DISCRETE-TIME MODELS FOR NONLINEAR AUDIO SYSTEMS

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ABSTRACT

A variety of computational models have been proposed for digital simulation of nonlinear systems with memory [1, 2, 3, 4]. They are dealing with different aspects of the problem, like methods for identification, avoiding aliasing and fast convolution algorithms. In this paper we shortly sum up some of the common approaches and present a straightforward method for bandlimited discrete-time realization of analog nonlinear audio effects, like tube amps, exciters etc., using off-time digital cross correlation measurements. From these measurements we obtain a rather inefficient Wiener representation of the unknown nonlinearity. We then reduce the number of required coefficients significantly on the basis of multi-dimensional Laguerre transformation of the related Volterra kernels to allow real-time implementation on a digital signal processor [5].

1. INTRODUCTION

In the beginning, the first major effort of digital music reproduction was to eliminate a number of technical artifacts produced by traditional audio systems so far. Today, twenty years later, it seems rather desirable to bring some of the old equipment's nonlinearities back again into the modern all digital studio environment because of their pleasing psychoacoustic properties. Due to the bandlimited nature of discrete-time signal processing special models are required for this task to avoid severe aliasing problems, resulting from nonlinear treatment otherwise, see Fig. 1.

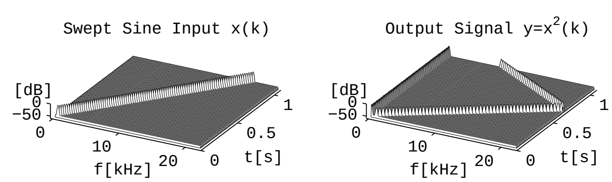

Figure 1: Aliasing of harmonics.

1.1. Oversampling

At first sight, the most obvious solution to the aliasing problem seems to be the application of oversampling, see Fig. 2 and 3. In this case the input signal is upsampled by a factor of L first, the

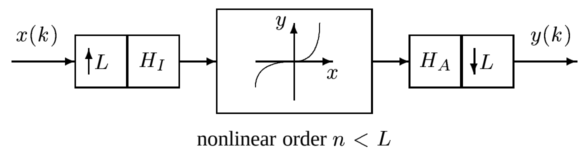

Figure 2: Oversampling to avoid aliasing.

images being then rejected by the interpolation filter $H_I(z)$. Now the nonlinearity of finite order $n < L$ spreads the spectrum of the oversampled input sequence by factor n (see Fig. 3). After passing an anti-aliasing filter $H_A(z)$, which reduces the bandwidth back to the Nyquist frequency, downsampling by a factor L finally leads to the output signal. Only the harmonic components that fall inside the Nyquist range remain in the output signal.

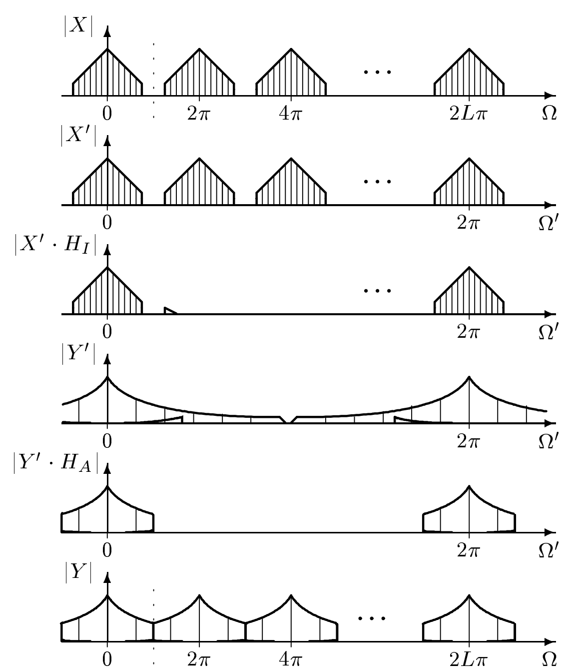

Figure 3: Oversampling from a spectral point of view.

A further examination of this idea reveals some practical drawbacks, the most important being the high computational requirements of oversampling techniques and the necessity to have analytic knowledge of the nonlinear model.

1.2. Harmonic Mixer

Another intuitive approach is to use a parallel bank of anti-aliasing filters to limit the bandwidth of the input signal to $f_S/2n$ and then apply individual nonlinear processing of $n-$th order, as shown in Fig. 4. This allows the simulation of almost any static nonlinearity (having no memory) inside the Nyquist range by adjusting the gain factors of the individual polynomial terms for the desired harmonic spectrum. In combination with further equalization a spectral shaping of the individual harmonics can also be performed.

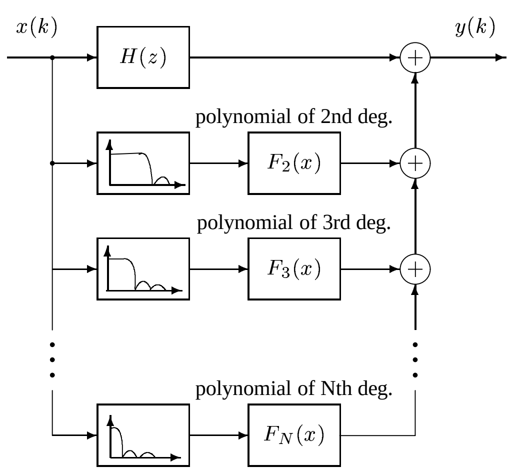

Figure 4: Harmonic mixer.

2. THEORY OF NONLINEAR SYSTEMS

If one is interested in the modeling of 'black boxes' with a nonlinear behavior, the Volterra and Wiener theories of nonlinear systems can be used to analyze and synthesize these systems [1]. This approach is especially useful if nothing about the internal structure of the nonlinear system is known.

2.1. Volterra Series

Nonlinear systems with memory (having frequency dependent behavior) can be described by a Volterra series expansion. This can be regarded either as a Taylor series with memory or as a multi-dimensional expansion of linear system theory. The multi-dimensional kernels $h_n(i_1, ..., i_n)$, called Volterra kernels, take the part of the linear impulse response for the higher order terms.

The system response can then be calculated by a sum

$$y(k) = \sum_{n=1}^{N} \mathrm{H}_n[x(k)] \tag{1}$$

of multi-dimensional convolutions, represented by the Volterra functionals

$$\mathrm{H}_n[x(k)] = \sum_{i_1=0}^{T_n-1} \cdots \sum_{i_n=0}^{T_n-1} h_n(i_1, ..., i_n) x(k-i_1)...x(k-i_n), \tag{2}$$

with the input signal $x(k)$, as shown in Fig. 5.

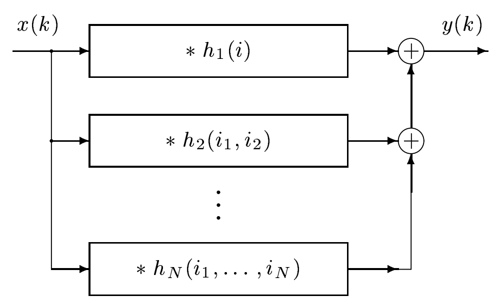

Figure 5: Discrete Volterra representation.

2.2. Wiener G-Functionals

The Wiener theory can be regarded as an orthonormal expansion of the Volterra representation of a system [1]. By dividing the system description into special subsystems $\mathrm{G}_n[k_n; x(k)]$, called G-functionals, the output can be described by

$$y(k) = \sum_{n=0}^{N} \mathrm{G}_n[k_n; x(k)], \tag{3}$$

where k_n are called Wiener kernels. These subsystems produce orthonormal output signals for white noise excitation. This allows the determination of the Wiener kernels k_n of the original system by using a cross correlation technique, as shown in Fig. 6. The G-functionals can then be synthesized by a number of special Volterra functionals K_n and $\mathrm{K}_{m(n)}$

$$\mathrm{G}_n[k_n; x(k)] = \mathrm{K}_n[x(k)] + \sum_{m=0}^{n-1} \mathrm{K}_{m(n)}[x(k)] \tag{4}$$

in which the so-called 'derived' Wiener kernels $k_{m(n)}$ are completely determinated by their 'leading' kernels k_n, which carry the entire information about the systems behavior [1].

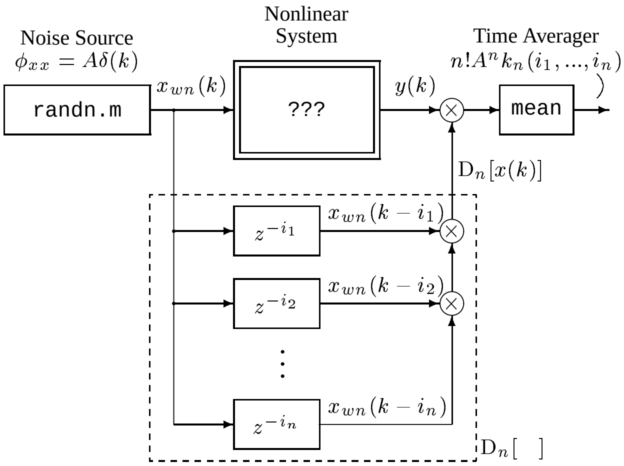

Figure 6: Identification of the Wiener kernels of an unknown discrete system by cross correlation.

2.3. Identification

The multiple cross correlation algorithm used to identify the leading Wiener kernels $k_n(i_1, \ldots, i_n)$ can be applied to all discrete systems. Aliasing-free measurement and modeling of nonlinear analog systems, which are not limited in bandwidth, can be accomplished by embedding the device under test (DUT) into synchronous DA and AD converters, see Fig. 7, thereby producing a bandlimited discrete system to be identified. Practically the measurement can be done by computing a digital white noise sequence which is then fed into a DA converter with a suitable sampling rate to provide the analog input signal to the DUT. The following AD converter is synchronized with the DA converter to gain a proper discrete-time system with synchronous input-/output relations. The discrete-time output signal is then again recorded by the PC for computing the cross correlations.

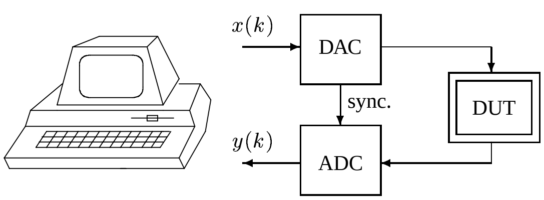

Figure 7: Aliasing-free discrete-time identification of unknown continuous-time system.

3. EFFICIENT IMPLEMENTATION

In linear system theory, natural systems can often be described more efficiently in terms of an orthonormal basis of ‘natural’ functions derived from simple recursive structures, like that of the Laguerre polynomials [1, 6]. By transforming the impulse response from time domain into the domain of these functions, the number of the required coefficients to describe the system behavior may be reduced, coming even close to the number of parameters needed in a pole-zero model of the original recursive structure. This method can also be applied to discrete-time multi-dimensional Volterra kernels of nonlinear systems [5], leading to a very efficient structure for the implementation of Wiener systems.

3.1. Discrete Laguerre Transformation

Wiener kernels k_p of the dimension $(T_p \times ... \times T_p)$ can be transformed into the Laguerre domain by using

$$c_{n_1 \cdots n_p} = \sum_{i_1=0}^{T_p-1} \cdots \sum_{i_p=0}^{T_p-1} k_p(i_1, \ldots, i_p) l_{n_1}(i_1) \cdots l_{n_p}(i_p), \quad (5)$$

where $l_{n_1} \ldots l_{n_p}$ are discrete Laguerre functions of a suitable time scale factor ξ. The inverse transform

$$k_p(i_1, \ldots, i_p) = \sum_{n_1=0}^{M_p-1} \cdots \sum_{n_p=0}^{M_p-1} c_{n_1 \cdots n_p} l_{n_1}(i_1) \cdots l_{n_p}(i_p) \quad (6)$$

leads again back to the Wiener kernels $k_p(i_1, \ldots, i_p)$. The discrete Laguerre transformation (DLT) gives a $(M_p \times ... \times M_p)$ matrix of Laguerre coefficients, in which the energy is often more concentrated in a smaller number of coefficients, especially for long nonlinear impulse responses T_p. Thus compared to the direct multi-dimensional convolution of a Volterra series representation a smaller number of operations is achievable. To take advantage

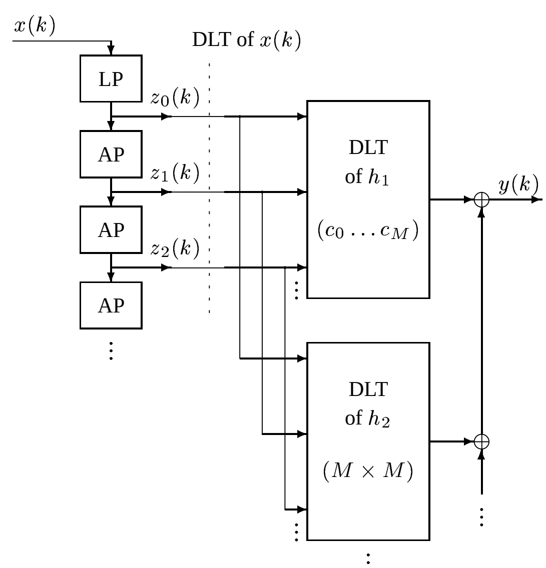

Figure 8: Efficient filter structure for real-time simulation of Wiener systems.

of this, the input signal has to be transformed into the Laguerre domain by means of a recursive filter structure, consisting of one first-order low-pass and $(M_p - 1)$ identical first-order all-pass filters. The entire original system can then be simulated quite efficiently by the configuration shown in Fig. 8. All required coefficients are obtained by multi-dimensional DLT according to Eq. (5) of the meta kernels h_p which are sums of all the systems leading or derived Wiener kernels having the same dimension p.

4. EXAMPLE

Figure 9 shows the measured first-order Wiener kernel (which is equivalent in this case to the linear impulse response) and the corresponding (linear) frequency response of a commercial analog audio processor used in studio environments to enhance the sound of recorded music.

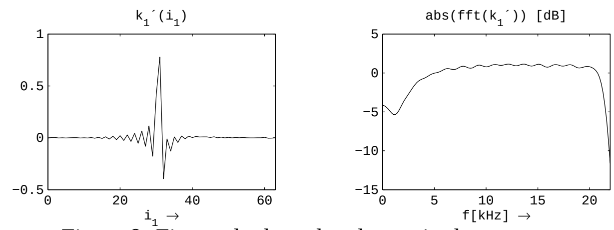

Figure 9: First-order kernel and magnitude response

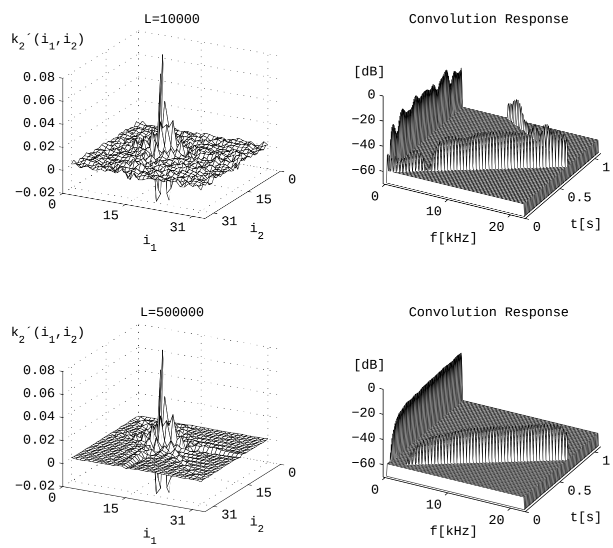

Figure 10: Second-order kernel after 10000 and 500000 correlated samples and convolutions with sine sweep.

The second-order kernel representing the DUT's intended frequency-dependent second-order nonlinearity is shown in Fig. 10. Discrete convolutions with swept sine input signals illustrate the necessity of allowing an appropriate correlation length. Finally the original system is compared to a discrete real-time DSP implementation in Fig. 11 using discrete Laguerre transformation to reduce the number of second-order coefficients significantly compared to the brute-force approach of plain convolution. The influence of the

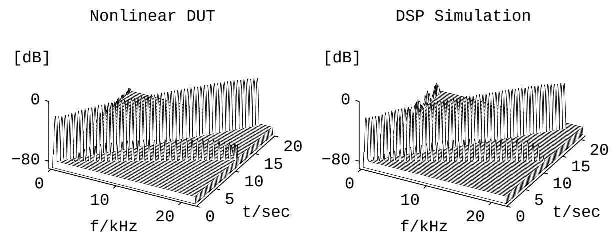

Figure 11: Nonlinear DUT simulation [5].

linear and the second-order effects can be observed accordingly in the amplitude of the swept sinusoid itself and its first harmonic.

5. CONCLUSION

Discrete-time models for static nonlinear audio systems can be obtained by simple oversampling techniques or more flexible by a parallel bank of low-pass filters followed by individual nonlinearities. The Volterra and Wiener theories of nonlinear systems are used for system identification and system approximation if an unknown system with nonlinearities and memory has to be modelled. Efficient real-time realizations of such models can be performed by means of Laguerre transformation of the input signal and a network configuration consisting of measured kernel coefficients transformed into Laguerre domain.

6. REFERENCES

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