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T2S NTS Algorithms Objectives

V 1.1

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TARGET2-SECURITIES

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1 Introduction

2 The T2S Night-time Settlement (NTS) Optimisation Algorithms Objectives document is part of the T2S scope
3 defining document, which aims at describing how settlement optimisation requirements of T2S URD are
4 implemented on the T2S platform into the NTS optimisation algorithms.

5 While the GFS presents the solution foreseen for T2S from a general perspective, this document provides a
6 specific view of the T2S settlement engine focused on settlement optimisation strategies and objectives
7 during NTS.

8 The NTS Optimisation Algorithms Objectives document aims at providing information on:

- 9 • Optimisation features (technical netting, prioritisation, partial settlement, auto-collateralisation
10 and optimisation algorithms);
 - 11 • Optimisation strategies implemented during night-time settlement;
 - 12 • NTS optimisation algorithms objectives;
 - 13 • Efficiency evaluation.
- 14

15 T2S NTS Algorithms Objectives Structure

16 The T2S NTS Algorithm Objectives document is a self-contained document, structured along two chapters.

17 **Chapter 1: Optimisation features of T2S**

18 The first chapter provides concise and pedagogic information on the T2S optimisation behaviour from a T2S
19 actor's perspective. After a short explanation of optimisation concepts (section 1.1), this chapter describes
20 T2S optimisation features (section 1.2) and provides a description of the common framework and the
21 optimisation strategy for night-time settlement (section 1.3). Finally, an overview of the settlement
22 optimisation algorithms families used during night-time is provided in section 1.4.

23 **Chapter 2: Optimisation and evaluation of settlement efficiency**

24 The second chapter provides a definition of the efficiency (section 2.1). It then describes settlement
25 optimisation's objectives (section 2.2) and proposes key indicators (metrics) to evaluate the settlement
26 optimisation strategies and algorithms efficiency (section 2.3).

27

1 Optimisation Features of T2S

1.1 What is Optimisation?

In mathematics and computer science, optimisation refers to the process of choosing the “best” element from a set of applicable solutions to a problem. In that respect, a problem is defined by a set of input data, one or more objectives usually translated into objective functions to be minimised or maximised, subject to a set of so-called constraints.

For a security settlement system, optimisation is the choice of a subset of transactions (i.e. settlement instructions, settlement restrictions or liquidity transfers eligible to settlement) which can be settled all together (all settleable subset are called a solution). The problem is defined by the transactions and available resources (cash or securities). The constraints are that neither security position nor cash balance can be negative and the objectives are to maximize the settlement ratio (in volume and/or in value).

Concretely speaking, as far as settlement optimisation is concerned, a problem can be illustrated via the following example:

- A list of transactions (DvP) between 3 participants owning a Securities Account (SA) and a T2S Dedicated Cash Account (DCA); the relevant transactions are detailed in the table below;
- The resources available on each account are also represented in the table below.

The constraint is that no position or balance can be negative.

The objective is to maximise the number of settled transactions and/or settled amount.

	SA1			SA2			SA3			DCA1	DCA2	DCA3
	P11	P12	P13	P21	P22	P23	P31	P32	P33			
Trx01	100			-100						-100000	100000	
Trx02		50						-50		-120000		120000
Trx03			-80			80				40000	-40000	
Trx04		-60			60					145000	-145000	
Trx05					-60			60			150000	-150000
Trx06			90						-90	-50000		50000
Trx07				80			-80				-90000	90000
Trx08	40						-40			-50000		50000
Trx09						-80			80		45000	-45000
Trx10			-25			25				15000	-15000	
Trx11						25			-25		-20000	20000
Trx12			30						-30	-20000		20000
Available	0	10	30	0	0	0	90	0	0	100000	95000	100000

The settlement attempt of all pending transactions is not a valid solution because there are not enough resources to settle all transactions. In this case, the provision check process shows four lacks:

PCK	140	0	45	-20	0	50	-30	10	-65	-40000	80000	255000
-----	-----	---	----	-----	---	----	-----	----	-----	--------	-------	--------

1 There are several solutions to this problem and the choice of the most relevant solution depends on the
2 optimisation objectives retained. Here are two applicable solutions:

- 3 • trx02, trx04, trx05, trx08 and trx10;
- 4 • trx02, trx04, trx05 and trx07.

5 The first one has a settlement ratio in volume equal to 42% and 57% in value. The second one has a
6 settlement ratio in volume equal to 33% and 60% in value. Without additional criteria, i.e. an optimisation
7 objective, it is not possible to determine which solution is better.

8 However, if it is agreed that the optimum balance between value and volume is defined by the mean of both
9 ratios, the first solution has a balanced ratio of 49% and the second of 47% and hence, the best solution is
10 unambiguously the first one.

11 In other words, this simple example highlights the need for precise optimisation objectives and the
12 importance of sharing a common view on T2S settlement optimisation process.

13 1.2 T2S Optimisation Features Overview

14 The general objective of T2S settlement, optimisation and recycling procedures is to maximise the volume
15 and value of settlement with the available securities and cash resources (T2S.08.010). For that purpose, T2S
16 shall employ three optimisation features (T2S.08.040, T2S.08.050):

- 17 • Technical netting;
- 18 • Partial settlement;
- 19 • Auto-collateralisation.

20 These three features are used by optimisation algorithms to create settleable sets of transactions.

21 1.2.1 Technical netting

22 In order to limit the resources needed for settlement, T2S may group transactions into a set and apply
23 technical netting on the set by calculating the net quantities and amounts to be settled on an all-or-none
24 basis.

25 These net quantities and amounts are the basis for the checks against the available resources and if needed
26 for the assessment of intraday credit to be provided.

27 This feature is useful to reduce resources needed and is the only way to solve gridlocks. It is more efficient if
28 settlement activity is concentrated on a limited number of accounts.

29 1.2.2 Partial settlement

30 T2S has a partial settlement capability (i.e. settling only a fraction of the original transaction) when full
31 settlement is not possible due to lack of resources. However, according to URD, different rules apply
32 depending on the type of transactions. More particularly, settlement restrictions and liquidity transfers may
33 be partially settled in case of lack of resources without any specific condition, whereas settlement
34 instructions can only be partially settled when there are insufficient securities to settle the full quantity and
35 provided the following conditions are met:

1 • The partial settlement window is currently running in real-time settlement period or the relevant
2 sequence (i.e. sequence X) is currently running in night-time settlement period (see below for
3 details on night-time settlement sequences);

4 • The Settlement Instructions are eligible for partial settlement (depending on T2S actors' choice).

5 This partial settlement feature is one of the most efficient to resolve insufficient resources without lowering
6 dramatically the settlement ratios. Currently, any limitation on the usage of the partial settlement facility
7 (restriction to lack of securities, high level of threshold) forbids reaching theoretical optimum.

8 **1.2.3 Auto-collateralisation**

9 T2S offers central banks and settlement banks with the capacity to provide intraday credit to credit
10 consumers via an automated process secured with eligible collateral. This credit aims at covering lacks of
11 cash on T2S dedicated cash accounts or insufficient external guarantee headroom on credit memorandum
12 balances. In addition, the auto-collateralisation process allows automatic release of collateral and automatic
13 substitution to cover lack of securities.

14 T2S automatically provides intraday credit through auto-collateralisation when necessary to settle settlement
15 instructions during the night-time and real-time periods through:

16 • The provision of additional cash on a T2S dedicated cash account for the central bank
17 collateralisation;

18 • The use of the client collateralisation headroom when the external guarantee limit is exhausted.

19 The auto-collateralisation is an important feature of T2S optimisation processes as it allows participants to
20 reduce their initial cash balance and limits the risk of failed settlement due to lack of cash.

21 **1.3 Optimisation strategies and framework**

22 **1.3.1 General approach**

23 T2S provides two types of settlement:

24 • Real time settlement;

25 • Batch settlement scheduled by cycles and sequences during the night.

26 T2S uses a common optimisation framework for both periods and resorts to first technical netting and after,
27 if necessary, to intraday credit for lacks of cash/external guarantee and partial settlement, if applicable, for
28 lacks of securities.

29 This common framework is events based. A settlement optimisation series is associated to each event and
30 each time such an event happens the related series is triggered.

31 A settlement optimisation series is an ordered list of algorithms and a set of optimisation features available
32 to those algorithms. This framework allows the definition of a list of series specialised by event type,
33 transaction type, volumes, etc.

34 During night-time settlement, a series is associated to each sequence.

1 1.3.2 Night-time settlement functional design

2 During the night-time settlement period, the T2S settlement is split into cycles and sequences defining the
3 settlement process perimeter and scheduling:

- 4 • Sequence 0 : Liquidity transfers and cash settlement restrictions;
- 5 • Sequence 1 : Settlement instructions related to corporate actions;
- 6 • Sequence 2 : Free of Payment (FOP) Settlement instructions for rebalancing purpose;
- 7 • Sequence 3 : Settlement instructions related to Central Bank Operations (CBO);
- 8 • Sequence 4: All type of settlement instructions, settlement restrictions and liquidity transfers;
- 9 • Sequence X: Sequence 4 with partial settlement available;
- 10 • Sequence Y: Liquidity transfers related to reimbursement of the “multiple liquidity providers”;
- 11 • Sequence Z: All liquidity transfers.

12 Sequence 1 to X accept also previous sequence perimeter.

13 Current scheduling of night-time is:

- 14 • Cycle 1: Sequence 0, Sequence 1, Sequence 2, Sequence 3 and Sequence 4;
- 15 • Last cycle: Sequence 4, Sequence X, Sequence Y and Sequence Z.

16 At the start of each sequence, T2S selects all pending eligible transactions and launches the series of
17 settlement optimisation algorithms associated to the current sequence. Each algorithm tries to find the
18 “best” (i.e. with the highest value of its objective function) set of transactions based of on its own
19 optimisation strategy and is executed in sequence to improve the previous settlement ratio the sequence in
20 a predefined time constraint.

1 1.4 NTS Optimisation algorithms

2 Optimisation algorithms identify and select pending transactions that are settleable when they are jointly
3 submitted to a settlement attempt.

4 They resort to intraday credit if necessary to fill a lack of cash or insufficient external guarantee and
5 eventually, if applicable, to partial settlement.

6 NTS optimisation algorithms are basically composed by three steps:

7 - A building step: mathematical optimisation algorithms helps to select rapidly a subset of Settlement
8 Transactions which may be part of a good solution;

9 - An improvement step: algorithms based on local search theory aim to enhance the previous
10 solution;

11 - A reparation step: ad hoc algorithm applies all T2S Business Rules to transform previous
12 "temporary" solution to a final solution (i.e. a settleable collection of Settlement Transactions).

13 Each step of the optimisation algorithms is guided by its own implementation of the optimisation objective
14 function to find the best reachable solution.

1 **2 Optimisation and evaluation of settlement efficiency**

2 **2.1 Definitions**

3 Efficiency: Efficiency is defined as the ratio between the actual output and the objective that the system is
4 designed to reach. For T2S, efficiency is defined as the ratio between the number/value of settled
5 transactions and the total number/value of transactions to be settled.

6 A more accurate evaluation may compare the results with a theoretical “optimum” solution. As this optimum
7 cannot be calculated for a real T2S NTS optimisation problem, the comparison is done with a calculated
8 optimum on a “relaxed” NTS problem.

9 Section 2.3 below provides some indicators to evaluate efficiency.

1 2.2 Settlement optimisation objectives

2 T2S Settlement optimisation process may find different solutions to the same problem. It therefore needs
3 other criteria to choose the “best” solution

4 URD provides rules whose have to be translated into settlement optimisation objectives (**T2S.08.010,**
5 **T2S.08.070, T2S.08.040, T2S.08.050 and T2S.08.060**):

- 6 • T2S settlement optimisation process shall favour the settlement of transactions with a higher
7 level of priority.
- 8 • When several transactions with the same level of priority compete for settlement, T2S
9 settlement optimisation process shall favour the settlement of transactions with the oldest
10 intended settlement date (ISD).
- 11 • When several pending transactions with the same level of priority and the same intended
12 settlement date compete for settlement, T2S settlement optimisation process shall favour the
13 settlement of transactions in a way that maximises the volume and value of settlement (in an
14 optimum balance).
- 15 • T2S settlement optimisation process shall use resources for oldest transactions first in order to
16 reduce the time during which a transaction remains unsettled beyond the intended settlement
17 date.
- 18 • During the night-time, T2S settlement optimisation process shall minimise the number and value
19 of auto-collateralisation operations.
- 20 • T2S settlement optimisation process shall minimise the number of transactions submitted to
21 partial settlement.

2.3 Metrics

The global settlement efficiency of the T2S platform is monitored by different indicators: Platform Settlement Efficiency Indicator (PSEI) and Market Settlement Efficiency Indicator (MSEI)¹.

The efficiency of NTS optimisation algorithms is difficult to evaluate due to multiple external parameters impacting efficiency (e.g. priority management, resources' scarcity), elapsed time (e.g. volumes). The efficiency indicators defined into this document only give information to compare different optimisation strategies and algorithms on the same problem.

Efficiency

In order to measure the efficiency based on the optimisation objectives, the basic indicators are the settlement ratio both in volume and value and they are detailed by priority, age (transaction age is defined below) and currency for the ratio in value.

Parameters p , a , c define respectively the priority, the age and the currency of transaction (t).

As a shortcut, we denote "transactions(p,a)" as the set of transactions of age " a " and priority " p ".

The priority " p " may take the following values, presented from the most important to the less one:

- 1: Reserved,
- 2: Top,
- 3: High,
- 4: Normal.

The age of a transaction is defined by the number of days between its ISD and the current business date.

Volume Ratio:

$$R_{Vol}(p, a) = \frac{1}{\text{card}(\text{transactions}(p, a))} \sum_{t \in \text{transactions}(p, a)} \frac{\text{Settled quantity}(t)}{\text{Quantity to be settled}(t)}$$

The volume ratio is calculated by priority, age and for each transaction is equal to the ratio between settled quantity and quantity to be settled (remaining quantity).

For example, three transactions with same priority (p) and age (a) are eligible to settlement:

- Trx1: Qty=100, Amount=10,000;
- Trx2: Qty=200, Amount=15,000;
- Trx3: Qty=200, Amount=10,000.

¹ The PSEI is proposed to assist in determining whether T2S is performing as expected from a platform perspective when considering: all transactions to be settled (Business Settlement Instructions, T2S Generated Settlement Instructions, Liquidity Transfers), all the technical means used to do so (settlement algorithms, system capacity), and the inability to settle linked to the lack of resources.

The MSEI is proposed for a market oriented approach, determining whether business objectives are being met and to assist in the setting of realistic targets for settlement efficiency (only business settlement instructions but no generated ones, considering on top of the lack of resources the inability to settle due to Party on Hold).

1 T2S proposed, during partial window, the solution:

- 2 • Trx1 settled;
- 3 • Trx2 unsettled;
- 4 • Trx3 partially settled (partial settled quantity=150, partial settled amount=7,500).

5 The volume ratio is equalled to:

- 6 • $R_{vol}(p, a) = 1/3 * (100/100 + 0/200 + 150/200) = 0.58$

7 **Value ratio:**

8 The value ratio is calculated using a weighted average by currency. First a ratio per currency is computed for
9 each couple priority, age:

$$R_{Val}(p, a, c) = \frac{\sum_{t \in transactions(p,a,c)} Settled\ Amount(t)}{\sum_{t \in transactions(p,a,c)} Amount\ to\ be\ settled(t)}$$

10

11 Then a global ratio is computed from the first one by priority and age. It is a weighted mean by currency
12 (i.e. the weight of one currency ratio is weighted by the number of transactions in this currency).

$$R_{Val}(p, a) = \frac{1}{card(transactions(p, a))} \sum_c card(transactions(p, a, c)) \times R_{val}(p, a, c)$$

13

14 For the previous example and considering all transactions are settled into the same currency (€), the value
15 ratio is calculated by:

- 16 • $R_{val}(p, a, \text{€}) = (10,000+0+7,500) / (10,000+15,000+20,000) = 0.39$

17

18 **Balanced ratio indicator (BRI):**

19 URD said that T2S settlement optimisation process shall favour the settlement of transactions in a way that
20 maximises the volume and value of settlement (in an optimum balance) for the same priority and ISD.

21 This requirement implies a multi-objective optimisation and increases significantly the complexity of the
22 optimisation algorithms. The implementation of this multi-objective has been done via the introduction of the
23 Balance Ratio Indicator (BRI).

24 The balance between volume and value is provided by the tuning the parameter $\lambda \in]0,1[$. The higher is λ ,
25 the heavier is the volume ratio in BRI.

$$BRI_{\lambda}(p, a) = \lambda R_{Vol}(p, a) + (1 - \lambda) R_{Val}(p, a)$$

26

27 If a same problem has two solutions (S1, S2) with $R_{vol}(S1)=0.75$, $R_{val}(S1)=0.55$, $R_{vol}(S2)=0.73$ and a
28 $R_{val}(S2)=0.57$, the BRI_{λ} may be:

λ	0.1	0.3	0.5	0.7	0.9
BRI (S1)	0.57	0.61	0.65	0.69	0.73

BRI (S2)	0.59	0.62	0.65	0.68	0.71
----------	------	------	------	------	------

1 With λ lower than 0.5, S2 is better but with λ greater than 0.5, it's the opposite. In this problem, with λ
 2 equal to 0.5, both solutions obtain the same BRI.

3 In T2S, since the go-live, the lambda value is set to 0.5.

4

5 As the URD require the optimisation to favour high priority and the oldest transactions these indicators must
 6 be computed with adequate breakdowns and shall be compared in a Lexicographical order.

7 Due to technical issue and time constraint, this lexicographical order has been translated into a linear
 8 objective function with different weights:

$$BRI_{\lambda} = \frac{1}{\sum_{p,a} Weight(p, a)} \sum_{p,a} Weight(p, a) \times BRI_{\lambda}(p, a)$$

9

10 With lambda set to 0.5, the $BRI_{0.5}$ for the previous example can be calculated:

- 11 • $BRI_{0.5}(p, a) = 0.5 \cdot R_{vol}(p, a) + (1-0.5) \cdot R_{val}(p, a) = 0.5 \cdot 0.58 + 0.5 \cdot 0.39 = 0.49$

12 The weight function proposed below offers the possibility to calculate a consolidated balance ratio indicator
 13 (BRI) but an analysis of each pair' (priority, ISD) BRI yields much more information.

14

15 To limit the risk of numerical instability and provide a good trade-off between duration and quality of the
 16 solution, the age of a Settlement Transaction is capped by 3 (i.e. $Age(t) = \text{Min}(BD - ISD(t), 3)$).

$$Weight(p, a) = \frac{1}{10^{2(4p-a-1)}}$$

17

18 The real weight for each pair priority and age is shown into
 19 the table below, which illustrates how T2S takes into account both the priority of the transactions (in
 20 decreasing order, reserved, top, high and normal priority) and their age (i.e. in decreasing order from most
 21 ancient transactions, i.e. transactions with an ISD equal or older to current business day – 3 days, to the
 most recent transactions, i.e. transactions with an ISD equal to current business day).

		Priority			
		Reserved	Top	High	Normal
A G E	0	0,000001	1E-14	1E-22	1E-30
	1	0,0001	1E-12	1E-20	1E-28
	2	0,01	1E-10	1E-18	1E-26
	3	1	0,00000001	1E-16	1E-24

22 Example:

23 A problem contains 100 transactions with a Normal priority and just 1 with a High priority. T2S finds 2
 24 solutions: the first one (S1) settles all Normal priority transactions; the second (S2) settles only the High
 25 priority transaction. In such a case, T2S has to choose the second one.

26 For the first solution $BRI_{S1}(\text{"High"},0)=0$ and $BRI_{S1}(\text{"Normal"},0)=1$, for the second $BRI_{S2}(\text{"High"},0)=1$ and
 27 $BRI_{S2}(\text{"Normal"},0)=0$.

1 $BRI_{S1} = (10^{-22} * BRI_{S1}(\text{"High"}, 0) + 10^{-30} * BRI_{S1}(\text{"Normal"}, 0)) / (1 + 10^{-22} + 10^{-30}) = 9.9 * 10^{-31}$

2 $BRI_{S2} = (10^{-22} * BRI_{S2}(\text{"High"}, 0) + 10^{-30} * BRI_{S2}(\text{"Normal"}, 0)) / (10^{-22} + 10^{-30}) = 9.9 * 10^{-23}$

3 The S2 solution is the "best" solution, in line with the priority management as described in the URD ($BRI_{S2} >$
4 BRI_{S1}) but this solution settles only 1 transaction out of 101.

5

6 **Integrity gap (IG):**

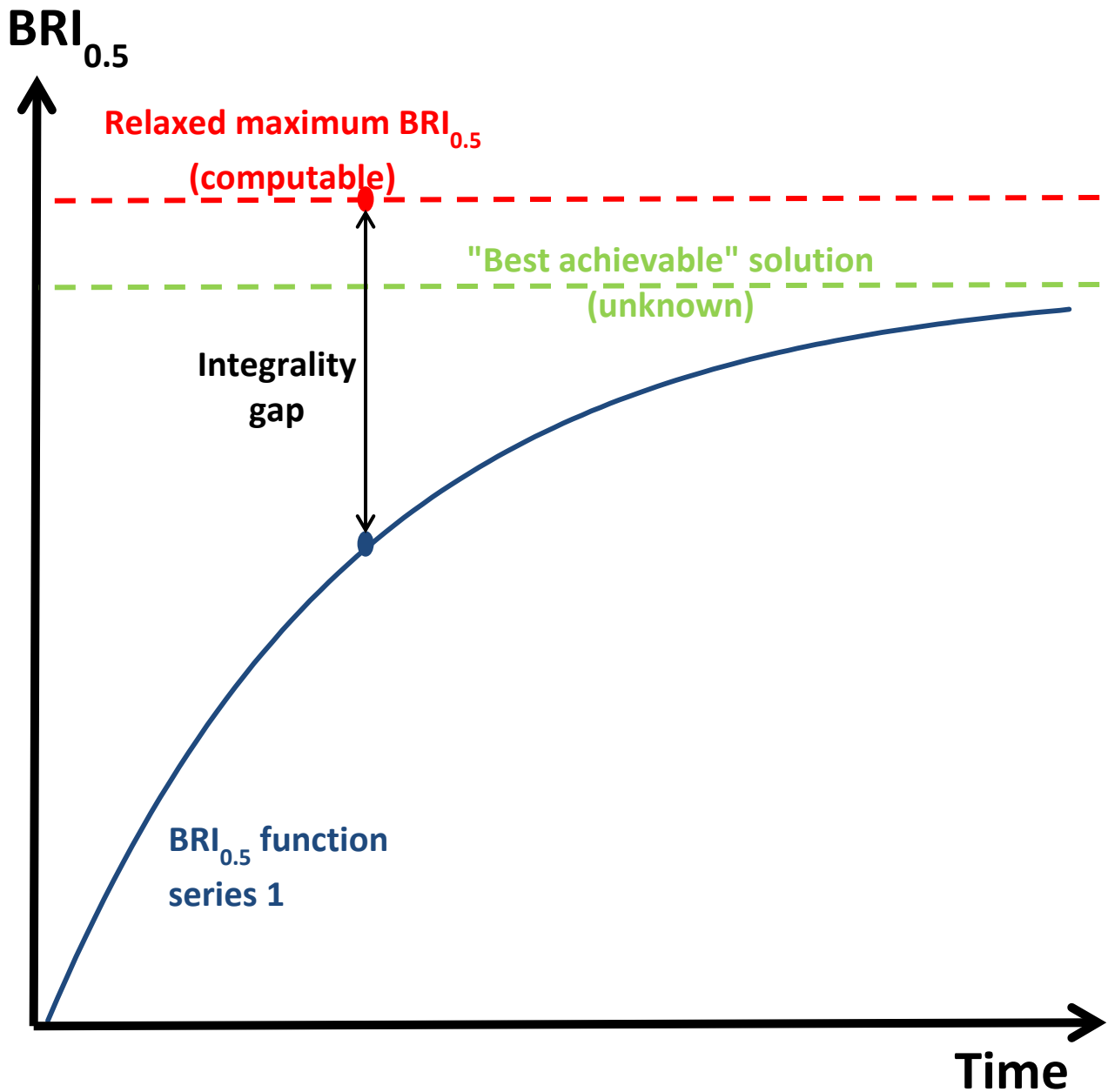
7 To evaluate and compare NTS optimisation algorithms, the 4CB determines first a calculated upper bound of
8 $BRI_{0.5}$ on a relaxed problem called "relaxed maximum". The relaxed maximum is the indicator value of a
9 solution computed without all constraints (e.. all transactions are eligible to partial settlement, links
10 constraints are deleted, etc.). This "relaxation" of the problem allows finding rapidly a solution which is
11 always better than any real solution. This relaxed solution can then be used to benchmark the possible
12 solutions found by the algorithms.

13 The indicator Integrity Gap (IG) is defined as the ratio between the T2S solution $BRI_{0.5}$ and relaxed one.

14 For example, if the "relaxed maximum" $BRI_{0.5}$ is equal to 0.93 and the $BRI_{0.5}$ of a solution found by T2S is
15 equal to 0.67, then the IG is equal to 0.72.

1

FIGURE 1 - INTEGRALITY GAP



2

3 A good efficiency indicator should be the function $IG(t)$ and its value for some identified values: $IG(15mn)$,
4 $IG(30mn)$ $IG(1h)$ and $IG(2h)$.

5 The experimental values hereafter show that the time constraint has a real impact to compare two series. If
6 the sequence has to stop the process after one hour, then the first series is the best. If the process can run
7 for one hour or longer, then the second series gives better results.

Series	$IG(15mn)$	$IG(30mn)$	$IG(1h)$	$IG(2h)$
T2S series 1	0.58	0.67	0.70	0.75
T2S series 2	0.55	0.65	0.75	0.80

1 These efficiency indicators that have been calculated for different series and data during experimentation
2 have helped the 4CB to choose optimisation series with regards to time available and the number of
3 transactions to be processed. These indicators are continuously monitored against real data.

4 **2.3.1 Volumes**

5 All things being equal, in night-time settlement, the number of transactions unique' influence is the time to
6 process the data. Even though the number of transactions should not be taken as the only factor explaining
7 the time taken to perform night-time settlement; there is nonetheless a tight relationship between those two
8 elements. Unfortunately, this relationship is not linear and fortunately enough it is not exponential.

9 Should the number of transactions with lacks be very low, the relationship between time and volumes would
10 be close to linear. On the other hand, a high number of lacks would result in a rapid increase of the time
11 needed for settlement optimisation (in the worse theoretical situation, the relationship between volume and
12 time is cubic).

13 As always, there is a trade-off to be made between the "quality" of the optimisation and the time taken to
14 perform it. We have several algorithms, each one having a different relationship to volumes (and opposite
15 relation to "quality") and we can switch from one to another if need be.

16 As we have limited time to process each sequence during night-time, we will need to reduce drastically the
17 "quality" of the optimisation when a peak volume is announced. The participant will have to alert when they
18 know they will send abnormal volume. The settlement ratio both in volume and value will be lowered during
19 a peak volume day.