The Gains from Resolving Debt Overhang: Evidence from a Structural Estimation*

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Abstract: What are the gains from resolving debt overhang for firm growth and aggregate welfare? To address this question, we develop a general equilibrium model where heterogeneous firms innovate to grow, can potentially default on their debt obligations, and can suffer from debt overhang. We estimate the model with data on U.S. nonfinancial public firms using indirect inference and obtain bounds on the extent to which debt overhang affects firm growth that are consistent with existing estimates in the corporate finance literature. We find that while the private gains to a firm from resolving debt overhang can be large if it faces sufficient default risk, the expected gains to the average entering firm are relatively modest. The social gains to long-run consumption and output from resolving debt overhang are smaller, as an endogenous rise in the real cost of innovation and the aggregate bankruptcy rate act as dampening forces. However, our model suggests the gains from resolving debt overhang over the business cycle can be large, as firm default risk rises significantly during the recession, which implies a significant decrease in innovation and subsequent firm growth.

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1 Introduction

Myers (1977) first laid out the debt overhang problem; put simply, existing debt can lead equity holders to underinvest, since part of the expected cash flow generated by the investment goes to debt holders, while equity holders bear its costs. Debt overhang has been empirically found to affect firm investment decisions and growth in the corporate finance literature.[1] In this paper, we ask: What are the gains from resolving debt overhang for nonfinancial firms, both for individual firms and for the aggregate economy in the long run?

To perform our analysis, we develop a general equilibrium model of firm dynamics where firms make endogenous entry and leverage decisions; can potentially default on their debt obligations; are heterogeneous in their investment opportunities; and endogenously innovate but can suffer from debt overhang when making innovation decisions. We estimate key model parameters using data on U.S. nonfinancial public firms, including the parameter that governs the extent to which debt overhang affects firm innovation decisions. We find bounds on the extent to which debt overhang affects firms that are consistent with existing estimates in the corporate finance literature. We find the expected private gains upon entry from resolving debt overhang are modest. The expected private gains to a single firm in the economy are larger than the long-run welfare gains from resolving this problem for all firms, as when all firms resolve debt overhang, an endogenous rise in the real cost of innovation and in the bankruptcy rate act to dampen the gains from resolving this problem. However, we find the private gains from resolving debt overhang are nonlinear and rise substantially for firms near default. When the distribution of firm default risk changes to the extent observed during the recent recession, our model implies significant year-ahead employment losses due to debt overhang, and, hence, the expected private gains from resolving debt overhang rise substantially.

To answer our question, we develop a general equilibrium model of firm dynamics. In our model, a firm is a monopoly producer of a differentiated product. The firm earns quasi-rents due to a constant markup of its price over marginal cost. Incumbent firms have an investment technology through which they can invest resources to lower their marginal cost of production and, hence, expand profits by expanding sales. We refer to this investment as *process innovation*. These incumbent firms differ in the productivity of this technology for investing to reduce marginal costs: for some firms it is cheap to invest to lower marginal cost and thus grow sales and profits, for others it is expensive to do so. Hence, firms differ in their *investment opportunities*.

At any time, new firms can pay a fixed cost to enter with a new differentiated product; we refer to this mechanism as *product innovation*. After entering, an intermediate good firm realizes its investment opportunities and its initial level of productivity. The firm then makes an initial debt decision in the face of a classical trade-off to maximize the joint value of equity holders and new creditors. Firms take out debt because it has a tax advantage, but do not fully finance themselves with debt because it can lead to costly bankruptcy. Firms have rational expectations, so they will take out less debt if they know

[1]See work by Lang et al. (1996), Giroud et al. (2012), Hennessy (2004), and Hennessy et al. (2007) for examples. Additionally, financial constraints more generally have been found to affect firm employment decisions; see work by Benmelech et al. (2013) and Chodorow-Reich (2014) for examples.

they will suffer more from debt overhang. The debt is perpetuity debt; it pays a fixed coupon every period, and there is no principal due. Conditional on its productivity, the firm makes production decisions using a constant returns-to-scale technology in labor. Each period, the firm enters with a productivity level, investment opportunities, and a coupon level. It has some probability of exiting exogenously. If it survives, equity holders *alone* decide whether or not to go bankrupt and then make the investment decision. Firm productivity follows a binomial process, and when firms invest, they invest in the drift of this process. The volatility of firm innovation represents firm business risk. If the firm does go bankrupt, creditors seize the firm (in effect, gaining full equity stake), the firm loses a fixed proportion of its productivity, and the firm makes a new leverage decision. Again, this decision is made to maximize the joint value of equity holders and creditors.

A competitive final good sector aggregates the differentiated products into a final good using a constant elasticity of substitution production function. Households derive utility from consumption of the final good, are risk-averse, and inelastically supply labor. All production and innovation requires labor. Households also hold the equity and debt of intermediate good firms and receive the dividend and coupon payments. We solve for a stationary competitive equilibrium of our model where all aggregates are in steady state.[2]

We use an indirect inference procedure to locally identify the parameters that govern the mechanisms in our model. The data moments we use to estimate the model come from annual data on an unbalanced panel of U.S. nonfinancial public firms from 1982 to 2012 and a balanced panel of manufacturing firms from 1992 to 1995. We rely on two measures, the first of which is employment growth. Across specifications of our estimation procedures, we control for different correlates of firm growth. The second measure we use is a measure of firm risk-adjusted leverage, distance-to-default, which is measured as the inverse of the firm's asset volatility times the natural logarithm of the value of a firm's assets relative to the book value of its debt. Distance-to-default is measured in units of the number of standard deviations of annual asset volatility by which the firm's assets must change to equal the firm's book value of debt. A value of 1 implies the firm is one standard deviation from its book value of debt exceeding its assets, 2 implies the firm is two standard deviations from its book value of debt exceeding its assets, and so on.

The moment conditions we specify relate to properties of firm growth and the significant relationship between firm distance-to-default and year-ahead growth we find in the data. Figure 1a shows that the average employment growth of U.S. public, nonfinancial firms close to default is almost 7% slower than firms further from default after controlling for year and industry fixed effects, and the relationship is nonlinear. In Figure 1b, we show this same relationship exists for sales and capital growth as well. The relationship is as strong for sales and stronger for capital. In Figure 1c, we show that even after controlling for year effects, industry effects, size, age, and access to external finance, this relationship exists, is still nonlinear, and only weakens slightly. Motivated by this finding, our estimation procedure uses moments that capture properties of the nonlinear relationship between firm growth and firm distance-to-default. In our preferred estimation procedure,

[2]Our model is in the spirit of the knowledge capital model of firm productivity of Griliches (1979); in particular, our model takes a one-country version of Atkeson and Burstein (2010) and embeds heterogeneity in investment opportunities and endogenous leverage decisions (with the potential for costly default) in such a general equilibrium model of firm dynamics with *process* and *product* innovation.

we derive an upper bound for the extent to which debt overhang affects firm growth by not controlling for unobserved heterogeneity at the firm level. We derive a reasonable lower bound in the context of our procedure by demeaning growth at the firm level but not firm distance-to-default (thus, leaving no room for debt overhang in explaining why a firm close to default has a potentially low average growth rate).

We estimate our model taking the distribution of distance-to-default as given from the data. Given the firm's investment opportunities, all moments we use to estimate our model are a function of how a firm will innovate conditional on its distance-to-default, which makes such an approach feasible. In turn, we only need to solve the problem of equity holders in order to estimate our model. Hence, although we specify assumptions on the general equilibrium environment and debt contract, it is only how these functional form assumptions enter the problem of equity holders that affect the identification of parameters in our model. An advantage of just focusing on the problem of equity holders in our model is that it has a tight closed-form approximation. We are thus able to demonstrate, with our closed-form approximate solution, how our moment conditions allow us to locally identify model parameters. With our estimated model, we then demonstrate that the bounds on our estimates imply that our range of estimates is consistent with a quasi-natural experiment in the literature, Giroud et al. (2012), and an important structural paper that relies on Q-theory, Hennessy (2004).

Although the distribution of distance-to-default is taken as given for our estimation procedure, the distribution is treated as endogenous for our welfare and firm value counterfactuals.[3] Firm entry and firm leverage decisions respond to the extent to which debt overhang affects firm investment decisions near default, which in turn affect aggregate welfare. The endogenous distribution of distance-to-default in our estimated model has distributional moments close to those from the data.[4]

We take as given the debt overhang problem can be resolved. We then assess the gains from resolving debt overhang were the firm as a whole, rather than equity holders alone, to make the process innovation decision. Although our estimated model implies expected firm value upon entry will only increase modestly for a firm resolving debt overhang, the firm will gain nonlinearly as it gets closer to default. In our estimated model, the shape of the distribution of distance-to-default at any given time can play a significant role in generating the expected future growth rate of firms. When the distribution of distance-to-default changes to the extent observed in the recent recession, absent compensating general equilibrium forces such as the real cost of innovation rising, the gains from resolving this problem are between about $1.5 - 3\%$ of annualized employment growth. In our general equilibrium exercise, the long-run welfare gains are more modest and are

[3]We do take the distribution of distance-to-default as given for our counterfactuals that assess the gains from resolving debt overhang in terms of expected year-ahead growth in the cross-section and over time.

[4]The distribution of distance-to-default when we estimate the model on our full sample of nonfinancial firms and control for firm heterogeneity in investment opportunities and other firm characteristics (version (6) of our estimation procedure), for example, has mean, standard deviation, skewness, and kurtosis of 5.24, 2.47, 0.48, and 2.45, respectively. The same moments in the data are 4.61, 2.82, 0.41, and 2.23, respectively. Although the estimated moments are similar to those in the data, by taking the distribution as given for our estimation procedure, there are massive computational gains.

dampened by the cost of innovation rising and a higher aggregate bankruptcy rate.

Related Literature Although debt overhang is a well-known problem, this paper is – to our knowledge – the first to study it through the lens of a classical model of innovation that can match many of the key features of the firm size distribution and firm growth as described by Luttmer (2007).[5]

The literature on debt overhang has continued to develop the theory since Myers (1977), examining the different margins through which this problem can affect firm investment; Diamond and He (2013) and Phillippon and Schnabl (2013) provide two important recent examples of such theoretical work. Diamond and He (2013) demonstrate how firm debt maturity interacts with the debt overhang problem, while Phillippon and Schnabl (2013) study a financial sector that suffers from debt overhang and ask under what conditions and how a government should engage in resolving debt overhang for that sector to improve welfare. Some studies, through the lens of Q-theory, demonstrate this problem exists for firms in the data; two important examples of this approach include the work of Hennessy (2004) and Hennessy et al. (2007). Our work is consistent with such work in that our estimates suggest debt overhang affects firm investment decisions. Moyen (2007) calibrates a simple model of the firm and studies the gains from resolving this problem for a firm in partial equilibrium. There have been a few papers that use quasi-natural experiments to show this problem exists and assess the gains from resolving it, such as Giroud et al. (2012). We demonstrate the extent to which our estimates are consistent with those of Giroud et al. (2012) and Hennessy (2004) in Section 4.

Chen and Manso (2010) demonstrate that the costs of debt overhang can be significantly higher in the presence of macroeconomic risk. Hence, given our parameter estimates, the gains from resolving this problem over the business cycle likely act as a lower bound in the context of their results were we to incorporate systematic risk into our framework. That said, such macroeconomic risks likely need to be considered in a full general equilibrium environment where there are possible dampening effects from resolving this problem. Gomes et al. (2014) examine how the costs of debt overhang can be exacerbated when inflation risk is present. Incorporating such additional risks into our framework will likely increase the gains from resolving this problem over the business cycle, depending on the nature of the general equilibrium environment.

Our work is consistent with work by Gourio (2014) who shows that firm default risk can play a significant role in driving employment losses in a recession. Our model does not speak to some of the externalities associated with bankruptcy that his paper discusses. The role of firm default and insolvency risk in the economy and its measurement is highlighted by Atkeson, Eisfeldt, and Weill (2013).

The rest of the paper follows as such. Section 2 builds the model in pieces, first outlining the elements of the model required for estimation as well as defining moment conditions in the model and how they can be used to locally identify parameters. We then outline a simple debt contract and general equilibrium environment, and define a

[5]The model will embed the discrete-time version of the model of Luttmer (2007) as a special case. Our model without debt and default is equivalent to a one-country version of the model of Atkeson and Burstein (2010).

stationary competitive equilibrium. Section 3 defines counterfactuals in the model. Section 4 describes the data we use, and the details and results of our estimation procedure. Section 5 describes the results from solving the counterfactuals defined in Section 3 under our estimates. Section 6 concludes.

2 Building a Theoretical Framework

First, we describe a standard production environment under which firms operate. We then define the problem of equity holders, and outline how the state variables and policy functions in their problem map into observables. We then define moment conditions we use to locally identify the parameters of their problem. Afterwards, we specify the problem of the debt holders and the general equilibrium environment, and define an equilibrium.

2.1 The Physical Environment

Time is discrete and indexed as $t = 0, 1, 2, ...$ There is a competitive final good sector and a monopolistically competitive intermediate good sector. The final good is produced from a continuum of differentiated intermediate goods that can be consumed by households (whose problem will be described in Subsection 2.6). Intermediate good firm productivities evolve endogenously through process innovation, and the measure of differentiated intermediate goods is determined endogenously through product innovation. All innovation and intermediate good firm production requires labor, which is paid wage w_t. Firms can issue both equity and debt to finance their operations, which are held and priced by households.

Production The final good is produced from intermediate goods with constant elasticity of substitution (CES) production function:

$$Y_t = \left(\int_i {y_t^i}^{1-\frac{1}{\rho}} di \right)^{\frac{\rho}{\rho-1}} , \tag{1}$$

where i indexes intermediate good firms, and $\rho > 0$. In our model, there is a standard inefficiency due to the monopoly markup in the production of intermediate goods. To undo this distortion, we allow for a per-unit subsidy, τ^s, on the production of the consumption good.[6] In equilibrium, standard arguments show prices must satisfy

$$(1 + \tau^s)P_t = [\int_i {p_t^i}^{1-\rho} di]^{\frac{1}{1-\rho}} , \tag{2}$$

where p_t^i is the price set by firm i, and P_t is the price set by final good firms. We choose the price of the final good to be numeraire. Thus, from profit maximization demand for intermediate goods is

$$y_t^i = (1 + \tau_s)^\rho {p_t^i}^{-\rho} Y_t , \tag{3}$$

[6] The per-unit subsidy on the intermediate good keeps the specification without the tax advantage of debt from being distorted from optimal production, so the tax advantage (and firm debt issuance) does not resolve this entry/production inefficiency.

given (1) and (2).

Firm i produces output, y_t^i, with labor, l_t^i, using the following constant returns to scale production function:

$$y_t^i = \exp(z_t^i)^{\frac{1}{\rho-1}} l_t^i\,. \tag{4}$$

The productivity of an intermediate good firm is $e^{z\frac{1}{\rho-1}}$. We rescale productivity by $\frac{1}{\rho-1}$ so that each firm's variable profits and labor decision are proportional in e^z.

2.2 The Problem of Equity Holders

At time t, equity holders are indexed by the natural log of their productivity, z_t, the size of their current liabilities, d_t, and aggregate states: the tax advantage of debt, τ_t^d, and the proportion of the firm's productivity retained upon bankruptcy, α_t, which potentially vary over time. We also define Γ_t as the measure of firms indexed across (z_t, d_t) to simplify notation. We summarize the aggregate state the firm enters each period with as $S_t = (\Gamma_t, \tau_t^d, \alpha_t)$.

We will, in turn, summarize the transition function of the aggregate state as

$$S_{t+1} = H(S_t). \tag{5}$$

The firm has an investment technology through which it can lower the marginal cost of production; the cost of such investment is convex. Productivity at the firm level evolves conditional on the investments the firm has made in improving its productivity and on idiosyncratic productivity shocks. We assume that firm productivity follows a binomial process and that the size of the step a firm can move up or down, Δ_z, is constant. At time t, with probability q_t the firm's productivity will improve, and with probability $1-q_t$ its productivity will decrease.

We assume that the cost function is proportional in its size and convex, such that:

$$\phi(q_t) = \exp(z_t) h \exp(b q_t), \tag{6}$$

for all firms, where $b > 0$ and $h > 0$.

We assume that each period, firms have the same value of d as the period before. Equity holders have to pay d to creditors, and d has a tax advantage relative to equity, τ^d. Equity holders exit if their discounted present value of profits falls below 0. When the firm holds debt, marginal benefits from a unit of investment for equity holders vs. equity holders and creditors combined differ, and b governs how much the investment decision responds to this difference. The more levered the firm is relative to business risk, the lower the value of equity. Hence, as b is lower, so that the marginal benefits from a unit of investment for equity holders and the firm as a whole differ by more, firms will invest less as they are more levered relative to their business risk. In this sense, b controls the extent of the debt overhang distortion.

At every time t, firm i solves

$$\pi(S_t, z_t^i) = \max_{y_t^i} p(S_t, y_t^i) y_t^i - w(S_t) \frac{y_t^i}{\exp(z_t^i)^{\frac{1}{\rho-1}}}\,, \tag{7}$$

subject to (2) and (4) to maximize profits. With standard arguments, firm i's p_t^i, y_t^i, and l_t^i can be derived as functions of only z_t^i and the aggregate state.

Given the production environment in Subsection 2.1, π_t, from (7), of a firm with productivity $\exp(z_t)$ can be written as the firm's productivity multiplied by a scaling factor, $\Pi(S_t)$, which is a function of aggregates and parameters. Formally, $\pi(z_t) = \exp(z_t)\Pi(S_t)$.

Firms have to pay corporate taxes, τ, on their operating profits. Innovation requires labor. Equity holders thus receive the following cash flows:

$$CF_E(S_t, z_t, d_t) = (1-\tau)\Big(\pi(S_t, z_t) - w(S_t)\phi(q_t)\Big) - (1-\tau_t^d)d_t. \tag{8}$$

Hence, the expected discounted present value of profits for equity holders of a firm with state (S_t, z_t, d_t) satisfies the following Bellman equation:

$$\begin{aligned} V_E(S_t, z_t, d_t) \quad = \quad & \max_{q_t}\Big\{0, CF_E(S_t, z_t, d_t) + \qquad (9) \\ & E_t[M_{t+1}\Big(q_t V_E(S_{t+1}, z_t + \Delta_z, d_t) + (1-q_t)V_E(S_{t+1}, z_t - \Delta_z, d_t)\Big)|S_t]\Big\}. \end{aligned}$$

where M_{t+1} is the pricing kernel that one can derive from the household's problem, and CF_E is defined in (8). In a steady state, $M_{t+1} = \beta$. The firm's exit threshold, $\overline{z_t}(S_t, z_t, d_t)$, is a function of the aggregate state and the firm's idiosyncratic state.

The optimal innovation decision of equity holders can thus be written as

$$q_t^*(S_t, z_t, d_t) = \frac{1}{b}\log\Bigg(\frac{E_t[M_{t+1}\Big(V_E(S_{t+1}, z_t + \Delta_z, d_t) - V_E(S_{t+1}, z_t - \Delta_z, d_t)\Big)|S_t]}{w_t b(1-\tau)h\exp(z_t)}\Bigg). \tag{10}$$

2.3 Heterogeneity in Investment Opportunities

One reason that firms grow at different rates is they differ in their investment opportunities. We model firms differing in their investment opportunities following Acemoglu et al. (2013). Firms will differ in the level of their cost function at any given time. Define θ_t at time t as the level of investment opportunities for a given firm. θ_t is positive and follows an AR1 process.

We can amend (11), such that the cost function is now

$$\phi(q_t, \theta_t) = \exp(z_t)\theta_t^{-b} h \exp(bq_t)\,. \tag{11}$$

With such a functional form, firms will still differ in their investment decision if $b \to \infty$, conditional on θ. Notice that the problem without such heterogeneity has $\theta = 1$ across firms over time.

Equity holders thus receive the following cash flows:

$$CF_E(S_t, z_t, d_t, \theta_t) = (1-\tau)\Big(\pi(S_t, z_t) - w(S_t)\phi(q_t, \theta_t)\Big) - (1-\tau_t^d)d_t. \tag{12}$$

Given previous assumptions and parameter restrictions, the expected discounted present value of profits for equity holders of a firm with state $(S_t, z_t, d_t, \theta_t)$ satisfies the following Bellman equation:

$$\begin{aligned} V_E(S_t, z_t, d_t, \theta_t) \;&=\; \max_{q_t}\Big\{0, CF_E(S_t, z_t, d_t, \theta_t) + \\ &\quad E_t[M_{t+1}\Big(q_t V_E(S_{t+1}, z_t + \Delta_z, d_t, \theta_{t+1}) + \\ &\quad (1-q_t)V_E(S_{t+1}, z_t - \Delta_z, d_t, \theta_{t+1}\Big)|S_t, \theta_t]\Big\}. \end{aligned} \tag{13}$$

Equity holders again have an exit threshold, $\overline{z_t}$, which is now a function of z_t, d_t, and θ_t.

The optimal innovation decision of equity holders is thus

$$\begin{aligned} &q_t^*(S_t, z_t, d_t, \theta_t) = \\ &\frac{1}{b}\log\Big(\frac{E_t[M_{t+1}\Big(V_E(S_{t+1}, z_t + \Delta_z, d_t, \theta_{t+1}) - V_E(S_{t+1}, z_t - \Delta_z, d_t, \theta_{t+1})\Big)|S_t, \theta_t]}{b(1-\tau)w_t h \exp(z_t)}\Big). \\ &+ \log(\theta_t). \end{aligned} \tag{14}$$

(14) makes it apparent why as $b \to \infty$ only θ affects the level of investment.

2.4 Model Properties Used in Estimation

We now discuss some properties of the model as described until this point, which are useful for its quantitative evaluation. We write up these properties as two propositions, one which highlights how both the state variable and policy function can be translated into observables, and one which outlines how the observables can be then translated into moment conditions. To simplify our analysis, given the model is estimated when all aggregates are in a steady state, we write up the propositions for the problem of equity holders in steady state. Similarly, since the model is estimated assuming that firm types are fixed over time, we also make that a working assumption throughout this subsection.

An additional parameter we will need is the period length, Δ. If $\Delta = 1$, then the period length is one year. If $\Delta > 1$, the period length is less than one year.

Proposition 1. *The problem of equity holders in a steady state when firm types are fixed has the following properties:*

(i) It can be reduced to two state variables: (1) the firm's investment opportunities, θ (2) the number of steps (where the step size is Δ_z) until the firm declares bankruptcy, n.

(ii) Expected annualized employment growth can be fully characterized by the firm's innovation decision, (14), and parameters.

(iii) Given the θ of a firm, the state variable n has a 1-1 mapping with firm distance-to-default defined as: $\frac{V_A(n,\theta)-V_B^*(\theta)}{\sigma_A}$, where $V_A(n,\theta)$ is the value of the firm and V_B^* is the firm's default point.[7]

Proofs. *See Appendix A.*

Proposition 1 implies that the model predictions can easily be compared to well-defined and oft-studied objects in the data. Distance-to-default is a much studied variable in corporate finance with well-established methods for its estimation.[8]

We also are able to characterize, in a steady-state equilibrium, a closed-form approximation to the value function of equity holders and their optimal innovation decisions, as functions of state variable n_t. In Section 5, we demonstrate the closed-form approximation to the problem of equity holders is tight. This closed-form approximation, outlined in Appendix A, allows us to demonstrate identification of key parameters for the problem of equity holders, using moments related to the properties of firm employment growth and its relationship to firm distance-to-default. The key model parameters we estimate are Δ_z, the size of steps in the binomial process, b, the convexity of the cost function, and h, the level of the cost function. We define four moments used for local identification of the parameters of the model: the average year-ahead employment growth rate of unlevered firms, the variance of firm employment growth, and regression coefficients from a regression of annualized employment growth on distance-to-default and the square of distance-to-default. The following proposition outlines that, with the closed-form approximation, we can characterize each of these four moments analytically. We also describe some of the properties of these moment conditions.

Proposition 2. *The closed-form approximation for the problem of equity holders, as derived in Appendix A, has the following properties:*

(i) The derivative of firm expected employment growth to distance-to-default can be characterized in closed form, and its magnitude is proportional in $\frac{1}{b}$.

(ii) The second derivative of firm expected employment growth to distance-to-default can be characterized in closed form, and its magnitude is proportional in $\frac{1}{b}$.

(iii) The expected annualized growth rate of the unlevered firm can be fully characterized as a function of Δ_z, h, and Δ, and is decreasing in h.

(iv) The variance of firm growth rates can be characterized as a function of the expected annualized average growth rate of firms and Δ_z, and is increasing in Δ_z holding the expected average growth rate of firms fixed.

Proofs. *See Appendix A.*

We provide further discussion and detail of the local identification of the parameters in Appendix A. We argue that Δ_z has a quantitatively first-order effect on the variance of

[7] For (iii) of Proposition 1, we assume that the choice of q of the firm is 0.5. Given we construct DD from daily returns, this is a reasonable assumption. All of our estimates of the q_∞ of the firm are close to 0.5 as well.

[8] We discuss how we estimate firm distance-to-default and how our measurement procedure compares to those in the literature in Appendix C.

growth rates, and b has a first-order effect on the derivatives of firm growth with respect to distance-to-default. Both h and b matter substantially for average firm growth rates, but h has little effect on the derivatives of firm growth with respect to distance-to-default.

2.5 The Debt Contract and Firm Value

The market value of the firm depends on the nature of the debt contract, as firm value is the sum of the value of equity holders and the value of debt holders. In this subsection, we now specify a debt contract consistent with the functional form assumptions in the problem of equity holders and define the problem of the firm.

Perpetuity Debt and Trade-off Theory Following Leland (1994), we will assume that the firm only holds perpetuity debt. The problem of equity holders is the same as in (13). Firms hold debt because it has a tax advantage, but do not fully finance themselves with debt because of the possibility of costly bankruptcy.

Timing of Bankruptcy and the Problem of the Firm At the start of each period, t, each incumbent firm has a probability, δ, of exiting, and a probability, $1-\delta$, of surviving to produce. There is also a discount rate of the firm, r. Notice, then, $e^{-r}(1-\delta)$ is the discount factor of the firm, which we defined to be β in (13).[9] If the firm survives, equity holders then choose whether to declare bankruptcy or continue to operate. If the firm declares bankruptcy, it loses a fixed proportion, $(1-\alpha_t)$, of its productivity, where $\alpha_t \in$ (0,1]. The existing creditors then gain full equity control of the firm and take out new debt to maximize the joint value of equity holders and new creditors.

If equity holders decide not to go bankrupt, the expected discounted present value of profits for the joint value of equity holders and creditors of a firm with idiosyncratic state variable (z_t, d_t, θ_t) satisfies the following Bellman equation:

$$\begin{aligned} V_A(S_t, z_t, d_t, \theta_t) \quad = \quad & (1-\tau)\Big(\pi(S_t, z_t) - w(S_t)e^{z_t}\theta_t^{-b}he^{bq_t}\Big) + \tau_t^d d_t + \\ & E_t[M_{t+1}\Big(q_t V_A(S_{t+1}, z_t+\Delta_z, d_t, \theta_{t+1}) + \\ & (1-q_t)V_A(S_{t+1}, z_t-\Delta_z, d_t, \theta_{t+1}\Big)|S_t, \theta_t]\Big\}\,. \end{aligned} \tag{15}$$

If equity holders decide to go bankrupt, the expected discounted value of the profits of the firm is

$$V_A(S_t, z_t, d_t, \theta_t) \quad = \quad \max_{d_{t+1}} \; V_A(S_t, z_t + \log(\alpha_t), d_{t+1}, \theta_t)\,. \tag{16}$$

Let $d^*(S_t, z_t, \theta_t)$ be the optimal choice of d_{t+1} that satisfies (16). The value of creditors, V_B, is defined as the difference between the value of the firm as a whole, (15) and (16), and the value of equity holders, (9); thus,

[9] If we calibrate the model such that there are multiple periods in a year (say one period is $\frac{1}{\Delta}$ of a year), then the discount factor is $e^{-r\frac{1}{\Delta}}(1-\delta)$.

$$V_B(S_t, z_t, d_t, \theta_t) = V_A(S_t, z_t, d_t, \theta_t) - V_E(S_t, z_t, d_t, \theta_t)\,.$$

2.6 The General Equilibrium Environment

We now fully flesh out a general equilibrium environment consistent with the functional form assumptions that enter the problems of debt holders and equity holders.

Free Entry We assume there is free entry into the economy. New firms are created by purchasing n_e units of labor; a purchase in period t yields a new firm in period $t+1$ with initial state variables z_t and θ_t drawn from a distribution G. After receiving z_t and θ_t, the firm makes an initial debt decision to maximize the value of equity holders and new creditors. In any period with a positive mass of entering firms, we have

$$w(S_t)n_e = E_t[M_{t+1} \int_\theta \int_z \underset{d_{t+1}}{max} V_{A,t+1}(S_{t+1}, z, d_{t+1}, \theta) G(z,\theta) dz d\theta | S_t]\,. \tag{17}$$

We define $\Gamma_{e,t}$ as the measure of new firms entering the economy at period t that start producing in period $t+1$.

Households Households are endowed with L units of time which they supply inelastically. After all (idiosyncratic and aggregate) shocks are realized, households make a consumption decision, C_t, get paid wages, $w(S_t)$, receive a lump sum transfer of dividends, D_t, and a pay a lump-sum tax, T_t.

The recursive problem for households is the following:

$$V^H(S_t) = \underset{C_t}{max}[\log(C_t) + e^{-r} E_t V^H(S_{t+1})|S_t] \tag{18}$$

subject to their budget constraint:

$$C_t = w(S_t)L + D_t - T_t, \tag{19}$$

and the aggregate law of motion for S_t, (5). The aggregate dividend is the sum of all after-tax profits from intermediate good firms net of the entry costs of all newly entering firms.

The Distribution of Firms The distribution of operating firms at time t, $\Gamma_t(z, d, \theta)$, evolves over time as a function of the exogenous exit rate, δ, the choices of q_t by incumbent firms, and the mass of entering firms each period, $\Gamma_{e,t}$. To simplify the definition of the mass of firms with state $(z_{t+1}, d_{t+1}, \theta_{t+1})$ in period $t+1$, we break it into four pieces. First, there is a mass of continuing firms who did not go bankrupt who could enter period $t+1$ with state (z_{t+1}, d_t, θ_t), which is a function of continuing firms with productivity $z_{t+1} - \Delta_z$ last period that drew positive productivity shocks and continuing firms with productivity $z_{t+1} + \Delta_z$ last period that drew negative productivity shocks:

$$\begin{aligned}\Gamma^C_{t+1}(z_{t+1}, d_{t+1}, \theta_{t+1}) \;=\; & (1-\delta)(1 - q_t(S_t, z_{t+1} + \Delta_z, d_{t+1}, \theta_{t+1}))\Gamma_t(z_{t+1} + \Delta_z, d_{t+1}, \theta_{t+1}) \\ & +(1-\delta) q_t(S_t, z_{t+1} - \Delta_z, d_{t+1}, \theta_{t+1})\Gamma_t(z_{t+1} - \Delta_z, d_{t+1}, \theta_{t+1})\,.\end{aligned} \tag{20}$$

Define $\tilde{\Gamma^C}_{t+1}$ to be the distribution, also accounting for the fact that we could transition from a different θ following the exogenous process for θ specified.

Second, Γ_{t+1} is also a function of the mass of entering firms who received productivity, z_{t+1}, and investment opportunities, θ_{t+1}, such that they chose coupon payment d_{t+1}:

$$\Gamma^E_{t+1}(z_{t+1}, d_{t+1}, \theta_{t+1}) = \Gamma_{e,t} G(z_{t+1}, \theta_{t+1}) \,. \tag{21}$$

Third, Γ_{t+1} is a function of the mass of firms who have productivity $z_{t+1} + \Delta_z - \log(\alpha_t)$ last period, with type θ_{t+1} and coupon payment d that drew negative productivity shocks, went bankrupt, and chose coupon payment d_{t+1}.[10]

$$\begin{aligned} \Gamma^B_{t+1}(z_{t+1}, d_{t+1}, \theta_{t+1}) \;\; = \;\; & (1-\delta) \int (1 - q_t(S_t, z_{t+1} + \Delta_z - \log(\alpha_t), d, \theta_{t+1})) * \\ & \Gamma_t(z_{t+1} + \Delta_z - \log(\alpha_t), d, \theta_{t+1}) dd \,. \end{aligned} \tag{22}$$

Define $\tilde{\Gamma^B}_{t+1}$ to be the distribution, also accounting for the fact that we could transition from a different θ following the exogenous process for θ specified. Hence, we can define $\Gamma_{t+1}(z_{t+1}, d_{t+1}, \theta_{t+1})$, as the sum of $\tilde{\Gamma^C}_{t+1}(z_{t+1}, d_{t+1}, \theta_{t+1})$, $\Gamma^E_{t+1}(z_{t+1}, d_{t+1}, \theta_{t+1})$, and $\tilde{\Gamma^B}_{t+1}(z_{t+1}, d_{t+1}, \theta_{t+1})$ using (20), (21), and (22).

Equilibrium In our simple setup, market clearing for the final good requires:

$$C(S_t) = Y(S_t) \,.$$

Market clearing for labor requires

$$\int \int \int l_t(z) \Gamma_t(z, d, \theta) dz dd d\theta + L_{r,t} = L \,,$$

where $\int \int \int l_t(z) \Gamma_t(z, d, \theta) dz dd d\theta$ is total employment used to produce the intermediate good, whereas $L_{r,t}$ denotes labor spent on process and product innovation.

We can write labor spent on research (process and product innovation) as

$$\Gamma_{e,t} n_e + \int \int \int e^z \theta^{-b} h e^{b q_t} \Gamma_t(z, d, \theta) dz dd d\theta = L_{r,t} \,.$$

A recursive competitive equilibrium in this economy is defined as follows. Given initial distribution, Γ_0, and initial aggregate shocks, α_0 and τ^d_0, a recursive equilibrium consists of policy and value functions of equity holders, creditors, and intermediate good firms, $\{l(S_t, z_t), V_E(S_t, s_t),\ \bar{z}(S_t, x_t),\ q^*(S_t, s_t),\ V_B(S_t, s_t),\ V_A(S_t, s_t),\ d^*(S_t, z_t, \theta_t)\}$ where x_t=(d_t, θ_t) and $s_t = (z_t, x_t)$, household policy functions for consumption, $C(S_t)$, aggregate prices, $\{P(S_t),\ w(S_t)\}$, the mass of new entrants, $\Gamma_e(S_t)$, and the aggregate states including the distribution of firms, S_t, which evolve according to transition function $H(S_t)$ such

[10] It is also possible for firms to have had productivity $z_{t+1} - \Delta_z - \log(\alpha_t)$ last period, type θ_{t+1} and debt load d, to go bankrupt and choose debt d_{t+1}, although this does not occur in a steady state, so we do not include this case.

that for all t: (i) the policy and value functions of intermediate good firms are consistent with the firm's optimization problem, (ii) the representative consumer's policy function is consistent with its maximization problem, (iii) debt and equity holders' value functions and decision rules are priced such that they break even in expected value, (iv) free entry holds (v) labor and final good markets clear, and (vi) the measure of firms evolves in a manner consistent with the policy functions of firms, households, and shocks.

A stationary competitive equilibrium is an equilibrium in which all aggregates, prices, and the distribution of firms are constant over time. In such an equilibrium, we say these aggregates are in steady-state. We focus only on equilibria with positive entry.

The Profit Scaling Factor and Aggregation in a Steady State Our aggregation is equivalent to a one-country version of Atkeson and Burstein (2010) with a per-unit subsidy, τ^s, on the production of the consumption good. We present our model aggregation in steady state below (hence, we remove all time subscripts). We first note that

$$\pi(z) = e^z Y(1+\tau^s)^\rho w^{-\rho} \frac{1}{\rho^\rho(\rho-1)^{1-\rho}} .$$

It is then useful to define

$$\Pi = Y(1+\tau^s)^\rho w^{-\rho} \frac{1}{\rho^\rho(\rho-1)^{1-\rho}} .$$

The choice of labor by the intermediate good firm is

$$l(z) = e^z Y(1+\tau^s)^\rho \left(\frac{\rho-1}{\rho}\right)^\rho w^{-\rho} .,$$

Define the steady-state scaled distribution of firms across states as $\tilde{\Gamma}(z,d,\theta)$. We then find scaled aggregate productivity as

$$\tilde{Z} = \int\int\int e^z \tilde{\Gamma}(z,d,\theta) dz dd d\theta . \tag{23}$$

Another useful aggregate to define is average expenditures per entering firm, which we denote by Υ:

$$\Upsilon = n_e + \int\int\int e^z \theta^{-b} e^{bq} \tilde{\Gamma}(z,d,\theta) dz dd d\theta . \tag{24}$$

Given Π, $\tilde{Z}$, and Υ, we can recover the following equilibrium objects:

$$W = (1+\tau^s)\frac{\rho-1}{\rho}(\Gamma_e \tilde{Z})^{\frac{1}{\rho-1}} .$$

$$Y = (\Gamma_e \tilde{Z})^{\frac{1}{\rho-1}}(L - L_r) .$$

$$L_r = \frac{1}{\rho\xi} L,$$

where $\xi = \frac{\Pi \tilde{Z}}{\Upsilon}$ is the ratio of total variable profits to total expenditures on the research good. Total aggregate productivity is then

$$Z = (\Gamma_e \tilde{Z})^{\frac{1}{\rho-1}} .$$

3 Defining Counterfactuals

We use our model to address the following questions: What are the expected private gains from resolving debt overhang for firms in the cross-section and over the business cycle, and for the average firm entering the economy. We also ask: What are the gains for the aggregate economy in the long run from resolving this problem for all firms? In this section, we define the objects that allow us to answer these questions. We discuss our partial and general equilibrium counterfactuals separately.

3.1 Partial Equilibrium Counterfactuals

We outline a series of counterfactuals that can be solved in partial equilibrium (taking as given prices, the labor allocation, and the distribution of firms) that allow us to quantify the gains to a single firm from resolving debt overhang. We break them into counterfactuals that can be solved just from solving the problem of equity holders, and those that require the problem of the firm.

3.1.1 Partial Equilibrium Counterfactuals that Use Information Only from the Problem of Equity Holders

We describe below two counterfactuals. First, with only the cross-sectional distribution of firm distance-to-default from the data and the problem of equity holders, we can compute a counterfactual that assesses the expected private gains from resolving debt overhang problem. We can also perform this same counterfactual over time using the cross-section of distribution of distance-to-default year by year. With our closed-form approximation to the policy function, we can get an approximate solution to this counterfactual in closed form.

Comparing Growth Rates of Firms that Do and Do Not Suffer from Debt Overhang In this counterfactual, we do not assess the gains from resolving debt overhang precisely but instead assess the gains were all firms to make the same innovation decision as the unlevered firm. We are, in turn, comparing the gains were the cost function inelastic where $b = \infty$ to those when the cost function is elastic to the extent that we estimate. We compute the policy function and associated implied annualized growth rate under our estimate for each value of distance-to-default in the data. We then compute the weighted-average value of q and respective implied expected annualized growth rate. We then compare the implied annualized growth rate to the implied annualized growth rate were all firms to make the same decisions as the unlevered firm.

Using the Time-Varying Distribution of Distance-to-Default in the Data for Firm Value Counterfactuals at Each Time t Given the fact that there is a distribution of firms in each year, we can perform the counterfactual above year by year.

3.1.2 Partial Equilibrium Firm Value Counterfactuals

Firm Value Counterfactuals Similar to the counterfactuals above, conditional on n or firm distance-to-default, we can compare two firms, one that suffers from debt overhang (equity holders make the investment decision) and one that does not (the firm as a whole makes the investment decision), and compare their value functions or expected annualized growth rates assuming prices and the mass of firms do not change.

The Bellman Conditional on Resolving the Agency Problem The Bellman equations can also be solved if the firm as a whole, rather than equity holders alone, were to make the investment decision. It is always equity holders, however, who choose the point at which the firm goes bankrupt. Were the firm as a whole to make the bankruptcy decision, it would never go bankrupt. We define the Bellman equation for equity holders when the firm as a whole makes the investment decision in steady state with fixed types, V_E^{ND}, below:

$$\begin{aligned} V_E^{ND}(z,d,\theta) \;\; &= \;\; max\Big\{0,(1-\tau)\Big(\pi(z)-we^{z}\theta^{-b}he^{bq}\Big)-d+\tau^{d}d+ \\ &\qquad e^{-r}(1-\delta)\Big(qV_E^{ND}(z+\Delta_z,d,\theta)+(1-q)V_E^{ND}(z-\Delta_z,d,\theta)\Big)\Big\}\,. \end{aligned}$$

We define the Bellman equation for equity holders and creditors combined, V_A^{ND}, below:

$$V_A^{ND}(z,d,\theta)=\max_{q}\begin{cases} \max\limits_{d'} \;\; V_A^{ND}(z+\log(\alpha),d',\theta) & V_E^{ND}(z,d,\theta)<0 \\ (1-\tau)\left(\pi(z)-we^{z}\theta^{-b}he^{bq}\right)+\tau^{d}d & else \\ +e^{-r}(1-\delta)qV_A^{ND}(z+\Delta_z,d,\theta) & \\ +e^{-r}(1-\delta)(1-q)V_A^{ND}(z-\Delta_z,d,\theta)\,. & \end{cases} \tag{25}$$

We then use the first-order condition from (25) to find q:

$$q^{*}=\frac{1}{b}\log\Big(\frac{e^{-r}(1-\delta)(V_A(z+\Delta_z,d,\theta)-V_A(z-\Delta_z,d,\theta))}{b(1-\tau)hwe^{z}}\Big)+\log(\theta)\,. \tag{26}$$

Notice, now, no matter the value of b, the firm does not suffer from debt overhang, as equity holders and creditors are jointly making the investment decision. Because they make the investment decision taking into account the possibility of bankruptcy, if b has any convexity, the firm will invest more as it is more levered relative to its business risk to avoid bankruptcy.

It is still the case, then, that the value of debt holders, $V_B^{ND}(z,d,\theta)$, is defined as the difference between the value of the firm as a whole and the value of equity; thus,

$$V_B^{ND}(z,d,\theta)=V_A^{ND}(z,d,\theta)-V_E^{ND}(z,d,\theta)\,.$$

Hence, we can compare $V^{ND}(n)$ and $V_A(n)$ given n or across firms to assess the gains from resolving this problem conditional on n or across n. We can also do the same exercise for the respective policy functions and expected annualized growth rates. It is useful to "convexify" the innovation decision between equity holders and the firm as a whole with a parameter ν. It is useful to define a convexified value function that combines the value of equity holders and the firm as a whole; define the convexified value function as $V_C = \nu V_E + (1-\nu)V_A$.[11]

We can then use the first-order condition from (25) to find q in the case where ν varies as between steady states in our counterfactual exercise:

$$q^* = \frac{1}{b}\log\Big(\frac{e^{-r}(1-\delta)(V_C(z+\Delta_z,d,\theta) - V_C(z-\Delta_z,d,\theta))}{b(1-\tau)hwe^z}\Big) + \log(\theta)\,.$$

The Gains from Resolving Debt Overhang for Firm Value Upon Entry Decomposed To recover the gains in terms of firm value upon entry, we hold fixed all general equilibrium effects that could affect firm value (prices, the labor allocation, and the supply of firms), and solve the model again, assuming the firm resolves the debt overhang problem. We then compare the percentage difference between the value function of the average entering firm if the firm does not and does suffer from debt overhang. Following Moyen (2007), we decompose these gains into the gains from operations, the gains from the tax advantage, and the losses from bankruptcy. The value from operations is the expected discounted present value of the firm's production and investment activities. The tax advantage of debt is the expected discounted present value of all interest deductions. The default cost is the expected discounted present value of the deadweight losses from bankruptcy.

3.2 General Equilibrium Counterfactuals

There are three distortions in the model we want to focus on: debt overhang, bankruptcy costs, and other equilibrium distortions caused by the tax advantage of debt and bankruptcy costs. We develop a decomposition of the social losses in our baseline model relative to the planner's problem to isolate the effects of these distortions based on counterfactual objects.

Planner's Problem The social planner chooses consumption, product innovation, process innovation, and the labor allocation to maximizes her discounted present value of utility such that the final good market clears, the labor market clears, and the law of motion for productivity is satisfied. In our setup, the planner's problem is the equivalent of setting $\tau^d = 0$ and $\tau = 0$ with a per-unit subsidy, τ^s, on production of the consumption good to undo the distortion from the efficient allocation from the markup in our model. The subsidy takes value $\tau^s = \frac{\rho}{\rho-1}$. When aggregating our model, we include the subsidy in all counterfactuals and in our base case. We also set $\tau = 0$ to focus on the distortion of interest, which is the social cost due to the tax advantage of debt, and the associated costs of debt overhang and bankruptcy.

[11] For our debt overhang counterfactuals where we transition between steady states, ν should be time-varying, follow a stochastic process, and be included in the aggregate state.

Social Loss Decomposition We define the planner's problem above. Define consumption from the planner's problem to be C^{EFF}. We define $LOSSES_{EFF}$ as the long-run differences in aggregate consumption between the planner's problem and our base case with debt overhang:

$$LOSSES_{EFF} = \frac{C^{EFF} - C}{C},$$

where C is consumption from our baseline estimation. We call these losses "social losses," and moving forward we describe welfare as differences in long-run consumption between steady states. To further decompose these social losses, we create two more consumption measures. To create our first additional consumption measure, we have the firm as a whole, rather than equity holders, make decisions as we describe in Subsection 3.1.2. We solve for a stationary competitive equilibrium given these decision rules and recover a counterfactual object, C^{ND}. We define the losses from debt overhang as

$$LOSSES_{DO} = \frac{C^{ND} - C}{C}.$$

We then create one more object to recover two more counterfactuals objects. We treat α purely as a financial cost. Thus, this cost is a transfer payment, but no productivity is lost in bankruptcy. We then recover a new consumption measure: $C^{NO\alpha}$. This object gives us the ability to create two counterfactuals, which along with $LOSSES_{DO}$ should add up to $LOSSES_{EFF}$. The first counterfactual object represents the effect of bankruptcy on the total mass of productivity:

$$LOSSES_{NO\alpha} = \frac{C^{NO\alpha} - C^{ND}}{C}.$$

The second is the remaining loss, which can be interpreted as the degree to which α and τ^d distort firm decisions relative to the social planner's choice:

$$LOSSES_{REM} = \frac{C^{EFF} - C^{NO\alpha}}{C}.$$

4 Estimation

In this section, we describe the data we use to estimate our model and provide the results from our overidentified indirect inference estimation strategy. We also discuss how we calibrate the remaining parameters in our model.

4.1 Data and Measurement

We use nonfinancial public firm data from 1982 to 2012. Equity market data come from CRSP, and annual and quarterly accounting statements come from Compustat. We discuss exact details of how our data is constructed along with some variable definitions in Appendix B. In our estimation procedure, we will rely heavily on properties of employment growth, especially its relationship with firm distance-to-default. If we define V_B as the book value of debt, V_A as the value of assets, and σ_A as the standard deviation of the value of the firm's assets, we can define our measure of firm distance-to-default as

$$\frac{ln(\frac{V_A}{V_B})}{\sigma_A}.$$[12]

We detail how we construct V_A and σ_A from the data in Appendix C. Distance-to-default is measured in units of the number of standard deviations of annual asset volatility by which the firm's assets must change to equal the firm's book value of its debt. We winsorize the measure to lie between 0 and 10 when we take it to the model. Employment growth is measured as log differences in employment from year to year. Figures 1a and 1b plot the relationship between distance-to-default and a detrended measure of average year-ahead growth, where the measure of year-ahead growth is the residuals from a regression of year-ahead employment growth on year and industry dummies. We plot a quadratic fit through the data to demonstrate that the regression, (47), discussed in Appendix A, will provide a good fit to the shape of the data. The shape is also similar using a Kernel-smoothing regression. In Panel 1c of Figure 1, we plot the residuals from a regression on industry dummies, year dummies, log number of employees at the firm, firm age, and the Whited-Wu index for the firm, which is an index of firms' external financing constraints against firm distance-to-default.[13] The independent variables are all measures that are known to be strongly correlated with growth rates. We see that the relationship looks similar were we to focus on sales growth or capital growth, which should assuage concerns about measurement error in employment in Compustat affecting our results.

The relationship we establish between firm growth rates and firm distance-to-default will likely exist even absent debt overhang affecting firms. We expect firms that are not growing are on average more likely to have higher leverage relative to their business risk, so, ex ante, we should expect distance-to-default and growth to have a monotonically increasing relationship. The estimation procedure finds reasonable bounds on the extent to which the relationships plotted in Figures 1a, 1b, and 1c could be driven by such a reverse causality argument in the context of our estimation procedure.

4.2 Estimation Implementation

We use an indirect inference approach to estimate key model parameters. Our moments are a function of the joint distribution of firm growth rates and firm distance-to-default. We first compute moments in the data, and then compute model-implied moments using a combination of the distribution of firm distance-to-default and firm characteristics in the data and model implied decision rules conditional on firm distance-to-default. This procedure allows us to exactly match the distribution of distance-to-default for each firm over time as parameters change, thus avoiding some biases associated with the debt contract in the model being misspecified. Further, we inherently correct for important sample characteristics and selection effects in Compustat for which we want to account.

[12] We can also compute distance-to-default as in Merton (1974), and we find very similar data moments.
[13] See Whited and Wu (2006) for how to construct the Whited-Wu index.

We implement our indirect inference procedure in the following standard way. Say our model moments are the $1 \times n$ vector, $\hat{M}(G)_t$, where G represents the tuple $(b,\ q_\infty,\ \Delta_z)$, and our data moments are the $1 \times n$ vector, $\hat{D}_{t},$. Define $\hat{g} = \hat{M}(G)_t - \hat{D}_t$. We want to minimize the following objective function over G:

$$\hat{g} W \hat{g}' \, .$$

We use the identity matrix as our weighting matrix W. Our estimates will, in turn, be consistent but not efficient. Given we do not simulate data, given our estimates we can then derive standard errors and the J-statistic in the usual way as if we were doing GMM when using pre-specified weighting matrices. For these objects, we need the efficient weighting matrix, which we recover by creating a variance-covariance matrix of the data using 15,000 bootstraps.

4.3 Estimation Specifications

We use a variety of specifications to estimate the model. They differ in the data samples used and the methods by which we control for correlates of firm growth and potential firm heterogeneity. We consider a sample of manufacturing firms that exist in 1992 and survive through 1995; this panel corresponds to Versions (1)-(3) and (7)-(9) of our estimation procedures.[14] We choose this panel following Hennessy (2004).[15] We also consider a large sample of nonfinancial public firms that exist at any point in the period 1982 to 2012; this panel corresponds to Versions (4)-(6) and (10)-(12) of our estimation procedures, cleaned as described in Appendix C. For each of these datasets, we estimate the model in six ways. For the three of them which correspond to our first estimation procedure, we estimate the model without heterogeneity in investment opportunities and vary how we account for investment opportunities in the data. In the other three which correspond to our second estimation procedure, we estimate the model with heterogeneity in investment opportunities and vary how we parameterize the joint distribution of distance-to-default and investment opportunities.

Recall, we locally identify model parameters b, Δ_z, and h by comparing moments in the data to moments in the model.[16] In our estimation procedure, the estimate for h is obtained by directly estimating a related object, the expected growth rate of the unlevered firm $q_{\infty,\theta=1}$.[17] Given the calibrated parameters and estimates for b and Δ_z, $q_{\infty,\theta=1}$ implies an estimate for h. We compare coefficients of a regression of a measure of firm employment growth on distance-to-default its square. Our other moments are the average growth rate of unlevered firms and the average standard deviation of employment growth. We take the distribution of distance-to-default from the data when we estimate our model; hence, it is only the implied innovation decisions conditional on the distribution that drive our estimates of parameters, not how these estimates would then feed back into changing the distribution of distance-to-default. The distribution for the full sample of nonfinancial firms we consider is plotted in Figure 2b.

[14] Manufacturing firms are defined as those firms with two-digit SIC codes between 20 and 39.

[15] We have slightly more firms than his panel due to different procedures for cleaning of the data.

[16] See Appendix A for the derivation of these moments in the model.

[17] This is done to improve numerical accuracy.

Estimation Procedure 1 We run three versions of the estimation procedure for each panel. In the model, we always run the same regression and compute the same moments. The only difference in procedures is how we treat employment growth. The first and fourth specifications demean growth by year and industry averages but do not control for the firm's average growth rate. The second and fifth specifications control for differences in firm investment opportunities by also demeaning growth at the firm level. The third and sixth specifications are the residuals from regressions where the dependent variable is the variable in the second and fifth specifications, respectively, and the independent variables are age, size, and a measure of firm's access to external finance.

Estimation Procedure 2 Given a guess of b, Δ_z, and q_∞, this procedure determines the joint distribution of θ and n before computing moments. We run three versions of the estimation procedure for each panel. As in the first estimation procedure, we always run the same regression and compute the same moments. The procedures differ in the data in how we treat employment growth. The procedures differ in the model in how we determine the joint distribution of θ and distance-to-default. First, we describe what we do to the data in each specification. The first, second, fourth, and fifth specifications demean growth by year and industry averages. The third and sixth specifications are the residuals from regressions where the dependent variable is the variable in the first and fourth specifications, respectively, and the independent variables are age, size, and a measure of firm's access to external finance.

For specifications one and four, we determine the joint distribution of θ and n by specifying it exogenously. We specify a nonlinear functional form for the distribution such that the distribution of θ is increasing and concave along the grid of firm distance-to-default. We first need to specify a minimum q_∞ that will correspond to a given value of θ, as we explain below. The maximum q_∞ will be the value of q_∞ we estimate. The minimum q_∞ is set exogenously to 0.47. We then define an evenly spaced grid between the minimum and maximum q_∞ of length ZT. For a given n, the implied q_∞ is then $\sqrt{n} * (ZT-1)/(\sqrt{10}) + 1$, which is increasing and concave along the firm distance-to-default. When we report the associated estimate of q_∞, we report the q_∞ corresponding to the annualized expected growth rate that we find matches the average employment growth of unlevered firms we estimate. When performing counterfactuals, we solve the model with no heterogeneity and this value of q_∞ for all unlevered firms. Given a value of h and a value of q_∞, when types are permanent, a value of θ can then be backed out analytically:

$$\theta = \frac{\Pi\beta(\exp(\Delta_z) - \exp(-\Delta_z))}{h\exp(bq_\infty) * (\beta * (\exp(\Delta_z) - \exp(-\Delta_z)) - b(1 - \beta(\exp(\Delta_z) - \exp(-\Delta_z))q_\infty - \beta\exp(-\Delta_z)))}. \tag{27}$$

When we estimate θ in Versions (2), (3), (5), and (6), for a given guess of parameters, we determine the value of θ that corresponds to a given firm in the data. Recall, we are assuming here that θ is permanent for a given firm. For each firm, we find the value of θ that minimizes the sum of squared differences between year-ahead growth in the model and the data, taking the firm's value of distance-to-default as given. So, a firm that exists in our sample for four years with values of distance-to-default of $\{1, 1, 2, 3\}$ with respective growth rates of $\{-1\%, -2\%, -.5\%, -.2\%\}$ in the data may have a θ equal to .9 (given a guess of b, Δ_z, and q_∞) that implies growth rates of $\{-1\%, -1\%, -.5\%, -.2\%\}$ for the same values of distance-to-default. If these values minimize the squared differences between growth rates in the model and data, we will then assign a value of θ of .9 to this firm. After finding the value of θ for each firm following this procedure, we compute the same three moments as computed in estimation procedure 1 with the dependent variables defined as above. Holding fixed Δ_z and q_∞, if the implied distribution of θ is increasing and concave along firm distance-to-default and not overly noisy, the value of b estimated will be higher than that were we not to estimate θ.

We ultimately refer to these different specifications in Estimation Procedure 2 as (7)-(12) respectively corresponding to (1)-(6) in our description here.

4.4 Remaining Calibration

In Table 3, we show our remaining calibration. We set τ_d to 0.2 to match the value chosen in Leland (1998). There exists a range of different estimates in the corporate finance literature. This number will not matter for our estimation procedure, as it will only matter for the choice of debt by the firm. We set the corporate tax rate, τ, to 0 in our procedure so that when we perform counterfactuals the tax advantage is a pure distortion. Had corporate taxes been positive, the tax advantage will further act as a subsidy to entry; such a policy is of less interest to this paper. The intermediate good firm's problem scales in taxes, so only aggregates will be different (not decision rules) had corporate taxes been positive. Hence, we get the same estimates of parameters no matter the level of τ. We choose α to be 0.8, which is the upper bound of bankruptcy costs found in Bris et al. (2006). α will also not affect the value of equity holders in our estimation procedure. The overall welfare losses are decreasing in α, since a higher α implies more productivity is lost in bankruptcy. The per-period entry cost and total labor supply are set to one, as these objects' values will not affect our results. We choose ρ to be 4 to match ρ in Atkeson and Burstein (2015). This parameter does not affect firm decisions, only aggregates. Assuming $\rho > 1$, holding all other parameters fixed, the welfare losses from debt overhang are decreasing in ρ. If $\rho \to \infty$, the CES production function becomes linear, and, in turn, the losses from debt overhang tend toward 0 in the limit.[18] Notice, none of the parameters discussed so far affect the estimates of parameters in our model.

We set the discount factor to 0.994. The discount factor will affect firm's decisions and play a role in the estimation procedure. We do not check our results across a range of discount factors; however, our choice fits in the range considered in the literature. The discount factor is $e^{-r\Delta}(1-\delta)$, which given Δ, a function of a discount rate r and an exogenous exit rate δ. We choose an exogenous exit rate high enough such that our problem admits a stationary equilibrium (so its value is 0.006), and the residual r becomes $\log(1.001)$.

[18]See Acemoglu (2008) Chapter 2 for further discussion of the properties of the CES production function.

4.5 Estimation Results and Discussion

We show the extent to which our parameters are locally identified in Figures 3a, 3b, and 3c. We hold fixed two of the parameters at their values when the objective function is minimized and vary the parameter noted in the title.[19] As one can see, for each parameter, we have a parabola with a clear minimum at our estimate of the value. We discuss our results from Estimation Procedures 1 and 2 separately below.

Estimation Procedure 1 Our results from our first estimation procedure are presented in Table 1. We argue the estimate we obtain when we demean firm employment growth by its average growth rate can be perceived as implying an upper bound of b, whereas when we do not control for unobserved heterogeneity at the firm level, we have a reasonable lower bound. The lower bound argument is clear in that by not controlling for firms having differences in their investment opportunities, we are overstating the role debt overhang plays in driving the relationship between distance-to-default and growth. The upper bound argument relies on the fact that debt overhang can affect the firm's average growth rate. Both heterogeneity and debt overhang can generate a positive relationship between distance-to-default and growth. Both have persistence, so many firms with lower average growth rates will also have lower average distance-to-default. By demeaning at the firm level, we are attributing all of the differences in average growth rates to firm heterogeneity. Therefore, in the regression where we demean firm growth rates, we have firms who have low average distance-to-default but thanks to demeaning, not lower average growth. This substantially weakens the relationship between distance-to-default and growth, and thus raises the estimate of b. Hence, for a given panel of firms, we argue we have reasonable upper and lower bound estimates of b.

Across versions of the estimation procedure, the clearest result is that, as expected, when one demeans growth at the firm level, the value of b increases (the convexity, and thus the extent to which debt overhang affects firm innovation decisions, decreases) in either subsample. The relationship between distance-to-default and growth can be explained to some extent by the fact that the firms that have not been growing are the firms with worse investment opportunities. However, given the functional form of the cost function we assumed and the parameter for b we estimate, the relationship still exists enough such that debt overhang is costly for firms especially as they near default, consistent with findings in the literature.

Estimation Procedure 2 Our results from our second estimation procedure are presented in Table 2. The second estimation procedure specifies the joint distribution of firm heterogeneity and firm distance-to-default given a guess of b, Δ_z, and q_∞. Versions (7) and (10), where we exogenously fix θ to be increasing and concave along the grid of distance-to-default, provide an upper bound in the context of the model, but likely an

[19] When we actually implement our procedure, we use the numerical solution to the model. To search 640,000 combinations of parameters takes less than four hours using a parallellized grid search. Using the closed-form approximation, we can do the same search in a manner of minutes, which helps inform the guess of where to search (although we greatly increase the size of the grid around our informed guess to ensure a global minimum).

unreasonable upper bound given that firm types are likely not correlated 100% with firm distance-to-default. Not surprisingly, then, we find a larger value of b than that implied by our more reasonable upper bound in estimation procedure 1. Nonetheless, we find it to be a useful exercise to demonstrate possible specifications of differences in firm investment opportunities that could explain most of the relationship between distance-to-default and growth.

We get a different result when we attempt to estimate θ using the joint distribution of distance-to-default and growth. We find that θ has a low correlation with firm distance-to-default, and hence our estimate of b is close to our estimates of b in our first estimation procedure in the specifications wherein we do not control for unobserved heterogeneity. We were not surprised we ended up with a noisy relationship between θ and firm distance-to-default. However, interestingly, we found that when the correlation between θ and firm distance-to-default was below 50%, the estimate of b decreases substantially. Hence, even a reasonable amount of noise in the relationship between distance-to-default and growth can drastically affect the estimate of θ using our procedure.

4.6 Comparing our Estimates to those in the Literature

We focus on two papers in comparing our results to those from the literature: a reduced-form paper with a quasi-natural experiment, Giroud et al. (2012), and a structural paper that uses Q-theory, Hennessy (2004). We only discuss results from estimation procedure one for brevity and because it is our preferred estimation procedure.

Giroud et al. (2012) use a quasi-natural experiment to estimate the impact of exogenous shocks to firm book leverage on returns, other measures of performance, and sales growth for a sample of highly leveraged firms. We construct a theoretical analog of an estimated regression coefficient that we can derive just from the problem of equity holders, the partial derivative of firm sales growth to exogenous changes in firm book leverage, by computing this derivative for firms with similar book leverage to those in their sample. Our estimation procedures on our full sample of firms that take firm heterogeneity into account generates estimates of this derivative consistent with their results for firms with leverage ratios close to those in their sample. Our estimation procedures that do not take exogenous firm heterogeneity into account imply a significantly stronger relationship between book leverage and firm growth. This suggests that controlling for firm heterogeneity is important for generating estimates of the effect of debt overhang consistent with existing studies. Nonetheless, we find we can "hit" their estimate no matter the estimation specification, but where on the grid we hit it varies depending on the estimation procedure. This is because the model-implied derivative changes along the margin of distance-to-default, due to the changing intensity of the debt overhang problem. Thus, while our microeconomic estimates correspond to the literature, our macroeconomic implications likely differ meaningfully from those which could be generated by extrapolating from the linear estimates found in the literature.

Hennessy (2004) uses a Q-theoretic approach to estimate the extent to which debt overhang affects firm investment. He uses the expectation of the market value of lenders total recovery claim (reflecting both the probability of default and how much lenders recover) scaled by the capital stock as a measure of firm's exposure to debt overhang. He then runs a regression of investment on this measure of debt overhang and controls and finds that this measure has a significant negative relation with investment. The resulting regression coefficient can be interpreted as an estimate of the derivative of investment with respect to this expected recovery claim. We compute, in our model, both the expected recovery claim as well as an analogue of investment, expenditures on innovation, to compute this derivative in our model. Again, this derivative is non-constant along the grid of firm distance-to-default. Our estimation procedures that take firm heterogeneity into account generate estimates of this derivative consistent with the results of Hennessy (2004) for firms with an average distance-to-default close to those in our balanced panel, the construction of which is based off of the data used by Hennessy (2004). The value of distance-to-default where we are closest to hitting this derivative decreases as our estimate of the convexity of the cost function increases.

Comparing our Estimates to Those from a Quasi-Natural Experiment We use the estimate of the derivative of a change in three-year-ahead sales growth to a change in book leverage in Table IV of Giroud et al. (2012) of -0.039. We can easily compute three-year-ahead sales growth in the model in a manner similar to how we compute year-ahead employment growth in (32). To compute book leverage in the model, we define a measure of book debt and a measure of book assets. First, we define book assets as $\kappa \exp(z)$ where κ is a free parameter that we calibrate such that the average value of book leverage matches the average value of book leverage in Compustat. Second, we define book debt in the model as short-term debt plus one-half long-term debt. With some algebra, we can derive book debt for a given value of n as:

$$\frac{\bar{a}}{(1-\tau^d)} \exp(z) \exp(-\Delta_z n) \frac{1-\frac{1}{2}\beta^{\Delta}}{1-\beta}\,, \tag{28}$$

where $\bar{a}$ is defined as in (34). Notice, $\exp(z)$ in book debt and book assets will cancel. We can then compute the ratio of changes in three-year-ahead sales growth to changes in book leverage at each point on the grid. We find values extremely close to the derivative in Versions (1)-(6) of our estimation procedure of -0.0389, -0.039, -0.0386, -0.039, -0.0377, -0.0416 at values of distance-to-default of 3.46, 2.31, 2.02, 2.89, 2.31, and 2.02, respectively. Two results stand out: (1) We can hit this derivative, if not exactly, almost exactly under all of our estimates. (2) As our estimate of b rises, the distance-to-default at which we are closest to this derivative falls.

We can also take as given the sample median in their paper for the book leverage of restructuring ski firms of 1.77, and compare the derivative implied by our value of book leverage closest to their number to their value of -0.039. In versions (4)-(6) of our estimation procedure, the values of distance-to-default closest to hitting this number are 2.89, 2.31, and 2.02, respectively, and the coefficients are -0.0553, -0.0454, and -0.0416. We get increasingly closer to hitting their number at their median book leverage for our estimate that controls for firm heterogeneity. The book leverage in their sample of highly levered ski firms corresponds to a low distance-to-default.

Comparing our Estimates to Those from a Structural Paper that Builds on Q-theory We compare our results to Table III, Column 4's estimate of the derivative of investment to the capital stock relative to the imputed market value of lenders recovery claim in default normalized by the capital stock (of -0.173) in Hennessy (2004). Our estimate of firm investment is defined as the firm's cost function (which scales in the firm's size, $\exp(z)$). The imputed market value of lender's recovery claim in default in the model is defined as

$$(1-\alpha)\exp(z)\exp(-\Delta_z n)E[\beta^{TD}]\,, \tag{29}$$

where $E[\beta^{TD}]$ is the discount rate at the firm's expected time of default, conditional on a firm's n. Because volatility follows a binomial process, we can exactly compute this value given the firm's policy function and the exogenous exit rate. The value of the firm upon entry is also in this term; however, since the cost of entry is one, this value will also be one in equilibrium. Given the capital stock's will cancel in both terms, as will $\exp(z)$, we can easily compute this derivative numerically as in the quasi-natural experiment by computing changes in investment relative to changes in the imputed market value of lender's recovery claim in default. We find the following: -0.178, -0.167, -0.175, -0.162, -0.168, and -0.167 at values of distance-to-default (backed out from n) of 6.93, 5.2, 4.33, 6.06, 5.2, and 4.91, respectively. Again, as b increases, the distance-to-default at which we hit this derivative falls. Notice, the average distance-to-default in our balanced panel of firms, the construction of which is based off of Hennessy (2004), is 4.81. Hence, for firms with similar characteristics as those in the balanced panel in Hennessy (2004), we are closest to the derivative of Hennessy (2004) in the procedures wherein we control for heterogeneity in firm growth rates.

5 Results from Counterfactuals under our Estimates

We now use our estimates to provide results from the counterfactuals defined in Section 3. We only discuss results from Versions (1)-(6) of our first estimation. Nonetheless, for the partial and general equilibrium counterfactuals for which there are figures for versions (1)-(6) (Figures 5, 7, 9, and 11), we include plots of our results for versions (7)-(12) in Figures 6, 8, 10, and 12, respectively.

5.1 Partial Equilibrium Counterfactuals

The first counterfactual we defined in Section 3 requires only the distribution of firm distance-to-default from the data and the solution to the problem of equity holders. This counterfactual compares growth rates of firms that do and do not suffer from debt overhang in our model under the estimates backed out from the estimation procedure described in Section 4.

Comparing Growth Rates of Firms that Do and Do Not Suffer from Debt Overhang In this counterfactual, we compare the estimated expected growth rate of firms across the distribution to the expected growth rate of firms were they to grow at the rate of firms that are unlevered. We find the following annual gains for versions (1)-(6) of our estimation procedures, respectively: 0.959%, 0.482%, 0.378%, 0.952%, 0.592%, and

0.515%. Given versions (1) and (4) do not demean at the firm level, we would expect that versions (2) and (3) would have values less than version (1), and versions (5) and (6) would have values less than version (4), and we find such a result. Notice, we are not testing the gains were all firms to have the population average of distance-to-default but rather what are the gains conditional on the distribution of distance-to-default. The result would be lower in the former case. We can intuit why from Figure 4a. The difference between q_∞ and q at a distance-to-default of 6.23, the average distance-to-default in Figure 2b, is small.

Given that we have already shown that the analytical approximation to the policy function for equity holders is an upper bound on the numerical version for all values we found, we expect the values for this counterfactual to be lower across the board. We find this to be this case: for versions (1)-(6) of our procedure, we find values of 0.806%, 0.435%, 0.348%, 0.831%, 0.541%, and 0.475%, respectively.

Using the Time-Varying Distribution of Distance-to-Default in the Data for Firm Value Counterfactuals at Each Time t In this next counterfactual, we take the distribution of distance-to-default year by year and perform the same counterfactual as above. In Figure 2, we show the distribution is usually close to that of the stationary distribution, but in 2008 it compresses. Hence, if we look at the average implied growth rate in 2008, we should expect it to fall.

Figure 7 tests for the potential gains from having firms all choosing the investment policy of the unlevered firm. As we expect, these gains increase during times when the distribution of distance-to-default compresses. Similarly, we find that the gains are higher when b is lower.

5.1.1 Partial Equilibrium Counterfactuals that Require the Debt Contract be Specified

Firm Value Counterfactuals Similar to the counterfactuals above, conditional on n, we can compare two firms, one that suffers from debt overhang (equity holders make the investment decision) and one that does not (the firm as a whole makes the investment decision), and compare their value functions or expected annualized growth rates assuming prices and the mass of firms do not change. The only difference now is that we assume a debt contract, so we can compute the value of the firm.

The Gains from Resolving Debt Overhang for Firm Value Upon Entry Decomposed Debt overhang in our model is a highly nonlinear problem. We demonstrate this in Figure 7 across versions of our estimation procedure. We plot the difference in expected annualized growth rates between two firms, one that does not suffer from debt overhang (the firm as a whole makes the investment decision), and one that does (equity holders make the investment decision) for a given value of distance-to-default, assuming prices and the mass of firms do not change.

We estimate that the expected gains from resolving debt overhang are modest for an entering firm (in terms of firm value). The blue bar on the left side in a given panel of Figure 9 presents the gains as a percent of firm value upon entry. Following Moyen (2007), we decompose these into the gains from operations, the gains from the tax advantage, and the losses from bankruptcy. The value from operations is the expected discounted present value of the firm's production and investment activities. The tax advantage of debt is the expected discounted present value of all interest deductions. The default cost is the expected discounted present value of the deadweight losses from bankruptcy. Most of the gains in partial equilibrium come from gains in terms of the value of operations because, in expectation, the firm makes better investment decisions near default. The firm also anticipates that it will suffer less from debt overhang, so it takes on more debt. In turn, the average entering firm gains more from the tax shield, but also goes bankrupt more often, and these two effects mostly offset.

In general equilibrium, the free entry condition implies that there will be no expected gains in terms of firm value upon entry from resolving this problem.

5.2 General Equilibrium Counterfactuals

We explain how to compute our general equilibrium counterfactual objects in Subsection 3.2. In Figure 11, we show that the social losses vary with the estimation procedure, which are expressed in terms of baseline consumption. More interesting in our case is the decomposition, especially the gains from resolving debt overhang. As we can see, they do not vary substantially with a changing b. The gains do not vary much as we vary b because in the long run when a large mass of firms increase their innovation decisions they raise the cost of labor, and production, entry, and process innovation all require labor. Further, the aggregate bankruptcy rate will rise, as firms will now have more leverage, on average, as they anticipate they will not suffer from debt overhang near default.

There are two other counterfactuals in this figure which we also explain how to compute in Subsection 3.2. The first other object is the effect of bankruptcy on the total mass of productivity. Recall, the interpretation of α in our model is that there is some mass of productivity that is being lost which will be costly to replace. We get rather large losses relative to the losses from debt overhang from this effect. There are a few points to make here given that there are other possible interpretations of α. One could interpret the costs of bankruptcy as not destroying any productivity, but being costly in terms of labor. In this case, we have a smaller, but still significant, blue bar. Another interpretation of α is that the cost of bankruptcy is a direct financial transfer; this will make the dark blue bar zero. In these two cases, the white bar, the social loss, will move close to proportionally with movements in the dark blue bar. Even though our estimation procedure will not change with these different interpretations, the interpretation of bankruptcy is very important in translating the costs of bankruptcy into social losses.

The light blue bar is the effect of taxes and bankruptcy on firm value. Even without debt overhang, there are losses from firm decisions being distorted by the tax advantage and bankruptcy, and these losses are comparable in size to the losses from debt overhang.

6 Conclusion

How do financial frictions affect the growth rate of firms, and what are the gains from resolving them over the business cycle and in the long run? This paper contributes to answering such a large question by developing and estimating a tractable structural model to assess the extent to which a specific financial friction, debt overhang, affects firm growth and aggregates. With our estimated model, we assess the expected private gains and long-run welfare gains from resolving debt overhang. Understanding the potential gains from resolving this problem over the business cycle conditional on the type of shock that hits the economy is the focus of current work in progress. An important complement to this line of research should be the development of realistic optimal contracts that could resolve debt overhang for firms.

Appendix A

In this appendix, we prove the statements in Propositions 1 and 2. The proofs in Proposition 2 first require we also derive the closed-form approximate solution to the problem of equity holders.

Proofs to Proposition 1

Recall, in Proposition 1 we outline properties of the problem of equity holders in a steady state where S_t is constant for all t, and assuming that firm types are constant for a given firm such that $\theta_t^i = \theta^i$ for all t for a given firm i. It is useful to define the Bellman function for equity holders in a steady state where θ, although heterogeneous across firms, is constant for a given firm:

$$\begin{aligned} V_E(z,d,\theta) \quad &= \quad \max_q \Big\{ 0, (1-\tau)(\exp(z)\Pi - w\exp(z)h\theta^{-b}\exp(bq)) - (1-\tau^d)d + \\ &\quad \beta\Big(qV_E(z+\Delta_z,d,\theta) + (1-q)V_E(z-\Delta_z,d,\theta)\Big)\Big\} . \end{aligned} \tag{30}$$

First, we will prove that the problem of equity holders can be reduced to two state variables: (1) the firm's investment opportunities, θ, and (2) the number of steps, Δ_z, until the firm declares bankruptcy, n.

Proof. Define $\hat{V_E} = \frac{V_E}{\exp(z)}$. We can thus redefine (30) as:

$$\begin{aligned} \hat{V_E}(n,\theta) \quad &= \quad \max_q \Big\{ (1-\tau)(\Pi - wh\theta^{-b}\exp(bq)) - \frac{(1-\tau^d)d}{\exp(z^*(\theta))}\exp(-n\Delta_z) + \\ &\quad \beta\Big(q\exp(\Delta_z)\hat{V_E}(n+,\theta) + (1-q)\exp(-\Delta_z)\hat{V_E}(n-1,\theta)\Big)\Big\} , \end{aligned} \tag{31}$$

where the firm goes bankrupt if $n < 0$ and $\exp(z^*(\theta))$ is the bankruptcy threshold of a given type, θ. Define $\bar{a}(\theta) = \frac{(1-\tau^d)d}{\exp(z^*(\theta))}$. Given θ and d constant over time for a given firm,

$\bar{a}(\theta)$ is constant for a given firm over time. It can be easily verified that $\bar{a}(\theta)$ does not vary in d, because $\exp(z^*(\theta))$ is proportional in d. $\square$

Next, we prove that expected year-ahead employment growth can be fully characterized by the firm's innovation decision, (14), and parameters.

Proof. A firm's expected period-ahead growth rate in the model is $2q(n)\Delta_z - \Delta_z$. We can annualize this growth rate to recover Δy_t^i for firm i between year t and year $t+1$ as:

$$(2q(n)\Delta_z - \Delta_z + 1)^{\Delta} - 1 \tag{32}$$

$\square$

Lastly for Proposition 1, we prove the state variable n has a 1-1 mapping with firm distance-to-default:

Proof. The state vector (n, θ) for equity holders is the same as that for the problem of the firm as a whole, $V_A(n, \theta)$. Thus, we can define firm distance-to-default conditional on θ, $DD(\theta)$ as: $\frac{V_A(n,\theta) - V_B^*(\theta)}{\sigma_A}$, where $V_B^*(\theta)$ is the firm's default threshold and $\Delta_z = \sigma_A\sqrt{\frac{1}{\Delta}}$.[20] Note that $n(\theta) = \frac{V_A(n,\theta) - V_B^*(\theta)}{\Delta_z}$. In turn, $n(\theta) = DD(\theta)/\sqrt{\Delta}$. $\square$

Closed-Form Approximation to the Problem of Equity Holders

To recover a closed-form approximation for the value function and the choice of q in (30) and (10), respectively, we take the following steps. We first solve for a closed-form solution for the value function with constant aggregates for the problem where firms do not optimize how much process innovation to undergo conditional on their leverage and always choose q as if they were the unlevered firm. We then plug in this solution into the optimal choice of q in the problem with optimally chosen process innovation. As this and the next subsection are solved in steady state, and are used to demonstrate identification and will not be referenced when defining an equilibrium, we drop all time subscripts.

Also, when firm's types are fixed, given h, we can solve for each type's Bellman individually given the choice of q of the unlevered firm (which we describe how to recover given θ in Section 4) and associated value of the Bellman of the unlevered firm, and noting that the bankruptcy threshold may be different conditional on θ. Hence, we solve for the closed-form approximation over only n, assuming that we are doing this solution procedure for each θ and that we have chosen the h to be the h were $\theta = 1$.

From (30), the optimal innovation decision of equity holders is

$$q^*(n) = \frac{1}{b} log\Big(\frac{\beta\Big(\exp(\Delta_z)V_E(n+1) - \exp(-\Delta_z)V_E(n-1)\Big)}{w(1-\tau)bh}\Big). \tag{33}$$

[20] Δ_z has a close relationship with asset volatility, σ_A. In particular, $\Delta_z = \sigma_A\sqrt{4q(1-q)\frac{1}{\Delta}}$, where q is the average q in the economy. If we assume this q is 0.5, which is true if we look at a high enough frequency of data, then $\Delta_z = \sigma_A\sqrt{\frac{1}{\Delta}}$.

The default threshold, $\exp(\overline{z})$, is proportional in debt outstanding. Thus, $(1-\tau^d)\frac{d}{\exp(\overline{z})}$ is a constant when aggregates are fixed. Define:

$$\overline{a} = (1-\tau^d)\frac{d}{\exp(\overline{z})}. \tag{34}$$

Now, consider the Bellman in (30) except where equity holders always invest as if they were unlevered. Call the innovation decision of the unlevered firm q_∞. Call the Bellman in this case, $\tilde{V_E}$.

$$\begin{aligned} \tilde{V_E}(n) &= (1-\tau)\Big(\Pi - w\phi(q_\infty)\Big) - \overline{a}\exp(-\Delta_z n) + \beta q_\infty \exp(\Delta_z)\tilde{V_E}(n+1) \\ &\quad + \beta(1-q_\infty)\exp(-\Delta_z)\tilde{V_E}(n-1). \end{aligned} \tag{35}$$

We also know

$$\tilde{V_E}(0) = 0. \tag{36}$$

We can easily solve for (35) with boundary condition (36), as this is a linear non-homogeneous second-order recurrence equation with a known solution:[21]

$$\begin{aligned} \tilde{V_E}(n) &= \frac{(1-\tau)\Big(\Pi - w\phi(q_\infty)\Big)}{1-\beta q_\infty(\exp(\Delta_z)-\exp(-\Delta_z))+\exp(-\Delta_z)}\Bigg(1 - \\ &\quad \overline{a}^{-\exp(\Delta_z)(n+1)}\frac{\beta(1-q_\infty)\exp(-\Delta_z) - 1 + \frac{1}{2}(1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)})}{\exp(-\Delta_z)(\beta(1-q_\infty) - 1 + \frac{1}{2}(1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}))} \\ &\quad - \frac{\beta(1-q_\infty)(1-\exp(-\Delta_z))}{(\beta(1-q_\infty) - 1 + \frac{1}{2}(1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}))}\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n\Bigg). \end{aligned} \tag{37}$$

Now, suppose we have a general cost function as in (6). We plug in (37), our closed-form approximation to the Bellman of equity holders, into the optimal policy function for q for the problem **with** optimally chosen process innovation, (33), to recover a closed-form approximation to (33).

In turn, we can define the choice of q as

$$\tilde{q}^*(n) = \frac{log\left(\frac{\beta\frac{(\tilde{\Pi}-\phi(q_\infty))}{1-\beta q_\infty(\exp(\Delta_z)-\exp(-\Delta_z))+\exp(-\Delta_z)}\left(\exp(\Delta_z)-\exp(-\Delta_z)-K\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty\exp(\Delta_z)}\right)^n\right)}{bh}\right))}{b}, \tag{38}$$

[21]Notice, also as $n \to \infty$, $V(\infty) = \frac{(1-\tau)(\Pi - whe^{bq_\infty})}{1-\beta q_\infty(\exp(\Delta_z)-\exp(-\Delta_z))+\beta\exp(-\Delta_z)}$.

where

$$K = \left(\exp(\Delta_z)\left(\frac{2\beta q_\infty \exp(\Delta_z)}{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}\right) - \exp(-\Delta_z)\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)\right)$$
$$* \frac{\beta(1-q_\infty)(1-\exp(-\Delta_z))}{(\beta(1-q_\infty)-1+\frac{1}{2}(1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}))} \tag{39}$$

and $\tilde{\Pi} = \frac{\Pi}{w}$.

Hence, we can recover $\tilde{q^*}$ as a function of n and parameters.

Proofs to Proposition 2

Recall, for Proposition 2 we will use the closed-form approximation to the problem of equity holders to define moments in closed form and prove some of their properties.

First, we show the derivative of firm expected employment growth to distance-to-default can be characterized in closed form, and its magnitude is proportional in $\frac{1}{b}$.

Proof. The derivative of q with respect to n is

$$\frac{\partial q}{\partial n} = \frac{1}{b}\frac{log\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n K}{(\exp(-\Delta_z)-\exp(\Delta_z)) + K\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n}, \tag{40}$$

where K is defined in (39). b does not enter into K. Hence, as $b \to \infty$, we find that $\frac{\partial q}{\partial n} \to 0$.

We can recover this derivative relative to DD by multiplying (40) by $\sqrt{\Delta}$. Both objects are proportional in $\frac{1}{b}$. Notice, to calculate the derivative of period-ahead growth with respect to n, $\frac{\partial 2q\Delta_z - \Delta_z}{\partial n}$ we just multiply $\frac{\partial q}{\partial n}$ by $2 * \Delta_z$. We can thus also recover the derivative of year-ahead growth (annualized from the period-ahead growth rate assuming $\Delta > 1$) with respect to n using $\frac{\partial 2q\Delta_z - \Delta_z}{\partial n}$.

$$\frac{\partial(2q\Delta_z - \Delta_z + 1)^\Delta - 1}{\partial n} = \frac{\partial 2q\Delta_z - \Delta_z}{\partial n}\Delta *$$
$$\left(\frac{2\Delta_z}{b} log\left(\frac{\beta}{bh}\frac{(\tilde{\Pi}-\phi(q_\infty))}{1-\beta q_\infty(\exp(\Delta_z)-\exp(-\Delta_z))+\exp(-\Delta_z)} *\right.\right.$$
$$\left.\left.\left(\exp(\Delta_z)-\exp(-\Delta_z) - K\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n\right)\right)\right)$$
$$\left.-\Delta_z\right)^{\Delta-1}. \tag{41}$$

Furthermore, we can find $\frac{\partial(2q\Delta_z - \Delta_z+1)^\Delta - 1}{\partial DD}$ by multiplying (41) by $\sqrt{\Delta}$. And we have the result that this derivative goes to 0 as $b \to \infty$. □

We now prove the second derivative of firm expected employment growth to distance-to-default can be characterized in closed form, and its magnitude is proportional in $\frac{1}{b}$.

Proof. We can find the second derivative of q with respect to n as:

$$\frac{\partial^2 q}{\partial n^2} = \frac{-K}{b}\frac{log\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^{2n}K}{\left((\exp(-\Delta_z)-\exp(\Delta_z))+K\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^{n}\right)^2}$$
$$+\frac{1}{b}\frac{log\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^2\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^{n}K}{(\exp(-\Delta_z)-\exp(\Delta_z))+K\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^{n}}. \tag{42}$$

We can recover this derivative relative to DD by multiplying (42) by $\sqrt{\Delta}$. Both objects are proportional in $\frac{1}{b}$. (42) is only decreasing in b when the first term is greater than the second. As before, we can find the second derivative of one-period growth with respect to n, $\frac{\partial^2 2q\Delta_z-\Delta_z}{\partial n^2}$, by multiplying $\frac{\partial^2 q}{\partial n^2}$ by $2\Delta_z$. And we can also thus recover the second derivative of year-ahead growth (annualized from the period ahead growth rate assuming $\Delta > 1$) with respect to n using $\frac{\partial 2q\Delta_z-\Delta_z}{\partial n}$:

$$\frac{\partial^2(2q\Delta_z-\Delta_z+1)^\Delta-1}{\partial n^2} = \frac{\partial^2 2q\Delta_z-\Delta_z}{\partial^2 n}\Delta *$$
$$\left(\frac{2\Delta_z}{b}log\left(\frac{\beta}{bh}\frac{\tilde{\Pi}-\phi(q_\infty)}{1-\beta q_\infty(\exp(\Delta_z)-\exp(-\Delta_z))+\exp(-\Delta_z)}*\right.\right.$$
$$\left.\left.\left(\exp(\Delta_z)-\exp(-\Delta_z)-K\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n\right)\right)\right.$$
$$\left.-\Delta_z\right)^{\Delta-1}+\left(\frac{\partial 2q\Delta_z-\Delta_z}{\partial n}\right)^2\Delta(\Delta-1)*$$
$$\left(\frac{2\Delta_z}{b}log\left(\frac{\beta}{bh}\frac{\Pi-w\phi(q_\infty)}{1-\beta q_\infty(\exp(\Delta_z)-\exp(-\Delta_z))+\exp(-\Delta_z)}*\right.\right.$$
$$\left.\left.\left(\exp(\Delta_z)-\exp(-\Delta_z)-K\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n\right)\right)\right.$$
$$\left.-\Delta_z\right)^{\Delta-2} \tag{43}$$

If (42) is decreasing in b, then (43) is decreasing in b, as the first term is greater in absolute value than the second term under the parameter restrictions we introduced. And again, we can find $\frac{\partial^2(2q\Delta_z-\Delta_z+1)^\Delta-1}{\partial DD^2}$ by multiplying (43) by $\sqrt{\Delta}$. □

We now prove the expected growth rate of the firm is decreasing in h, and the expected growth rate of the unlevered firm can be fully characterized as a function of Δ_z, h, and Δ.

Proof. The expected growth rate of the unlevered firm is:

$$(2q_\infty\Delta_z-\Delta_z+1)^\Delta-1\,, \tag{44}$$

which is clearly increasing q_∞. Since q_∞ is decreasing in h, expected year-ahead growth for the unlevered firm is thus decreasing in h. □

We now show the variance of annualized firm growth rates can be characterized as a function of the expected average growth rate of firms across the economy and Δ_z, and is increasing in Δ_z.

Proof. Denote the relative mass of firms at a given state (z, d, θ) as: $F(z, d, \theta) = \frac{\Gamma(z,d,\theta)}{\int\int\int \Gamma(z,d,\theta) dz dd d\theta}$ where $\Gamma(z, d, \theta)$ denotes the mass of firms for a given (z, d, θ) in steady state. We could also write this problem in terms of just two states: n and θ. We can then write the average q of the economy, $\bar{q}$, as

$$\bar{q} = \int\int\int F(z, d, \theta) q(z, d, \theta) dz dd d\theta. \tag{45}$$

The per-period variance of growth rates is then

$$= \int\int\int F(z, d, \theta) q(z, d, \theta) \Big(\Delta_z - \Big(2\bar{q}\Delta_z - \Delta_z\Big)\Big)^2 dz dd d\theta +$$
$$\int\int F(z, d, \theta)(1 - q(z, d, \theta)) \Big(-\Delta_z - \Big(2\bar{q}\Delta_z - \Delta_z\Big)\Big)^2 dz dd d\theta,$$

from the formula $V(x) = E[x - \bar{x}]^2$.

$$= \int\int\int F(z, d) q(z, d) \Big(\Delta_z - \Big(2\bar{q}\Delta_z - \Delta_z\Big)\Big)^2 dz dd d\theta$$
$$+ \int\int\int F(z, d)(1 - q(z, d)) \Big(-\Big(2\bar{q}\Delta_z\Big)\Big)^2 dz dd d\theta.$$

$$= 4\Delta_z^2 \Big(1 - \bar{q}\Big)^2 \int\int\int F(z, d) q(z, d) dz dd d\theta$$
$$+ 4\Delta_z^2 \bar{q}^2 \int\int\int F(z, d)(1 - q(z, d)) dz dd d\theta.$$

Now notice that we can use the definition of $\bar{q}$ to simplify it further:

$$= 4\Delta_z^2 \Big(1 - \bar{q}\Big)^2 \bar{q} + 4\Delta_z^2 \bar{q}^2 \Big(1 - \bar{q}\Big).$$

$$= 4\Delta_z^2 \bar{q}(1 - \bar{q}) \Big((1 - \bar{q}) + \bar{q}\Big).$$

$$= 4\Delta_z^2 \bar{q}(1 - \bar{q}). \tag{46}$$

$4\Delta_z^2 \bar{q}(1 - \bar{q})$ is clearly increasing in Δ_z holding $\bar{q}$ fixed.

□

Discussion of Local Identification So that we can eventually compare the model to data, we will recover regression coefficients in the model from the following regression:

$$\Delta y_{i,t} = \alpha + \beta_1 DD_{i,t} + \beta_2 DD_{i,t}^2 + \epsilon_{i,t} \tag{47}$$

Notice, $\frac{\partial \Delta y_{i,t}}{\partial DD_{i,t}}$ is equal to $\beta_1 + 2\beta_2 DD_{i,t}$, and $\frac{\partial^2 \Delta y_{i,t}}{\partial DD_{i,t}^2}$ is equal to $2\beta_2$, so given these two derivatives and given $y_{i,t}$ and $DD_{i,t}$ are known, we can back out regression coefficients β_1 and β_2.

We can estimate b, q_∞ (and h), and Δ_z with the moments above (the average growth rate of unlevered firms, the coefficients in (47), and the variance of employment growth rates), as β_1 and β_2 from (47), are proportional in $\frac{1}{b}$, the standard deviation of growth rates across firms outlined in (46) is proportional in Δ_z^2 and expected average growth of zero default risk firms, (44), is proportional in q_∞. We will need to estimate all parameters at once, as b affects the average growth rate in the economy in (46), as does q_∞, q_∞ enters into β_1 and β_2 as does Δ_z, and Δ_z enters into the average growth rate of zero default risk firms (although b does not). When we estimate the model, we will take as given the distribution of firms across distance-to-default from the data. By taking the distribution as given, we can estimate the parameters of the model with the solution to the problem of equity holders and avoid simulation of data. As we show in Section 4, our moments are locally identified, and driven by the expected parameters.

Additional Moment Conditions with Heterogeneity When we explicitly model firm heterogeneity, we argue that we can use the same moment conditions as above to locally identify b, Δ_z, and q_∞, with one additional step. Given a guess of b, Δ_z, and q_∞, we can use information on the joint distribution of firm distance-to-default and either firm growth rates or a measure of market to book to back out estimates of θ. In the model, two firms can have the same value of market to book but differ in their investment decision. It is only when we also condition on a firm's distance-to-default that we can obtain the firm's policy function. We describe the procedure we use to implement this approach in Section 4.

Appendix B

Data Construction

As described in Section 4, our empirical analysis relies on data from U.S. nonfinancial public firms. We take daily stock returns and other equity market data from CRSP and merge them with annual and quarterly accounting data from Compustat. We use the linking table from the CRSP/Compustat merged database to merge the datasets.

For the core sample of firms, we keep only firms with two-digit SIC codes that are not between 60 and 69, are less than 90, and are not equal to 49, following Hennessy and Whited (2007), as our model is not necessarily representative of regulated, financial, or public service firms. Following Hennesy and Whited (2005), we trim each series at the 2nd percentile except measures that are inherently bounded in nice ranges.

Variable Definitions

Market capitalization is defined as closing price times shares outstanding, and is the data equivalent of the value of equity in the model. To create our measure of distance-to-default, we require the book value of debt, which we define as short-term debt + one-half times long-term debt, where short-term debt is the max of debt in current liabilities (data item 34) and total current liabilities (data item 5), and long-term debt is data item 9. Employment is data item 29. We define book leverage as the book value of debt relative to the book value of assets (data item 6). We ask the reader to consult Whited and Wu (2006) for how to construct the Whited-Wu index.

To create our age measure, we download the entire time series for stock returns for each firm from CRSP. For each date, for each firm, dating back to 1926, we then state the age of a firm is 1 if it is the first date that shows up for the given firm. The age will then be 2 the next year, and so on.

Appendix C

Methodology for Computing V_A and σ_A

We follow a procedure consistent with Bharath and Shumway (2008) and Gilchrist and Zakrajsek (2012) in measuring firm V_A and σ_A, whose procedures are in the spirit of Merton (1974). V_A is the value of assets, V_B is the value of debt, μ_A is the mean rate of asset growth, and σ_A is asset volatility. We provide a detailed description of how we measure and clean the data in Appendix B. We recover V_A and σ_A from the data closely following the procedure outlined by Gilchrist and Zakrajsek (2012). For each firm, we linearly interpolate our quarterly value of debt to a daily frequency. We use daily data on the market value of equity; call this V_E. We guess a value of asset volatility, $\sigma_A = \sigma_E \frac{V_B}{V_E+V_B}$, where the standard deviation of the value of equity is calculated as the square root of the annualized 21-day moving average of squared returns for a firm. Here, we differ from Gilchrist and Zakrajsek (2012) in that they choose a 252-day horizon for the moving average.

Given our guess of σ_A, we use the following equation from Merton (1974):

$$V_E(t) = V_A(t)\Phi(d_1) - e^{-r(T-t)} * V_B\Phi(d_2)$$

where $d_1 = \frac{log(\frac{V_A}{V_B})+(r+\frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}}$ and $d_2 = d_1 - \sigma_A\sqrt{T}$ to recover the value of assets. We define r to be the one-year Treasury-constant maturity, which we take from the Federal Reserve's H.15 report. After converging on V_A for the given σ_A, we recompute σ_A from our implied V_A using the same methodology we use to compute σ_E. We ultimately converge on σ_A through a slow-updating procedure.[22]

[22] We iterate on both σ_A and V_A until they converge to a tolerance of 1e-5. We choose updating parameters for the slow-updating procedure on V_A and σ_A, .25 and .15, respectively, such that 100% of firms converge.

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Table 1
Estimation Procedure 1: Data Moments and Parameter/Model Moment Estimates Across Specfications

	Manufacturing Balanced Panel			Larger Core Sample		
	Demean by Yr. and Industry (1)	(1)+Demean by Firm —— (2)	(2)+Addl. Controls —— (3)	Demean by Yr. and Industry (4)	(4)+Demean by Firm —— (5)	(5)+Addl. Controls —— (6)
Data Moments						
β_1: From Eq. (47)	0.0165	0.00902	0.00709	0.0202	0.0121	0.0105
β_2: From Eq. (47)	-0.00123	-0.000722	-0.000614	-0.00136	-0.000856	-0.000735
Avg. Gr. for High DD Firms	-0.0136	-0.00389	-0.00351	0.0166	0.00776	0.00735
Avg. Std. Dev. of Emp. Gr.	0.165	0.152	0.146	0.173	0.159	0.154
Model Moments						
β_1: From Eq. (47)	0.0165	0.00903	0.0071	0.0202	0.0121	0.0105
β_2: From Eq. (47)	-0.0012	-0.000663	-0.000522	-0.00153	-0.000912	-0.000792
Avg. Gr. for High DD Firms	-0.0135	-0.00391	-0.00353	0.0166	0.0078	0.00732
Avg. Std. Dev. of Emp. Gr.	0.165	0.152	0.146	0.173	0.159	0.154
Parameter Ests. (S.E.'s)						
$\tilde{b}$	45.4	79.9	101	41.9	70.6	81.3
	(0.0293)	(0.037)	(0.00519)	(0.00274)	(0.002)	(0.0142)
$\tilde{\Delta}_z$	0.165	0.152	0.146	0.173	0.159	0.154
	(5.65e-06)	(4.07e-06)	(2.7e-05)	(4.89e-05)	(4.07e-05)	(1.56e-05)
q_∞	0.488	0.496	0.497	0.514	0.507	0.507
	(1.69e-05)	(1.94e-05)	(1.84e-05)	(6.79e-08)	(3.45e-05)	(2.47e-05)
Data Sample Properties						
#Firms	894	894	894	5650	5650	5650
Avg. # Employees	5130	5130	5130	5520	5520	5520
Avg. Distance to Default	4.81	4.81	4.81	4.61	4.61	4.61

Notes: Columns (1)-(3) present results from a balanced panel of manufacturing firms that exist between 1992 to 1995. In Column (1), employment growth is demeaned by year and industry average growth rates. In Column (2), employment growth is demeaned by year, industry, and firm average growth rates. In Column (3), employment growth as defined in Column (2) is regressed on the Whited-Wu index (a measure of access to external finance), firm age, and the natural logarithm of the number of employees in the firm. The residuals are the new measure of employment growth. Columns (4)-(6) present results from the larger unbalanced panel of nonfinancial firms from 1982 to 2012. In Column (4), employment growth is demeaned by year and industry average growth rates. In Column (5), employment growth is demeaned by year, industry, and firm average growth rates. Column (6) regresses employment growth as defined in Column (5) on the Whited-Wu index, firm age, and the natural logarithm of the number of employees in the firm as report in Compustat. The residuals are the new measure of employment growth. High DD is defined as a DD greater than 8. $\tilde{\Delta}_z$ is equal to $\frac{\Delta_z}{\sqrt{\Delta}}$, and $\tilde{b}$ is $\frac{b}{2\Delta_z\Delta}$.

Table 2

Estimation Procedure 2: Data Moments and Parameter/Model Moment Estimates Across Specfications

	Manufacturing Balanced Panel			Larger Core Sample		
	Fixed θ Demean by Yr. and Ind. (7)	Estimated θ Demean by Yr. and Ind. (8)	Estimated θ (7)+Addl. Controls (9)	Fixed θ Demean by Yr. and Ind. (10)	Estimated θ Demean by Yr. and Ind. (11)	Estimated θ (10)+Addl. Controls (12)
Data Moments						
β_1: From Eq. (47)	0.0165	0.0165	0.0127	0.0202	0.0202	0.0179
β_2: From Eq. (47)	-0.00123	-0.00123	-0.000841	-0.00136	-0.00136	-0.00119
Avg. Gr. for High DD Firms	-0.0136	-0.0136	0.01	0.0166	0.0166	0.0154
Avg. Std. Dev. of Emp. Gr.	0.165	0.165	0.161	0.173	0.173	0.171
Model Moments						
β_1: From Eq. (47)	0.0165	0.0164	0.0126	0.0203	0.0213	0.0182
β_2: From Eq. (47)	-0.00108	-0.00122	-0.000907	-0.00125	-0.00157	-0.00133
Avg. Gr. for High DD Firms	-0.0146	-0.014	0.0106	0.0151	0.0129	0.0135
Avg. Std. Dev. of Emp. Gr.	0.165	0.165	0.161	0.174	0.173	0.171
Parameter Ests. (S.E.'s)						
$\tilde{b}$	54.3	40.6	50.8	66	31	40
	(0.315)	(0.0401)	(0.276)	(0.972)	(0.971)	(0.341)
$\tilde{\Delta}_z$	0.165	0.165	0.161	0.174	0.173	0.171
	(9.6e-05)	(0.0165)	(8.6e-05)	(0.000637)	(0.000333)	(0.000372)
q_∞	0.487	0.488	0.51	0.512	0.511	0.511
	(0.000833)	(0.000842)	(0.00129)	(0.000867)	(0.0175)	(0.00802)
Data Sample Properties						
#Firms	894	894	894	5650	5650	5650
Avg. # Employees	5130	5130	5130	5520	5520	5520
Avg. Distance to Default	4.81	4.81	4.81	4.61	4.61	4.61

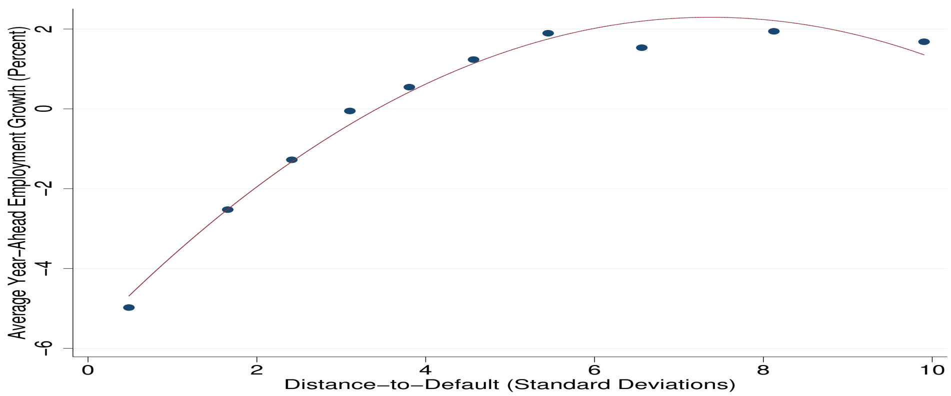
Notes: Columns (1)-(3) present results from a balanced panel of manufacturing firms that exist between 1992 to 1995. In Columns (1) and (2), employment growth is demeaned by year and industry average growth rates. In Column (3), employment growth demeaned by year and industry average growth rates is regressed on the Whited-Wu index (a measure of access to external finance), firm age, and the natural logarithm of the number of employees in the firm as reported in Compustat. The residuals are the new measure of employment growth. Columns (4)-(6) present results from the larger unbalanced panel of nonfinancial firms from 1982 to 2012. The estimates in these columns were estimated on a coarser grid for $\tilde{b}$, which is why they are less well estimated and $\tilde{b}$ is always a whole number. In Columns (4) and (5), employment growth is demeaned by year and industry averages. Column (6) regresses employment growth as defined in Columns (4) and (5) on the Whited-Wu index, age, and the natural logarithm of the number of employees in the firm as report in Compustat. The residuals are the new measure of employment growth. High DD is defined as a DD greater than 8. $\tilde{\Delta}_z$ is equal to $\frac{\Delta_z}{\sqrt{\Delta}}$, and $\tilde{b}$ is $\frac{b}{2\Delta_z\Delta}$.

Table 3

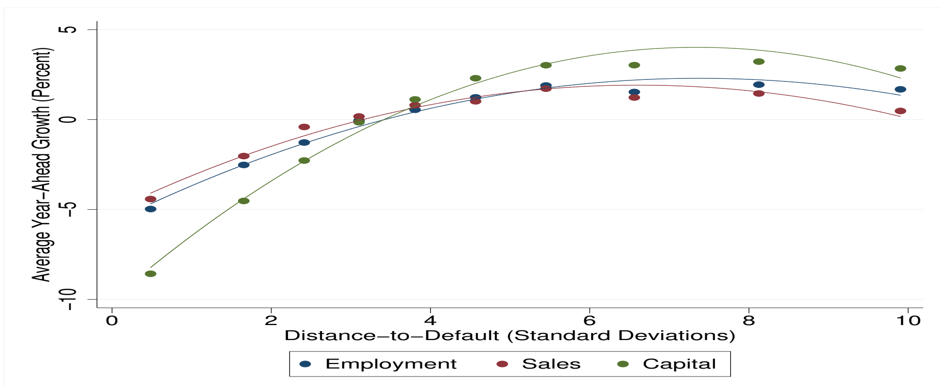
Remaining Parameterization

Parameters	Value
Affects Estimation	
Discount factor, β	0.994
Period length, Δ	$\frac{1}{12}$
Does not Affect Estimation	
Tax advantage of debt, τ_d	0.2
Retained value of the firm after bankruptcy, α	0.8
Elasticity of substitution across intermediate goods, ρ	4
Per-period entry cost, $n_e\Delta$	1
Total labor supply, L	1

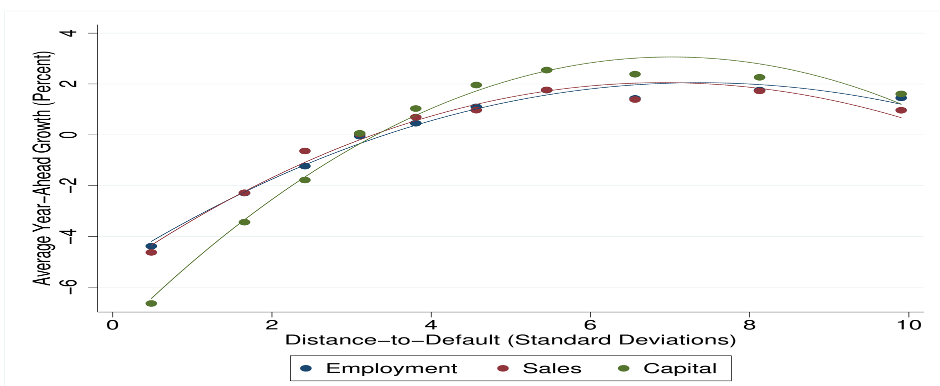
Figure 1 The Relationship between Distance-to-Default and Growth across U.S. Nonfinancial Public Firms



(a) Employment growth vs. Distance-to-default controlling for year and industry effects



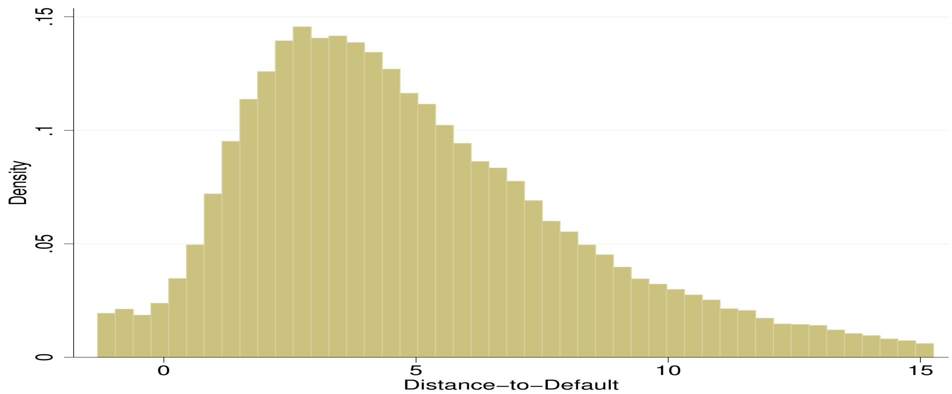
(b) Employment, Sales, and Capital growth vs. Distance-to-default controlling for year and industry effects



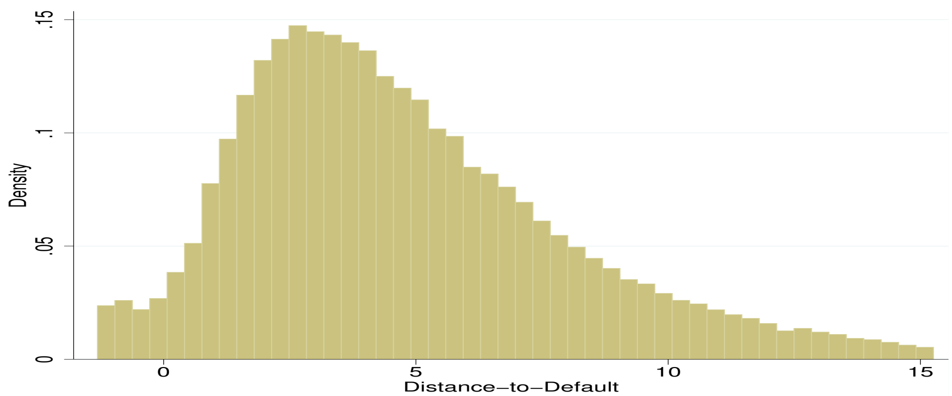
(c) Employment, Sales, and Capital growth vs. Distance-to-default controlling for year effects, industry effects, firm size, firm age, and a measure of firm access to external finance

Sample Period: 1982 to 2012. These figures present binned scatter plots (binned into 10 bins) of a residualized measure of year-ahead growth vs. firm distance-to-default. We also plot a quadratic fit line derived from the underlying data. The y-axis is the residuals from a linear regression on controls, whereas the x-axis is not controlled. Distance-to-default is measured using the methodology described in Appendix C.

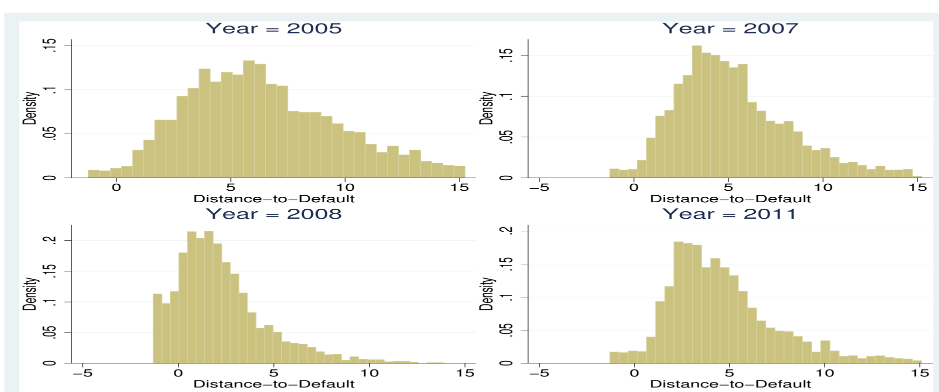
Figure 2 The Distribution of Distance-to-Default across U.S. Nonfinancial Public Firms



(a) The distribution of distance-to-default across firm-years excluding NBER recession years



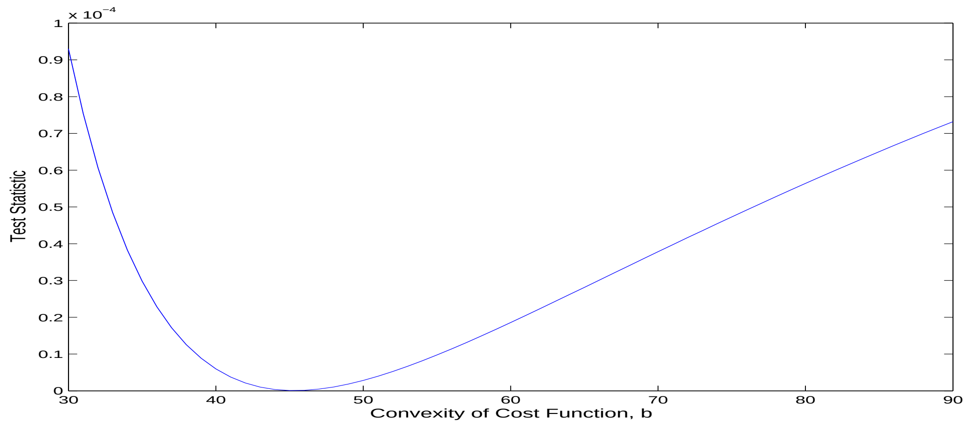
(b) The distribution of distance-to-default across firm-years including NBER recession years



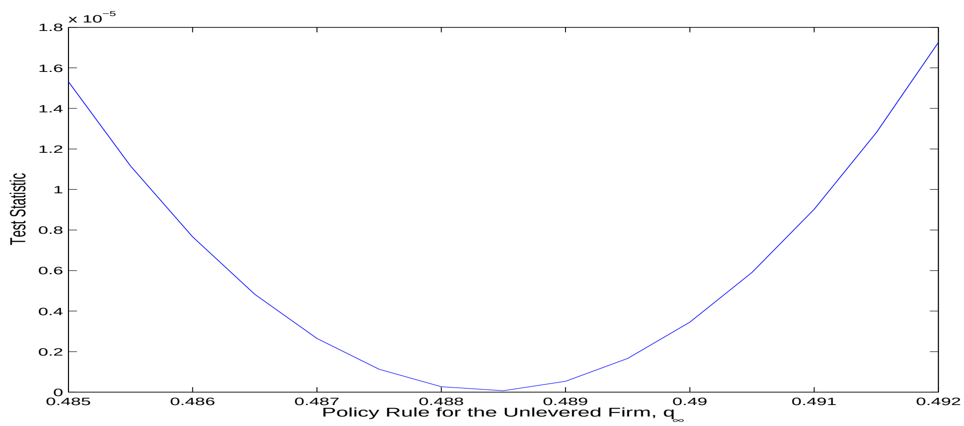
(c) The distribution of distance-to-default across firms in a given year

Sample Period: 1982 to 2012. The top subfigure plots the distribution outside of NBER recession years. The middle subfigure plots the distribution including NBER recession years. A given panel in subfigure 2 plots a histogram of the distribution of distance-to-default for a given year. Distance-to-default is measured as described in Appendix C.

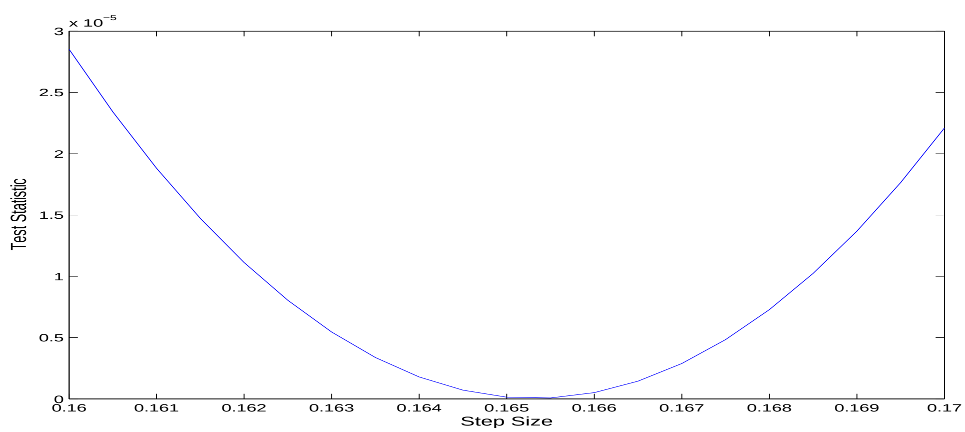
Figure 3 Local Identification of Parameters for Version 1 of the Estimation Procedure



(a) Holding Fixed Other Parameters at their Estimates, Varying $\tilde{b}$



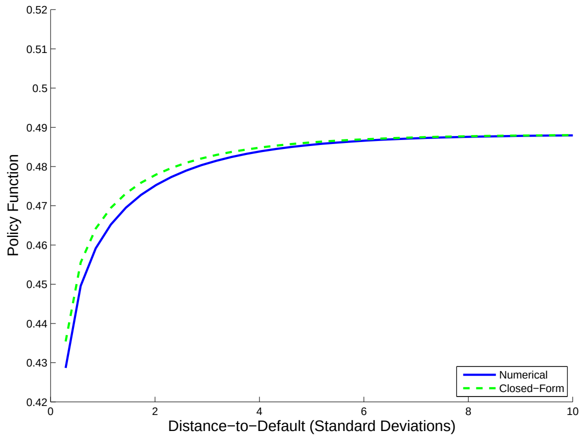
(b) Holding Fixed Other Parameters at their Estimates, Varying q_∞



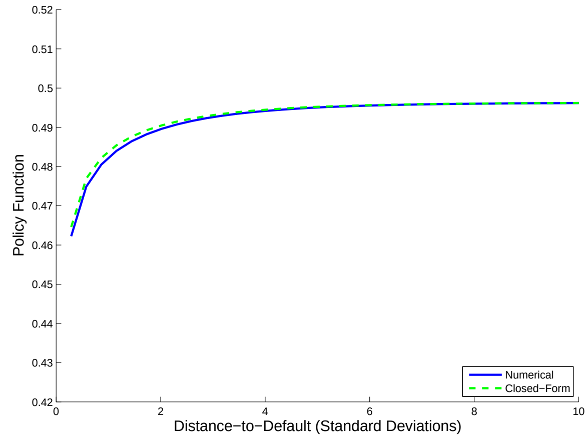
(c) Holding Fixed Other Parameters at their Estimates, Varying $\tilde{\Delta_z}$

In this plot, we hold fixed all parameters and vary one of the parameters, $\tilde{b}$, q_∞, and $\tilde{\Delta_z}$ around its local minimum. $\tilde{\Delta_z}$ is equal to $\frac{\Delta_z}{\sqrt{\Delta}}$, and $\tilde{b}$ is $\frac{b}{2\Delta_z\Delta}$. The line is the statistic we minimize over for our estimation procedure.

Figure 4 Policy Functions Compared across Estimates: Closed-form vs. Numerical



(a) Version 1



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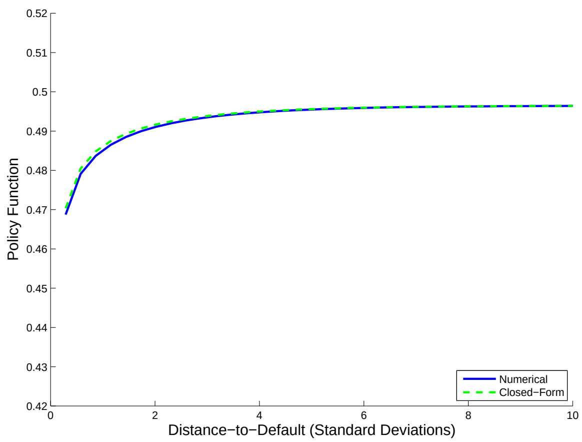
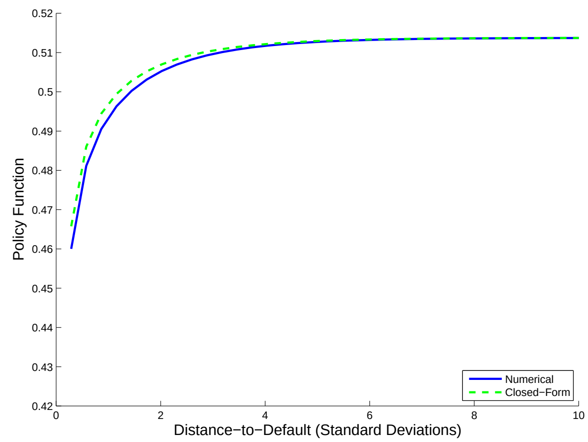


(c) Version 3



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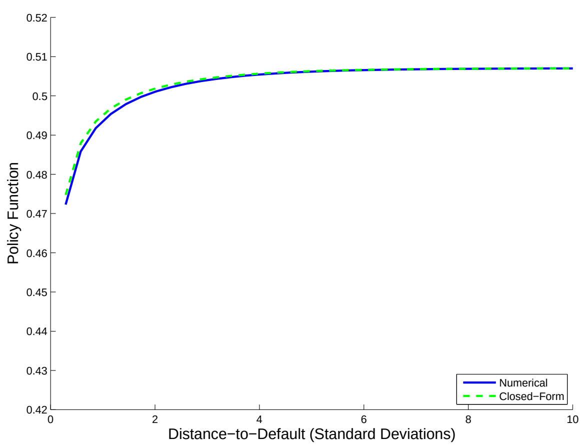
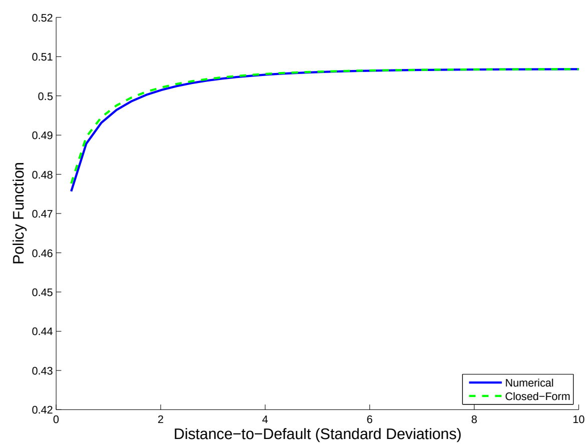


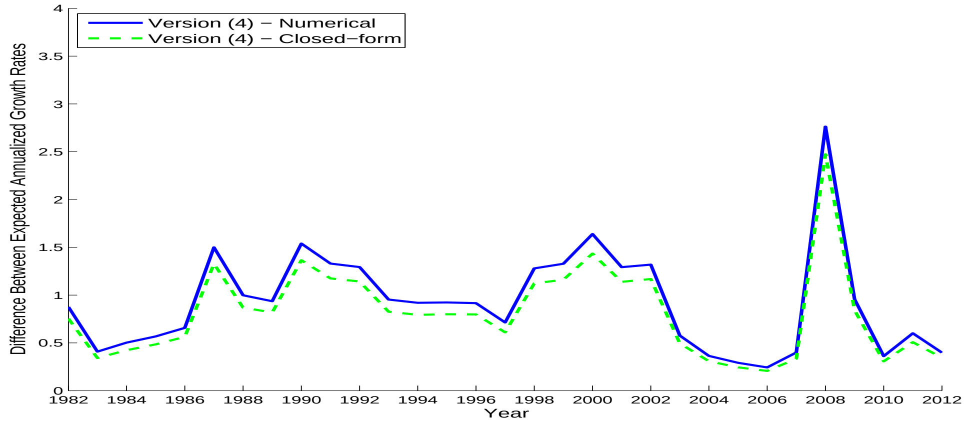
(e) Version 5



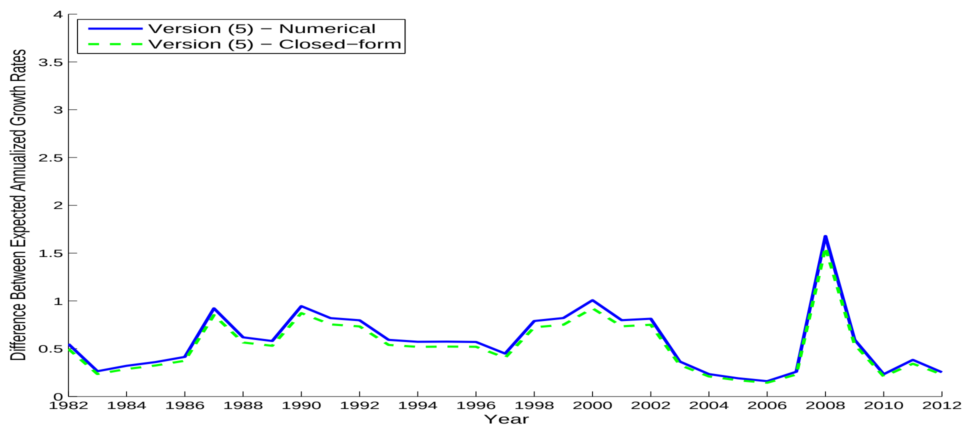
(f) Version 6

The figures above compare the closed-form approximation and the numerically solved policy function for the firm's innovation decision, q, in the model across the estimates in Table 1.

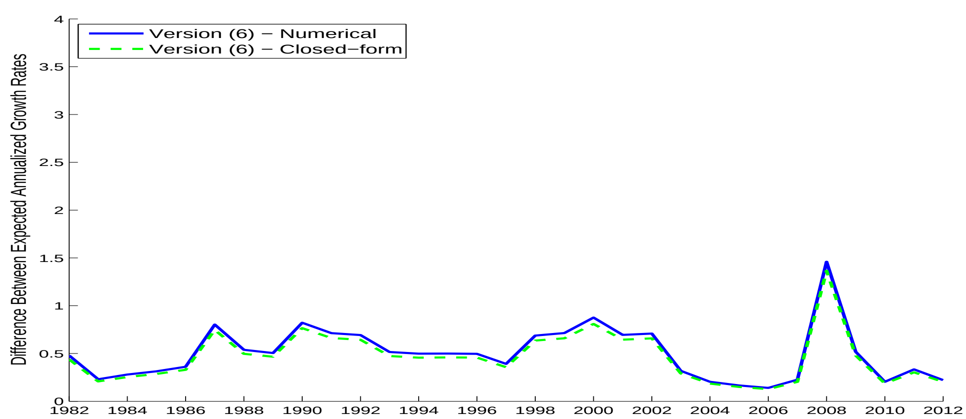
Figure 5 Estimation Procedure 1: Partial Equilibrium Gains from Resolving Debt Overhang Conditional on the Observed Distribution of Distance-to-Default in the Data in a given Year



(a) Version (4)



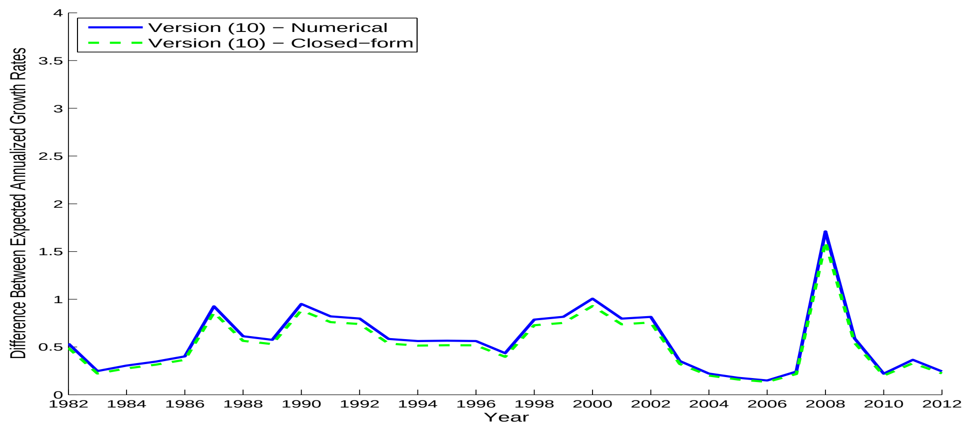
(b) Version (5)



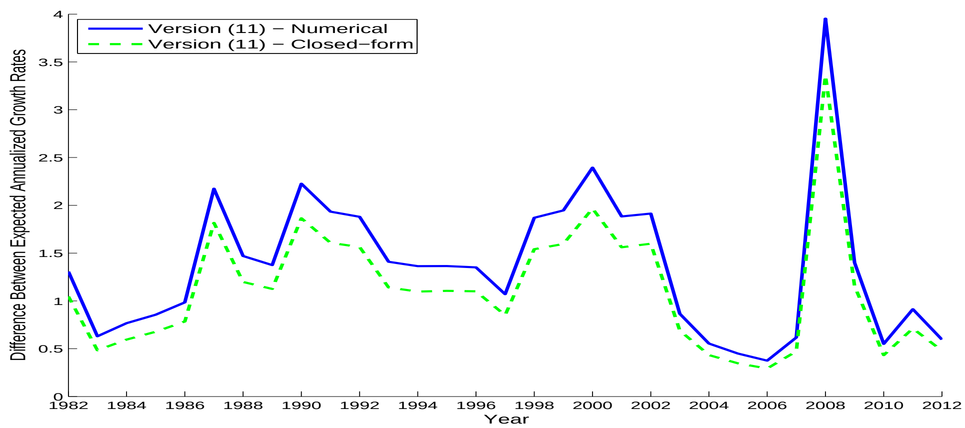
(c) Version (6)

For a given set of estimates in Table 1, the lines in a given panel above show the difference between the expected annualized growth rate of firms (in percentage terms) were all firms unlevered and the expected annualized growth rate of firms conditional on the observed distribution of distance-to-default among U.S. nonfinancial public firms in a given year. The green and blue lines respectively show the closed-form approximate and numerical solutions.

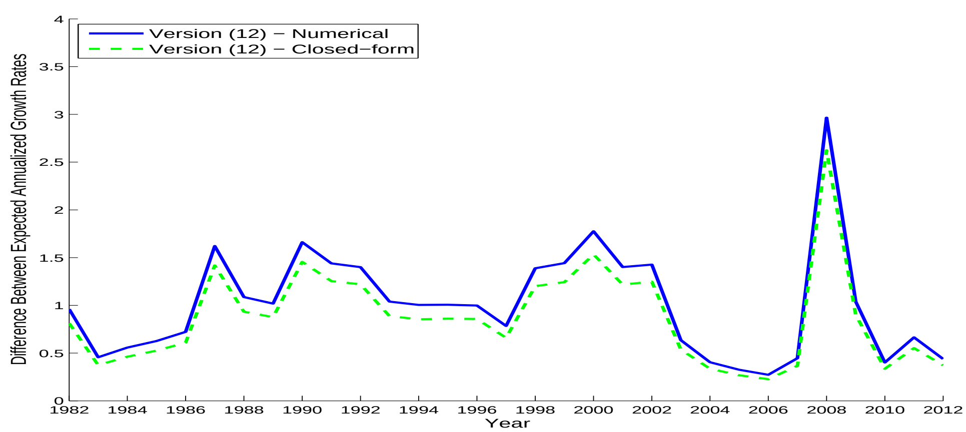
Figure 6 Estimation Procedure 2: Partial Equilibrium Gains from Resolving Debt Overhang Conditional on the Observed Distribution of Distance-to-Default in the Data in a given Year



(a) Version (10)



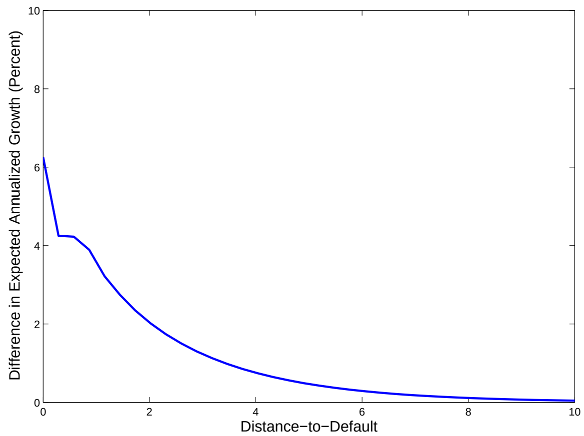
(b) Version (11)



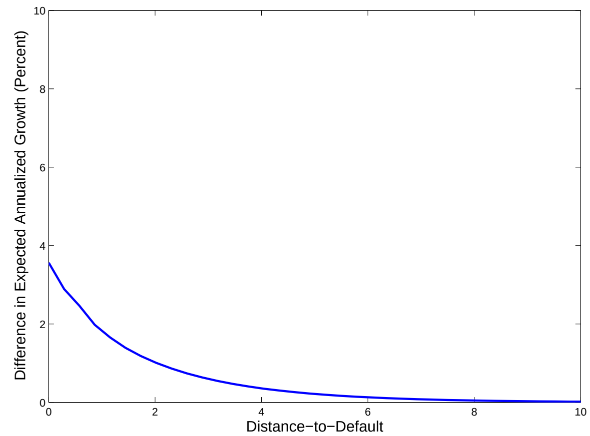
(c) Version (12)

For a given set of estimates in Table 2, the lines in a given panel above show the difference between the expected annualized growth rate of firms (in percentage terms) were all firms unlevered and the expected annualized growth rate of firms conditional on the observed distribution of distance-to-default among U.S. nonfinancial public firms in a given year. The green and blue lines respectively show the closed-form approximate and numerical solutions.

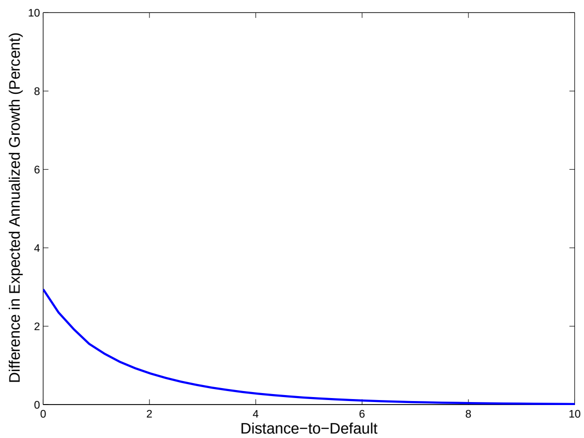
Figure 7 Estimation Procedure 1: Percentage Difference in Expected Annualized Growth between Debt Overhang and No Debt Overhang Case



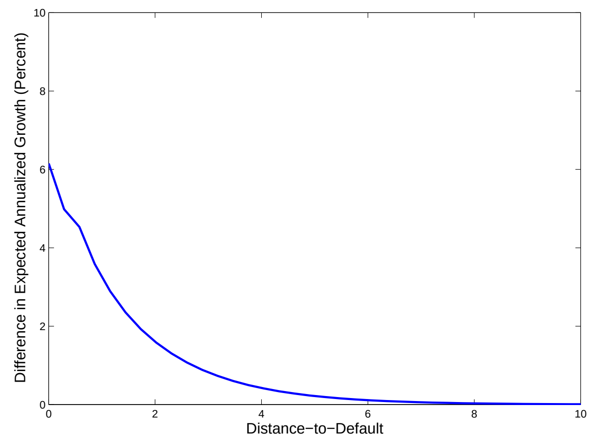
(a) Version 1



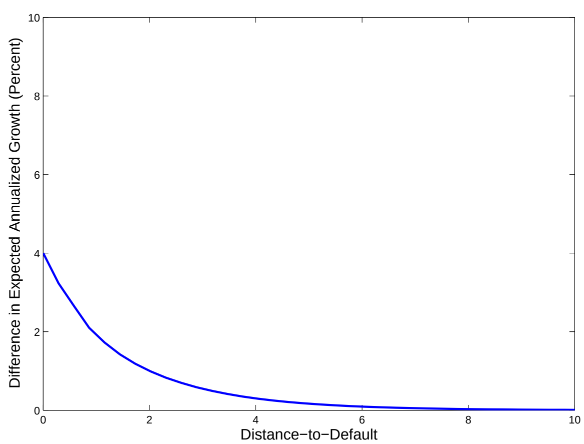
(b) Version 2



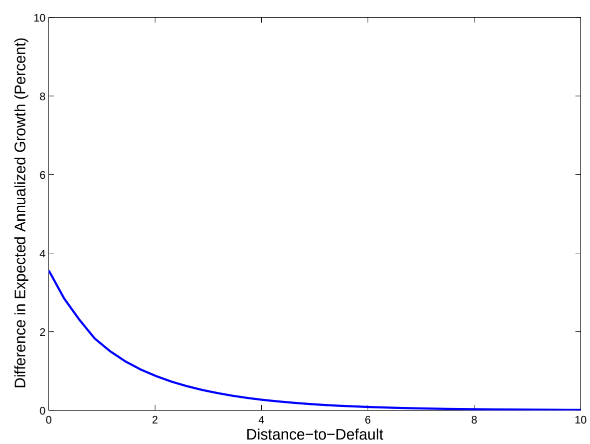
(c) Version 3



(d) Version 4



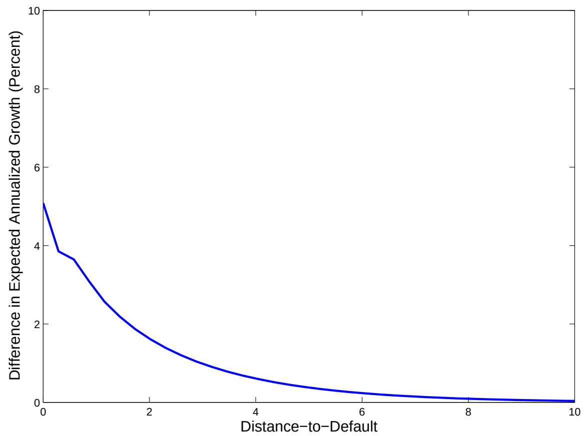
(e) Version 5



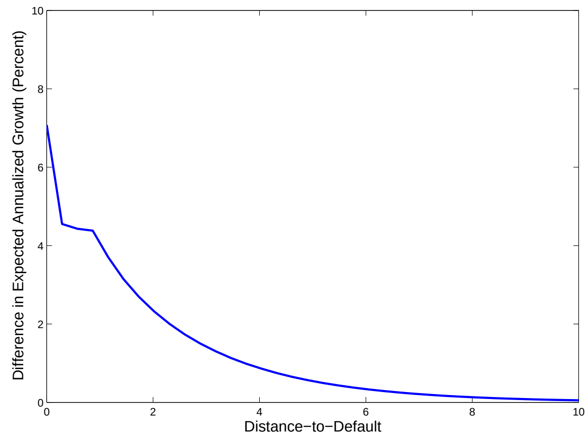
(f) Version 6

The blue line in a given panel is the difference in expected annualized growth in percentage terms between a firm that does not suffer from debt overhang and a firm that does conditional on firm distance-to-default. There exists a kink in most of the panels near default, because the default threshold changes between cases. The counterfactuals above are solved in partial equilibrium (prices and the mass of firms remain constant) and each panel refers to a set of estimates in Table 1.

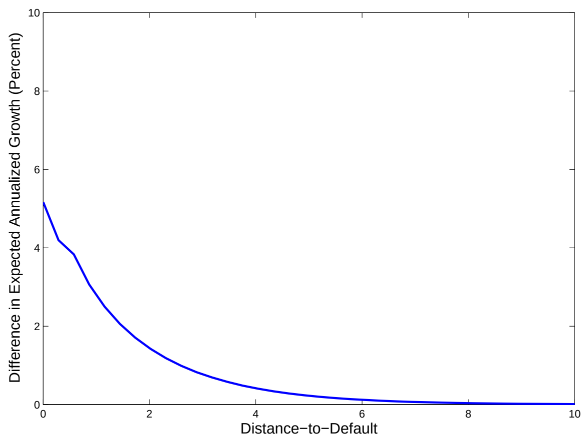
Figure 8 Estimation Procedure 2: Percentage Difference in Expected Annualized Growth between Debt Overhang and No Debt Overhang Case



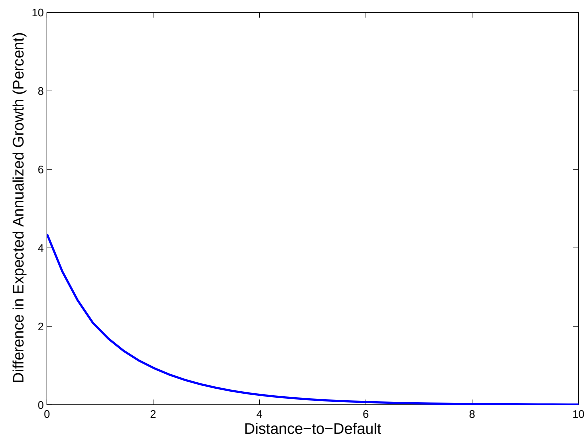
(a) Version 7



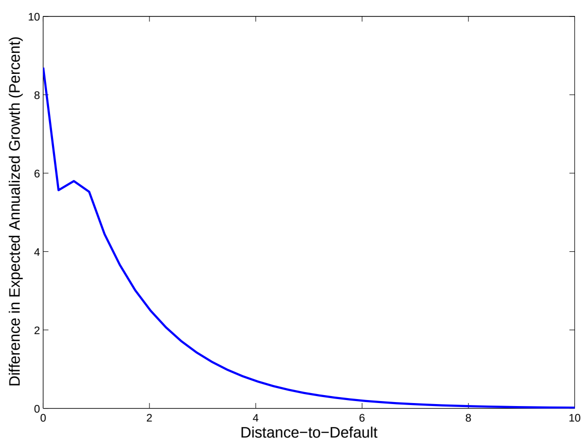
(b) Version 8



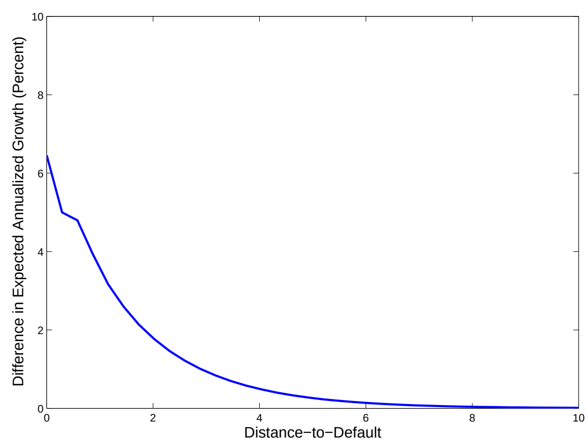
(c) Version 9



(d) Version 10



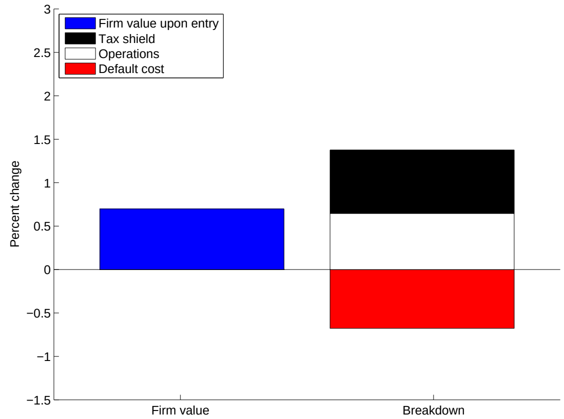
(e) Version 11



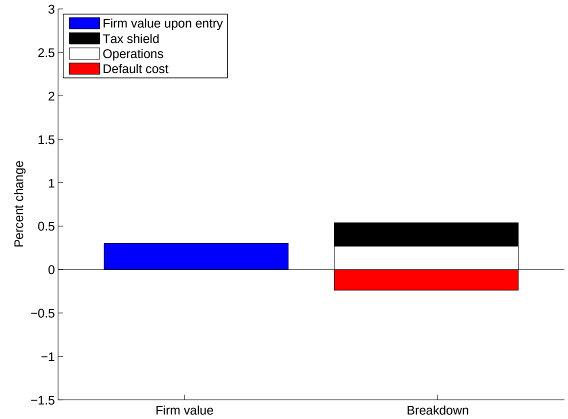
(f) Version 12

The blue line in a given panel the difference in expected annualized growth in percentage terms between a firm that does not suffer from debt overhang and a firm that does conditional on firm distance-to-default. There exists a kink in most of the panels near default, because the default threshold changes between cases. The counterfactuals above are solved in partial equilibrium (prices and the mass of firms remain constant) and each panel refers to a set of estimates in Table 2.

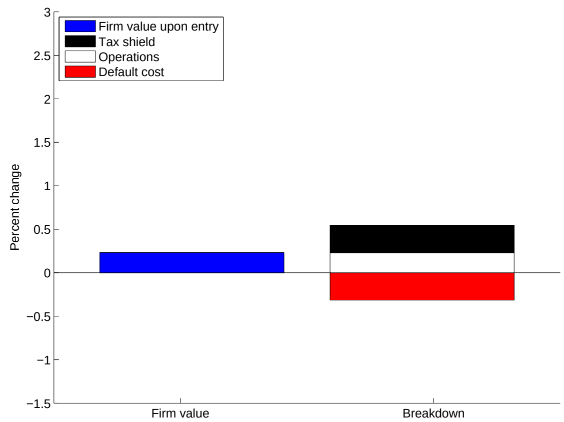
Figure 9 Estimation Procedure 1: Firm Value Decomposition Across Estimation Methods



(a) Version 1



(b) Version 2



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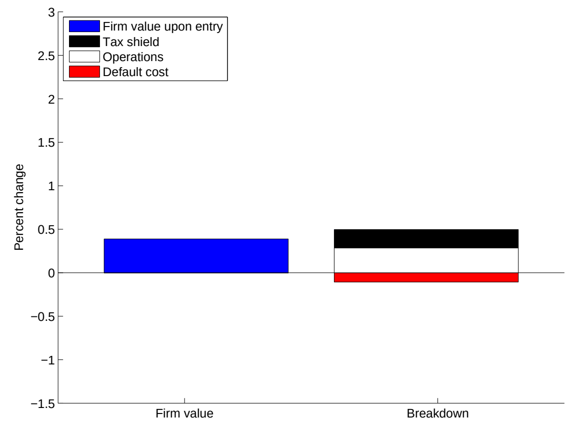

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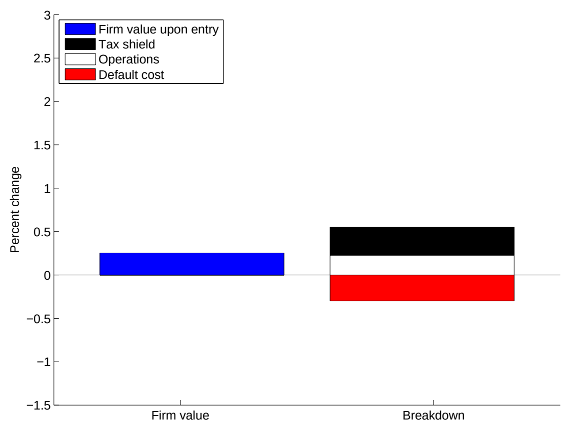
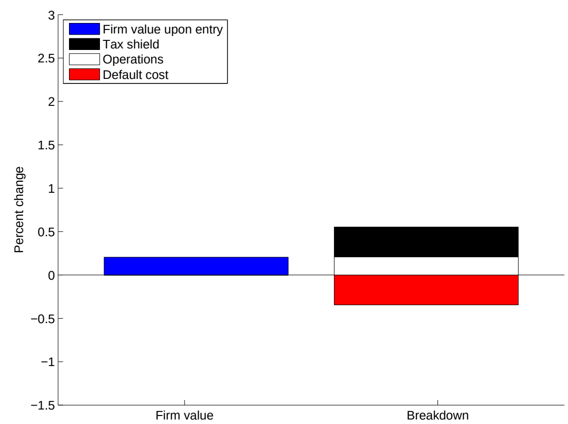


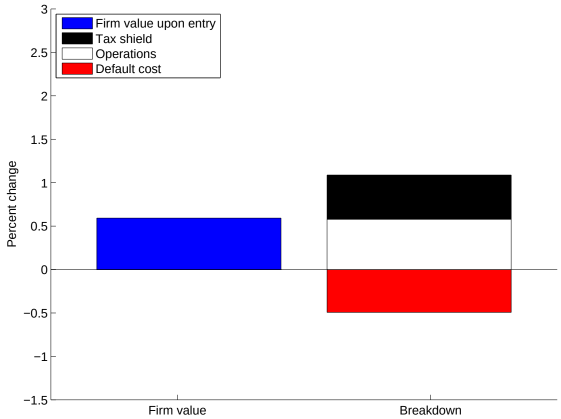
(e) Version 5



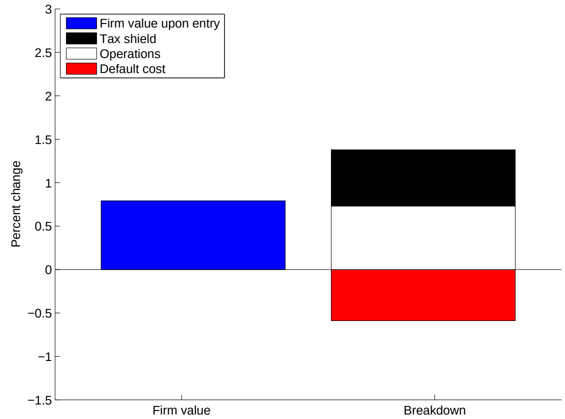
(f) Version 6

The counterfactuals above assess and decompose the partial equilibrium percent change in firm value upon entry between a firm that does not and a firm that does suffer from debt overhang across the estimates in Table 1. The value from operations is the expected discounted present value of the firm's production and investment activities. The tax shield is the expected discounted present value of all interest deductions. The default cost is the expected discounted present value of the deadweight losses from bankruptcy. The value of operations, the tax shield, and the default cost add up to the gains in firm value upon entry.

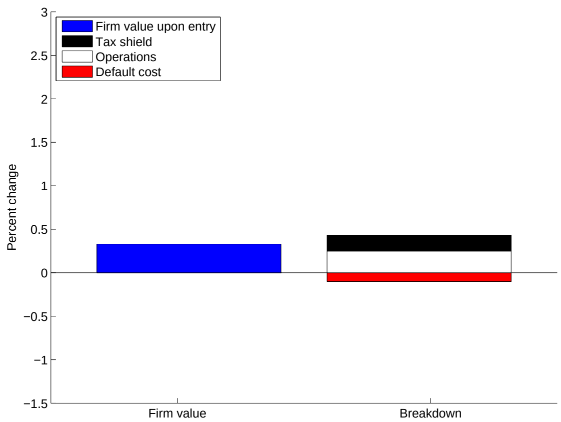
Figure 10 Estimation Procedure 2: Firm Value Decomposition Across Estimation Methods



(a) Version 7



(b) Version 8



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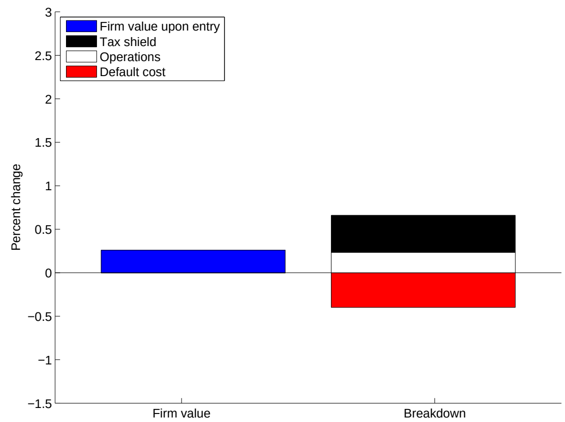

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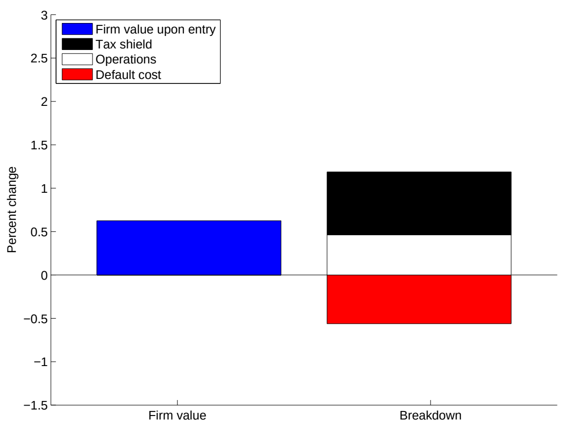
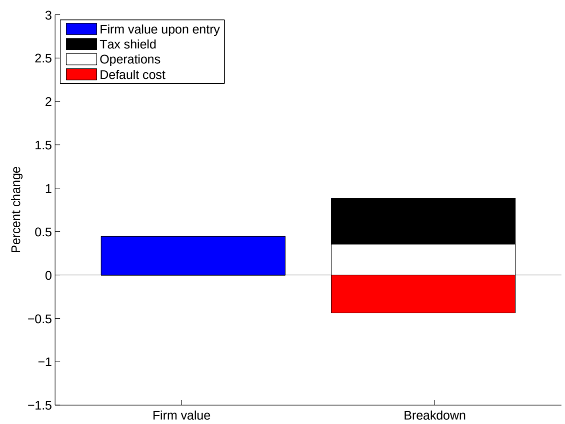


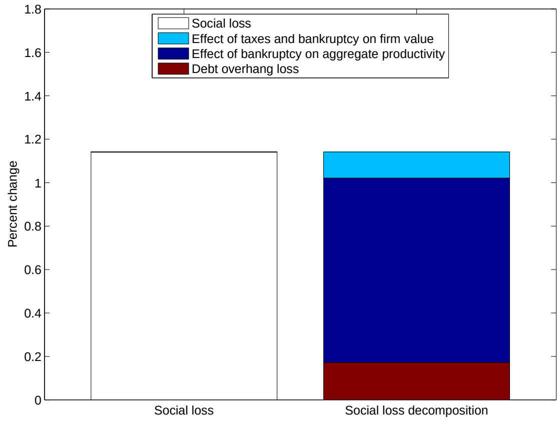
(e) Version 11



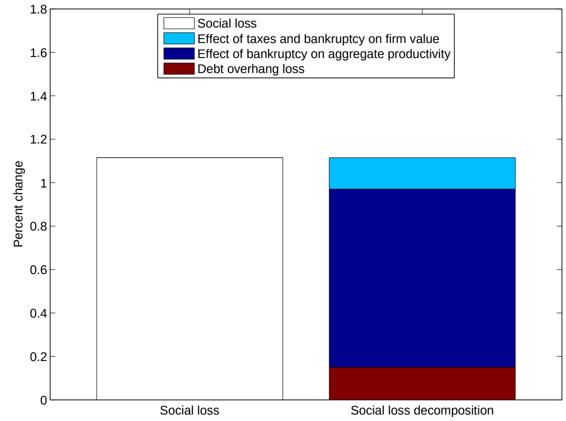
(f) Version 12

The counterfactuals above assess and decompose the partial equilibrium percent change in firm value upon entry between a firm that does not and a firm that does suffer from debt overhang across the estimates in Table 2. The value from operations is the expected discounted present value of the firm's production and investment activities. The tax shield is the expected discounted present value of all interest deductions. The default cost is the expected discounted present value of the deadweight losses from bankruptcy. The value of operations, the tax shield, and the default cost add up to the gains in firm value upon entry.

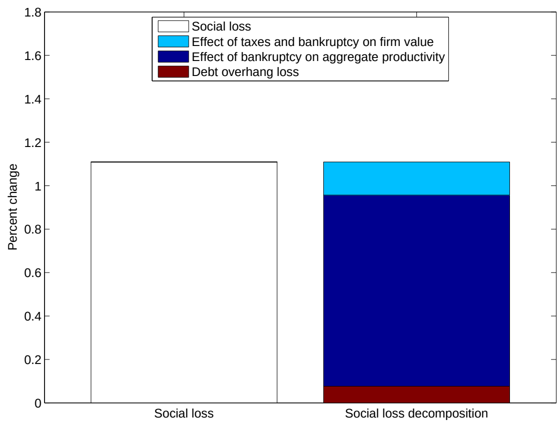
Figure 11 Estimation Procedure 1: Welfare Analysis Across Estimation Methods



(a) Version 1



(b) Version 2



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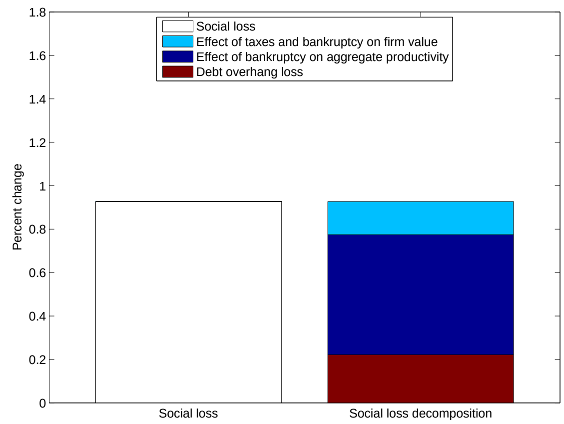

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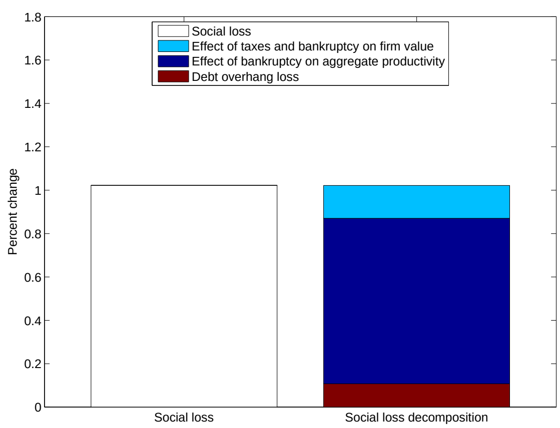

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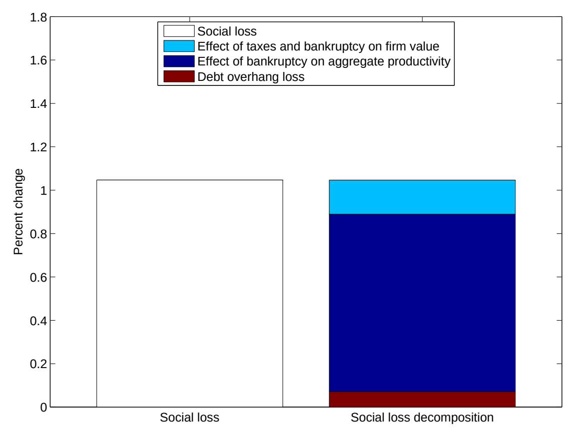
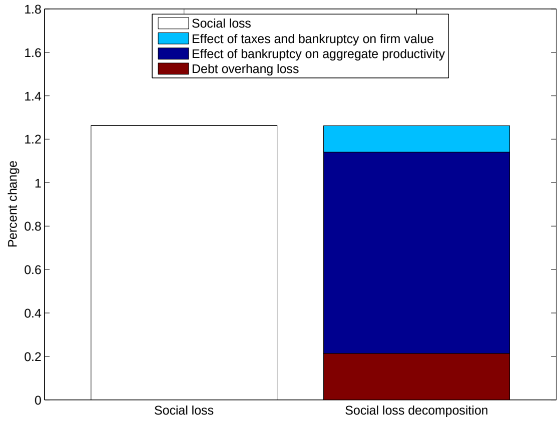


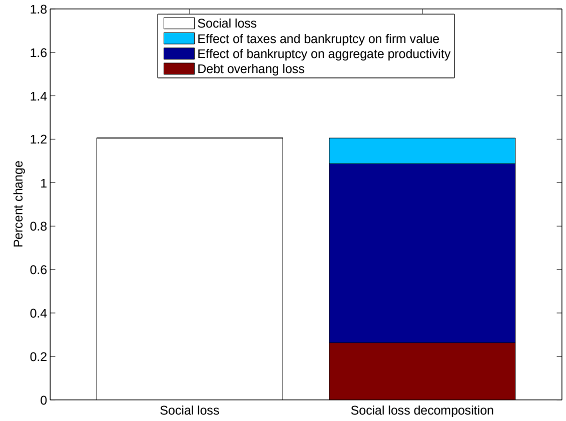
(f) Version 6

The counterfactuals above assess and decompose the long-run percent change between efficient and baseline consumption (social loss) across the estimates in Table 1. Debt overhang loss is the percent change between consumption in the steady state where debt overhang does not affect firm's investment decisions and steady-state baseline consumption. The effect of bankruptcy on aggregate productivity is the percentage change between steady-state consumption if debt overhang does not affect firm investment and firms do not lose $1-\alpha$ of their productivity in bankruptcy and the steady-state consumption if debt overhang does not affect firm investment decisions. The effect of taxes and bankruptcy on firm value is the percent change between the social loss and the sum of the two previous counterfactuals.

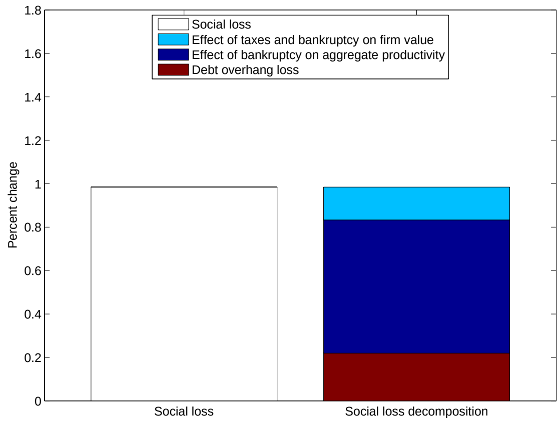
Figure 12 Estimation Procedure 2: Welfare Analysis Across Estimation Methods



(a) Version 7



(b) Version 8



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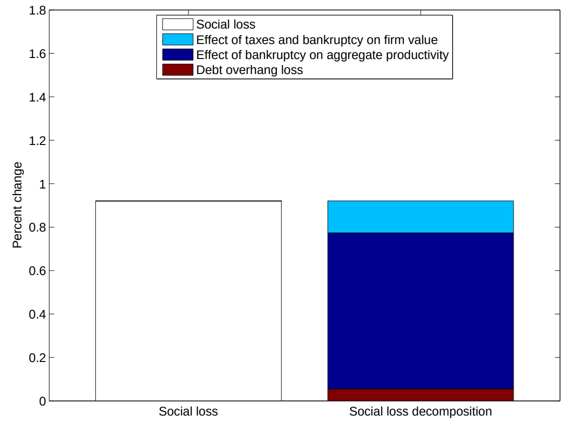

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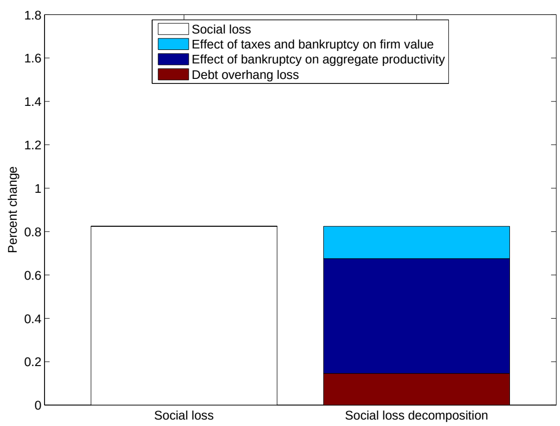

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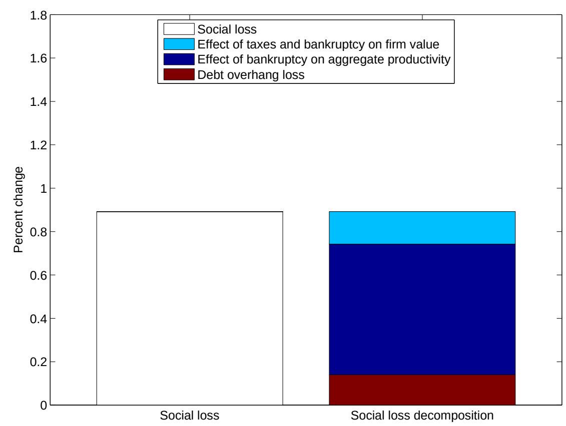

(f) Version 12

The counterfactuals above assess and decompose the long-run percent change between efficient and baseline consumption (social loss) across the estimates in Table 2. Debt overhang loss is the percent change between consumption in the steady state where debt overhang does not affect firm's investment decisions and steady-state baseline consumption. The effect of bankruptcy on aggregate productivity is the percentage change between steady-state consumption if debt overhang does not affect firm investment and firms do not lose $1-\alpha$ of their productivity in bankruptcy and the steady-state consumption if debt overhang does not affect firm investment decisions. The effect of taxes and bankruptcy on firm value is the percent change between the social loss and the sum of the two previous counterfactuals.