiased compared to that used for calibration.

Changing the spatial scale of the model computations raises other issues than just the effect on the model calculations. The main reason to use a finer spatial scale operationally is to take advantage of higher resolution data that may be available and thus better model the spatial variation of the snow cover. The areal depletion curve determined for a watershed or elevation zone during calibration is unlikely to apply to each grid of a distributed version of the model. When running SNOW-17 in a distributed mode the areal depletion curve may not even be used, but instead the areal snow cover pattern would be defined by which grids contain snow at any given point in time.

VI. Summary of Snow Model Parameters and State Variables

The SNOW-17 model has 12 parameters. This counts the areal depletion curve as one parameter though it is input as a series of 9 values used to define the shape of the curve. Some of the parameters have more influence on the simulation results than others. The most influential, or major parameters, are those that typically have to be determined through calibration even though some guidelines are available to obtain initial estimates

[*Anderson* (2002)]. The others, the minor parameters, typically can be assigned values based on the climatological conditions at the location being modeled. These parameters have a much smaller effect, in general, on the results and seldom need to be altered from their initially assigned value. There are no parameters associated with the snow depth computations in SNOW-17. There are several potential parameters, but all of these have been set to a fixed value based on comparisons of simulated versus observed snow depth and water equivalent at selected locations that had quality observations of both quantities.

The major parameters for the SNOW-17 model are:

1. **SCF** - The multiplying factor which adjusts precipitation that is determined to be in the form of snow. **SCF** primarily accounts for gage catch deficiencies, but also implicitly includes the net effect of vapor transfer (sublimation and condensation, including from intercepted and blowing snow) and transfers across areal divides.

2. **MFMAX** - Maximum melt factor during non-rain periods – assumed to occur on June 21st (mm·°C^{-1}· 6 hr^{-1}).

3. **MFMIN** - Minimum melt factor during non-rain periods – assumed to occur on December 21st (mm·°C^{-1}· 6 hr^{-1}).

4. **UADJ** - The average wind function during rain-on-snow periods (mm·mb^{-1}). **UADJ** is only a major parameter when there are fairly frequent rain-on-snow events with relatively warm temperatures.

5. **SI** - The mean areal water equivalent above which there is always 100 percent areal snow cover (mm). **SI** is not a major parameter when the model is applied at a point location or when significant bare ground appears soon after melt begins no matter the magnitude of the snow cover.

6. Areal Depletion Curve – Curve that defines the areal extent of the snow cover as a function of how much of the original snow cover remains after significant bare ground shows up. The areal depletion curve also implicitly accounts for the reduction in the mean areal melt rate that occurs as less of the area is covered by snow. Generally not needed for a point location.

The minor parameters for the SNOW-17 model are:

1. **NMF** - Maximum negative melt factor (mm·°C^{-1}·6 hr^{-1}). The negative melt factor has the same seasonal variation as the non-rain melt factor, thus the maximum value is assumed to occur on June 21st.

2. **TIPM** - Antecedent temperature index parameter (real – range is 0.01

to 1.0). Controls how much weight is put on temperatures from previous time intervals when computing ATI. The smaller the value of **TIPM**, the more previous time intervals are weighted.

3. **PXTEMP** - The temperature that separates rain from snow (˚C). If the air temperature is less than or equal to **PXTEMP**, the precipitation is assumed to be in the form of snow. The **PXTEMP** parameter, as defined for SNOW-17, is not used if a rain-snow elevation time series is used to determine the form of precipitation.

4. **MBASE** - Base temperature for snowmelt computations during non-rain periods (˚C). Typically a value of 0˚C is used.

5. **PLWHC** - Percent liquid water holding capacity (decimal fraction). Indicates the maximum amount of liquid water, as a fraction of the ice portion of the snow, that can be held against gravity drainage (maximum allowed value is 0.4).

6. **DAYGM** - Constant daily amount of melt which takes place at the snow-soil interface whenever there is a snow cover ($mm \cdot day^{-1}$).

The SNOW-17 model has 14 state variables. These are as follows:

1. W_i - water equivalent of the ice portion of the snow cover (mm),
2. D - heat deficit (mm),
3. ATI - antecedent temperature index (˚C),
4. W_q - liquid water held by the snow (mm),
5. W_{max} - the maximum amount of water equivalent that existed during an accumulation period (mm),
6. W_{ns} - the water equivalent when new snowfall first occurs on a partly bare area, i.e. the water equivalent at the point where the areal cover leaves the depletion curve (mm),
7. A_{ns} - the areal cover when new snowfall occurs on a partly bare area, i.e. the depletion curve value at the point where the areal cover leaves the curve (decimal fraction),
8. W_{100} - the amount of water equivalent where the areal cover drops below 100% when melt occurs after new snowfall takes place on a partially bare area (mm),
9. S - amount of lagged excess liquid water in storage (mm),
10. A_{adj} – A_i value computed for use in depletion curve computations after an adjustment to the areal extent of snow cover – allows the water equivalent to remain the same as before the adjustment (mm),
11. E_l - average hourly lagged excess water for each precipitation time interval – $5/\Delta t_p+2$ values (mm),
12. H - depth of the total snow cover (cm),
13. T_s - average snow cover temperature (˚C), and

14. $T_{a,t-\Delta t}$ – air temperature for the previous computational time interval (˚C).

Acknowledgement

The initial algorithms for computing snow depth were added to the model by Victor Koren. The ability to compute snow depth, in addition to water equivalent, is very helpful when modeling in regions where only observed depth measurements are available. This is generally the case in much of the United States east of the Rocky Mountains.

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SNOW-17 Model – Appendix A
Physics of Snow Cover Energy Exchange

Eric Anderson, January 2006

Even though using a conceptual snow model, such as SNOW-17, that uses air temperature as the sole index to energy exchange and includes simplified representations of the physical processes that control the accumulation and ablation of the snow cover, it is important to have a basic understanding of the physics that control the snow cover energy balance. It is important to understand the energy exchange process in order to know the limitations of a conceptual, index model. Only when one has this understanding is it possible to make logical decisions involving real time adjustments to model computations given the current meteorological and snow cover conditions. This appendix gives a brief overview of the energy balance of a snow cover. A more complete description of the energy exchange of a snow cover can be found in *Anderson* (1976).

Energy Balance Equation

The energy balance equation for a snow cover can be expressed as (the units of each term are cal·cm^{-2}):

$$Q_i - Q_r + Q_a - Q_s + Q_h + Q_e + Q_m + Q_g = \Delta Q \qquad \text{(A-1)}$$

where: Q_i = incident solar radiation,
Q_r = reflected solar radiation,
Q_a = incoming atmospheric and terrestrial longwave radiation,
Q_s = longwave radiation emitted by the snow cover,
Q_h = sensible heat transfer,
Q_e = latent heat transfer,
Q_m = heat transfer due to mass changes,
Q_g = heat transfer at the snow-soil interface, and
ΔQ = change in the heat storage of the snow cover.

Figure A-1 illustrates the various terms in the energy balance equation for a snow cover. Most of the energy exchange occurs at or near the snow-air interface. This is where the vast majority of the snowmelt occurs, thus it is important to understand the surface energy exchange process in order to properly evaluate model estimates of melt.

The change in heat storage term, ΔQ, consists of the energy used to melt the ice portion of the snow cover, freeze liquid water within the snow cover, and change the internal temperature of the snow. All of these processes don't always occur during a given time interval. Water in both the solid and liquid phase can exist when snow is at 0˚C. The liquid water is the result of melting or rainfall. The liquid water may refreeze or remain in storage. When the snow cover is isothermal at 0 ˚C and saturated with liquid water (condition commonly referred to as a 'ripe' snow cover) any excess liquid water produced by melting or rainfall will move through the snow and become outflow from the snow cover.

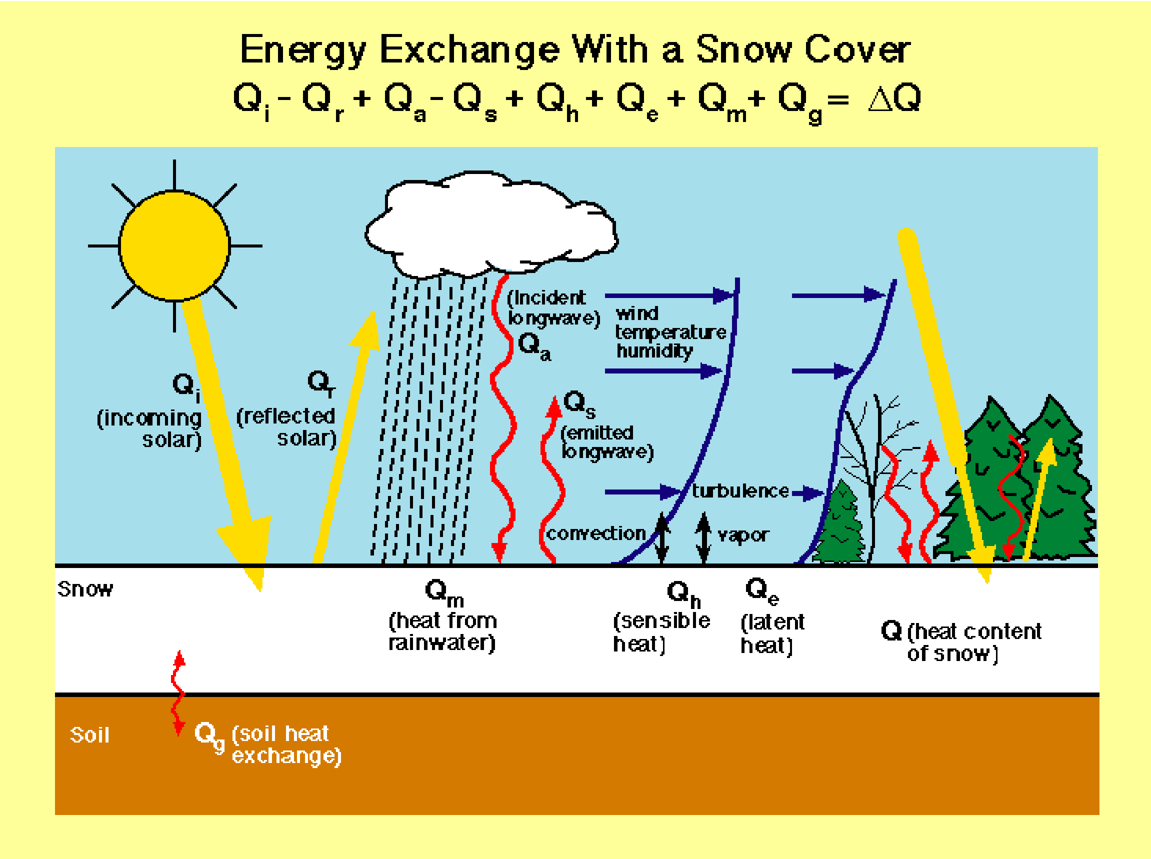

Figure A-1. Summary of snow cover energy exchange

<u>Radiation Transfer</u>

All objects emit radiation as a function of their surface temperature. The warmer the object, the more radiation is generated and the shorter the wave length of that energy. For snow radiation transfer is separated into shortwave (solar) and longwave (atmospheric or terrestrial) radiation due to differences in the source of the radiation and differences in how snow reacts to radiation as a function of wave length.

Shortwave Radiation

Shortwave or solar radiation is emitted from the sun. The wave length of solar radiation varies from 0 to 4 microns. The visible range, i.e. what humans can see, is from about 0.4 to 0.7 microns. Below 0.4 microns is referred to as ultraviolet and above 0.7 microns is infrared. The peak intensity of solar radiation is at about 0.5 microns. Over 75% of the solar energy occurs between about 0.2 and 1.1 microns. Incoming solar radiation on a clear day varies with the time of year and latitude as shown in Figure A-2. Above the artic circle there are periods in the winter with no solar radiation since the sun never rises above the horizon. Incoming solar radiation obviously also varies during the day with the peak intensity typically around noon local time. The amount of incoming solar radiation that reaches the earth's surface is dependent primarily on cloud cover, vegetation cover, and the slope and aspect of the surface. A dense cloud cover will greatly reduce the amount of incoming solar radiation reaching a snow cover as will a dense conifer forest. The maximum amount of solar radiation is received on a surface that is oriented perpendicular to the sun's rays during the time of the day when solar radiation is at its peak.

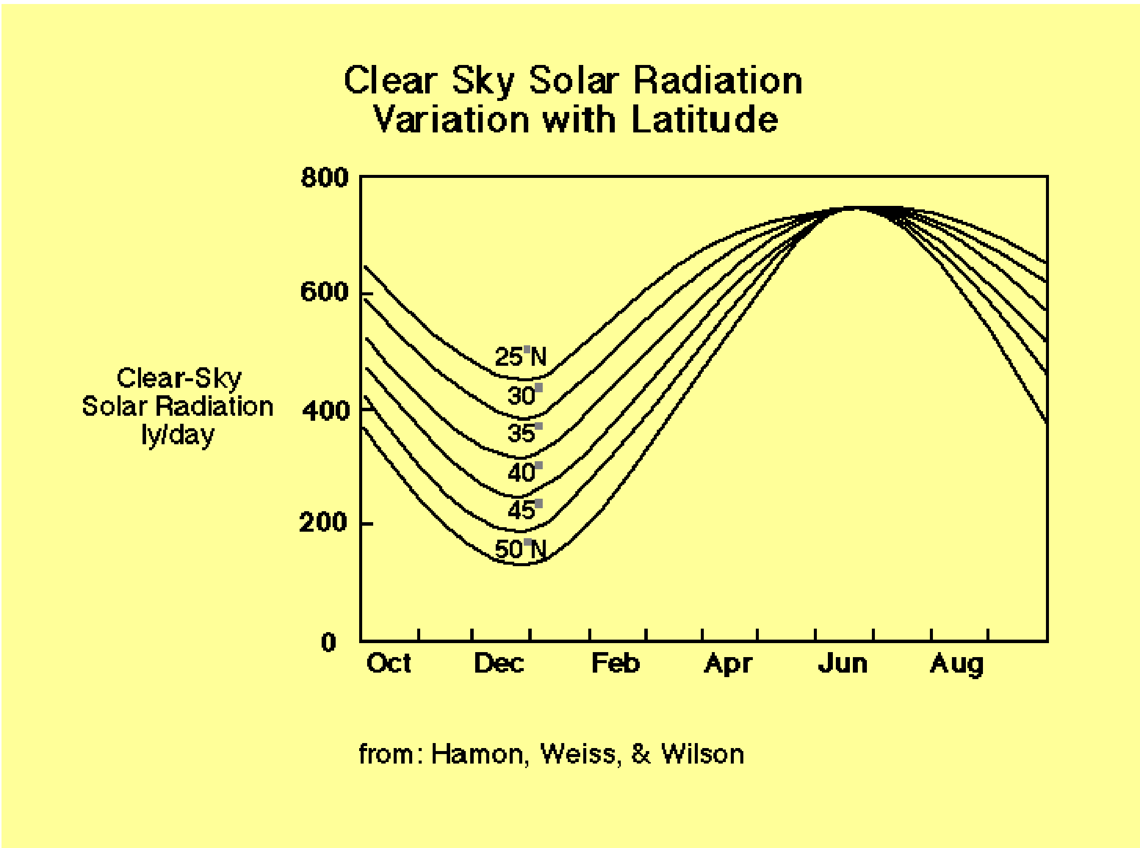

Figure A-2. Variation of clear sky solar radiation with time of year and latitude.

Snow is highly reflective to solar radiation. Figure A-3 shows how reflectivity varies with wave length over the major portion of the shortwave portion of the spectrum for fresh, wind blown snow. The reflectively of fresh snow is especially high over the range where most of the solar energy occurs, including the visible range. The total reflectance over the shortwave portion of the solar spectrum is referred to as albedo. Fresh snow has an albedo of around 90%, indicating that most of the solar radiation is reflected from fresh snow rather than being absorbed. The albedo of snow varies with the surface properties of the snow cover. While fresh snowfall typically reflects around 90% of the incoming solar radiation, well aged snow generally has an albedo in the range of 40-50%. The albedo of snow primarily varies with the grain size of the snow crystals near the surface. Grain size is fairly well correlated with the density. Thus, there is good relationship between albedo and the density near the snow surface. The snow crystals age more rapidly during warmer periods, thus albedo decreases faster during melt periods than when no melt is taking place. The solar radiation that is absorbed by the snow is not all absorbed right at the surface. Shortwave radiation will penetrate a short distance into a snow cover, though about 90% is absorbed in roughly the upper 7.5 cm. Figure A-4 summarizes how the albedo of a snow cover varies and gives some figures on the penetration of shortwave radiation into snow as a function of surface density.

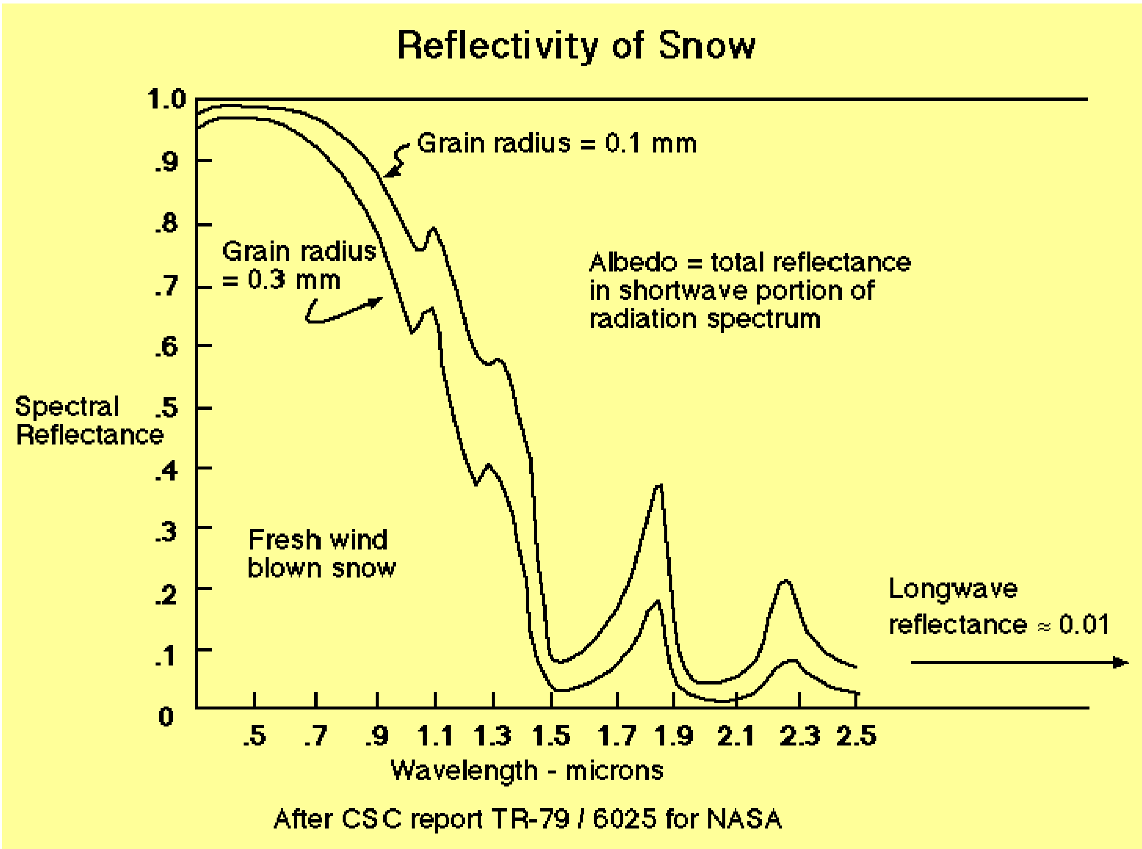

Figure A-3. Variation in reflectivity of snow as a function of wave length.

Longwave Radiation

Longwave radiation is emitted by the atmosphere and terrestrial objects. The wave length of longwave radiation varies from about 4 to 100 microns. The peak intensity varies with the temperature of the object, but is generally in the range of 10-12 microns. The total amount of energy emitted by a terrestrial object is given by Stefan's law:

$$Q_L = \varepsilon \cdot \Delta t \cdot \sigma \cdot T^4 \qquad \text{(A-2)}$$

where: Q_L = Total emitted longwave radiation ($\text{cal}\cdot\text{cm}^{-2}$),
Δt = time interval (hours),
ε = emissivity (decimal fraction, range 0.0 – 1.0),
σ = Stefan-Boltzmann constant ($4.878\cdot10^{-9}$ $\text{cal}\cdot\text{cm}^{-2}\cdot{}^\circ\text{K}^{-4}\cdot\text{hr}^{-1}$), and
T = temperature (°K).

An object that emits the maximum amount of radiation for a given temperature ($\varepsilon = 1.0$) is referred to as a blackbody. Emissivity also equals absorption, thus a blackbody not only emits the maximum amount of radiation but also absorbs all longwave radiation incident upon it. Reflectivity is equal to $1.0 - \varepsilon$, i.e. the radiation that is not absorbed, is reflected. As shown in Figure A-3 the reflectance of snow in the longwave portion of the spectrum is about 0.01, thus the emissivity of snow is about 0.99 indicating that snow is nearly a perfect blackbody with respect to longwave radiation. This means that a snow surface absorbs nearly all the longwave radiation it receives and emits longwave radiation at nearly 100% efficiency based on the snow surface temperature.

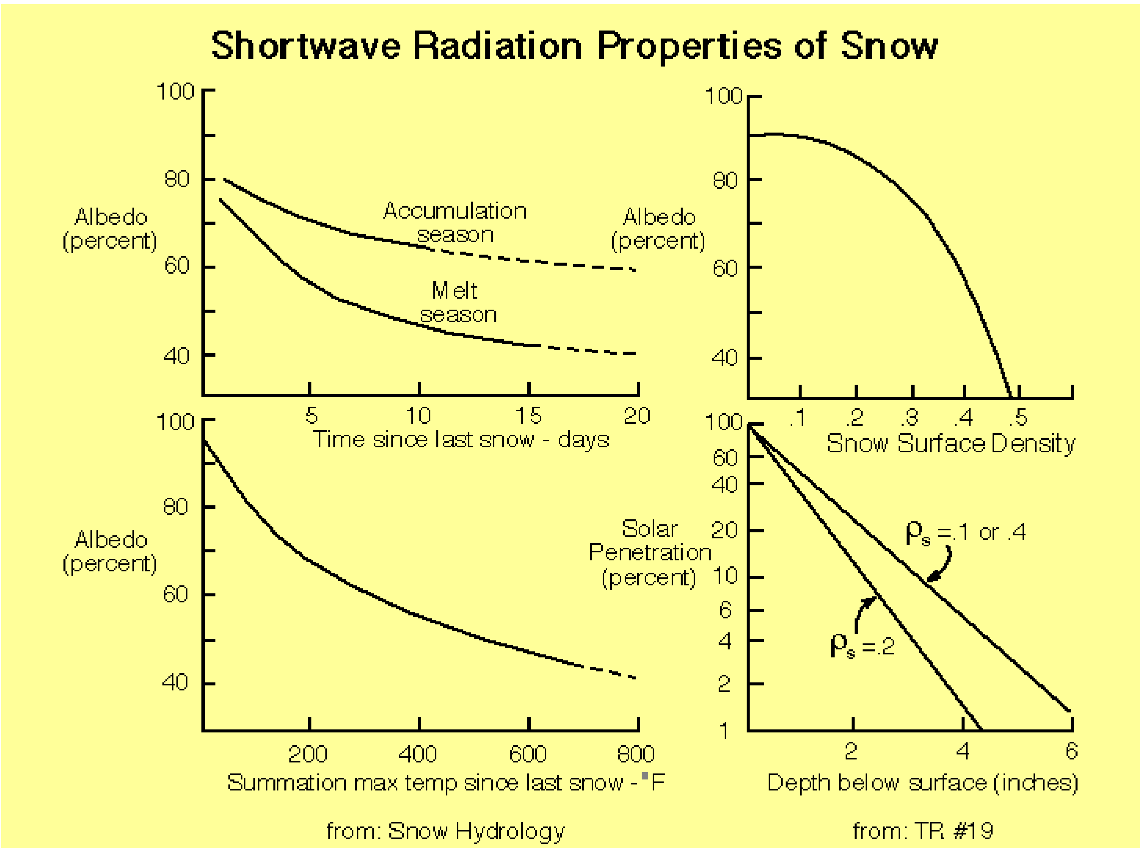

Figure A-4. Shortwave radiation properties of snow.

The amount of longwave radiation generated by the atmosphere varies based on atmospheric conditions, including temperature. For a clear sky, longwave radiation is emitted by various particles in the air and varies primarily with the amount of water vapor and temperature. The effective emissivity [$Q_L/(\Delta t \cdot \sigma \cdot T^4)$ where T is the air temperature at ground level] for a clear sky typically varies from about 0.6 to 0.8, depending on temperature and humidity. When a thick layer of clouds cover the sky, the effective emissivity of the atmosphere is close to 1.0. Clear sky and overcast conditions define the range of the amount of longwave radiation generated by the atmosphere. Some measure of cloud cover can be used along with these limits to produce a daily estimate of incoming atmospheric radiation. Such a relationship typically assumes that the temperature of the cloud base is the same as the surface air temperature during overcast conditions and that there is fairly constant relationship between surface and upper air temperatures when the sky is clear. Figure A-5 shows a typical relationship relating the daily amount of atmospheric radiation generated during overcast and clear skies to surface air temperature and relative humidity. Also shown in Figure A-5 is the amount of longwave radiation produced by a melting snow surface. Thus for an open site, i.e. no forest cover, the net longwave radiation exchange would be positive under overcast conditions whenever the air temperature is above freezing, while under clear skies the net amount of longwave radiation would be negative until the air temperature rises above 20-25 ˚C.

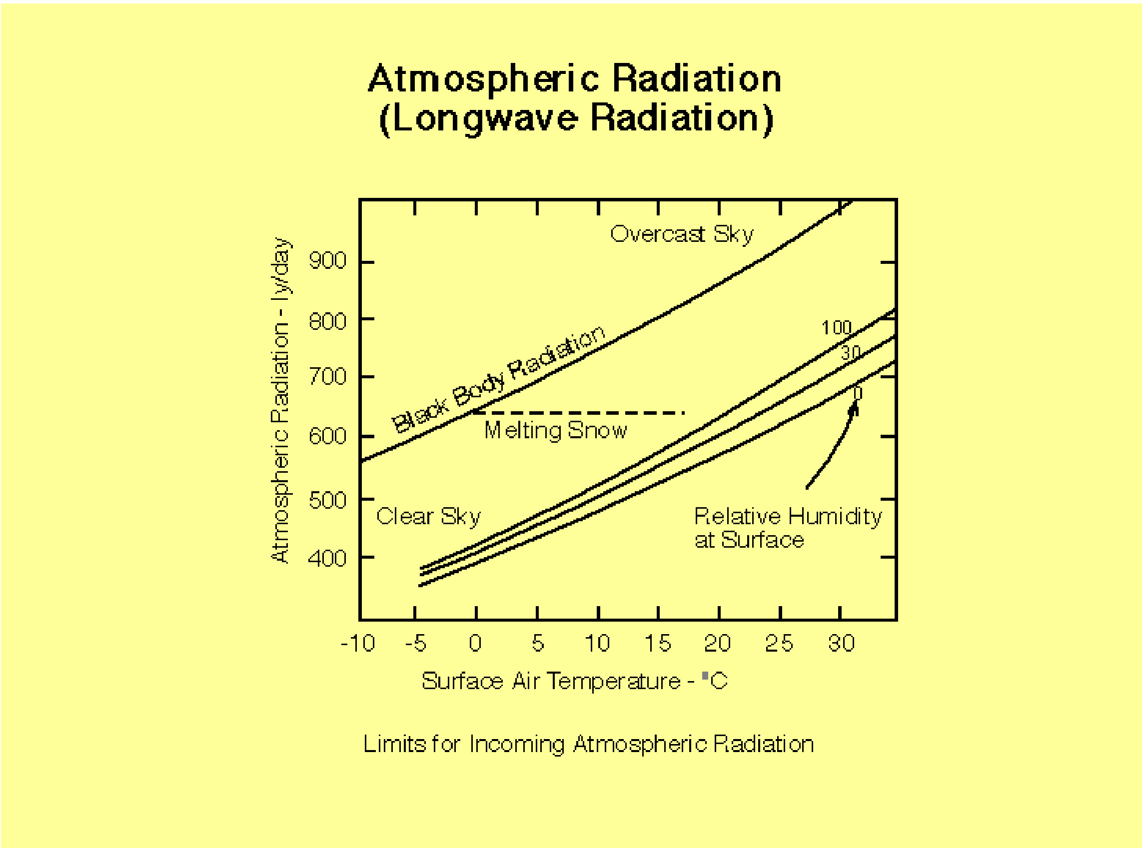

Figure A-5. Atmospheric radiation as a function of temperature and sky conditions.

The amount of incoming longwave radiation to a snow cover is not only determined by the amount of incoming atmospheric radiation, but also by terrestrial radiation produced primarily by vegetation. The extreme case is a very dense conifer forest, so dense that very little sunlight penetrates the canopy. In this case the amount of incoming longwave radiation would be essentially blackbody radiation at the canopy temperature which should be close to the air temperature measured within the forest. In such a forest, air movement would be minimal even when strong winds existed above the canopy, thus the solar radiation, sensible heat, and latent heat terms in Equation A-1 would be negligible and longwave exchange would almost completely dominate the energy balance.

Net Radiation

The net radiation transfer for a snow cover (i.e. the Q_i, Q_r, Q_a , and Q_s terms in Equation A-1) can be expressed as:

$$Q_n = Q_i \cdot (1.0 - A) + Q_a - \left[\Delta t \cdot 0.99 \cdot \sigma \cdot (T_{sur} + 273.16)^4\right] \qquad \text{(A-3)}$$

where: Q_n = net radiation ($cal \cdot cm^{-2}$),
A = albedo (decimal fraction),
T_{sur} = snow surface temperature (˚C), and
273.16 = 0˚C on the Kelvin scale.

<u>Turbulent Exchange</u>

Both latent and sensible heat transfers are turbulent exchange processes, i.e. they are a function of an exchange produced by the turbulence created by wind. Latent heat exchange involves the transfer of water vapor and a phase change at the snow surface. Sensible heat exchange involves a transfer of energy based on differences in the temperatures of the air and the snow surface.

Latent Heat Exchange

The direction of latent heat exchange is based on the vapor pressure gradient in the air just above the snow surface. If the vapor pressure is less at the snow surface than in the overlying air, vapor will move from the air to the snow surface and vice versa. The rate at which the water vapor is transferred depends on the turbulence of the air. The amount of turbulence is related to the wind speed. Water vapor transfer can be expressed in an equation attributed to Dalton as:

$$V = f(u_a) \cdot (e_a - e_{sur}) \qquad \text{(A-4)}$$

where: V = amount of vapor transferred ($mm \cdot hr^{-1}$),
u_a = wind speed ($km \cdot hr^{-1}$) at height z_a (cm) above the surface,
$f(u_a)$ = wind function ($mm \cdot mb^{-1} \cdot hr^{-1}$),
e_a = vapor pressure of the air at z_a (mb), and
e_{sur} = vapor pressure at the surface (mb) (saturated vapor pressure at the snow surface temperature).

The heat transfer occurs when the water vapor either reaches the snow surface and condenses, thereby releasing latent heat, or the water vapor sublimates from the snow surface (ice converted to vapor) which requires energy, thus removing heat from the snow. The heat transfer is equal to the amount of water vapor multiplied by the latent heat of sublimation. Thus latent heat transfer can be expressed as:

$$Q_e = \frac{L_s \cdot \rho_w}{10} \cdot \Delta t \cdot f(u_a) \cdot (e_a - e_{sur}) \qquad \text{(A-5)}$$

where: L_s = latent heat of sublimation – 677 cal·gm^{-1}, and
ρ_w = density of water – 1.0 gm·cm^{-3}.

Sensible Heat Exchange

Sensible heat transfer is related to the heat content of the air. The direction of heat transfer is determined by the air temperature gradient just above the snow surface. Heat moves from warmer to cooler temperatures. Since the snow surface is generally cooler than the overlying air, especially during melt, heat is normally being transferred from the air to the snow cover. In addition to the temperature gradient, the rate of sensible heat transfer, similarly to water vapor transfer, depends on the turbulence of the air. Studies have shown that it is realistic to assume that the turbulent transfer coefficients for heat and water vapor are equal. This is reasonable for all atmospheric stability situations that have been investigated. With this assumption the ratio of Q_h/Q_e (commonly referred to as Bowen's ratio) can be expressed as:

$$\frac{Q_h}{Q_e} = \frac{P_a \cdot c_p}{0.622 \cdot L_s} \cdot \frac{(T_a - T_{sur})}{(e_a - e_{sur})} \qquad \text{(A-6)}$$

where: P_a = atmospheric pressure (mb),
c_p = specific heat of dry air – 0.24 cal·gm^{-1}·°C^{-1},
0.622 = molecular weight ratio of water vapor to dry air, and
T_a = air temperature (˚C).

Substituting Equation A-5 into A-6 sensible heat transfer can be expressed as:

$$Q_h = \frac{L_s \cdot \rho_w}{10} \cdot \Delta t \cdot \gamma \cdot f(u_a) \cdot (T_a - T_{sur}) \qquad \text{(A-7)}$$

where: $$\gamma = \frac{c_p \cdot P_a}{0.622 \cdot L_s} \qquad \text{(A-8)}$$

The units of γ are mb·˚C^{-1}. γ is sometimes referred to as the psychrometric constant. It can be treated as a constant for a given location by assigning P_a a value based on the 'standard atmosphere' altitude versus pressure relationship:

$$P_a = 33.86 \cdot \left(29.9 - 0.335 \cdot H_e + 0.00022 \cdot H_e^{2.4}\right) \qquad \text{(A-9)}$$

where: H_e = elevation (meters).

Wind Function

The wind function is often defined empirically using the equation:

$$f(u_a) = a + b \cdot U_a \tag{A-10}$$

where: U_a = wind travel (wind speed multiplied by the time interval) (km),
b = an empirical constant ($mm \cdot mb^{-1} \cdot km^{-1}$), and
a = an empirical constant which indicates the amount of vapor transfer with no wind ($mm \cdot mb^{-1}$).

Theoretically a=0.0 and the coefficient b varies with the stability of the air just above the surface. Stable conditions exist when the air temperature is warmer than the surface temperature, i.e. the temperature gradient impedes the flow of heat or vapor. Unstable conditions exist when the air is cooler than the surface. In this case the warmer air at the surface wants to rise and assist with the transfer of heat or vapor, e.g. cool air over a warm water surface results in a significant increase in the evaporation rate. The air over a snow cover is predominantly stable as the air temperature is almost always warmer than the snow surface temperature. This is especially true during periods when melt is occurring at the surface since the temperature of the snow cannot rise above 0˚C.

Neutral stability exists when the air and the surface have the same temperature. Under neutral conditions the coefficient b can theoretically be expressed as:

$$b = \frac{\rho_a \cdot 0.622}{P_a \cdot \rho_w} \cdot 10^6 \cdot \frac{k^2}{\left[\ln \frac{z_a}{z_0}\right]^2} \tag{A-11}$$

where: ρ_a = density of the air ($gm \cdot cm^{-3}$),
k = von Karman's constant (typically = 0.40), and
z_o = the roughness length of the surface (cm).

Thus theoretically under neutral conditions the wind function varies primarily based on the wind speed and the roughness of the surface. The roughness of a snow surface can vary from one location to another and throughout the snow season depending on climatological conditions. There is a tendency for the surface to become smoother as the snow season progresses with values of z_o in the range of 0.01 to 0.25 during the melt season.

The degree of stability of the air can be defined using the Richardson number. When considering the temperature gradient between a specified level above the surface and the surface itself, the bulk Richardson number is defined as:

$$(R_i)_B = \frac{2 \cdot g \cdot z_a \cdot (T_a - T_{sur})}{(T_a + T_{sur}) \cdot (27.78 \cdot u_a)^2} \quad \text{(A-12)}$$

where: $(R_i)_B$ = bulk Richardson number, and
g = acceleration of gravity ($cm \cdot sec^{-2}$).

Positive Richardson numbers indicate stable conditions and negative values indicate instability. The critical Richardson number, Ri_{cr}, is the value beyond which turbulent conditions no longer exist, i.e. the air is so stable that there is no turbulence. Experimental data suggests that Ri_{cr} is in the range from 0.15 to 0.25. Under stable conditions the reduction in the theoretical wind function can be expressed as $(1.0 - \alpha \cdot (R_i)_B)^2$ where $\alpha = Ri_{cr}^{-1}$. When the Richardson number indicates unstable conditions, the increase in the wind function over neutral conditions is more complex and varies with surface roughness in addition to the degree of instability. Results from the NOAA-ARS cooperative snow research station near Danville, Vermont [*Anderson*(1976)] indicate that the bulk Richardson number varied between 0.05 and 0.10 during most hours when net turbulent heat transfer causes significant snowmelt.

An empirical wind function is often used in the case of snow because of the difficulty in measuring the values needed to compute a theoretical value, the variability of the surface roughness, and the small range in atmospheric stability during periods when latent and sensible heat are most important for computing melt rates. An empirical wind function can be determined by carefully placing clear plastic pans full of snow flush with the snow surface and determining the change in weight and the wind travel over some period of time. Such measurements when adjusted to a 1.0 meter wind measurement height produced values of the constant 'b' in Equation A-10 that varied from 0.0019 to 0.0042. The constant 'a' was zero in all cases except one (case when b=0.0019). An empirical wind function can also be determined by calibration of an energy balance model at a location where good quality measurements of the energy budget variables and observed snow conditions exist. For the Danville, Vermont site values of a=0.0 and b=0.002 were determined by calibration [*Anderson*(1976)].

<u>Heat Transfer due to Mass Changes</u>

The mass balance of a snow cover can be expressed as:

$$P - O_s + \Delta t \cdot V + V_g = \Delta W \quad \text{(A-13)}$$

where: P = water equivalent of precipitation (mm),
O_s = liquid water outflow from the snow cover (mm),
V_g = vapor transfer between the snow and soil (mm), and
ΔW = change in water equivalent of the snow cover (mm).

If the temperature of the snow cover outflow is assumed to be 0˚C and the heat content of the transferred vapor is assumed negligible, then only the heat transferred by precipitation needs to be considered. The wet-bulb temperature should be a good approximation of the temperature of precipitation because of the analogy between falling precipitation and a ventilated wet-bulb thermometer. Thus, the heat transfer due to mass changes can be expressed as:

$$Q_m = \frac{c_r \cdot \rho_w \cdot P}{10} \cdot T_w \qquad \text{(A-14)}$$

where: c_r = specific heat of precipitation ($cal \cdot gm^{-1} \cdot {}^{\circ}C^{-1}$) (when precipitation is rain, =1.0, specific heat of water, c_w, and when precipitation is snow, =0.5, specific heat of ice, c_i), and
T_w = wet-bulb temperature (˚C).

When rain occurs on snow, the amount of melt due to the rainwater is that generated by the heat that is released as the rain is cooled to 0˚C:

$$M_w = \frac{Q_m \cdot 10}{L_f \cdot \rho_w} = \frac{c_w \cdot P \cdot T_w}{L_f} = 0.0125 \cdot P \cdot T_w \qquad \text{(A-15)}$$

where: M_w = melt due to rain (mm).

Thus if the temperature of the rain is 10˚C, the amount of melt produced by the rainwater is 12.5% of the amount of precipitation and if the temperature of the rain is 20˚C, the melt is 25% of the precipitation. It is a misconception that rain causes considerable snowmelt. This misconception likely came about because the depth of snow often decreases significantly during a rain event even with the temperature just slightly above freezing. This reduction in depth is due to rain speeding up the metamorphism processes which cause the density to increase and thus the depth to decrease even though the water equivalent stays about the same. Significant melt can occur during a rain event if the temperature is quite warm and there is a strong wind. Most of the melt in that case is due to latent and sensible heat transfer.

Heat Transfer at the Snow-Soil Interface

Heat transfer across the snow-soil interface is primarily a function of the temperature gradient in the upper soil layer, i.e. the difference in temperature between the interface and some distance down into the soil. The thermal conductivity of soil varies depending on soil type and moisture content. Some heat can also be transferred across the snow-soil interface due to vapor movement. When there is a temperature gradient, water vapor will move from warmer to colder temperatures. The amount of water vapor in the soil is a function of the amount of soil moisture. The greatest amount of heat transfer across the snow-soil interface typically occurs when snowfall first occurs in late fall or early winter. In this case the soil is still relatively warm. As the soil cools over the winter, the amount of heat transfer across the interface decreases. In permafrost regions or whenever there is frozen soil below the snow, the amount of heat transfer is negligible. Even when the soil is not frozen, the amount of heat transfer across the snow-soil interface is minimal compared to the heat transfer between the snow and the atmosphere except when small amounts of snow fall on warm ground.

Change in the Heat Storage of the Snow Cover

When there is an exchange of heat between the snow and the air, and to a minor extent with the underlying soil, changes take place internally within the snow cover. The snow surface temperature attempts to change so that there is a balance between the energy exchange with the atmosphere and the heat flow within the snow cover. Since the ice in the snow cover cannot get warmer than 0˚C, when the surface temperature reaches 0˚C any additional heat from the atmosphere is converted into melt. Surface melt or rain water will then move down into the snow cover. If the temperature of the snow below the surface is below freezing then at least some of the percolating water will refreeze, releasing heat in the process. If the entire snow cover is isothermal at 0˚C, then the water will percolate all the way to the bottom of the snow cover and become outflow.

When the heat exchange with the atmosphere becomes negative, the snow surface temperature will drop below 0˚C and a temperature gradient will develop within the snow cover. Heat flow within the snow cover is due to conduction and vapor transfer, both of which are a function of the temperature gradient at any point within the pack. The amount of heat transferred by conduction is a function of the temperature gradient and the thermal conductivity of the snow. The thermal conductivity for snow is generally referred to as the effective thermal conductivity to indicate the combined effects of conduction through ice grains, conduction through the air in the void spaces, and radiant energy exchange across the void spaces (longwave radiation exchange due to differences in temperature from one grain to another). The lower the density of the snow, the more voids and thus more entrapped air and the better the insulating effect. Low density snow is a very good insulator. Figure A-6 shows how the effective thermal conductivity of snow varies with density. It can be seen that the effective thermal conductivity for very low density snow approaches that for air and values for very dense snow approach the thermal conductivity of ice.

Heat can also be transferred within the snow due to vapor movement. When a temperature gradient develops vapor will sublimate from warmer snow grains, requires heat, and condense on adjacent cooler grains, releasing heat. This results in a transfer of heat and is a function of the temperature gradient and a coefficient referred to as the effective diffusion coefficient.

Calculating Energy Exchange for a Snow Cover

In general terms the solution of the energy balance equation involves having measurements or estimates of the atmospheric variables and then solving Equation A-1 for the snow surface temperature and the temperature profile within the snow cover. Typically an iterative solution technique is used. The atmospheric variables that are needed to compute the energy balance are:

- incoming solar radiation,
- incoming atmospheric radiation,
- air temperature,
- dew point,
- wind speed, and
- precipitation amount.

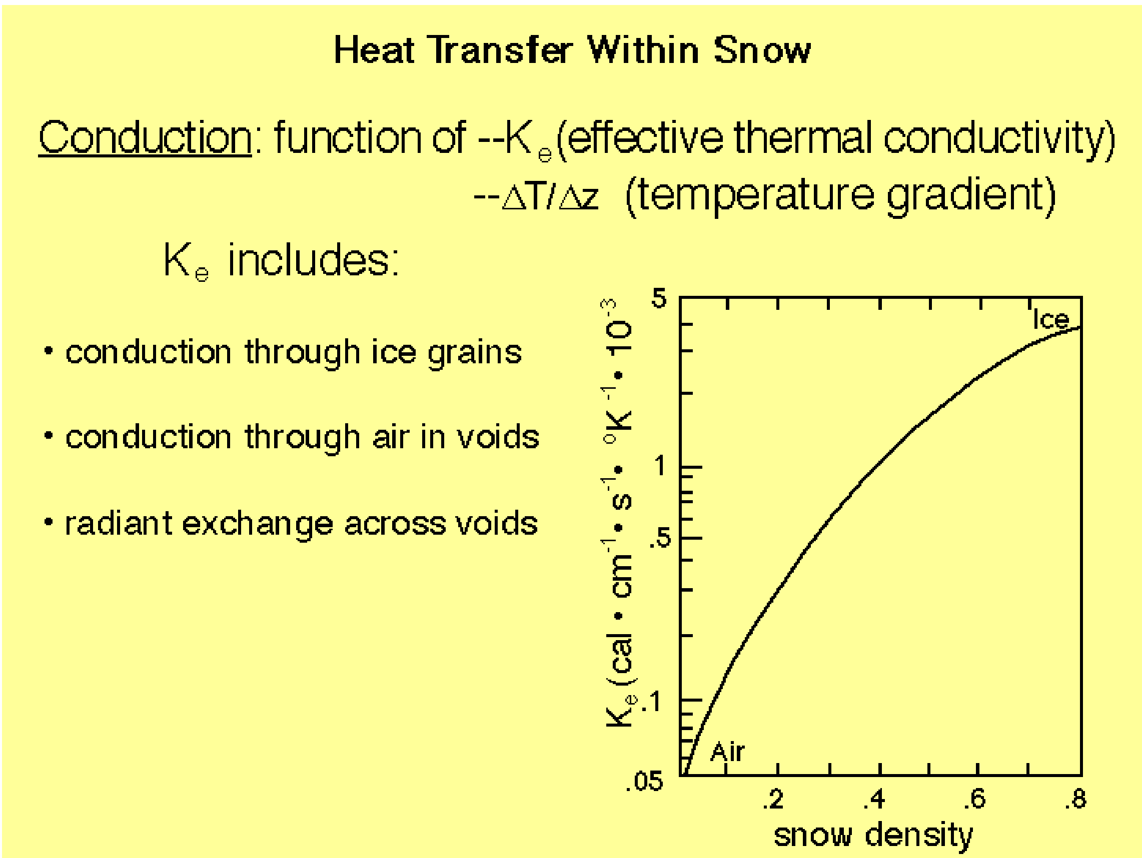

Figure A-6. Variation in effective thermal conductivity with density.

In addition some estimate of the temperature below the soil surface is needed. The albedo of the snow could be obtained by measuring reflected solar radiation in addition to incoming, but is typically related to some snow surface property, such as surface density, or some other variable. The solution model must also typically calculate the density profile of the snow cover since the effective thermal conductivity and to a lesser extent the effective diffusion coefficient vary with density. The complexity of solving the energy budget equation becomes even more involved when the available values for the atmospheric variables don't correspond specifically to the exact location where the computations are being made. In such cases adjustments may need to be applied to the variables to take into account factors such as vegetation cover, slope, aspect, and the elevation difference between the location and the elevation associated with the values of the variables.

There is one case when the energy exchange at the snow-air interface can be directly calculated. That is the case when the entire snow cover is isothermal at 0˚C, any precipitation is rain, and there is a positive energy exchange. Substituting Equations A-3, A-5, A-7, and A-14 into Equation A-1, inserting the proper values for any coefficients, assuming Q_g is negligible, and expressing ΔQ in terms of the amount of melt the result is:

$$\begin{aligned} M &= 0.125 \cdot [Q_i \cdot (1.0 - A) + Q_a] - 3.37 \cdot \Delta t + 0.0125 \cdot P \cdot T_w \\ &+ 8.5 \cdot \Delta t \cdot f(u_a) \cdot [(e_a - 6.11) + 0.00057 \cdot P_a \cdot T_a] \end{aligned} \tag{A-16}$$

where: M = surface melt (mm).

Of course even in that case good values of the atmospheric input variables and albedo must be available for the location where the computations are being preformed in order to get an accurate estimate of the amount of snowmelt.

Reference

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SNOW-17 Model – Appendix B
List of Symbols

Symbol	Description	Units
A	Albedo of snow	decimal fraction
A_{adj}	A_i value after A_s is adjusted	mm
A_i	Areal index	mm
A_s	Areal extent of snow cover	decimal fraction
A_v	Seasonal melt variation adjustment	real
A_{ns}	Areal extent when new snowfall first occurs on a partially bare area	decimal fraction
ATI	Antecedent temperature index	˚C
D	Heat deficit	mm
ΔD_p	Change in heat deficit due to snowfall	mm
ΔD_t	Change in heat deficit due to temperature	mm
E	Excess liquid water in the snow cover	mm
E_l	Average hourly lagged excess liquid water	inches
E_{ls}	Average E_l over the snow covered area	inches
H	Depth of the total snow cover	cm
H_e	Elevation	m
H_n	Depth of new snowfall	cm
H_x	Depth of existing snow	cm
K_e	Effective thermal conductivity of snow	$cal \cdot cm^{-1} \cdot sec^{-1} \cdot {}^\circ K^{-1}$
L	Lag time for excess liquid water	hr
L_f	Latent heat of fusion	$cal \cdot gm^{-1}$
L_s	Latent heat of sublimation	$cal \cdot gm^{-1}$
M	Melt at snow surface	mm
M_f	Melt factor	$mm \cdot {}^\circ C^{-1} \cdot \Delta t_t^{-1}$
M_g	Ground melt	mm
M_{nr}	Surface melt during non-rain periods	mm
M_r	Surface melt during rain-on-snow periods	mm
M_w	Melt due to rain water	mm
N	Day number since March 21st	integer
NM_f	Negative melt factor	$mm \cdot {}^\circ C^{-1} \cdot \Delta t_p^{-1}$
O	Total outflow over area (O_s + rain-on-bare-ground)	mm
O_g	Snow cover outflow due to ground melt	mm
O_{mr}	Snow cover outflow from melt or rain-on-snow	mm
O_s	Snow cover outflow (total)	mm
P	Total amount of precipitation	mm
P_a	Atmospheric pressure	mb
P_n	Water equivalent of new snowfall	mm
ΔQ	Change in heat storage of the snow cover	$cal \cdot cm^{-2}$
Q_a	Incoming longwave radiation	$cal \cdot cm^{-2}$
Q_e	Latent heat transfer	$cal \cdot cm^{-2}$
Q_f	Liquid water that refroze within the snow	mm

Symbol	Description	Units
Q_g	Heat transfer at snow-soil interface	$cal \cdot cm^{-2}$
Q_h	Sensible heat transfer	$cal \cdot cm^{-2}$
Q_i	Incoming solar radiation	$cal \cdot cm^{-2}$
Q_L	Total emitted longwave radiation	$cal \cdot cm^{-2}$
Q_m	Heat transfer due to mass changes	$cal \cdot cm^{-2}$
Q_n	Net radiation	$cal \cdot cm^{-2}$
Q_r	Reflected solar radiation	$cal \cdot cm^{-2}$
Q_s	Longwave radiation emitted by snow	$cal \cdot cm^{-2}$
Q_w	Liquid water available at snow surface	mm
R_1	One hour withdrawal rate of excess water	hr^{-1}
$(R_i)_B$	Bulk Richardson's number	dimensionless
Ri_{cr}	Critical Richardson's number	dimensionless
S	Amount of lagged excess water in storage	mm
S_v	Seasonal sine curve melt variation	real
T	Temperature	˚K
T_a	Air Temperature	˚C
ΔT_a	Change in air temperature from previous period	˚C
T_n	Temperature of new snowfall	˚C
T_r	Temperature of rain water	˚C
T_s	Average temperature of the total snow cover	˚C
T_{sur}	Temperature of the snow surface	˚C
T_w	Wet bulb temperature	˚C
T_x	Average temperature of existing snow	˚C
U_a	Wind travel	km
V	Vapor transfer	$mm \cdot hr^{-1}$
V_g	Vapor transfer at snow-soil interface	
W	Water equivalent (ice + liquid water storage, W_q)	mm
ΔW	Change in water equivalent	mm
W_i	Water equivalent of ice portion of snow cover	mm
W_{is}	Average W_i over the snow covered area	inches
W_{ix}	Water equivalent of ice portion of existing snow	mm
W_{max}	Maximum 'W' during accumulation period	mm
W_q	Liquid water storage contents	mm
W_{qt}	Total liquid water (storage + transmission)	mm
W_{qx}	Liquid water storage capacity	mm
W_t	Total water equivalent (ice + all liquid, W_{qt})	mm
W_{ns}	Value of W when new snowfall first occurs on a partially bare area	mm
W_{100}	Water equivalent when the areal extent will first drop below 100% as new snow on a partially bare area melts	mm
a	Empirical wind function constant	$mm \cdot mb^{-1}$
b	Empirical wind function constant	$mm \cdot mb^{-1} \cdot km^{-1}$
c	Effective specific volumetric heat capacity of snow	$watts \cdot sec \cdot m^{-3} \cdot {}^{\circ}C^{-1}$

Symbol	Description	Units
c_a	Volumetric heat capacity of air	$watts \cdot sec \cdot m^{-3} \cdot {}^{\circ}C^{-1}$
c_c	Volumetric heat capacity of ice	$watts \cdot sec \cdot m^{-3} \cdot {}^{\circ}C^{-1}$
c_i	Specfic heat of ice	$cal \cdot gm^{-1} \cdot {}^{\circ}C^{-1}$
c_p	Specific heat of dry air	$cal \cdot gm^{-1} \cdot {}^{\circ}C^{-1}$
c_q	Volumetric heat capacity of water	$watts \cdot sec \cdot m^{-3} \cdot {}^{\circ}C^{-1}$
c_r	Specific heat of precipitation	$cal \cdot gm^{-1} \cdot {}^{\circ}C^{-1}$
c_w	Specific heat of water	$cal \cdot gm^{-1} \cdot {}^{\circ}C^{-1}$
c_x	Destructive metamorphism decay factor	dimensionless
c_1	Fractional increase in density per cm of load at 0˚C	$cm^{-1} \cdot hr^{-1}$
c_2	Compaction parameter	$cm^{3} \cdot gm^{-1}$
c_3	Fractional settling rate at 0˚C for $\rho_x<\rho_d$	hr^{-1}
c_4	Settling parameter	$^{\circ}C^{-1}$
c_5	Increase in settling rate when liquid water present	real
e_a	Vapor pressure of air	mb
e_{sat}	Saturated vapor pressure at T_a	mb
e_{sur}	Vapor pressure of snow surface	mb
f_r	Fraction of precipitation in the form of rain	decimal fraction
f_s	Fraction of precipitation in the form of snow	decimal fraction
$f(u_a)$	Wind function	$mm \cdot mb^{-1} \cdot hr^{-1}$
g	Acceleration of gravity	$cm \cdot sec^{-2}$
k	Von Karman constant	dimensionless
u_a	Wind speed	$km \cdot hr^{-1}$
Δt	Time interval	hr
Δt_p	Time interval of precipitation data	hr
Δt_t	Time interval of temperature data	hr
z_a	Height above snow surface	cm
z_o	Surface roughness	cm
θ_q	Fraction of liquid water in the snow	decimal fraction
α	Ri_{cr}^{-1}	dimensionless
γ	Psychrometric constant	$mb \cdot {}^{\circ}C^{-1}$
ε	Emissivity	decimal fraction
λ	Thermal conductivity of snow	$watts \cdot m^{-1} \cdot {}^{\circ}C^{-1}$
ρ	Density of ice portion of the total snow cover	$gm \cdot cm^{-3}$
σ	Stefan-Boltzmann constant	$cal \cdot cm^{-2} \cdot {}^{\circ}K^{-4} \cdot hr^{-1}$
ρ_a	Density of air	$gm \cdot cm^{-3}$
ρ_d	Threshold settling rate density	$gm \cdot cm^{-3}$
ρ_n	Density of new snowfall	$gm \cdot cm^{-3}$
ρ_w	Density of water	$gm \cdot cm^{-3}$
ρ_x	Density of ice portion of existing snow cover	$gm \cdot cm^{-3}$
σ	Stefan-Boltzman constant	$mm \cdot {}^{\circ}K^{-1} \cdot hr^{-1}$