Common Drifting Volatility in Large Bayesian VARs

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May 4, 2012

Introduction

Motivation:

- Larger BVARs tend to forecast better (lower RMSEs, higher scores) than smaller BVARs
  - Banbura, et al. (2010), Carriero, et al. (2011), Koop (2012)
- Allowing stochastic volatility improves the accuracy of both point and density forecasts
  - Clark (2011), D'Agostino, et al. (2012)

Introduction

Problem: Computation becomes too time-consuming with more than 3-5 variables

- Root of challenge is the $n(np+1) \times n(np+1)$ dimension of the coefficient variance matrix
- No Kroneker structure with conventional stochastic volatility:

$$\bar{\Omega}_{\Pi}^{-1} = \underline{\Omega}_{\Pi}^{-1} + \sum_{t=1}^{T} (\Sigma_t^{-1} \otimes X_t X_t')$$

Introduction

We develop a BVAR with a single, common stochastic volatility process that is much faster to estimate

- Time-varying volatility driven by single multiplicative factor
    - Stochastic discount factor model described in Jacquier, Polson, and Rossi (1995)
- Exploits evidence of fairly strong commonality in volatilities
- Prior takes a particular form that permits the essential Kroneker factorization

Introduction

For VARs of different sizes, we compare CPU time, volatility estimates, model fit, and forecast accuracy (point and density)

Models include:

- VAR with constant volatilities
- VAR with independent stochastic volatilities
  - Cogley and Sargent (2005), Primiceri (2005), Clark (2011)
- Our proposed model with common stochastic volatility

Our results cover:

- 4 and 8-variable models for the U.S., with real-time forecasts
- 15-variable model for the U.S.
- 4 and 8-variable models for the U.K.

Introduction

Findings:

- CSV much more efficient than independent st. vols.
- CSV volatility estimate looks like principal component of independent volatility estimates
- CSV improves the accuracy of real-time point forecasts and density forecasts
  - CSV accuracy comparable to independent SV accuracy

Outline

1. BVAR-CSV specification and implementation
2. Data and forecasting design
3. Results
   - Full sample
   - Forecasting

BVAR-CSV specification and implementation

$$
\begin{aligned}
y_t &= \Pi_0 + \Pi(L) y_{t-1} + v_t, \\
v_t &= \lambda_t^{0.5} A^{-1} S^{1/2} \epsilon_t, \ \epsilon_t \sim N(0, I_n), \\
\log(\lambda_t) &= \log(\lambda_{t-1}) + \nu_t, \ \nu_t \sim \text{iid } N(0, \phi)
\end{aligned}
$$

- Identification: first variable's loading on λ_t is 1
- Diagonal S allows the variances of the variables to differ by a factor that is constant over time
- Choleski structure of A
- $\text{var}(v_t) \equiv \Sigma_t \equiv \lambda_t A^{-1} S A^{-1\prime}$

BVAR-CSV specification and implementation

Prior distributions:

$$\begin{aligned}
\text{vec}(\Pi)|A, S &\sim N(\text{vec}(\underline{\mu}_{\Pi}), \underline{\Omega}_{\Pi}) \\
a_i &\sim N(\underline{\mu}_{a,i}, \underline{\Omega}_{a,i}),\ i = 2, \ldots, n \\
s_i &\sim IG(d_s \cdot \underline{s}_i, d_s),\ i = 2, \ldots, n \\
\phi &\sim IG(d_\phi \cdot \underline{\phi}, d_\phi) \\
\log \lambda_0 &\sim N(\underline{\mu}_{\lambda}, \underline{\Omega}_{\lambda})
\end{aligned} \tag{1}$$

- To obtain a Kroneker structure, we use a prior for Π conditional on $\tilde{A} = S^{-1/2}A$:

$$\underline{\Omega}_{\Pi} = (\tilde{A}'\tilde{A})^{-1} \otimes \underline{\Omega}_0 \tag{2}$$

- $\underline{\Omega}_0$ corresponds to the typical Minnesota-style prior variance

BVAR-CSV specification and implementation

Posterior distributions:

- Conditional posteriors with, in most cases, same forms as priors
- Metropolis-Gibbs algorithm
- Posterior for VAR coefficients: Define $\tilde{y}_t = \lambda_t^{-0.5} y_t$, $\tilde{X}_t = \lambda_t^{-0.5} X_t$.

$$
\begin{aligned}
\text{vec}(\Pi)|A, S, \phi, \Lambda, y &\sim N(\text{vec}(\bar{\mu}_\Pi), \bar{\Omega}_\Pi) \\
\bar{\mu}_\Pi &= \left(\tilde{X}'\tilde{X} + \underline{\Omega}_0^{-1}\right)^{-1} \left(\underline{\Omega}_0^{-1}\underline{\mu}_\Pi + \tilde{X}'\tilde{y}\right) \\
\bar{\Omega}_\Pi &= \left(\tilde{A}'\tilde{A}\right)^{-1} \otimes \left(\underline{\Omega}_0^{-1} + \tilde{X}'\tilde{X}\right)^{-1}
\end{aligned}
$$

BVAR-CSV specification and implementation

Treatment of volatility:

$$\tilde{v}_t = A(y_t - \Pi_0 - \Pi(L)y_{t-1})$$
$$w_t = n^{-1}\tilde{v}_t' S^{-1}\tilde{v}_t$$

- Conditional posterior due to Jacquier, et al. (1995):

$$f(\lambda_t|\lambda_{t-1}, \lambda_{t+1}, \ldots) \sim \lambda_t^{-1.5} \exp\left(\frac{-w_t}{2\lambda_t}\right) \exp\left(\frac{-(\log \lambda_t - \mu_t)}{2\sigma_c^2}\right)$$

- Estimation proceeds as in Cogley and Sargent (2005), with single process using w_t instead of n processes using $y_{i,t}^2$

BVAR-CSV specification and implementation

Prior settings:

- Π: prior means = 0; overall shrinkage of 0.2; st. dev's. from AR estimates
- A: uninformative
- S_i: mean from ratios of residual standard deviations; 3 degrees of freedom
- $\log \lambda_0$: mean from training sample error variances; variance = 4
- ϕ: mean = 0.035; 3 degrees of freedom

Other models

BVAR-SV:

$$\begin{aligned} y_t &= \Pi_0 + \Pi(L) y_{t-1} + v_t, \\ v_t &= A^{-1} \Lambda_t^{0.5} \epsilon_t,\ \epsilon_t \sim N(0, I_n),\ \Lambda_t = \text{diag}(\lambda_{1,t}, \dots, \lambda_{n,t}), \\ \log(\lambda_{i,t}) &= \log(\lambda_{i,t-1}) + \nu_{i,t},\ \nu_{i,t} \sim\ N(0, \phi_i),\ i = 1, n \end{aligned}$$

BVAR:

$$y_t = \Pi_0 + \Pi(L) y_{t-1} + v_t,\ v_t \sim N(0, \Sigma) \tag{3}$$

- Normal-diffuse prior and posterior, as in Kadiyala and Karlsson (1997)

Data and forecasting design

8 variables: GDP growth, PCE growth, BFI growth, employment growth, unemployment, GDP inflation, 10-year Treasury yield, and funds rate

- Real-time data series: GDP, PCE, BFI, employment, and GDP inflation
- Final vintage series: unemployment, bond yield and funds rate

4 variables: GDP growth, unemployment, GDP inflation, and funds rate

Data and forecasting design

Starting point of the model estimation sample is always 1965:Q1

Forecast horizons: 1Q, 2Q, 1Y, 2Y

Sample of forecasts: 1985-2010:Q4

Actuals in evaluating forecasts: 2nd available estimate in FRB Philadelphia RTDSM

- Romer and Romer (2000), Sims (2002), Croushore (2005), and Faust and Wright (2009) do the same
- Forecasters normally can't foresee large changes of annual or benchmark revisions.

Results

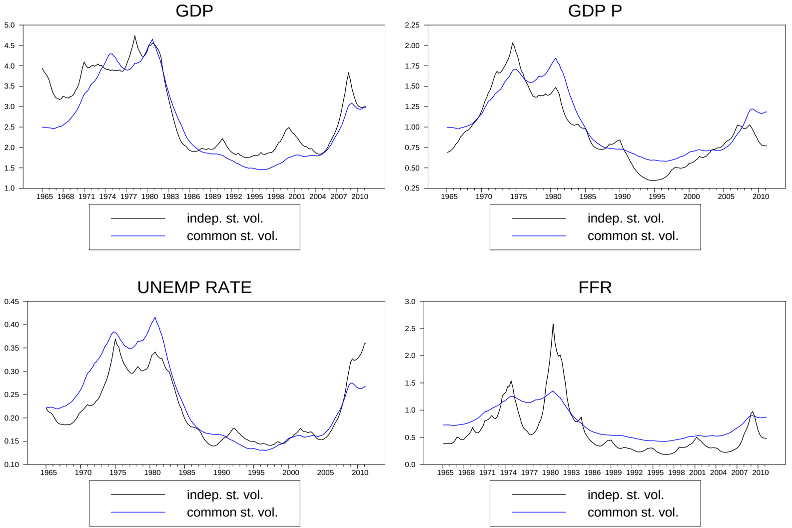
Table 2. CPU time requirements

model	CPU time (minutes)
4 variables, independent stochastic volatility	83.6
8 variables, independent stochastic volatility	879.5
4 variables, common stochastic volatility	16.4
8 variables, common stochastic volatility	46.9

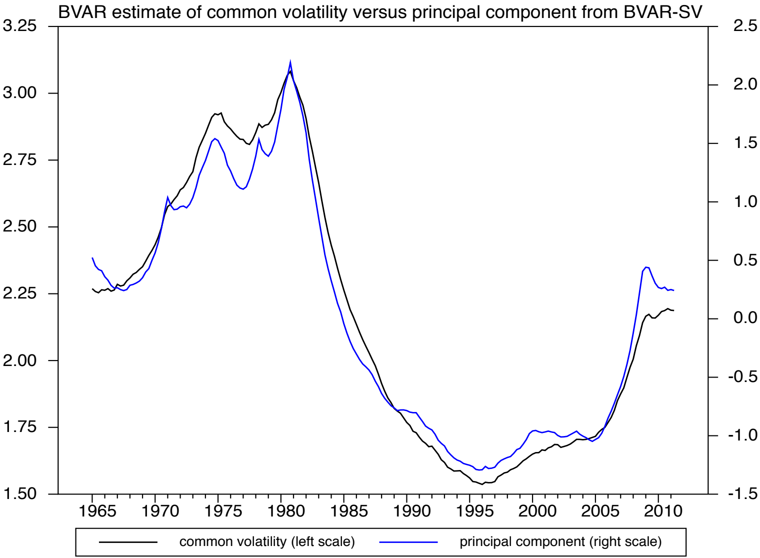
- models with 4 lags
- 105,000 draws

Results

Volatility estimates: indep. vs. common st. vol.



Results


BVAR estimate of common volatility versus principal component from BVAR-SV
3.25
3.00
2.75
2.50
2.25
2.00
1.75
1.50
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
common volatility (left scale)
principal component (right scale)

Results

Table 3. Log predictive likelihoods, 1980:Q1-2011:Q2

model	log PL
4 variables, constant volatility	-656.578
4 variables, independent stochastic volatility	-550.363
4 variables, common stochastic volatility	-569.269
8 variables, constant volatility	-1545.288
8 variables, common stochastic volatility	-1464.062

Results

Table 4. Real-Time Forecast RMSEs, 4-variable BVARs, 1985:Q1-2010:Q4

(RMSE ratios relative to const. vol. BVAR)

	$h = 1Q$	$h = 2Q$	$h = 1Y$	$h = 2Y$
BVAR with independent stochastic volatilities				
GDP growth	0.908 ***	0.908 ***	0.899 **	1.005
Unemployment	0.948 ***	0.932 **	0.929 *	0.975
GDP inflation	0.939 ***	0.913 ***	0.838 ***	0.791 ***
Fed funds rate	0.905 ***	0.936 *	0.953	0.945 *
BVAR with common stochastic volatility				
GDP growth	0.881 ***	0.881 ***	0.867 **	1.036
Unemployment	0.877 ***	0.868 **	0.882 *	0.960
GDP inflation	0.930 ***	0.875 ***	0.778 ***	0.725 ***
Fed funds rate	0.984	0.987	0.957	0.926 **

- Allowing independent stochastic volatilities lowers RMSEs
- Making volatility common lowers RMSEs a bit more

Results

Table 5. Real-Time Forecast RMSEs, 8-variable BVARs, 1985:Q1-2010:Q4

(RMSE ratios relative to const. vol. BVAR)

	$h = 1Q$	$h = 2Q$	$h = 1Y$	$h = 2Y$
BVAR with common stochastic volatility				
GDP growth	0.960 *	0.940 **	0.931 *	1.028
Consumption	0.964 **	0.971 *	0.942 *	1.038
BFI	0.991	0.993	1.000	1.013
Employment	0.867 ***	0.870 ***	0.872 **	0.957
Unemployment	0.931 **	0.921 *	0.923 *	0.968
GDP inflation	0.956 ***	0.904 ***	0.831 ***	0.766 ***
Treasury yield	0.991	1.032	1.031	0.979
Fed funds rate	1.002	1.028	0.993	0.960

- Larger BVAR more accurate than smaller (not shown)
- Adding common volatility lowers RMSEs

Table 6. Average log predictive scores, 4-variable BVARs, 1985:Q1-2010:Q4

(differences in scores vs. benchmark BVAR)

	$h = 1Q$	$h = 2Q$	$h = 1Y$	$h = 2Y$
BVAR with independent stochastic volatilities				
All variables	0.810 ***	0.690 **	0.633	-0.166
GDP growth	0.149 ***	0.080	-0.062	-0.180
Unemployment	0.187 ***	0.147	-0.098	-0.639
GDP inflation	0.089 ***	0.109 ***	0.186 ***	0.196 ***
Fed funds rate	0.504 ***	0.261 **	0.010	-0.101
BVAR with common stochastic volatility				
All variables	0.678 ***	0.739 ***	0.704 **	0.165
GDP growth	0.196 ***	0.132 *	-0.070	-0.173
Unemployment	0.230 ***	0.207 **	0.076	-0.314
GDP inflation	0.090 ***	0.124 ***	0.222 ***	0.266 ***
Fed funds rate	0.267 ***	0.191 ***	0.088	0.000

- Allowing independent st. vol. improves scores
- Making volatility common raises scores a bit more

Results

Table 7. Average log predictive scores, 8-variable BVARs, 1985:Q1-2010:Q4

(differences in scores vs. benchmark BVAR)

	$h = 1Q$	$h = 2Q$	$h = 1Y$	$h = 2Y$
BVAR with common stochastic volatility				
All variables	0.449 ***	0.368 **	-0.072	-0.590
GDP growth	0.100 **	0.074	-0.120	-0.118
Consumption	0.025	0.012	-0.035	-0.142
BFI	0.029	-0.034	-0.137	-0.190
Employment	0.162 ***	0.111 **	0.104	-0.107
Unemployment	0.115 ***	0.056	-0.111	-0.272
GDP inflation	0.032 *	0.064 ***	0.113 ***	0.158 ***
Treasury yield	0.044 ***	-0.006	-0.017	-0.022
Fed funds rate	0.113 ***	0.067 ***	0.018	-0.014

- Larger BVAR more accurate than smaller (not shown)
- Adding common volatility improves scores at shorter horizons

Conclusions

We develop a BVAR with a single, common stochastic volatility process that can be estimated relatively quickly

- Time-varying volatility driven by single multiplicative factor
- Prior takes a particular form that permits the essential Kroneker factorization

Findings:

- CSV much more efficient than independent st. vols.
- CSV captures most volatility movement and improves full-sample model fit
- CSV improves the accuracy of real-time point forecasts and density forecasts
  - Macro models with 4, 8, and 15 variables, in U.S. and U.K. data
  - CSV accuracy comparable to independent SV accuracy