Networks of Common Asset Holdings : Aggregation and Measures of Vulnerability

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Introduction : Financial linkages

Financial networks have been used to model the propagation of distress due to insolvency.

It is now believed that the most important sources of financial distress come from illiquidity and price mediated contagion effects.

Linkages that transmit price feedback effects : not direct claims but overlap in portfolio holdings.

Since they are not direct claims, they are not directly observable form data.

Research question : How to measure the linkages due to common assets ?

Network model

$\mathbf{N} = \{1, \dots, N\}$ is a set of portfolios
$\mathbf{K} = \{1, \dots, K\}$ is a set of stocks
$\mathbf{S} = (s_1, \dots, s_K)$ is a vector of stock prices
$\mathbf{B} = [\beta_{ki}] \qquad i \in \mathbf{N}, k \in \mathbf{K}$ the holdings matrix : β_{ki} represents the number of shares of stock k owned by portfolio i.
The value of portfolio i can be written as

$$P_i = \sum_{k=1}^{K} \beta_{ki} s_k = \beta_i \cdot S, \qquad \text{where } \beta_i = (\beta_{1i}, \dots, \beta_{Ki}).$$

Network model : portfolios (funds) represent nodes. The strength of the links (edge weights) between any two portfolios are defined by answering the following question : what effect will the liquidation of fund i have on fund j ?

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- Initial shock : Uniform redemptions / forecast of redemptions / contemporaneous redemptions ;
- Behavior of a fund : Assumption : proportional liquidation ;
- Impact on other funds : Need to model the liquidity of each stock and define the edge weights ;

Edge weights under linear price impact

The price impact function is linear and of the form :

$$PI_k(x) = \frac{x}{\lambda_k},$$

where λ_k is such that buying/selling $\frac{\lambda_k}{100}$ stocks will move the price of the asset up/down by 1%. The parameter λ_k captures the *market depth* of stock k (Kyle '85), with

$$\lambda_k = \frac{1}{\tilde{\lambda}} \frac{ADV_k}{\sigma_k},$$

where ADV_k is the average daily volume of trades, σ_k is the daily returns standard deviation of stock k and $\tilde{\lambda}$ is an invariant across stocks (Kyle & Obizaheva 2011)

When portfolio i liquidates its shares of asset k, the price of the asset s_k drops by $\frac{\beta_{ki}}{\lambda_k} s_k$.

The value of portfolio j decreases by $\beta_{kj} \frac{\beta_{ki}}{\lambda_k} s_k$.

The total loss experienced by j if portfolio i liquidates defines the **edge weight**

$$w_{ij} = \sum_{k=1}^{K} \frac{\beta_{ki}}{\lambda_k} \beta_{kj} s_k .$$

Data

Quarterly mutual fund holdings data from the CRSP Mutual Fund database ranging from 01/2003 - 12/2012.

We use only US equity funds and portfolios with TNA above 100 millions.

Fund flows are calculated using

$$Flow_t = \frac{TNA_t - (1 + r_t)TNA_{t-1}}{TNA_{t-1}},$$

where TNA_t is the total net assets of a portfolio in period t and r_t is the return of the portfolio in period t.

To calculate stock market depths we use daily stock data from the CRSP US Stock Database.

vulnerability measure

We define **the Vulnerability measure** as the cumulative first order effects on node *i*'s loss that are imposed by its neighbors :

$$VI_i = \frac{1}{P_i} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_{ji}.$$

Interpretation : εVI_i represents the fraction by which portfolio *i*'s value will decrease (increase) if all its neighbors liquidate (expand) their portfolios by a factor of ε.

The most significant drops in cumulative TNA occured in Q4 2008, shortly after the collapse of the Lehman Brothers, and in Q3 2011 during which S&P downgraded the U.S. credit rating.

TABLE: Fund returns during quarter t regressed against fund vulnerability at the start of the quarter.

	2008Q4	2011Q3
Constant	-0.2117***	-0.1239***
	(-58.56)	(-47.26)
VI_t	-0.0270***	-0.0801***
	(-7.14)	(-20.57)
Adj. R^2	0.0179	0.1012
Observations	2748	3749

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

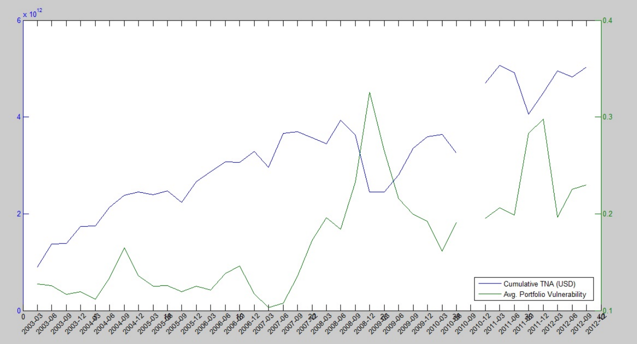

FIGURE: Average portfolio vulnerability VI plotted alongside the cumulative TNA of all portfolios. Data for 09/2010 is missing.

This suggests that *increases in average vulnerability precede significant drops in total net assets.*

Two refinements

F_i : the flow *to be experienced* by fund i over the current quarter.

1. The **flow-adjusted vulnerability (FAV) measure** for fund i

$$FAV_i = \frac{1}{P_i} \sum_{j=1}^{N} F_j w_{ji},$$

2. The measure of **vulnerability to order imbalance**.

$$FAV_i^* = \frac{1}{P_i} \sum_{k=1}^{K} \beta_{ki} s_k \left(\left(\frac{\hat{N}_{B,k}}{\hat{N}_{S,k}} \right)^{\alpha} - 1 \right).$$

where for stock k, we set the estimators for the number of buyers/sellers

$$\hat{N}_{B,k} = \sum_{i=1}^{N} \beta_{ki} F_i \mathbf{1}_{\{F_i > 0\}},$$

$$\hat{N}_{S,k} = \sum_{i=1}^{N} \beta_{ki} F_i \mathbf{1}_{\{F_i < 0\}}.$$

Results

TABLE: Fama-Macbeth regressions of future fund returns with Newey-West corrections of four lags. The dependent variable is fund return over quarter t, the FAV^* and FAV measures are computed using the asset holdings at the start of quarter t and the fund flows over quarter t.

	(1)	(2)	(3)
FAV_t^*		0.0683^{***}	0.0705^{***}
		(4.85)	(4.36)
FAV_t	0.463^{***}		-0.00706
	(4.38)		(-0.08)
$\log(TNA_{t-1})$	0.00101	0.000635	0.000623
	(0.69)	(0.41)	(0.39)
$\log(Shares_{t-1})$	-0.000800	-0.000352	-0.000333
	(-0.57)	(-0.24)	(-0.22)

	(1)	(2)	(3)
$flow_t$	0.0230***	0.0162**	0.0155**
	(4.55)	(3.49)	(3.34)
$return_{t-1}$	0.000742	-0.0233	-0.0251
	(0.02)	(-0.51)	(-0.56)
Constant	0.0227	0.0205	0.0203
	(1.48)	(1.29)	(1.23)
Sample Size	87257	87257	87257
R^2	0.1746	0.2032	0.2105

t statistics in parentheses

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Conclusions

- The network representation allows us to derive measures of vulnerability of funds to the shocks held by their neighbors in the network.
- We find, using mutual fund data, that the vulnerability index is useful in predicting returns in periods of mass liquidations. In such periods, we can identify vulnerable funds based on asset holdings and the liquidity characteristics of the stocks.
- The flow-adjusted measure of vulnerability to order imbalance, based on our model for the price impact of trading, is shown to be correlated with returns throughout all our sample period, not only during periods of mass liquidations.