# Financial Heterogeneity and Monetary Union[*]

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**Abstract**

We analyze the business cycle and welfare consequences of monetary union among countries that face varying degrees of financial market distortions and whose firms compete in customer markets. In the absence of devaluation, firms experiencing an adverse liquidity shock in financially weak countries (the periphery) have an incentive to raise prices—and sacrifice their market share in order to overcome a temporary liquidity squeeze—while firms in countries with greater financial capacity (the core) lower prices, undercutting their financially constrained foreign competitors and gaining their market share. Because the latter do not internalize the effects of a price cut on union-wide aggregate demand, a monetary union among countries with heterogeneous financial capacity can create a tendency toward internal devaluation for core countries, leading to chronic current account deficits in the periphery. While a risk-sharing arrangement between the core and the periphery can undo the distortion brought about by the currency union, such an arrangement involves large wealth transfers from the core to the periphery. Depending on the degree of pecuniary externality not internalized by the predatory pricing strategies of individual firms, unilateral fiscal devaluation carried out by the peripheral countries can improve joint welfare of the union.

JEL Classification: E31, E32, F44, F45
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# 1 Introduction

The growing consensus in both academic and policy circles is that the eurozone's recent economic woes stem from a classic balance-of-payment crisis, which can be traced to the toxic mix of excessive credit growth and loss of competitiveness in the euro area "periphery." Following the introduction of the euro in early 1999, periphery countries such as Greece, Ireland, Italy, Spain, and Portugal went on a borrowing spree, the proceeds of which were used largely to finance domestic consumption and housing investment. Foreign investors' widespread reassessment of risks during the 2008–09 global financial crisis, along with a growing recognition of an unsustainable fiscal situation in Greece, precipitated a sharp pullback in private capital from the euro area periphery in early 2010. This further tightening of credit conditions significantly exacerbated the already painful process of deleveraging, through which the periphery economies were attempting to bring domestic spending—both government and private—back into line with domestic incomes.[1]

As shown by Figure 1, this narrative accords well with the empirical evidence. The solid line in panel (a) shows that the median current account deficit in the euro area periphery reached almost 10 percent of GDP on the eve of the global financial crisis, with some countries running current account deficits as high as 15 percent of their GDP.[2] The evidence of overheating that led to the crisis is provided in panels (b) and (c). Between 1999 and 2007, periphery economies saw their real GDP growing persistently above its potential, whereas their counterparts in the core registered a much more balanced pattern of economic growth (panel (b)). As a result, prices in the periphery increased at a much faster pace during this period compared with those in the core countries (panel (c)). Given these developments, real exchange rates in the periphery appreciated substantially (panel (d)), eroding the countries' competitiveness and producing large trade deficits, which were easily financed by foreign capital inflows against the backdrop of the convergence in domestic interest rates across the euro area.

In a currency union comprised of countries in a dramatically different economic condition and limited labor mobility and no common fiscal policy, the crisis had to be resolved largely through a significant downward adjustment of the overvalued real exchange rates in the periphery. In the euro area, however, this adjustment has occurred very slowly. Although the periphery has endured significant disinflation since 2010, a noticeable gap remains, on balance, between the general level of prices in the core and periphery (panel (c) of Figure 1). As a result, real effective exchange

[1] As emphasized by Auer (2014) and Higgins and Klitgaard (2014), the tightening of credit conditions was not as severe as might have been expected given the scale of capital flight from the periphery. The withdrawal of capital was tempered importantly by cross-border credits to deficit countries' central banks, extended by other euro area central banks through the so-called TARGET2 system, a mechanism for managing payment imbalances among eurozone member countries. In combination with policies to supply liquidity to banks in the periphery, this balance of payments financing helped offset the drain of funds abroad.

[2] Throughout the paper, we use the following definition of the euro area core and periphery. Core countries: Austria, Belgium, Finland, France, Germany, and Netherlands. Periphery countries: Greece, Ireland, Italy, Portugal, and Spain. We omit the Eastern European countries (Estonia, Latvia, Lithuania, Slovakia, and Slovenia) from the periphery because they adopted the euro relatively recently. Our analysis also excludes Cyprus, Luxembourg, and Malta because of limited data in some instances and because of their very specialized economies. All told, our sample of countries accounts for about 95 percent of the eurozone's total GDP.

rates in the periphery generally remain above those of the core euro-area countries (panel (d) of Figure 1).

What economic forces are responsible for such a slow adjustment in the price levels between the core and periphery countries? Why have firms in the periphery—given the degree of resource underutilization in these economies—been so slow to cut prices? By the same token, why have firms in the core been reluctant to increase prices, despite an improvement in the economic outlook and highly stimulative monetary policy? In fact, some prominent commentators have argued that it is the core countries that are exporting deflationary pressures into the periphery, a dynamic contrary to that needed to reverse the real exchange rate appreciation that has eroded the periphery's competitiveness (Krugman, 2014). The following quote from Sergio Marchionne, the CEO of Fiat Chrysler, at the nadir of the crisis in mid-2012 offers an intriguing insight:

> Mr. Marchionne and other auto executives accuse Volkswagen of exploiting the crisis to gain market share by offering aggressive discounts. "It's a bloodbath of pricing and it's a bloodbath on margins," he said.
>
> *The New York Times*, July 25, 2012

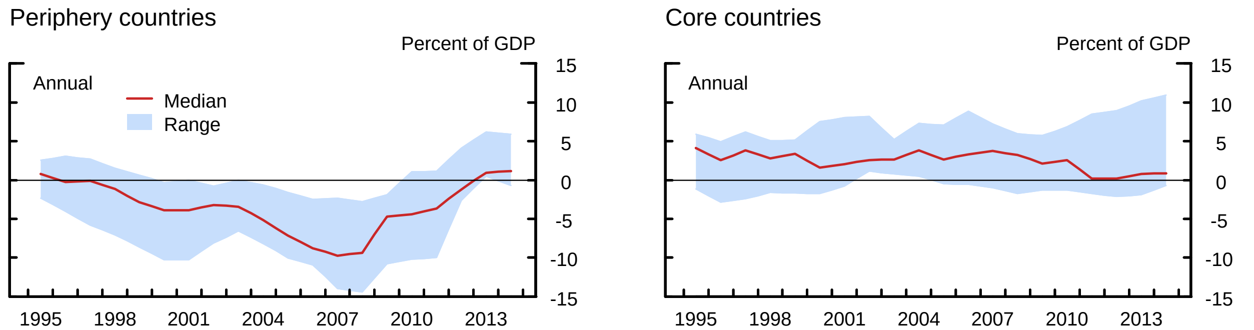
In this paper, we attempt to answer the above questions by formalizing Mr. Marchionne's observation within a multi-country dynamic stochastic general equilibrium model, featuring customer markets and financial market distortions.[3] Specifically, building on our earlier work (Gilchrist, Schoenle, Sim, and Zakrajšek, 2015), we study the macroeconomic consequences of creating a currency union among countries, where firms operate in customer markets—both domestically and abroad—price to market, and face differing degrees of financial frictions.[4] We show that in such an environment, firms from the "core"—that is, firms with relatively ample financial capacity—have a strong incentive to expand their market share at home and abroad by undercutting prices charged by their "periphery" competitors, especially when the latter are experiencing a liquidity crunch. Periphery firms in financial distress, by contrast, have an incentive to increase markups in order to ensure adequate internal liquidity, even though doing so means forfeiting some of their market share in the near term.

The idea that firms setting prices to actively manage current versus future expected demand are confronted—when experiencing a liquidity shortfall—with a tradeoff between maintaining current profits and the long-run consideration of their market share is not new to macroeconomics (Gottfries, 1991; Chevalier and Scharfstein, 1996). When applied to the euro area crisis, this mechanism helps explains why the periphery countries have managed to avoid a potentially devastating

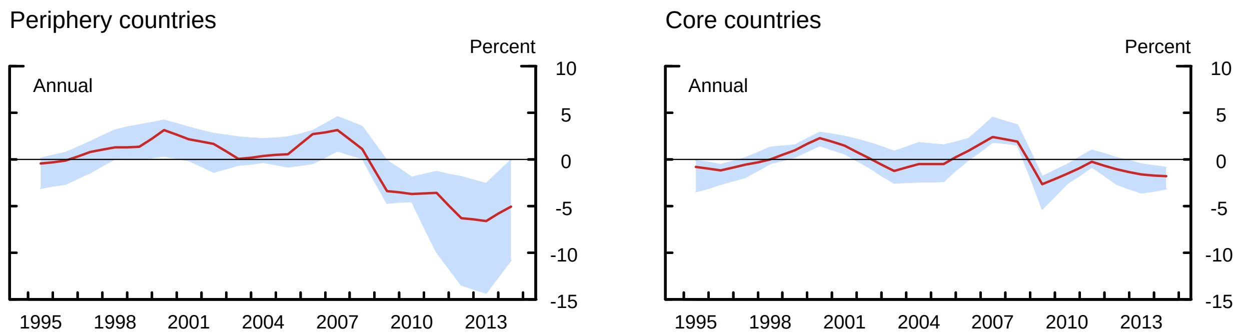
[3]By customer markets, we mean markets in which a customer base is "sticky" and thus an important determinant of firm's assets and its ability to generate profits. Various microeconomic mechanisms that can lead to sticky customer base include costly switching (Klemperer, 1987), costly search (Hall, 2008), or idiosyncratic preferences (Bronnenberg, Dube, and Gentzkow, 2012). As emphasized by Phelps and Winter (1970) and Bils (1989), firms' pricing decisions in such environment are a form of investment that builds the future customer base.

[4]The assumption of pricing to market—that is, of price discrimination across geographical areas—is strongly supported by empirical studies of firms' pricing-setting behavior in the euro area (Fabiani, Loupias, Martins, and Sabbatini, 2007).

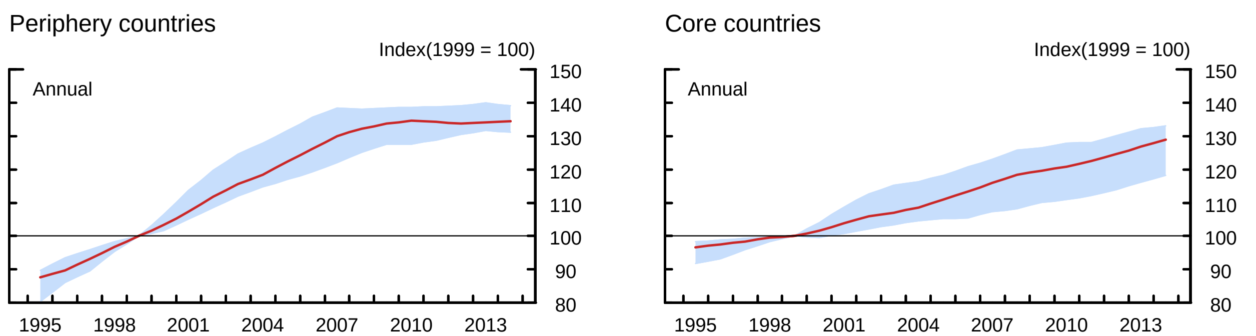
Figure 1: Macroeconomic Performance of the Euro Area, 19952014



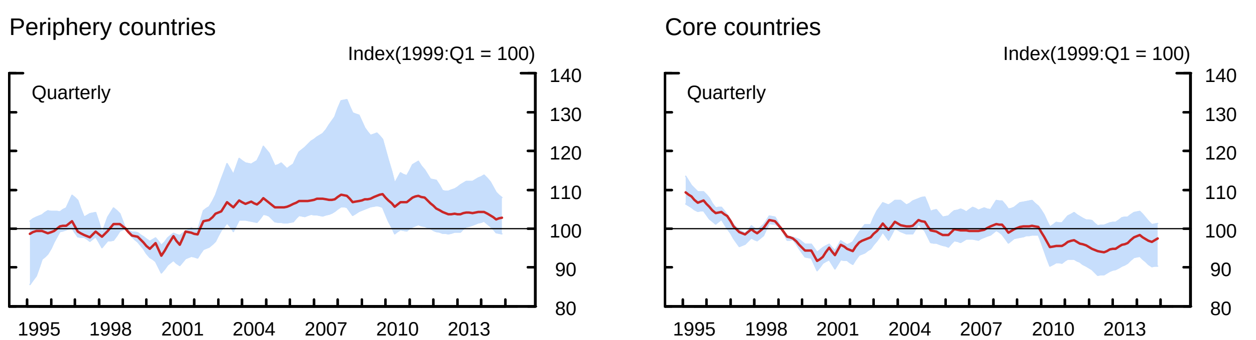

(a) Current accounts




(b) Output gaps




(c) Prices (GDP deflators)




(d) Real exchange rates

Note: The solid line in each panel depicts the time-series evolution of the cross-sectional median of the spec- ified macroeconomic series, while the shaded band denotes the corresponding cross-sectional range. Periphery countries: Greece, Ireland, Italy, Portugal, and Spain. Core countries: Austria, Belgium, Finland, France, Germany, and Netherlands.

Fisherian debt-deflation spiral in the face of massive and persistent economic slack. It also helps us to understand the chronic stagnation in the euro area periphery and how the "price war" between the core and periphery has impeded the adjustment process through which the latter economies have been trying to regain their competitiveness.

Our general equilibrium framework also allows us to compare macroeconomic outcomes under alternative exchange rate arrangements. We show that under a flexible exchange rate regime, monetary authorities in the periphery can to a large extent offset the effects of an asymmetric financial shock by cutting policy interest rates. The easing of monetary policy leads to a significant depreciation of nominal exchange rates in the periphery. And although the price levels between the core and periphery move in opposite directions because of customer markets considerations, the policy-induced currency devaluations can be sufficiently large to cause real exchange rates to depreciate, thereby boosting exports of firms in the periphery and helping to stabilize the contraction in output (see Friedman, 1953).

In a currency union, this policy option is, of course, not available. The pricing behavior of firms in the core in response to an asymmetric financial shock in the periphery implies a real exchange rate depreciation vis-à-vis the periphery, causing a small export-driven boom in the core countries and a deepening of the recession in the periphery.[5] The divergent economic trajectories between the core and periphery present a problem for monetary authorities because monetary policy cannot be targeted to any one region. According to our simulations, common monetary policy in a situation when members of a union are experiencing different economic conditions can lead to an endogeneous increase in macroeconomic volatility that is double that obtained in the case of flexible exchange rates, even when the volatilities of shocks hitting the economies are the same. This translates into a welfare loss for the union as a whole, with the loss borne disproportionately by the periphery.

Given the currency union's problem with a "one-size-fits-all" monetary policy, we consider two fiscal policy alternatives: a fiscal union and a unilateral fiscal devaluation by the periphery. First, we show how a complete risk-sharing arrangement among union members can remedy the distortion brought about by fixed exchange rates. In principle, such cross-country risk-sharing arrangement can be achieved by forming a fiscal union, a point emphasized by Farhi and Werning (2014). However, our numerical simulations indicate that such a union involves a large transfer of wealth from the core to the periphery, casting doubt on its political feasibility.

As an alternative, we consider the macroeconomic implications of a fiscal devaluation. Recent work by Adao, Correira, and Teles (2009) and Farhi, Gopinath, and Itskhoki (2014a) has explored the stabilization properties of certain fiscal policy mixes, intended to replicate the effects of nominal devaluation in a fixed exchange rate regime. As emphasized by these authors, what makes such fiscal devaluation policies desirable is the fact that they can be implemented unilaterally by the

[5] Broadly speaking, this pattern is also consistent with the stylized facts of the euro area crisis. Although Germany was hit severely by the 2008–09 financial turmoil, its economy rebounded quickly on the back of resurging current account surpluses. Importantly, as documented by Bibow (2013), the increase in Germany's surplus position was primarily vis-à-vis non-EU trading partners, as the euro area sovereign debt crisis and the concomitant deep recession squeezed Germany's surplus position within Europe.

periphery countries encountering economic weakness. However, it is not clear why the core countries should welcome such unilateral policy interventions—in many instances, core countries have joined the currency union precisely to avoid the manipulation of nominal exchange rates by the monetary authorities in the periphery. A natural question that then emerges is can the periphery carry out unilateral fiscal devaluations without worrying about a retaliatory reaction from the core countries?

Our simulations show that a fiscal devaluation by the periphery can be beneficial even to the core, provided that the aggregate demand externality generated by the international predatory price war is not remedied by the currency union's policymakers. When firms in the core countries slash prices to expand their market shares, they do not internalize the pecuniary externality—in which driving out their foreign competitors by undercutting prices to an excessive degree—can also reduce aggregate demand for their own products. As shown by Farhi and Werning (2014), in such situations, a distortionary taxation can help core firms internalize this externality, and fiscal devaluations provide an effective means of achieving this goal. Furthermore, we show that benefits to the core resulting from a unilateral fiscal devaluation by the periphery increase with the degree of financial market distortions that generate the pecuniary externality.

The remainder of the paper is organized as follows. In Section 2, we provide some motivating empirical evidence, which shows that inflation dynamics in the eurozone periphery during the recent financial crisis were influenced importantly by financial strains. Section 3 presents our model and discusses its calibration. Section 4 contains our baseline simulation results, while section 5 explores the welfare implications of a fiscal union and fiscal devaluations. Section 6 concludes.

## 2 An Empirical Motivation

Through more than half of a century of evolution, the Phillips curve has provided macroeconomists with an increasingly coherent determination of aggregate inflation dynamics. In all of its reincarnation, one of its key predictions is that a high level of resource underutilization should cause inflation to decline over time. However, the absence of a potentially catastrophic self-reinforcing deflationary spiral in the euro area periphery during the recent financial crisis and its aftermath poses a significant empirical challenge to this central tenet of the most widely used macroeconomic models.[6] In this section, we provide some empirical evidence, which shows that inflation dynamics in the euro area periphery between 2008 and 2013 were influenced importantly by the extraordinary financial turmoil that swept through the area during this period.

[6] Wieland and Wolters (2014), on the other hand, argue that the absence of a pernicious deflation cum recession spiral in the euro area is not that puzzling because inflation forecasts based on standard *time-invariant* Phillips-curve specifications likely overstate the effect of resource underutilization on the firms' price-setting behavior. Their conclusion is based in large part on an apparent statistical regularity, according to which the effect of the resource slack on inflation declines along with trend inflation (see De Veirman, 2009).

Table 1: Panel-Data Estimates of the Phillips Curve, 1970–2014

| Explanatory Variables | APC | NKPC |
|---|---|---|
| $(u_{it} - \bar{u}_{it})$ | $-0.142^{**}$ | . |
| | (0.069) | |
| $(y_{it} - \bar{y}_{it})$ | . | $0.104^{**}$ |
| | | (0.048) |
| $\pi_{i,t-1}$ | $0.912^{***}$ | $0.533^{***}$ |
| | (0.027) | (0.049) |
| $E_t[\pi_{i,t+1}]$ | . | $0.449^{***}$ |
| | | (0.051) |
| $R^2$ (within) | 0.824 | . |
| Pr > $J_N$[a] | . | 0.424 |
| Obs. | 403 | 429 |

NOTE: Sample period: annual data from 1970 to 2014; No. of countries = 11 (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain). In each specification, the dependent variable is $\pi_{it}$, inflation rate of country $i$ in year $t$, measured by the log-difference of the (implicit) GDP price deflator from year $t-1$ to year $t$. The entries denote the estimates of the coefficients associated with $(u_{it} - \bar{u}_{it})$, unemployment rate gap in year $t$; $(y_{it} - \bar{y}_{it})$, output gap in year $t$; $\pi_{i,t-1}$, inflation in year $t-1$; and $E_t[\pi_{i,t+1}]$, expected inflation in year $t+1$. The APC is estimated by OLS, and the NKPC is estimated by GMM, treating $(y_{it} - \bar{y}_{it})$ and $E_t[\pi_{i,t+1}]$ as endogenous and instrumented with lags 1 to 3 of $(y_{it} - \bar{y}_{it})$ and $\pi_{it}$, and lags 0 to 2 of the log-difference of commodity prices. Both specifications include country fixed effects (not reported). Heteroskedasticity- and autocorrelation-consistent asymptotic standard errors reported in parentheses are robust to arbitrary spatial correlation and are computed according to Driscoll and Kraay (1998): * $p < .10$; ** $p < .05$; and *** $p < .01$.
[a] $p$-value for the Hansen (1982) of the over-identifying restrictions.

We consider two widely used Phillips curve specifications:

$$\pi_{it} = \rho\pi_{i,t-1} + \phi(u_{it} - \bar{u}_{it}) + \eta_i + \epsilon_{it}; \tag{1}$$

and

$$\pi_{it} = \gamma_b\pi_{i,t-1} + \gamma_f E_t[\pi_{i,t+1}] + \lambda(y_{it} - \bar{y}_{it}) + \eta_i + \epsilon_{it}, \tag{2}$$

where $i$ indexes country and $t$ time (in years); $\pi_{it}$ is inflation as measured by the log-difference of the (implicit) GDP price deflator; $u_{it}$ is the unemployment rate; $\bar{u}_{it}$ denotes an estimate of the NAIRU; $y_{it}$ is the log-level of real GDP; and $\bar{y}_{it}$ denotes the log-level of potential output.[7]

Specification (1) is the traditional "accelerationist" Phillips curve (APC), which assumes that inflation expectations are proportional to past inflation.[8] Although Phillips curves of this sort tend to fit the data quite well, their major theoretical shortcoming involves the assumptions of adaptive inflation expectations. Accordingly, we also consider its widely used New Keyne-

[7] All the data used in the analysis were obtained from the OECD database.

[8] Note that in equation (1) we do not restrict the coefficient $\rho$ to be equal to one, as it is often done in practice. Imposing this restriction, however, had a negligible effect on all the results reported in the paper.

sian variant (NKPC)—equation (2)—which incorporates into the process of inflation determination both rational expectations as well as more explicit microfoundations (Galí and Gertler, 2000; Galí, Gertler, and López-Salido, 2001).

Table 1 summarizes our estimation results, based on a panel of 11 euro area countries covering the period from 1970 to 2014. In both specifications, the degree of resource slack is an important—both economically and statistically—determinant of inflation dynamics. The estimates of the NKPC point to a significant forward-looking component in the euro area inflation, though the inflation processes appear to be also characterized by substantial inertial behavior, results consistent with those of Benigno and López-Salido (2006).

Our interest is not in these estimates per se. Rather, we are interested in the "fit" of these Phillips curves during the recent financial crisis and whether inflation deviations are systematically related to the degree of financial strains. To examine this question, we estimate the following regression:

$$\hat{\epsilon}_{i,t+1} = \beta_P \big( \log \mathrm{CDS}_{it} \times \mathbf{1}[i \in \text{Periphery}] \big) + \beta_C \big( \log \mathrm{CDS}_{it} \times \mathbf{1}[i \in \text{Core}] \big) + \nu_{i,t+1}, \tag{3}$$

where $\hat{\epsilon}_{it}$ denotes the inflation residual from one of the estimated Phillips curves in Table 1, $\mathrm{CDS}_{it}$ is the spread on the *sovereign* (5-year, dollar-denominated) credit default swap (CDS) contract of country $i$, and $\mathbf{1}[\cdot]$ is a 0/1-indicator function, indicating whether country $i$ is in the periphery or core.

We use sovereign CDS spreads to measure the degree of financial strains in each country and, by extension, the severity of financing constraints faced by businesses and households.[9] As emphasized by Lane (2012), the European sovereign debt crisis originated over concerns related to the solvency of national banking systems in the periphery. Accordingly, the level of sovereign CDS spreads likely provides a timely and accurate gauge of pressures faced by the national banking systems in the eurozone during the recent financial crisis. Given the bank-centric nature of the euro area financial system, variation in the sovereign CDS spreads across countries should also accurately reflect differences in the tightness of credit conditions faced by businesses and households.

The results of estimating regression (3) over the 2008–2013 period are detailed in Table 2.[10] As evidenced by the entries in the table, the level of CDS spreads in the eurozone periphery—our proxy for the degree of financial frictions—is systematically related to the departures of inflation from the dynamics implied by the Phillips curve-type relationships.

[9] A CDS—a financial derivative used extensively by investors for hedging of and investing in credit risk (both corporate and sovereign)—is simply an insurance contract between two parties: a protection buyer, who makes fixed, periodic payments; and a protection seller, who collects these premiums in exchange for making the protection buyer whole in case of default. We uses premiums implied by the 5-year, dollar-denominated contracts because they are the most liquid segment of the credit derivatives market. We convert daily CDS quotes to annual frequency by averaging the daily quotes over the trading days of the fourth quarter of each year.

[10] We estimate equation (3) by OLS. However, the associated statistical inference that relies on the usual asymptotic arguments is likely to be unreliable, given a relatively small number of observations, especially in the time-series dimension. Accordingly, we report the 95-percent confidence intervals for coefficients $\beta_P$ and $\beta_C$, based on the time-clustered wild bootstrap procedure of Cameron, Gelbach, and Miller (2008), which is designed for situations in which the number of clusters or the number of observations within each cluster is relatively small.

Table 2: Phillips Curve Prediction Errors and Financial Strains, 2008–2013

| Explanatory Variables | APC | NKPC |
|---|---|---|
| $\log \text{CDS}_{it} \times \mathbf{1}[i \in \text{Periphery}]$ | 0.548 | 0.427 |
| | $[0.119, 0.978]$ | $[0.229, 0.652]$ |
| $\log \text{CDS}_{it} \times \mathbf{1}[i \in \text{Core}]$ | 0.103 | 0.101 |
| | $[-0.145, 0.350]$ | $[-0.109, 0.311]$ |
| $R^2$ | 0.142 | 0.142 |
| Obs. | 54 | 54 |

NOTE: Sample period: annual data from 2008 to 2013; No. of countries = 11. In each specification, the dependent variable is $\hat{\epsilon}_{i,t+1}$, inflation residual of country $i$ in year $t+1$, implied by the Phillips curve specification in Table 1. The entries denote the OLS estimates of the coefficients associated with the log-level of sovereign CDS spread of country $i$ at the end of year $t$, interacted with a 0/1-variable indicating whether country $i$ is in the euro area periphery or core. Periphery countries: Greece, Ireland, Italy, Portugal, and Spain. Core countries: Austria, Belgium, Finland, France, Germany, and Netherlands. All specifications include a separate constant for the periphery and core countries. The 95-percent confidence intervals reported in brackets are based on the empirical distribution of coefficients across 5,000 replications, using the wild bootstrap clustered in the time dimension (see Cameron, Gelbach, and Miller, 2008).

The 95 percent confidence intervals bracketing the point estimates of the coefficients associated with the periphery countries in both cases firmly exclude zero, an indication that this relationship is statistically significant at conventional levels. It is also significant in economic terms. Using the midpoint of the two estimates—about 0.50—as a benchmark and evaluating the implied semi-elasticity at the median of the CDS spreads for the periphery countries during the 2008–2013 period, our estimates indicate that an increase in the CDS spreads of 1 percentage point at the end of year $t$ is associated with a rate of inflation in year $t+1$ that is a full percentage point higher than that implied by the estimated Phillips curves in Table 1. For the core euro area countries, by contrast, there appears to be no systematic relationship between inflation residuals and financial strains.

# 3 Model

## 3.1 Preferences

We focus on a two-country setting, where foreign country variables are denoted by asterisks. In each country, there exists a continuum of households, indexed by $j \in N_c \equiv [0, 1]$. Each household consumes two types, $h$ and $f$ of different varieties of consumption goods, indexed by $i \in N_h \equiv [1, 2]$ in the home country and by $i \in N_f \equiv [2, 3]$ in the foreign country. As is standard, we assume that the home country only produces $h-$ type and the foreign country only produces $f-$ type. For instance, $c^j_{i,f,t}$ denotes home country consumer $j$'s consumption of product $i$ of type $f$ whereas $c^{j*}_{i,f,t}$ denotes the foreign counterpart. Note that $c^j_{i,f,t}$ is consumption of an imported good by a home country consumer whereas $c^{j*}_{i,f,t}$ is consumption of a domestically produced good by a foreign consumer.

For simplicity, we assume that labor is perfectly immobile. The preferences of the home country are given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U(x_{t+s}^j - \delta_{t+s}, h_{t+s}^j) \text{ for } j \in [0,1]. \tag{4}$$

The period utility function $U(\cdot,\cdot)$ is strictly increasing and concave in its first argument, and is strictly decreasing and concave in its second argument. The consumption/habit aggregator $x_t^j$ is defined as an Ravn-Schmitt-Grohé-Uribe-Armington aggregator,

$$x_t^j \equiv \left\{ \sum_{k=h,f} \omega_k \left[ \int_{N_k} (c_{i,k,t}^j / s_{i,k,t-1}^{\theta})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}} \right\}^{1/(1-1/\varepsilon)}, \sum_{k=h,f} \omega_k^{\varepsilon} = 1 \tag{5}$$

where $\eta$ and $\varepsilon$ are the elasticities of substitution within a type and between types. $\omega_k$ is a parameter that governs the consumption home bias in consumption basket in the steady state. $s_{i,k,t}$ denotes *good-specific* habit, which evolves according to

$$s_{i,k,t} = \rho s_{i,k,t-1} + (1-\rho) \int_0^1 c_{i,k,t}^j dj \text{ for } k = h, f. \tag{6}$$

We define $c_{i,k,t}$ as an average consumption level of good $i$, that is, $c_{i,k,t} \equiv \int_0^1 c_{i,k,t}^j dj$ for $k = h, f$. When $\theta < 0$, the stock of habit formed by past consumption positively affects the utility from today's consumption, making the customer desire the same good more. This creates an incentive for firms to lower their prices to build market share. Note that it is not the individual level of consumption that matters for the habit, but the level of average consumption. Hence, the preferences can be considered "Catching Up with Jonesses" at the goods level rather than at the aggregate consumption level. $\delta_t$ is a demand shock that alters the marginal utility of consumption, a device that will be exploited to generate demand shock later.

The cost minimization associated with (4) implies the following demand functions:

$$c_{i,k,t}^j = \left( \frac{P_{i,k,t}}{\tilde{P}_{k,t}} \right)^{-\eta} s_{i,k,t-1}^{\theta(1-\eta)} x_{k,t}^j \tag{7}$$

where the welfare-based price index and consumption/habit basket are defined as

$$\tilde{P}_{k,t} \equiv \left[ \int_{N_k} (P_{i,k,t} s_{i,k,t-1}^{\theta})^{1-\eta} di \right]^{1/(1-\eta)} \tag{8}$$

$$\text{and } x_{k,t}^j \equiv \left[ \int_{N_k} (c_{i,k,t} / s_{i,k,t-1}^{\theta})^{1-1/\eta} di \right]^{1/(1-1/\eta)} \tag{9}$$

for $k = h, f$. $\tilde{P}_{k,t}$ and $x_{k,t}^j$ should be interpreted as habit adjusted price index, and habit adjusted consumption/habit basket of goods of type $k$ in the home country. The appendix presents the details of the derivations of (7) to (9). It also shows that the consumption/habit basket (9) in equilibrium is determined as

$$x^j_{k,t} = \omega^\varepsilon_k \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t}\right)^{-\varepsilon} x^j_t \text{ for } k = h, f \tag{10}$$

$$\text{where } \tilde{P}_t = \left[\sum_{k=h,f} \omega_k \tilde{P}^{1-\varepsilon}_{k,t}\right]^{1/(1-\varepsilon)}, \tag{11}$$

denotes the welfare-based aggregate price index of the home country. Due to the symmetric structure of the two countries, the foreign counterparts can be expressed simply with asterisks placed in (7)∼(11). For later use, we also define the CPI as

$$P_t \equiv \left[\sum_{k=h,f} \omega_k P^{1-\varepsilon}_{k,t}\right]^{1/(1-\varepsilon)} \tag{12}$$

where

$$P_{k,t} \equiv \left[\int_{N_k} (P_{i,k,t})^{1-\eta} di\right]^{1/(1-\eta)} \text{ for } k = h, f$$

is defined as a type-specific CPI.

### 3.2 Technology

Following our earlier work (Gilchrist, Schoenle, Sim, and Zakrajsek (2013)), we assume that the production technology is specified as

$$y_{i,t} = \left(\frac{A_t}{a_{i,t}} h_{i,t}\right)^\alpha - \phi, \quad 0 < \alpha \leq 1 \tag{13}$$

$$\text{and } y^*_{i,t} = \left(\frac{A^*_t}{a^*_{i,t}} h^*_{i,t}\right)^\alpha - \phi^*, \quad 0 < \alpha \leq 1 \tag{14}$$

where $A_h$ and $A^*_t$ are country specific aggregate technology shocks, possibly correlated with each other, and $a_{i,t}$ and $a^*_{i,t}$ are idiosyncratic technology shocks to home and foreign firms. We assume that the idiosyncratic shocks follow symmetric iid log-normal distributions, $\log a_{i,t} \sim N(-0.5\sigma^2, \sigma^2)$ and $\log a^*_{i,t} \sim N(-0.5\sigma^2, \sigma^2)$.

In this setup, $\phi$ and $\phi^*$ are country-specific fixed costs of operation, which make it possible for the firms to have negative income, and hence a liquidity problem if external financing is costly. As shown by Gilchrist, Schoenle, Sim, and Zakrajsek (2013), the fixed costs can work as a parsimonious proxy for heterogeneous degrees of financial friction, including fixed payments to long term bonds, etc. However, to make $\phi$ and $\phi^*$ important for liquidity concerns of firms, we need to introduce some frictions to the flow of funds constraint of the firms.

### 3.3 Pricing Frictions and Financial Distortions

To allow for nominal rigidities, we assume that the firms face a quadratic cost of adjusting nominal prices as specified in (Rotemberg (1982)):

$$\frac{\gamma}{2}\left(\frac{P_{i,h,t}}{P_{i,h,t-1}} - \bar{\pi}\right)^2 c_t + \frac{\gamma^*}{2}\frac{S_t P_t^*}{P_t}\left(\frac{P_{i,h,t}^*}{P_{i,h,t-1}^*} - \bar{\pi}\right)^2 c_t^* \tag{15}$$

where $S_t$ is the nominal exchange rate. We allow the degree of nominal rigidity to differ in home and foreign countries as indicated by separate notations for $\gamma$ and $\gamma^*$. We assume that price adjustment costs are proportional to local consumptions, that is, $c_t$ and $c_t^*$.[11] After dividing the numerators and denominators by type specific price indices $P_{h,t}$ and $P_{h,t}^*$, we can express the price adjustment costs as

$$\frac{\gamma}{2}\left(\frac{p_{i,h,t}}{p_{i,h,t-1}}\pi_{h,t} - \bar{\pi}\right)^2 c_t + \frac{\gamma^*}{2}q_t\left(\frac{p_{i,h,t}^*}{p_{i,h,t-1}^*}\pi_{h,t}^* - \bar{\pi}^*\right)^2 c_t^*.$$

where $p_{i,h,t} \equiv P_{i,h,t}/P_{h,t}$, $p_{i,h,t}^* \equiv P_{i,h,t}^*/P_{h,t}^*$ and $q_t \equiv S_t P_t^*/P_t$, the real exchange rate.

We adopt a specific timing convention, which plays an important role in creating uncertainty and liquidity problem for pricing firms. Specifically, we assume that in the first half of the period, the firms collect information about the aggregate economy. Based on this aggregate information, the firms post prices, take orders from customers and plan production based on expected marginal cost. In the second half of the period, the firms realize actual marginal cost and hire labor to meet demand. Ex-post, profits may be too low to cover the total cost of production in which case the firms must raise external finance. Without loss of generality, we focus on equity finance.[12] We assume that ex-post equity finance involves a constant per-unit dilution cost $\varphi$, $\varphi^* \in (0, 1)$.

The firm problem maximizes the present value of dividend flows, $\mathbb{E}_t[\sum_{s=0}^{\infty} m_{t,t+s} d_{i,t+s}]$ where $d_{i,t} \equiv D_{i,t}/P_t$ denotes real dividend payouts when positive, and equity issuance when negative. We assume that the firms are owned by the households, and discount future cashflows with the stochastic discounting factor of the representative household, $m_{t,t+s}$. The dilution cost implies that when a firm issues a notional amount of equity $d_{i,t}(< 0)$, actual cash inflow from the issuance is reduced to $-(1-\varphi)d_{i,t}$.

An implicit assumption is that the equity markets in the two countries are segmented: only domestic households invest in the shares of domestic firms. This assumption can be justified if the information asymmetry underlying the dilution phenomenon is disproportionately larger for cross-border equity holdings. The home bias in equity holdings is well documented by empirical researchers in fiance: French and Poterba (1991), Tesar and Werner (1995) and Obstfeld and Rogoff (2000). In fact, in our model, even an arbitrarily small disadvantage associated with cross-border financing is large enough to generate perfect home bias because the perceived supply of domestic funding at the given cost of dilution is infinitely elastic. Hence the stream of dividends is discounted

[11]This is for the homogeneity of the problem as usual with no first-order consequences for model outcomes.

[12]As shown by Gomes (2001) and Stein (2003), other forms of costly external financing can be replicated by properly parameterized equity dilution costs.

with the stochastic discounting factor of the representative household in the local country.

Another important assumption we adopt is that the two countries are different in terms of the degree of capital market imperfection. In particular, we assume that the dilution cost is strictly greater in the home country than in foreign country: $0 \leq \varphi^* < \varphi$. Despite this difference, we assume that the firms in home country cannot avoid the higher external financing cost by issuing equities overseas. Together with the higher operating costs in the home country, the higher external financing cost exposes the firms in the home country to a greater degree of liquidity risk. We explore the implication of such relative financial vulnerability on pricing under various macroeconomic environments such as fixed vs floating exchange rate on the one hand, and complete vs incomplete risk sharing between the two countries on the other hand. To save space, we describe the firm problem from the viewpoint of home country.

### 3.4 Profit Maximization Problem

The firm problem is subject to the following flow of funds constraint:

$$\begin{aligned} d_{i,t} = {} & p_{i,h,t} p_{h,t} c_{i,h,t} + q_t p^*_{i,h,t} p^*_{h,t} c^*_{i,h,t} - w_t h_{i,t} + \varphi \min\{0, d_{i,t}\} \\ & - \frac{\gamma}{2}\left(\frac{p_{i,h,t}}{p_{i,h,t-1}}\pi_{h,t} - \bar{\pi}\right)^2 c_t - \frac{\gamma^*}{2} q_t \left(\frac{p^*_{i,h,t}}{p^*_{i,h,t-1}}\pi^*_{h,t} - \bar{\pi}^*\right)^2 c^*_t \end{aligned} \tag{16}$$

where $p_{h,t} \equiv P_{h,t}/P_t$, $p^*_{h,t} \equiv P^*_{h,t}/P^*_t$, and $w_t \equiv W_t/P_t$ is real wage rate. The firm problem is also subject to a demand constraint:

$$\left(\frac{A_t}{a_{i,t}} h_{i,t}\right)^{\alpha} - \phi \geq c_{i,h,t} + c^*_{i,h,t} \tag{17}$$

The firm problem can then be expressed with the following Lagrangian:

$$\begin{aligned} \mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} m_{0,t} \Bigg\{ & d_{i,t} + \kappa_{i,t}\left[\left(\frac{A_t}{a_{i,t}} h_{i,t}\right)^{\alpha} - \phi - (c_{i,h,t} + c^*_{i,h,t})\right] \\ & + \xi_{i,t}\Bigg[p_{i,h,t} p_{h,t} c_{i,h,t} + q_t p^*_{i,h,t} p^*_{h,t} c^*_{i,h,t} - w_t h_{i,t} - d_{i,t} + \varphi \min\{0, d_{i,t}\} \\ & \qquad - \frac{\gamma}{2}\left(\frac{p_{i,h,t}}{p_{i,h,t-1}}\pi_{h,t} - \bar{\pi}\right)^2 c_t - \frac{\gamma^*}{2} q_t \left(\frac{p^*_{i,h,t}}{p^*_{i,h,t-1}}\pi^*_{h,t} - \bar{\pi}^*\right)^2 c^*_t\Bigg] \\ & + \nu_{i,h,t}\left[(p_{i,h,t})^{-\eta} \tilde{p}^{\eta}_{h,t} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t} - c_{i,h,t}\right] \\ & + \nu^*_{i,h,t}\left[(p^*_{i,h,t})^{-\eta} \tilde{p}^{*\eta}_{h,t} s^{*\theta(1-\eta)}_{i,h,t-1} x^*_{h,t} - c^*_{i,h,t}\right] \\ & + \lambda_{i,h,t}\left[\rho s_{i,h,t-1} + (1-\rho) c_{i,h,t} - s_{i,h,t}\right] \\ & + \lambda^*_{i,h,t}\left[\rho s^*_{i,h,t-1} + (1-\rho) c^*_{i,h,t} - s^*_{i,h,t}\right] \Bigg\} \end{aligned}$$

where $\tilde{p}_{h,t} \equiv \tilde{P}_{h,t}/P_{h,t}$, $\tilde{p}^*_{h,t} \equiv \tilde{P}^*_{h,t}/P^*_{h,t}$, and $\kappa_{i,t}$, $\xi_{i,t}$, $\nu_{i,h,t}$, $\nu^*_{i,h,t}$, $\lambda_{i,h,t}$ and $\lambda^*_{i,h,t}$ are shadow values of the constraints (17), (16), (7) and (6). The efficiency conditions of the problem are given by the following:

$$d_{i,t} : \xi_{i,t} = \begin{cases} 1 & \text{if } d_{i,t} \geq 0 \\ 1/(1-\varphi) & \text{if } d_{i,t} < 0 \end{cases} \tag{18}$$

$$h_{i,t} : \xi_{i,t} w_t = \alpha \kappa_{i,t} \left( \frac{A_t}{a_{i,t}} h_{i,t} \right)^{\alpha - 1} \tag{19}$$

$$\text{where} \quad h_{i,t} = \frac{a_{i,t}}{A_t} \left( \phi + c_{i,h,t} + c^*_{i,h,t} \right)^{1/\alpha} \tag{20}$$

$$c_{i,h,t} : \mathbb{E}^a_t[\nu_{i,h,t}] = \mathbb{E}^a_t[\xi_{i,t}] \; p_{i,h,t} p_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho)\lambda_{i,h,t} \tag{21}$$

$$c^*_{i,h,t} : \mathbb{E}^a_t[\nu^*_{i,h,t}] = \mathbb{E}^a_t[\xi_{i,t}] \; q_t p^*_{i,h,t} p^*_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho)\lambda^*_{i,h,t} \tag{22}$$

$$s_{i,h,t} : \lambda_{i,h,t} = \rho \mathbb{E}_t[m_{t,t+1}\lambda_{i,h,t+1}] + \theta(1-\eta)\mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}^a_{t+1} \left[ \nu_{i,h,t+1} \frac{c_{i,h,t+1}}{s_{i,h,t}} \right] \right\} \tag{23}$$

$$s^*_{i,h,t} : \lambda^*_{i,h,t} = \rho \mathbb{E}_t[m_{t,t+1}\lambda^*_{i,h,t+1}] + \theta(1-\eta)\mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}^a_{t+1} \left[ \nu^*_{i,h,t+1} \frac{c^*_{i,h,t+1}}{s^*_{i,h,t}} \right] \right\} \tag{24}$$

$$\begin{aligned} p_{i,h,t} : \eta \frac{\mathbb{E}^a_t[\nu_{i,h,t}]}{p_{i,h,t}} c_{i,h,t} &= \mathbb{E}^a_t[\xi_{i,t}] \left[ p_{h,t} c_{i,h,t} - \gamma \frac{\pi_{h,t}}{p_{i,h,t-1}} \left( \pi_{h,t} \frac{p_{i,h,t}}{p_{i,h,t-1}} - \bar{\pi} \right) c_t \right] \\ &+ \gamma \mathbb{E}_t \left[ m_{t,t+1} \mathbb{E}^a_{t+1}[\xi_{i,t+1}] \pi_{h,t+1} \frac{p_{i,h,t+1}}{p^2_{i,h,t}} \left( \pi_{h,t+1} \frac{p_{i,h,t+1}}{p_{i,h,t}} - \bar{\pi} \right) c_{t+1} \right] \end{aligned} \tag{25}$$

$$\begin{aligned} p^*_{i,h,t} : \eta \frac{\mathbb{E}^a_t[\nu^*_{i,h,t}]}{p^*_{i,h,t}} c^*_{i,h,t} &= \mathbb{E}^a_t[\xi_{i,t}] \left[ q_t p^*_{h,t} c^*_{i,h,t} - \gamma^* \frac{q_t \pi^*_{h,t}}{p^*_{i,h,t-1}} \left( \pi^*_{h,t} \frac{p^*_{i,h,t}}{p^*_{i,h,t-1}} - \bar{\pi}^* \right) c^*_t \right] \\ &+ \gamma^* \mathbb{E}_t \left[ m_{t,t+1} \mathbb{E}^a_{t+1}[\xi_{i,t+1}] q_{t+1} \pi^*_{h,t+1} \frac{p^*_{i,h,t+1}}{p^{*2}_{i,h,t}} \left( \pi^*_{h,t+1} \frac{p^*_{i,h,t+1}}{p^*_{i,h,t}} - \bar{\pi}^* \right) c^*_{t+1} \right] \end{aligned} \tag{26}$$

We omit the presentation of associated complementary slackness conditions for the sake of space. Note that (20) is the labor demand conditional on the level of output, which, in turn, is determined by demand given price. We combine (19) and (20), and express the efficiency condition for labor hours as

$$\kappa_{i,t} = \xi_{i,t} a_{i,t} \frac{w_t}{\alpha A_t} \left( \phi + c_{i,h,t} + c^*_{i,h,t} \right)^{\frac{1-\alpha}{\alpha}} . \tag{27}$$

### 3.5 Symmetric Equilibrium and International Price Wars

The last six FOCs describe the efficiency conditions for the decisions made prior to the realization of the idiosyncratic cost shock. These first-order conditions involve the expected value of internal funds $\mathbb{E}^a_t[\xi_{i,t}] \equiv \int_0^\infty \xi_{i,t}(a_{i,t}) dF(a)$ where the information set of the expectations operator $\mathbb{E}^a_t[\cdot]$ includes all aggregate information up to time $t$ except the realization of the idiosyncratic shock. In contrast, the realized values $\xi_{i,h,t}$ and $a_{i,h,t}$ enter the efficiency conditions (18) and (19) without the expectation operator since equity issuance and labor hiring decisions are made after the realization of the idiosyncratic shock. In (21) and (22), $\nu_{i,h,t}$ and $\nu^*_{i,h,t}$ also enter the efficiency conditions with the expectation operater because $\nu_{i,h,t}$ and $\nu^*_{i,h,t}$ measure the value of marginal sales in the home and foreign markets and the firm does not know their values in choosing $c_{i,h,t}$ and $c^*_{i,h,t}$ until the idiosyncratic shock realizes and it gets to know its financial condition.

With the assumption of risk-neutrality and i.i.d. idiosyncratic shocks, the timing convention adopted above implies that firms are identical ex ante. Hence we focus on a symmetric equilibrium whereby all monopolistically competitive firms choose identical relative prices ($p_{i,h,t} = 1$ and $p^*_{i,h,t} = 1$), production scales ($c_{i,h,t} = c_{h,t}$ and $c^*_{i,h,t} = c^*_{h,t}$), habit stocks ($s_{i,h,t} = s_{h,t}$ and $s^*_{i,h,t} = s^*_{h,t}$), and shadow values of habit ($\lambda_{i,h,t} = \lambda_{h,t}$ and $\lambda^*_{i,h,t} = \lambda^*_{h,t}$). However, the distributions of labor hours, dividend payouts, equity issuance and the realized shadow values of internal funds ($\xi_{i,t}$), marginal cost ($\kappa_{i,t}$) and marginal sales ($\nu_{i,h,t}$ and $\nu^*_{i,h,t}$) are non-degenerate and depend on the realization of idiosyncratic shocks.

Note that the symmetric equilibrium condition $p_{i,h,t} = 1$ and $p^*_{i,h,t} = 1$ imply that the firms in the home country choose the same price levels in home and in foreign markets vis a vis other competitors from the same origin.[13] However, this symmetric equilibrium condition does not imply that the firms in foreign country make the same pricing decision as home country firms in the same market, even when they share the same fundamentals, that is, preference and aggregate technology level. In other words, foreign firms make the same pricing decisions among themselves both in domestic and export markets such that $p_{i,f,t} = 1$ and $p^*_{i,f,t} = 1$, but the price levels chosen by home firms and and foreign firms in a given market differ from each other, and as a result, $p_{h,t} = P_{h,t}/P_t \neq 1$, $p^*_{h,t} = P^*_{h,t}/P^*_t \neq 1$, $p_{f,t} = P_{f,t}/P_t \neq 1$, $p^*_{f,t} = P^*_{f,t}/P^*_t \neq 1$, and $p_{h,t} \neq p_{f,t}$ and $p^*_{h,t} \neq p^*_{f,t}$ in general. This is because the firms in two countries face different degrees of capital market distortions, which create different liquidity conditions for home and foreign firms and lead these two groups of firms to follow different mark-up strategies. In particular, as will be shown below, the relatively poorer financial conditions lead home firms to maintain higher markups and higher prices in the neighborhood of the non-stochastic steady state such that $p_h > p_f$ and $p^*_h > p^*_f$.

From the vantage point of foreign firms, the same phenomenon can be thought of as them engaging in a price war, exploiting the financial vulnerability of home firms to expand market shares both in their home and in their export markets. The stronger the long-run relationship in customer markets, the greater the incentive of foreign firms to undercut the prices of home firms. If the exchange rate is floating, however, the depreciation of home currency, assisted by easy monetary policy of home country, can greatly improve the liquidity conditions for home country firms, providing an effective defense against the foreign firms' aggressive invasion on their markets. By joining a currency union, the home country essentially surrenders this armor.

To see how the financial distortion affects the optimal pricing strategy, consider for a moment flexible price environment for simplicity. Even when prices and wages are completely flexible, the model's implication for markup decision can be analyzed in the same way. Under the flexible price assumption, after imposing the symmetric equilibrium condition, the internal funds of the firm is

[13] Remind that $p_{i,h,t}$ and $p^*_{i,h,t}$ are relative prices against average prices charged by the firms of home country. These are different from the relative prices against local and foreign CPIs, which are average prices of domestic and imported goods.

given by the revenue minus cost:

$$p_{ht}c_{ht} + q_t p^*_{ht} c^*_{ht} - w_t \underbrace{\frac{a_t}{A_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{1/\alpha}}_{h(a_t)\text{: conditional labor demand}}.$$

It is then clear that external financing is required if and only if the realized idiosyncratic cost shock is greater than the following equity issuance threshold:

$$a_{i,t} \geq a^E_t \equiv \frac{A_t}{w_t} \cdot \left[\frac{p_{h,t}c_{h,t} + q_t p^*_{h,t}c^*_{h,t}}{(\phi + c_{h,t} + c^*_{h,t})^{1/\alpha}}\right]. \tag{28}$$

Using the equity issuance trigger, we can rewrite the FOC for dividends as

$$\xi(a_{i,t}) = \begin{cases} 1 & \text{if } a_{i,t} \leq a^E_t \\ 1/(1-\varphi_t) & \text{if } a_{i,t} > a^E_t \end{cases}. \tag{29}$$

This condition simply states that the realized shadow value of internal funds jumps to $1/(1-\varphi_t)$ due to the costly external financing when the idiosyncratic cost shock is greater than the threshold value. Let $z^E_t$ denote the standardized value of $a^E_t$, that is, $z^E_t = \sigma^{-1}(\log a^E_t + 0.5\sigma^2)$. From (29), the expected shadow value of internal funds is

$$\mathbb{E}^a_t[\xi_{i,t}] = \int_0^{a^E_t} 1dF(a) + \int_{a^E_t}^{\infty} \frac{1}{1-\varphi_t}dF(a) = 1 + \frac{\varphi_t}{1-\varphi_t}[1 - \Phi(z^E_t)] \geq 1 \tag{30}$$

where $\Phi(\cdot)$ denotes the cdf of the standard normal distribution. The expected shadow value is strictly greater than unity as long as equity issuance is costly ($\varphi_t > 0$) and future costs are uncertain ($\sigma > 0$). As emphasized in Gilchrist, Schoenle, Sim, and Zakrajsek (2013), this makes the firms de facto risk averse in their pricing decision: setting the price too low and taking an imprudently large number of orders by lowering the price too much exposes the firm to liquidity risk under costly external finance. Note that our approach encompasses the polar case of $\varphi_t = 1$, in which case outside funding is impossible. This is indeed the case of Brunnermeier and Sannikov (2014).

In this sense, $\mathbb{E}^a_t[\xi_{i,t}]$ properly measures the liquidity condition of the firms because $\mathbb{E}^a_t[\xi_{i,t}]$ is the ex ante value of *sure* one dollar. The reason why this deviates from a dollar is because marginal valuation of such cash flow is uncertain to the firm. In this regard, there is another important measure of liquidity, $\mathbb{E}^a_t[\xi_{i,t}a_{i,t}]$, which plays an important role. Remind that $\mathbb{E}^a_t[a_{i,t}] = 1$. $\mathbb{E}^a_t[\xi_{i,t}a_{i,t}]$ then measures the ex ante value of *uncertain* one dollar. Since $\mathbb{E}^a_t[a_{i,t}] = 1$,

$$\begin{aligned} \mathbb{E}^a_t[\xi_{i,t}a_{i,t}] - \mathbb{E}^a_t[\xi_{i,t}] &= \text{Cov}[\xi_{i,t}, a_{i,t}] \\ &= \frac{\varphi_t}{1-\varphi_t}[\Phi(z^E_t) - \Phi(z^E_t - \sigma_z)] > 0. \end{aligned} \tag{31}$$

The analytical expression in the second line can be derived based on the properties of lognormal distribution (see Johnson, Kotz, and Balakrishnan (1994)). (31) shows that the discrepancy between the two measures of liquidity is equivalent to the covariance between the realized values of idiosyncratic shock and the shadow value of internal funds, and more importantly the covariance is strictly positive, meaning that the firm's internal valuation of liquidity is higher when the firm falls short of liquidity due to an adverse cost shock, implying de facto risk-aversion.

Such risk-aversion plays an important role in the determination of equilibrium markup. For a sharper characterization, consider a case with no deep habits, so that $\theta = 0$, in which case $\lambda_{i,h,t} = \lambda^*_{i,h,t} = 0$. By combining the first-order condition (19)$\sim$(22), (25) and (26), we can express the pricing rules at home and at export markets as

$$p_{i,h,t} = \frac{\eta}{\eta - 1} \frac{\mathbb{E}^a_t[\xi_{it} a_{it}]}{\mathbb{E}^a_t[\xi_{it}]} \underbrace{\left[ \frac{W_t / P_{h,t}}{\alpha A_t} (c_{ht} + c^*_{h,t} + \phi)^{\frac{1-\alpha}{\alpha}} \right]}_{\text{marginal cost deflated by domestic price}} \tag{32}$$

$$p^*_{i,h,t} = \frac{\eta}{\eta - 1} \frac{\mathbb{E}^a_t[\xi_{it} a_{it}]}{\mathbb{E}^a_t[\xi_{it}]} \underbrace{\left[ \frac{W_t / (S_t P^*_{h,t})}{\alpha A_t} (c_{ht} + c^*_{h,t} + \phi)^{\frac{1-\alpha}{\alpha}} \right]}_{\text{marginal cost deflated by export price}} \tag{33}$$

where the real wage is constructed by deflating the nominal wage by local market prices charged by home firms.[14] The last two expressions show that optimal pricing follows the conventional markup pricing rule (a constant markup over real marginal cost) except one thing: the discrepancy between the two liquidity measures intervenes as an additional markup term. When the de facto risk-aversion of the firms as measured by $\mathbb{E}^a_t[\xi_{it} a_{it}]/\mathbb{E}^a_t[\xi_{it}]$ increases during an economic downturn, it creates a direct pressure on markup. Note that in the symmetric equilibrium, the left hand sides are equalized to one. What it means is that to satisfy (32) and (33) in the symmetric equilibrium, the marginal cost has to go down, or equivalently, accounting markup, defined as $\mu_t \equiv [((W_t/P_{h,t})/\alpha A_t)(c_{ht} + c^*_{h,t} + \phi)^{\frac{1-\alpha}{\alpha}}]^{-1}$, has to go up, implying countercyclical markup dynamics. For later use, we define financially adjusted markup $\tilde{\mu}_t$ (as opposed to the accounting markup) as

$$\tilde{\mu}_t \equiv \frac{\mathbb{E}^a_t[\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t} a_{i,t}]} \frac{\alpha A_t}{w_t} \left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}} = \frac{\mathbb{E}^a_t[\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t} a_{i,t}]} \mu_t. \tag{34}$$

As mentioned above, under the flexible price environment, any cahnges in the two components of the financially adjusted markup are always completely offset by each other. Under nominal rigidity, however, this is no longer the case and the financially adjusted markup plays an essential role.

[14]To derive this, we use the following identities:

$$w_t/p_{h,t} = (W_t/P_t)/(P_{h,t}/P_t) = W_t/P_{h,t}$$
$$\text{and } w_t/(q_t p^*_{h,t}) = (W_t/P_t)(P_t/(S_t P^*_t))/(P^*_{h,t}/P^*_t) = W_t/P^*_{h,t}.$$

### 3.6 Financial Friction and Phillips Curves

We now recover the assumptions of nominal rigidity and deep habits to explore the full implications of the model. Imposing the symmetric equilibrium conditions to the FOCs for pricing decisions yields the following Phillips curves:

$$\begin{aligned}\gamma\pi_{h,t}(\pi_{h,t}-\bar{\pi}) &= p_{h,t}\frac{c_{h,t}}{c_t}-\eta\frac{\mathbb{E}^a_t[\nu_{i,h,t}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{c_{h,t}}{c_t}\\ &\quad+\gamma\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\pi_{h,t+1}(\pi_{h,t+1}-\bar{\pi})\frac{c_{t+1}}{c_t}\right] \qquad (35)\\ \text{and } \gamma q_t\pi^*_{h,t}(\pi^*_{h,t}-\bar{\pi}^*) &= q_t p^*_{h,t}\frac{c^*_{h,t}}{c^*_t}-\eta\frac{\mathbb{E}^a_t[\nu^*_{i,h,t}]}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{c^*_{h,t}}{c^*_t}\\ &\quad+\gamma^*\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}q_{t+1}\pi^*_{h,t+1}(\pi^*_{h,t+1}-\bar{\pi})\frac{c^*_{t+1}}{c^*_t}\right] \qquad (36)\end{aligned}$$

The first terms in the Phillips curves are different from 1 since $p_{h,t}\neq 1$ and $c_{h,t}\neq c_t$. This is because (35) and (36) describe 'sectoral' inflation dynamics rather than aggregate inflation. Overall inflation dynamics in the two countries determined by

$$\pi_t=\left[\sum_{k=h,f}\omega_k(p_{k,t-1}\pi_{k,t})^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \qquad (37)$$

$$\text{and } \pi^*_t=\left[\sum_{k=h,f}\omega^*_k(p^*_{k,t-1}\pi^*_{k,t})^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \qquad (38)$$

More importantly, (35) and (36) show an important departure from New Keynesian Phillips curve in that the dynamic liquidity condition plays an essential role in inflation dynamics. This can be seen from the fact that the ratio of the shadow value of internal funds today vs tomorrow ($\mathbb{E}^a_{t+1}[\xi_{i,t+1}]/\mathbb{E}^a_t[\xi_{i,t}]$) works as an additional discounting factor. However, at least in linear dynamics, this does not create any difference as the coefficient of the term in the linearized model will be zero as $\pi^*_{h,t+1}-\bar{\pi}=0$ in the steady state.

An essential difference from a canonical NK Phillips curve is the presence of $\mathbb{E}^a_t[\nu_{i,h,t}]/\mathbb{E}^a_t[\xi_{i,t}]$. Without financial friction, in which case $\mathbb{E}^a_t[\xi_{i,t}]=1$, and without deep habits, $\mathbb{E}^a_t[\nu_{i,h,t}]$ simply measures one period real marginal profit as the first-order condition (21) becomes $\mathbb{E}^a_t[\nu_{i,h,t}]=p_{i,h,t}p_{h,t}-\mathbb{E}^a_t[\kappa_{i,t}]$ where $\mathbb{E}^a_t[\kappa_{i,t}]$ is the expected marginal cost.[15]

With the introduction of habit, however, $\mathbb{E}^a_t[\nu_{i,h,t}]$ and $\mathbb{E}^a_t[\nu^*_{i,h,t}]$ measure not only today's profit but also future profits due to consumption habit formed by today's consumption. In addition, financial friction creates two additional layers in the dynamic optimization. First, the stream of cash flows created by the changes in market share needs to be evaluated by firms' liquidity conditions, properly measured by the stream of expected value of internal funds. Second, the sequence of current and future covariance terms between the internal liquidity condition and cash

[15] In this environment, the timing convention becomes neutral and even the expectation operator becomes irrelevant.

flow need to be properly incorporated in the evaluation of current and future markups. In the appendix, we show that the first-order conditions (19)∼(24) jointly imply

$$\frac{\mathbb{E}_t^a[\nu_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_t} + \chi \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \tilde{\beta}_{t,s} \frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]} \left( p_{h,s} - \frac{1}{\tilde{\mu}_s} \right) \right] \tag{39}$$

$$\text{and } \frac{\mathbb{E}_t^a[\nu^*_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} = q_t p^*_{h,t} - \frac{1}{\tilde{\mu}_t} + \chi \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \tilde{\beta}^*_{t,s} \frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]} \left( q_s p^*_{h,s} - \frac{1}{\tilde{\mu}_s} \right) \right] \tag{40}$$

where $\chi \equiv (1-\rho)\theta(1-\eta)$ and the growth-adjusted, compounded discount rates $\tilde{\beta}_{t,s}$ and $\tilde{\beta}^*_{t,s}$ are defined as

$$\tilde{\beta}_{t,s} \equiv m_{s,s+1} g_{h,s+1} \cdot \prod_{j=1}^{s-t} (\rho + \chi g_{h,t+j}) m_{t+j-1,t+j} \text{ with } g_{h,t} \equiv \frac{s_{h,t}/s_{h,t-1} - \rho}{1-\rho} \tag{41}$$

$$\tilde{\beta}^*_{t,s} \equiv m_{s,s+1} g^*_{h,s+1} \cdot \prod_{j=1}^{s-t} (\rho + \chi g^*_{h,t+j}) m_{t+j-1,t+j} \text{ with } g^*_{h,t} \equiv \frac{s^*_{h,t}/s^*_{h,t-1} - \rho}{1-\rho}. \tag{42}$$

Note that $\mathbb{E}_t^a[\nu_{i,h,t}]/\mathbb{E}_t^a[\xi_{i,t}]$ and $\mathbb{E}_t^a[\nu^*_{i,h,t}]/\mathbb{E}_t^a[\xi_{i,t}]$ enter the Phillips curves with negative signs. Hence when the present values given by the right hand sides of (39) and (40) go up, the firm will lower its prices. In our model, holding the sequence of profits fixed, this essentially depends on two competing discounting factors: $\tilde{\beta}_{t,s}$ ($\tilde{\beta}^*_{t,s}$, if the relevant market is the foreign country) vs $\mathbb{E}_s^a[\xi_{i,s}]/\mathbb{E}_t^a[\xi_{i,t}]$. $\tilde{\beta}_{t,s}$ measures how much the firm's market share can be expanded over time by marginally lowering today's product price. Hence when $\tilde{\beta}_{t,s}$ is expected to grow, the firm lowers the current price. $\mathbb{E}_s^a[\xi_{i,s}]/\mathbb{E}_t^a[\xi_{i,t}]$ measures how much direr tomorrow's liquidity condition is expected to be relative to today's liqudity condition. As a consequence, when $\mathbb{E}_s^a[\xi_{i,s}]/\mathbb{E}_t^a[\xi_{i,t}]$ is expected to go down, the firm discounts more heavily the values of future market share and increases the current price to secure cash flow today.

## 3.7 Household Problem

To analyze how different risk sharing arrangements between the two countries affect macroeconomic allocations, we consider the problem of household under two different arrangements for international finance: (i) complete risk sharing through trading of state-contingent bonds; (iii) incomplete risk sharing with only state noncontingent bonds.

### 3.7.1 Complete Risk Sharing Under Floating

To streamline notation, we omit the household index $j$. The representative household in the home country works $h_t$, It saves by investing in the shares of the firms of home-country, state-contingent bonds that are traded internationally and non-contingent government bonds that are available in zero net supply. The household budget constraint is given by

$$0 = W_t h_t + B(s^t) + R_{t-1}B_t^G + \int_{N_h} [\max\{D_{it}, 0\} + P^S_{i,t-1,t}]s^S_{i,t}di$$
$$- \sum_{k=h,f} \int_{N_k} P_{i,k,t}c_{i,k,t}di - B^G_{t+1} - \int_{\mathcal{S}} M_t(s_{t+1}|s^t)B(s^{t+1})ds - \int_{N_h} P^S_{i,t}s^S_{i,t+1}di \tag{43}$$

where $s^t \equiv s_0, \ldots, s_t$. A unit of state-contingent bond $B(s^{t+1})$ pays out one unit of home currency upon the realization of state $s_{t+1}$ and $M_t(s_{t+1}|s^t)$ is the price of the bond at time $t$. $B^G_{t+1}$ is the government bond, and $r_t$ is the corresponding interest rate. Using the accounting identity, $\int_{N_k} P_{i,k,t}c_{i,k,t}di = \tilde{P}_{k,t}x_{k,t}$ for $k = h, f$ (see the appendix), we can simplify the last term in the budget constraint as $\sum_{k=h,f} \tilde{P}_{k,t}x_{k,t}$. $s^S_{i,t}$ denotes the outstanding shares of home country firm $i$, $P^S_{i,t-1,t}$ is the time $t$ value of the shares outstanding at time $t-1$ and $P^S_{i,t}$ is the ex-dividend value of shares at time $t$. The last two terms are related via the accounting identity, $P^S_{i,t} = P^S_{i,t-1,t} + E^S_{i,t}$ where $E^S_{i,t}$ is the value of new shares issued at time $t$. The costly equity finance assumption implies that $E^S_{i,t} = -(1-\varphi)\min\{D_{i,t}, 0\}$. Using this relationship, we can express the budget constraint only in terms of $P^S_{i,t}$:

$$0 = W_t h_t + B(s^t) + R_{t-1}B_t^G + \int_{N_h} (\tilde{D}_{i,t} + P^S_{i,t})s^S_{i,t}di$$
$$- \sum_{k=h,f} \tilde{P}_{k,t}x_{k,t} - B^G_{t+1} - \int_{\mathcal{S}} M(s_{t+1}|s^t)B(s^{t+1})ds - \int_{N_h} P^S_{i,t}s^S_{i,t+1}di \tag{44}$$

where $\tilde{D}_{i,t} \equiv \max\{D_{i,t}, 0\} + (1-\varphi)\min\{D_{i,t}, 0\}$.

The above expression makes it clear that costly equity finance takes the form of sales of new shares at a discount in general equilibrium. Since the owners of old and new shares are the same entity, there is no direct wealth effect associated with costly equity financing: the losses of the old shareholders exactly offset the gains of the new shareholders. Denoting the multiplier for the budget constraint by $\Lambda_t$ and maximizing (4) subject to (44) yields

$$x_{h,t} : \Lambda_t \tilde{P}_{h,t} = \omega_h \frac{x_t}{x_{h,t}} U_{x,t} \tag{45}$$
$$x_{f,t} : \Lambda_t \tilde{P}_{f,t} = \omega_f \frac{x_t}{x_{f,t}} U_{x,t} \tag{46}$$
$$h_t : \Lambda_t W_t = -U_{h,t} \tag{47}$$
$$B(s^{t+1}) : \Lambda_t M(s_{t+1}|s^t) = \beta \Pr(s_{t+1}|s^t)\Lambda(s_{t+1}|s^t) \tag{48}$$

$$B^G_{t+1} : \Lambda_t = \beta \mathbb{E}_t[\Lambda_{t+1}R_t] \tag{49}$$
$$s^S_{i,t+1} : \Lambda_t = \beta \mathbb{E}_t\left[\Lambda_{t+1}\left(\frac{\mathbb{E}^a_{t+1}[\tilde{D}_{i,t+1}] + P^S_{t+1}}{P^S_t}\right)\right] \tag{50}$$

where we use $P^S_t = P^S_{i,t}$ in our symmetric equilibrium. (45) and (46) imply $\Lambda_t \sum_{k=h,f} \tilde{P}_{k,t}\ x_{k,t} = x_t U_{x,t}$. Since $\sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} = \tilde{P}_t x_t$ holds from an accounting identity (see the appendix), (45) and (46) are equivalent to $\tilde{P}_t \Lambda_t = U_{x,t}$. Combining this condition with (48) yields

$$M_t(s_{t+1}|s^t) = \beta \frac{U_x(s^{t+1})/\tilde{P}_{t+1}}{U_{x,t}/\tilde{P}_t} \Pr(s_{t+1}|s^t) = \beta \frac{U_x(s^{t+1})/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{P_t}{P_{t+1}} \Pr(s_{t+1}|s^t) \tag{51}$$

In symmetric equilibrium, $s_{i,k,t-1} = s_{k,t-1}$, and thus it is straightforward to show that $\tilde{p}_t = [\sum_{k=h,f} \omega_k p_{k,t}^{1-\varepsilon} s_{k,t-1}^{\theta(1-\varepsilon)}]^{1/(1-\varepsilon)}$(see the appendix, (A.16)). A condition similar to (51) holds for the foreign representative household, that is,

$$M_t(s_{t+1}|s^t) = \frac{S_t P^*_t}{S(s^{t+1}) P^*_{t+1}} \beta \frac{U^*_x(s^{t+1})/\tilde{p}^*_{t+1}}{\tilde{U}^*_{x,t}/\tilde{p}^*_t} \Pr(s_{t+1}|s^t) \tag{52}$$

Then, equations (51) and (52) jointly imply the risk sharing condition:

$$q_t = \kappa \frac{\tilde{U}^*_{x,t}}{\tilde{U}_{x,t}} \text{ where } \kappa \equiv q_0 \frac{\tilde{U}_{x,0}}{\tilde{U}^*_{x,0}} \text{ and } \tilde{U}_{x,t} \equiv U_{x,t} \left[ \sum_{k=h,f} \omega_k p_{k,t}^{1-\varepsilon} s_{k,t-1}^{\theta(1-\varepsilon)} \right]^{-1/(1-\varepsilon)}. \tag{53}$$

Summing the prices of all state-contingent bonds yields the discount factor of firms

$$\int_{\mathcal{S}} M(s_{t+1}|s^t) ds = \mathbb{E}_t[m_{t,t+1}/\pi_{t+1}] = R_t^{-1} \text{ where } m_{t,t+1} \equiv \beta \tilde{U}_{x,t+1}/\tilde{U}_{x,t}.$$

We assume that the monetary authorities of the two countries control the prices of the government bonds using an identical Taylor-type rule:

$$R_t = R^{1-\rho_R} \left[ R_{t-1} \left( \frac{y_t}{y} \right)^{\rho_c} \left( \frac{\pi_t}{\pi} \right)^{\rho\pi} \right]^{\rho_R} \tag{54}$$

The foreign counterpart of (54) is symmetrically specified as

$$R^*_t = R^{1-\rho_R} \left[ R^*_{t-1} \left( \frac{y^*_t}{y^*} \right)^{\rho_c} \left( \frac{\pi^*_t}{\pi^*} \right)^{\rho\pi} \right]^{\rho_R}. \tag{55}$$

The risk sharing condition implies that the FOC of the home investor for the government bond can be rewritten as follows:

$$1 = \beta \mathbb{E}_t \left[ \frac{\tilde{U}_{x,t+1}}{\tilde{U}_{x,t}} \frac{R_t}{\pi_{t+1}} \right] = \beta \mathbb{E}_t \left[ \frac{\tilde{U}^*_{x,t+1}}{\tilde{U}^*_{x,t}} \frac{R_t}{\pi_{t+1}} \frac{q_t}{q_{t+1}} \right]. \tag{56}$$

Similarly, for the FOC of the foreign investor for the foreign government bond, the following holds:

$$1 = \beta \mathbb{E}_t \left[ \frac{\tilde{U}^*_{x,t+1}}{\tilde{U}^*_{x,t}} \frac{R^*_t}{\pi^*_{t+1}} \right] = \beta \mathbb{E}_t \left[ \frac{\tilde{U}_{x,t+1}}{\tilde{U}_{x,t}} \frac{R^*_t}{\pi^*_{t+1}} \frac{q_{t+1}}{q_t} \right]. \tag{57}$$

(56) and (57) show that the assumption of non cross-border holdings of government bonds are innocuous because the risk sharing condition makes the assumption irrelevant.

### 3.7.2 Incomplete Risk Sharing Under Floating Exchange Rate

To analyze the effects of incomplete risk sharing, we consider an alternative environment where the two countries trade state-noncontingent bonds. As in Ghironi and Melitz (2005), we assume that there are portfolio rebalancing costs associated with changing the level of capital accounts. These costs are a short-cut to real frictions that hinder efficient borrowing/lending across borders.[16] We denote home country's holdings of international bonds issued in home and foreign currency units by $B_{h,t+1}$ and $B_{f,t+1}$.[17] $B^*_{h,t+1}$ and $B^*_{f,t+1}$ denote the foreign counterparts. The interest rates are denoted by $R_t$ and $R^*_t$, respectively.[18]

The portfolio rebalancing costs are specified as $(\tau/2)P_t[(B_{h,t+1}/P_t)^2 + q_t(B_{f,t+1}/P^*_t)^2]$. Under these assumptions, the marginal cost of borrowing in home currency is given by $R_t/(1+\tau B_{h,t+1}/P_t)$, which is strictly greater than $R_t$ if $B_{h,t+1} < 0$. The marginal return on foreign lending in home currency is given by $R_t(S_t/S_{t+1})/(1+\tau B^*_{h,t+1}/P^*_t)$, which is strictly less than $R_t(S_t/S_{t+1})$ if $B^*_{h,t+1} > 0$. Thus, $(1 + \tau B_{h,t+1}/P_t)^{-1}$ represents a welfare loss not only to borrowers but also for lenders. As pointed out by Ghironi and Melitz (2005), the role of such cost function is simply to pin down the steady state levels of international bond holdings. Varying $\tau$ does not modify the model dynamics in any significant way. The intertemporal budget constraint of home country household is now given by

$$\begin{aligned}0 = W_t h_t + R_{t-1}B_{h,t} + S_t R^*_{t-1}B_{f,t} + \int_{N_h}(\tilde{D}_{i,t} + P^S_{i,t})s^S_{i,h,t}di \\ -\tilde{P}_t x_t - B_{h,t+1} - S_t B_{f,t+1} - \frac{\tau}{2}P_t\left[\left(\frac{B_{h,t+1}}{P_t}\right)^2 + q_t\left(\frac{B_{f,t+1}}{P^*_t}\right)^2\right] - \int_{N_h} P^S_{i,t}s^S_{i,t+1}di \end{aligned} \quad (58)$$

The efficiency conditions of the household problem do not change from (45), (46) and (47) except for international bond holdings, which are given by

$$B_{h,t+1} : \Lambda_t(1 + \tau b_{h,t+1}) = \beta\mathbb{E}_t\left[\Lambda_{t+1}R_t\right] \quad (59)$$

$$B_{f,t+1} : \Lambda_t(1 + \tau b_{f,t+1}) = \beta\mathbb{E}_t\left[\Lambda_{t+1}R^*_t\frac{S_{t+1}}{S_t}\right] \quad (60)$$

where $b_{h,t+1} \equiv B_{h,t+1}/P_t$ and $b_{f,t+1} \equiv B_{f,t+1}/P^*_t$. The monetary policy is specified in an identical

[16] One of the fundamental barriers to efficient allocation of international funds is sovereign default. Thus, employing a nonlinear framework such as Eaton and Gersovitz (1981) type would be ideal. However, given the large state space, such a nonlinearity is too costly for the current analysis. We leave that for future work.

[17] $B_{h,t+1} + B^*_{h,t+1} = 0$, where $B_{h,t+1}$ and $B^*_{h,t+1}$ are issued in home currency as denoted by the subscripts, and are held by home and foreign countries as denoted by asterisk or lack thereof.

[18] For the imcomplete risk sharing model, we assume away government bond. However, this does not create any material difference in equilibrium because we assume zero net supply of such bond even for the complete risk sharing environment. Such bond is employed just for valuation purpose.

way to (54). Substituting $\tilde{P}_t \Lambda_t = U_{x,t}$ in (59) and (60), one can rewrite them as

$$1 + \tau b_{h,t+1} = \beta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \tag{61}$$

$$\text{and } 1 + \tau b_{f,t+1} = \beta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{q_{t+1}}{q_t} \frac{R^*_t}{\pi^*_{t+1}} \right] \tag{62}$$

The bond market clearing conditions are given by

$$0 = b_{h,t+1} + b^*_{h,t+1} \tag{63}$$

$$\text{and } 0 = b_{f,t+1} + b^*_{f,t+1} \tag{64}$$

where foreign holdings of international bonds in home and foreign currencies $b^*_{h,t+1}$ and $b^*_{f,t+1}$ satisfy

$$1 + \tau b^*_{h,t+1} = \beta \mathbb{E}_t \left[ \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t+1}/\tilde{p}^*_{t+1}} \frac{q_t}{q_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \tag{65}$$

$$\text{and } 1 + \tau b^*_{f,t+1} = \beta \mathbb{E}_t \left[ \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t+1}/\tilde{p}^*_{t+1}} \frac{R^*_t}{\pi^*_{t+1}} \right]. \tag{66}$$

for foreign investors. Assuming that the portfolio rebalancing cost is transferred back to the household in lump sum, imposing the stock market equilibrium condition $s^S_{i,h,t} = s^S_{i,h,t+1} = 1$, and finally dividing the budget constraint through by $P_t$, one can rewrite it as a law of motion for bond holdings, that is,

$$b_{h,t+1} + q_t b_{f,t+1} = \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} q_t b_{f,t} + w_t h_t + \tilde{d}_t - \tilde{p}_t x_t \tag{67}$$

where $\tilde{d}_t \equiv \tilde{D}_t / P_t$ and $\tilde{d}^*_t \equiv \tilde{D}^*_t / P^*_t$. One can derive a similar law of motion for the foreign country,

$$q_t^{-1} b^*_{h,t+1} + b^*_{f,t+1} = \frac{R_{t-1}}{q_t \pi_t} b^*_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} b^*_{f,t} + w^*_t h^*_t + \tilde{d}^*_t - \tilde{p}^*_t x^*_t. \tag{68}$$

After multiplying (68) by $q_t$, subtracting (68) from (67) and imposing the bond market clearing conditions (63) and (64) yields

$$b_{h,t+1} + q_t b_{f,t+1} = \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} q_t b_{f,t} + \frac{1}{2}(w_t h_t - q_t w^*_t h^*_t) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}^*_t) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}^*_t x^*_t). \tag{69}$$

This condition replaces the risk sharing condition (53) in equilibrium.

### 3.7.3 Complete Risk Sharing Under Currency Union

We now consider the situation where the two countries bilaterally agree to adopt a single currency. One can think of the situation in the following way. Once the two countries adopt a single currency, all products and financial assets are denominated in the single currency unit. As a result, the nominal exchange rate is not defined. Also as a result, a single monetary authority sets the monetary policy rate, denoted by $R_t^U$, and all investors, regardless of their countries of origin and current locations, earn the same nominal return.

However, depending on the reference location of investors, the *real* return on international bond holdings differs. This is due to two factors: first, the two countries have different consumption baskets in the long-run owing to heterogeneous home biases; second, at any point in given time, the law of one price is violated as any two consumers residing in different countries have accumulated heterogeneous degrees of habits for an identical product, and as a consequence, firms, in general, price their products to markets, so called "pricing to habits" (see Ravn, Schmitt-Grohé, and Uribe (2007)). Hence, inflation rates are not equalized in the two countries despite the adoption of a single currency and monetary policy, and the real returns on international bonds are not equalized either.

We assume that the common monetary policy is specified so as to reflect the economic fundamentals of both countries:

$$R_t^U = (R^U)^{1-\rho_R} \left[ R_{t-1}^U \left( \frac{y_t^U}{y^U} \right)^{\rho_c} \left( \frac{\pi_t^U}{\pi^U} \right)^{\rho_\pi} \right]^{\rho_R} \tag{70}$$

where the union-wide variables are constructed as a weighted average with the weights given by the steady state share of output,[19] that is,

$$y_t^U = y_t \left( \frac{y}{y + qy^*} \right) + q_t y_t^* \left( \frac{qy^*}{y + qy^*} \right) \tag{71}$$

$$\text{and } \pi_t^U = \pi_t \left( \frac{y}{y + qy^*} \right) + \pi_t^* \left( \frac{qy^*}{y + qy^*} \right). \tag{72}$$

As a complete risk sharing regime, the currency union continues to ensure that the risk sharing condition (53) holds that prevails under the floating exchange rate regime. However, under the currency union, only one of the two consumption Euler equations (56) and (57) can be included in the system of equations that constitute the equilibrium. This is because the combination of the single monetary policy and the risk sharing condition makes the two consumption Euler equations linearly dependent. Hence, only the following efficiency condition enters the system with (53):[20]

$$1 = \beta \mathbb{E}_t \left[ \frac{\tilde{U}_{x,t+1}}{\tilde{U}_{x,t}} \frac{R_t^U}{\pi_{t+1}} \right] \tag{73}$$

[19]We have tried different formulae, real time or lagged weights, and have found no material difference in model dynamics.

[20]Otherwise, the two consumption Euler equations held within the system together with the common monetary policy and risk sharing condition would imply $\pi_{t+s} = \pi_{t+s}^*$ for all $s$, which cannot be satisfied.

### 3.7.4 Incomplete Risk Sharing Under Currency Union

Under the combination of incomplete risk sharing and currency union, the combined law of motion for international bond holdings (69) is replaced with

$$b_{h,t+1} = \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{1}{2}(w_t h_t - q_t w_t^* h_t^*) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}_t^*) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}_t^* x_t^*) \tag{74}$$

as there is no longer any distinction between international bonds issued in home or foreign currency. In addition, the bond market clearing condition drops out of the system of equations. Furthermore, there are only two, instead of four, Euler equations characterizing the efficiency in the international bond markets:

$$b_{h,t+1} \ : \ 1 + \tau b_{h,t+1} = \beta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{R_t^U}{\pi_{t+1}} \right] \tag{75}$$

$$\text{and } b_{h,t+1}^* \ : \ 1 + \tau b_{h,t+1}^* = \beta \mathbb{E}_t \left[ \frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*} \frac{q_t}{q_{t+1}} \frac{R_t^U}{\pi_{t+1}} \right] \tag{76}$$

Note that using the definition of the real exchange rate, we have as an identity

$$\frac{q_t}{q_{t+1}} = \frac{S_t}{S_{t+1}} \cdot \frac{\pi_{t+1}}{\pi_{t+1}^*}. \tag{77}$$

However, the currency union implies $S_t/S_{t+j} = 1$ for $j \geq 1$ permanently,[21] and (76) becomes equivalent to

$$1 + \tau b_{h,t+1}^* = \beta \mathbb{E}_t \left[ \frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*} \frac{R_t^U}{\pi_{t+1}^*} \right] \tag{78}$$

While an identical nominal return enters (75) and (78), their real and nominal discounting factors differ.

# 4 Results

## 4.1 Calibration

Our calibration strategy closely follows that of Gilchrist, Schoenle, Sim, and Zakrajsek (2013), expanding the set of parameters as needed for the international environment. There are three sets of parameters in the model: parameters related to preferences and technology; parameters governing the strength of nominal rigidities and monetary policy; parameters determining the strength of financial market frictions, including portfolio rebalancing costs.

We set the time-discounting factor equal to 0.995. The deep habit parameter $\theta$ is set equal to -0.9 similar to the choice of Ravn, Schmitt-Grohé, and Uribe (2005). The key tension between the market share maximization and cash flow maximization does not exist when $\theta = 0$. In this

[21] We assume that it is impossible to exit the union unilaterally. See Alvarez and Dixit (2014)'s real option approach for a theoretical consideration of the break up of a currency union.

Table 3: Baseline Calibration

| Description | Calibration |
|---|---|
| Preferences and production | |
| Time discounting factor, $\beta$ | 0.99 |
| Constant relative risk aversion, $\gamma_x$ | 2.00 |
| Deep habit, $\theta$ | $-$ 0.90 |
| Persistence of deep habit, $\rho$ | 0.95 |
| Elasticity of labor supply, $1/\gamma_h$ | 0.30 |
| Elasticity of substitution, $\eta$ | 2.00 |
| Armington elasticity, $\varepsilon$ | 1.50 |
| Home bias, $\omega_h^{\varepsilon}$ | 0.60 |
| Persistence of technology shock, $\rho_A$ | 0.90 |
| Returns to scale, $\alpha$ | 1.00 |
| Fixed operation cost, $\phi$, $\phi^*$ | 0.30, 0.00 |
| Nominal rigidity and monetary policy | |
| Price adjustment cost, $\gamma_p$ | 10.0 |
| Wage adjustment cost, $\gamma_w$ | 30.0 |
| Monetary policy inertia, $\rho^R$ | 0.85 |
| Taylor rule coefficient for inflation gap, $\rho^{\pi}$ | $0.25/(1-\rho^R)$ |
| Taylor rule coefficient for inflation gap, $\rho^{\pi}$ | $1.50/(1-\rho^R)$ |
| Financial Frictions | |
| Equity issuance cost, $\bar{\varphi}$, $\bar{\varphi}^*$ | 0.30, 0.30 |
| Idiosyncratic volatility (a.r.), $\sigma$ | 0.10 |
| Persistence of shocks, $\rho_A$, $\rho_{\varphi}$, $\rho_{\delta}$ | 0.90, 0.90, 0.90 |

environment, the financial shock we consider has considerably small effects on real outcome. It is in this sense that the current model owes a lot to customer market settings such as the "deep habit" model. We choose a fairly persistent habit formation such that only 10 percent of the habit stock is depreciated in a quarter. This highlights the firms' incentives to compete on market share. The CRRA parameter is then set equal to one given that the deep habit specification provides a strong motive to smooth consumption. We set the elasticity of labor supply equal to 1/3. For the persistence of exogenous shock processes, we specify the same degree of 0.90 for all three shocks.

The elasticity of substitution ($\eta$) is a key parameter in the customer-markets model as the greater the market power the firm has, the greater the incentive to invest in customer capital. We set the elasticity equal to 2 to be consistent with Broda and Weinstein (2006), who provide a set of point estimates for the elasticity of substitution for the U.S. economy. The estimates hover around 2.1$\sim$4.8, depending on the characteristics of products (commodities vs differentiated goods) and sub-samples (before 1990 vs after 1990). Our choice is virtually identical with the point estimate of Broda and Weinstein (2006) for the median value of the elasticity of differentiated goods of 2.1 for a sub-sample period since 1990. Differentiated products are the relevant category for the deep habit model considered in this paper. The choice is also broadly consistent with Ravn, Schmitt-Grohe, Uribe, and Uuskula (2010)'s point estimate of 2.48 using their structural estimation method.

Regarding $\omega_h$ and $\omega_f$, the weights of home and foreign goods in the utility function, we set these such that the share of imported goods in steady state consumption basket $(p_f c_f / \sum_{k=h,f} p_k c_k)$ is equal to 0.4. 0.4 is in the middle range of import/GDP ratios of European countries since 2000. For instance, Germany has 0.46 while Spain, Italy and Greece have 0.35 on average in 2012. Note that $\omega_f$ itself is not equal to the imported goods' share, and is set such that $\omega_f^{\varepsilon} = p_f c_f / \sum_{k=h,f} p_k c_k$. As for the Armington elasticity, we choose 1.5 to stay close to the near-unit elasticity estimated by Feenstra, Luck, Obstfeld, and Russ (2014). However, as long as it is greater than 1, a value lower than our choice does not affect our main results. For instance, lowering this value closer to 1.0 reduces the impact of a financial shock on the aggregate output under currency union to 2/3 of the baseline calibration. This is because the lower elasticity of cross-border substitution implies a lower degree of price war among countries. However, even in this polar case, the qualitative features of the equilibrium remain the same.

Another important parameter is the fixed operating cost, $\phi$. This parameter is jointly determined with the returns to scale parameter $\alpha$. We set $\alpha$ first, then choose $\phi$ such that dividend payout ratio (relative to operating income) hits the post war mean value of 2.5 percent in U.S. Decreasing returns to scale enhances the link between the financial market friction and the pricing decision. For this reason, we chose $\alpha = 0.8$ in our earlier work. However, in the current paper, we choose $\alpha = 1.0$ to be consistent with the convention in international macroeconomics literature. With the chosen $\alpha$, $\phi$ and $\eta$, the average mark-up is determined as 1.19. We then experimented with various values for $\phi^*$ to explore the implication of heterogenous financial frictions for the member countries of the currency union. To emphasize this aspect, we set $\phi^* = 0$, that is, the foreign country in the model does not face any fixed operating costs. To calibrate the financial friction, we set the dilution cost $\varphi = \varphi^* = 0.30$ as in Cooley and Quadrini (2001). $\varphi^* = 0.30$ alone does not create a sufficient degree of financial friction for the foreign country because the probability of external financing is vry low when $\phi^* = 0$. We discuss the influence of these choices on model outcomes below. The volatility of the idiosyncratic shock is calibrated to be 0.10 at an annual frequency, a moderate degree of idiosyncratic uncertainty.

For the parameters related to nominal rigidity, we set the adjustment costs of nominal price $\gamma_p = 10.0$. In our model presentation, we proceed as if nominal wages were flexible. In our actual simulation, we introduce nominal rigidity for wages along the line of Bordo, Erceg, and Evans (2000) and Erceg, Henderson, and Levin (2000). In particular, in symmetry to the nominal price rigidity, we assume market power for households that supply labor to production firms, and a quadratic cost of adjusting nominal wage. In this case, and under the assumption of a separable, constant elasticity of labor supply, $U_{h,t} = -h_t^{1/\zeta}$, the efficiency condition (47) is modified into

$$\eta_w \frac{h_t^{1/\zeta}/U_{x,t}}{w_t/\tilde{p}_t} = \eta_w - 1 + \gamma_w(\pi_{w,t} - \pi_w)\pi_{w,t} - \beta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \gamma_w (\pi_{w,t+1} - \pi_w) \pi_{w,t+1} \frac{\pi_{w,t+1}}{\pi_{t+1}} \frac{h_{t+1}}{h_t} \right] \tag{79}$$

where $\pi_{w,t} \equiv W_{t+1}/W_t$, $\gamma_w$ is the coefficient of nominal wage adjustment cost, and $\eta_w$ is the elasticity of substitution of labor. We choose $\eta_w = 3$ and $\gamma_w = 30$. Our choice of nominal rigidity for both price and wage are very close to the point estimates of $\gamma_p = 14.5$ and $\gamma_w =$

41.0 by Ravn, Schmitt-Grohe, Uribe, and Uuskula (2010), who show that the deep habit model substantially enhances the persistence of inflation dynamics without the help of implausibly large amount of adjustment friction in nominal prices. While nominal wage rigidity does not modify the main conclusions of the paper in any important way, it does help create a greater volatility of the real exchange rate. This is because the countercyclical markup of the country under a financial crisis, which is essential to the main conclusions of the paper, is achieved more by an immediate fall in nominal wage than by an increase in product price in a flexible wage environment. The more stable final product prices then lead to a less volatile real exchange rate, which runs counter to intention, in addition to being unrealistic.

Finally, we set the inertial coefficient of Taylor rule at a conventional level of 0.85 and the coefficient on the inflation gap as 1.5, following Taylor (1993). The long run coefficient for the output gap is less obvious. In traditional New Keynesian literature, this coefficient does not play an important role due to so called "divine coincidence". As a consequence, a strong reaction to inflation often makes the response to the output gap redundant or even inefficient. However, this is not the case in our current paper. As shown by Gilchrist, Schoenle, Sim, and Zakrajsek (2013), the specific combination of customer market setting and financial friction breaks the divine coincidence in the sense that a strong negative demand shock under a severe financial strain may lead to higher inflation pressure as firms under financial distress may find it optimal to raise prices in order to secure short-term liquidity, giving up on long-run market share. For this reason, we take a 50:50 prior by choosing a middle value between 0 and 0.5 suggested by Taylor (1993).

## 4.2 Currency Regime and Impacts of Financial Shock

In this section, we study the macroeconomic consequences of adopting a common currency, and hence a single monetary policy – in an environment where member countries face heterogeneous degrees of financial frictions. We assume that member countries neither have a complete risk-sharing arrangement through cross-border transfers of funds nor cross-border labor mobility. We compare the international macroeconomic dynamics under two environments: a floating exchange rate regime and a currency union. We will consider the effects of establishing the risk sharing arrangement later.

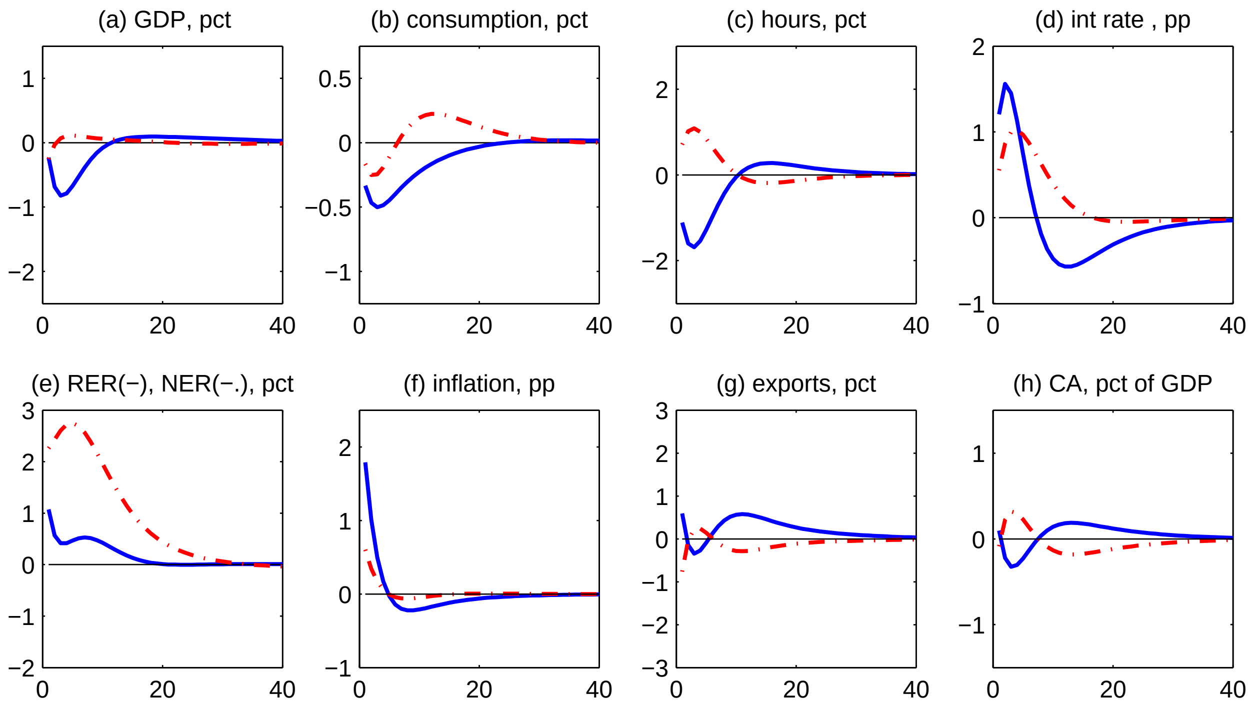
### 4.2.1 Impact of Financial Shocks

To study the effects of financial instability under various currency regimes, we impose a financial shock that elevates the cost of outside equity capital for firms in the member countries. More specifically, we subject the cost of issuing new equities to random shocks, and we call it the cost of capital shock:

$$\varphi_t = \bar{\varphi} f_t, \quad \log f_t = \rho_f \log f_{t-1} + \epsilon_{f,t}, \quad \epsilon_{f,t} \sim N(-0.5\sigma_f^2, \sigma_f^2) \tag{80}$$

To create an asymmetric financial environment for the two countries, $\epsilon_{f,t}$ such that the cost of capital is elevated to $2\bar{\varphi}$ on impact for the home country, and gradually comes down to the normal

Figure 2: Financial Shock to Peripheral Country Under Floating




Note: The blue, solid line depicts the peripheral country and red, dash-dotted line the core country. The shock assumes that the dilution cost for the peripheral countries go up by 100% from its normal level.

level $\bar{\varphi}$ while we keep the cost of capital for the financially strong, "foreign" country at $\varphi_t^* = \bar{\varphi}$ for all $t$.[22] The shock is designed to elevate the expected shadow value of internal funds for the firms in the home country. Without the shock, the expected shadow value of internal funds in the model is calibrated to 1.16. The shock immediately increases the expected shadow value to 1.50.

As we mentioned earlier, when the shadow cost of capital is elevated, firms in the model rebalance the benefit of investing in their customer base by lowering prices and the benefit of increasing current cashflow by temporarily raising prices. The former strategy is profitable in the long run, but in the short run, exposes the firms to liquidity risk.

Figure 2 displays the impact of a financial shock under a floating exchange rate regime. Blue, solid line shows the reaction of the home country and red, dash-dotted line the reaction of foreign country. Panel (f) of Figure 2 shows that the firms in the two countries indeed increase their prices in response to the financial shock. These results are consistent with the pattern we demonstrated for a closed-economy setting in our earlier paper (Gilchrist, Schoenle, Sim, and Zakrajsek (2013)). Since the shock here affects the home country directly and only indirectly the foreign country, the response of inflation is disproportionately larger for the home country. In our empirical section, we provide some evidence for exactly this pattern in the data in the context of the European financial crisis during the period of 2008-2012.

If nominal exchange rate does not respond to the shock, the differential responses of inflation

[22]However, in evaluating the consequences of different currency regimes, we assume that the foreign country is also subject to its own financial shocks given by $\varphi_t^* = \bar{\varphi} f_t^*$, $\log f_t^* = \rho_f \log f_{t-1}^* + \epsilon_{f,t}^*$, $\epsilon_{f,t}^* \sim N(-0.5\sigma_f^2, \sigma_f^2)$.

rates of the two country would imply a substantial appreciation of real exchange rate for home country. However this is not the case under a floating exchange rate regime. As shown in panel (3) of the figure, the nominal exchange rate, shown by red, dash-dotted line, strongly depreciates. In fact, the depreciation is strong enough that the real exchange rate also depreciates despite the price differential moving in the opposite direction. As in the data, the real exchange rate dynamics in the short run is dominated by the nominal exchange rate rather than price changes. Note that in panel (e), the deviation of nominal exchange rate looks like disappearing in the long run. However, this is simply a coincidence. The New Keyinesian framework adopted in the current paper does not have a prediction for the level of nominal exchange rate just as it does not have one for price level.[23] There is no reason why the levels of price index and nominal exchange rate should converge to specific levels.

The short-run depreciation of nominal exchange rate explains why GDP of home country is affected only mildly despite the large size of the financial shock.[24] Panel (a) shows that GDP of home country drops about one percent in the trough. The drop in aggregate consumption is even milder: only about a half percent. This is owing to relatively strong, initial gains in export, shown in panel (g), which is driven by the depreciation of real exchange rate. While short lived, the depreciation of nominal exchange rate helps firms avoid having to increase the relative prices of export products in foreign market too much with a view to increasing short-run liquidity precisely because the nominal depreciation does the job partially. This keeps them from losing their export market shares too much and thus limits the overall downside risk of the economy.

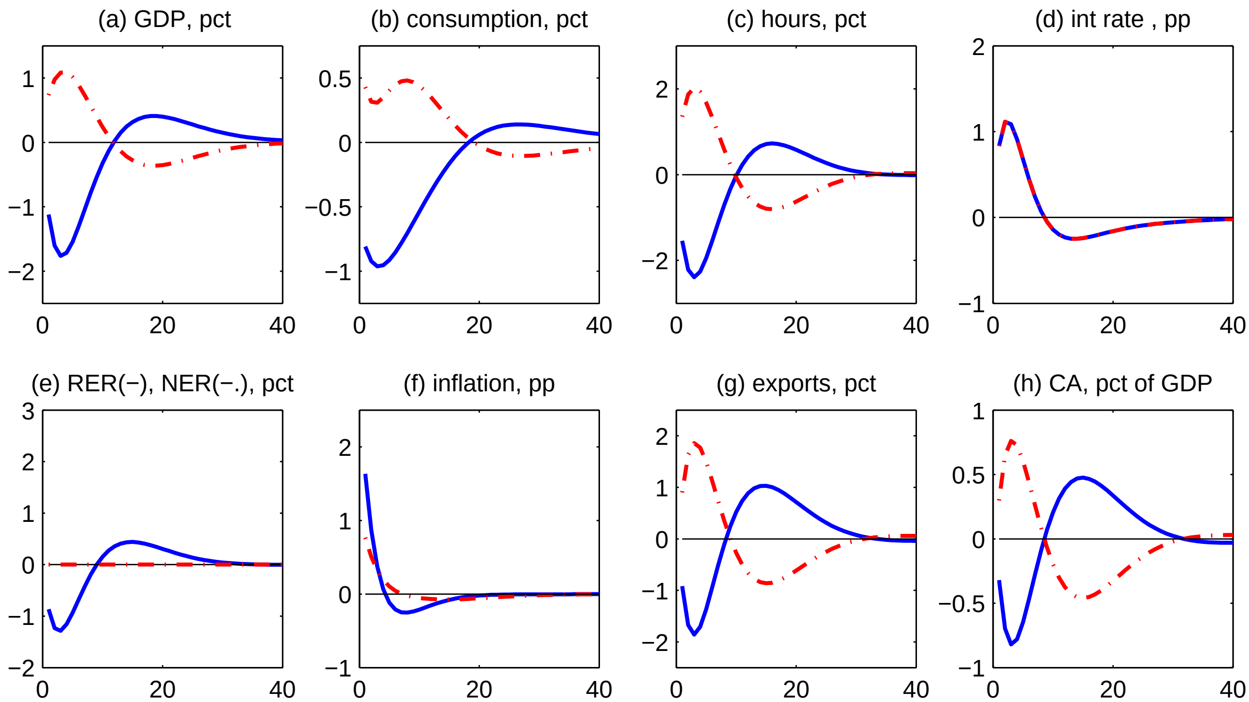
Under a currency union, adjustment looks dramatically different: Figure 3 shows the fundamentally different pattern of international macroeconomic adjustment ensuing the financial shock. From Panel (a) through (c), one can see that the drop in GDP, consumption and hours of the home country are almost two times greater under the currency union than under the floating exchange rate regime. In panel (g), the trough of exports and the current account deficit (relative to GDP) of the home country are almost 7 times and three times greater under the currency union. More strikingly, the financial crisis of home country is coupled with a modest boom in the foreign country: GDP, consumption and hours go up by 1, 0.5 and 2 percent each. The export and current account surplus (relative to GDP) of the foreign country to the home country go up by 2 and 3/4 percent, respectively.

The mechanism behind the stark difference in international macroeconomic adjustment patterns can be seen from Panels (d) through (f) in Figure 3. Owing to the financial friction, the firms in the country hit by the financial shock have a greater incentive to raise their prices more than their foreign counterparts. This pattern in the inflation differential following the financial shock is shown in Panel (f) of Figure 2 and 3, and similar across the two currency regimes. However, what is different under the currency union are the real exchange rate dynamics. Under a floating

[23]The figure assumes that the initial value of nominal exchange rate is one, which is an arbitrary, but innocuous assumption. However, the inflation rate of nominal exchange rate is well defined and is one of the model variables.

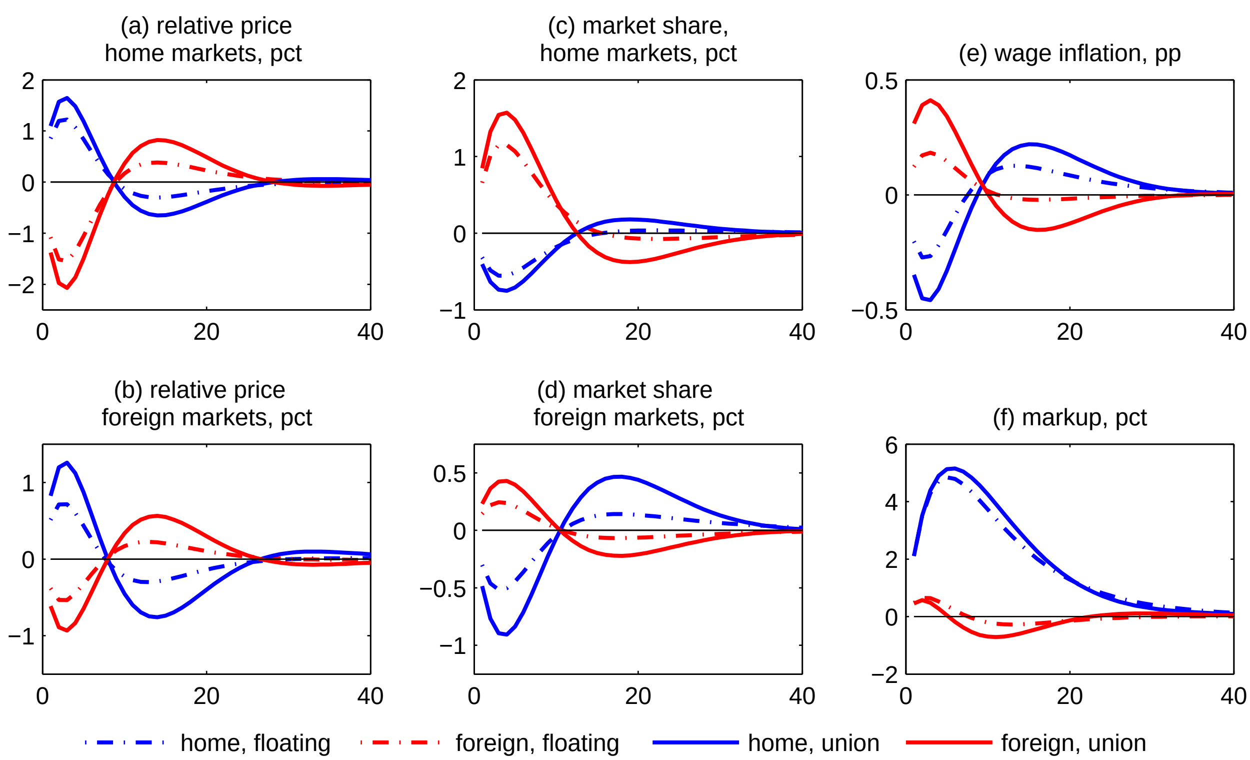
[24]GDP in the model is defined as domestic consumption plus export minus import, i.e., $p_t c_t + q_t p^*_{h,t} c^*_{h,t} - p_{f,t} c_{f,t}$. This is not equal to the volume index of aggregate output, $y_t$.

Figure 3: Financial Shock to Peripheral Country Under Monetary Union




Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country. The shock assumes that the dilution cost for the peripheral countries go up by 100% from its normal level.

Figure 4: Price Wars and Market Shares During A Financial Crisis




Note: Solid lines depict the cases of the currency union with blue and red indicating the periphery and the core, respectively. Dash-dotted lines depict the case of the floating exchange rate regime with the same color convention.

exchange rate regime, the international bond holding conditions (61) and (62) imply the following no-arbitrage condition:

$$\tau(b_{h,t+1} - b_{f,t+1}) = \mathbb{E}_t\left[m_{t,t+1}\left(\frac{R_t}{\pi_{t+1}} - \frac{q_{t+1}}{q_t}\frac{R_t^*}{\pi_{t+1}^*}\right)\right]. \tag{81}$$

In equilibrium with a relatively small cost of portfolio rebalancing, the left side is close to zero up to a first-order approximation. This means $R_t/\pi_{t+1} - (q_{t+1}/q_t)(R_t^*/\pi_{t+1}^*) \approx 0$ in expectation. As shown in panel (d) and (f) of Figure (2). The nominal interest differential between the home and the foreign country is smaller than inflation differential, that is, the real interest rate is lower in the home country than in the foreign country. This is intuitive given the recession in home country. The *absence of capital control* then implies that the real exchange rate should appreciate over time ($q_{t+1}/q_t < 1$) to avoid the exodus of capital from the home to the foreign country. This requires that the nominal exchange rate should *depreciate today* such that $q_{t+1}/q_t < 1$. This restriction from the free capital account is exactly what is missing in the currency union. The bond market efficiency conditions (75) and (78) impose no restriction on the dynamics of the real exchange rate. While the real interest rate differential still exists as in the case of the floating exchange regime, the differential does not have to be compensated by expected changes in nominal exchange rate: one can never exit from one currency to the same currency. (75) and (78) jointly require[25]

$$1 = \mathbb{E}_t\left[\frac{1}{2}\left(\frac{m_{t,t+1}}{\pi_{t+1}} + \frac{m_{t,t+1}^*}{\pi_{t+1}^*}\right)R_t^U\right], \tag{82}$$

This implies that the single policy rate $R_t^U$ should be set according to the *average* fundamental of the two economies regardless of the coefficients of the monetary policy reaction function (70).

As a consequence, any differential in inflation rates is directly translated into a movement of the real exchange rate. Since the model's financial frictions imply that the financially more vulnerable firms optimally choose higher relative prices, the real exchange rate appreciates substantially. This causes the export of the home country to contract severely, and so does GDP. Consumption does not contract as much as GDP since international borrowing, while subject to the costly rebalancing friction, allows consumers in the home country to smooth out the effects of the financial shock to a certain degree. The boom in economic activity of the foreign country is simply a mirror image of home country's economic plight. The contrast between the two countries is quite reminiscent of the European dichotomy between central and peripheral countries during the recent financial crisis.

One perhaps surprising result is that despite the harsher financial environment under the currency union, neither the level of the home country inflation nor the difference from foreign country are very different from the floating exchange rate regime. This is because international price wars are creating offsetting dynamics for overall inflation rates. Figure 4 provides a more detailed account of these international price wars. The solid lines, both blue and red, show the case of the currency

[25]One can derive this condition by simply adding the efficiency conditions and imposing the bond market clearing condition.

union with blue and red lines representing home and foreign countries in the crisis. The dash-dotted lines show the case of the floating exchange rate regime with blue and red lines representing home and foreign countries.

Panel (a) and (b) show the endogenous dispersion of relative prices. Regardless of the exchange rate regime, firms in the country hit by the financial shock increase their relative prices both in their domestic (panel (a)) and export markets (panel (b)). In contrast, the firms in the financially strong country substantially lower their relative prices. Firms in the home country raise their relative prices to secure short-term liquidity, sacrificing market share, both in home and foreign markets (panel (c) and (d)). The firms in the foreign country follow the opposite strategy, and lower their relative prices to gain market share. Interestingly, these firms slash their prices more in the home country (export prices) than in foreign country (domestic prices). Note that the intensity of price war, in terms of the dispersion of prices, is much greater under the currency union. This is because the firms in the home country can no longer rely on the depreciation of their currency to improve their cashflow. Since the higher prices of the home firms (domestic prices in the home country) are offset by the lower prices of the foreign firms (import prices in the home country), the overall inflation rate of the home country is not greatly affected by the currency regime. This explains the seemingly surprising result. Finally, note that the markup is strongly countercyclical in the home country regardless of currency regime during the financial crisis (see panel (e) and (f)).[26]

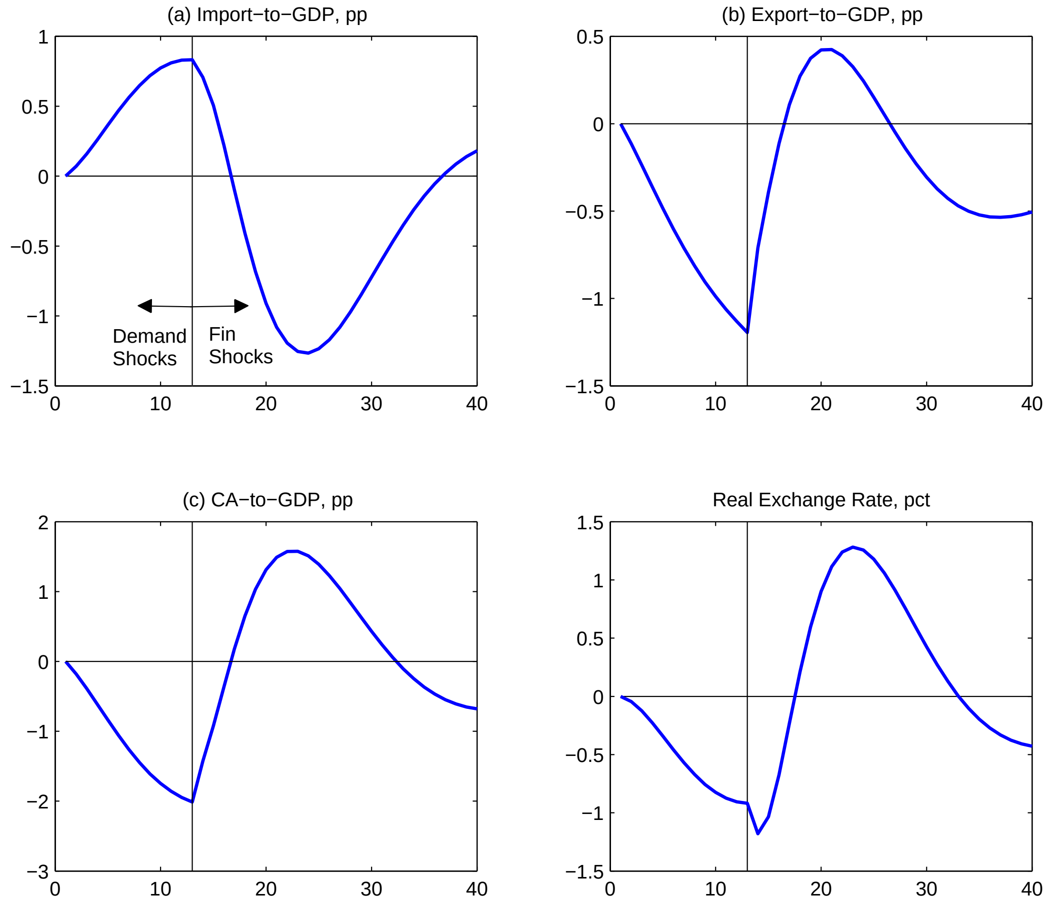
#### 4.2.2 Replicating the Boom-Bust Cycle

One aspect of macroeconomic adjustment dynamics shown in figure 3 may appear unfit to the sequence of the events characterizing the European crisis. Prior to the crisis, the peripharal countries were borrowing heavily, accumulating substantial amounts of current account deficits. During the crisis, the current account balances of the peripheral countries started improving rather rapidly due to the decline of imports. However, panel (g) and (h) show somewhat different dynamics: imports to the periphery increase first before they decline and the current account balances decrease before they turn positive.

However, this discrepancy in the timing patterns of the events in the model and in the European crisis does not imply that the model fails to replicate the crisis dynamics. The impulse response functions shown in figure 3 simply show the deviations from the steady state. In other words, the impulse response functions assume that the economies are at their steady states, which is not the situation right before the Europe has entered the crisis. It is very likely that the economic optimism since the adoption of the Euro substantially boosted aggregate and import demand in the periphery, and as a consequence, the inflation rates went up faster in the periphery, which then appreciated the real exchange rate for the periphery, aggravating the balance of payment situation even further. The financial shock then hits the European continent, which completely reverses the

[26] We have not shown the effects of technology shock for the sake of space. However, the appendix shows that the same conclusion is reached: the volatilities of macroeconomic variables under an aggregate technology shock are endogenously increased by the currency union. See figure 13 and 14.

Figure 5: Countercyclical Capital Flows



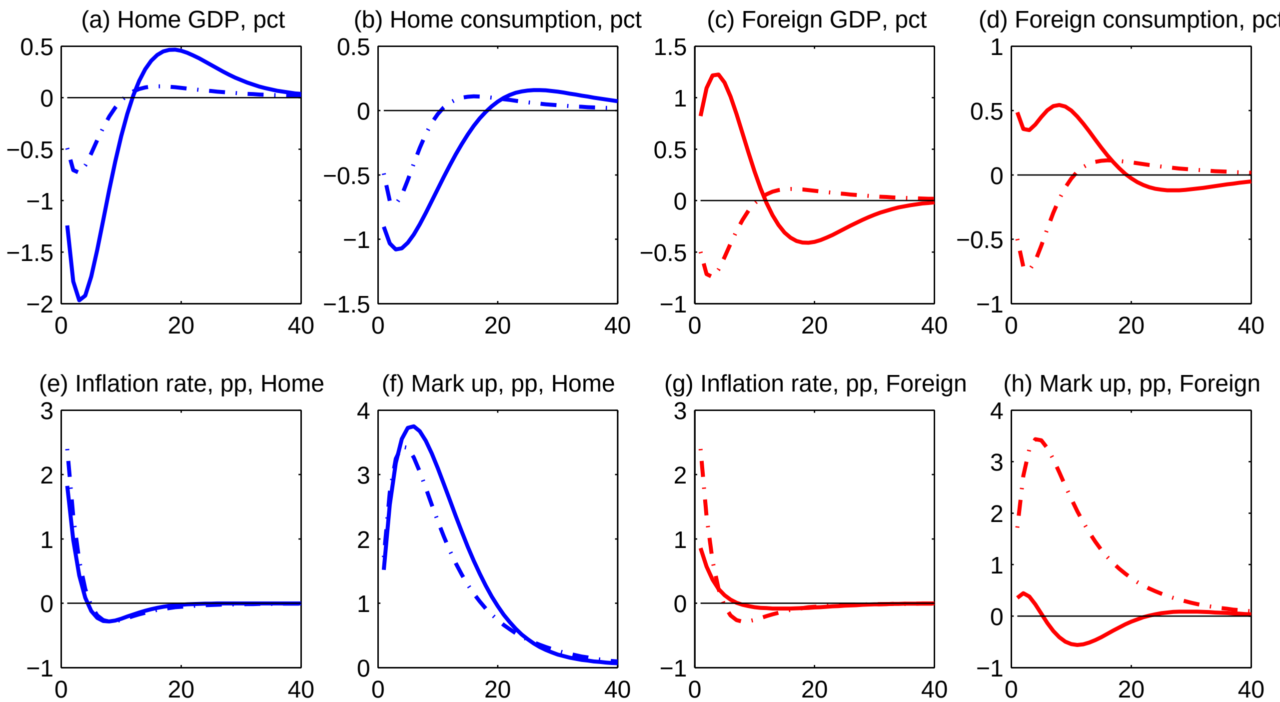

Note: The simulation assumes a gradually increasing positive demand shocks for the initial 12 quarters. The shocks are calibrated such that $\delta_t$ gradually go up from 1 percent to 5 percent from its steady state value. The economy then experience a one period financial shock that initially elevates the cost of capital from 0.3 to 0.5.

course of the events.

Such current account reversal dynamics can be easily engineered by mixing positive demand shocks before the home country is hit by the financial shock. The demand shocks are supposed to capture the effects of the economic optimism that probably presided in the periphery before the crisis. To that end, in figure 5, we peform an experiment in which there is a sequence of gradually increasing positive demand shocks for the initial 12 quarters and a single financial shock hits the home country in the 13th quarter. The shocks are calibrated such that $\delta_t$ gradually go up from 1 percent to 5 percent from its steady state value. The economy then experience a one period financial shock that initially elevates the cost of capital from 0.3 to 0.5.

In panel (a), import-to-GDP ratio of the periphery gradually increases in the period leading up to the occurence of the financial shock, which is indicated by the vertical black line. In contrast, export-to-GDP ratio continues to decline before the financial shock in panel (b) as the appreciation in the real exchange rate, shown in panel (d) weakens the competitiveness of the peripheral economy.

Figure 6: Heterogeneity as a Propagation Channel




Note: Solid lines describe the baseline case with asymmetric financial condition and shocks and dash-dotted lines show the alternative case with symmetric financial condition and shocks.

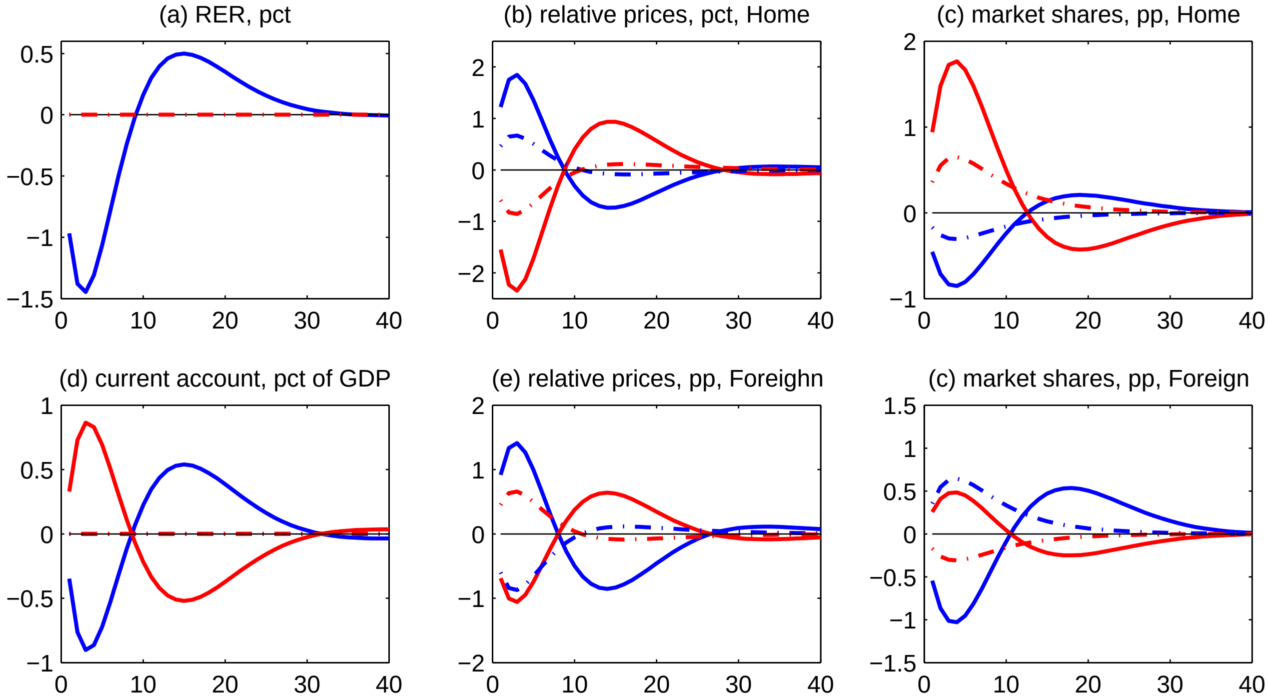
As a consequence, the current account deficit increases up to 3 percent of GDP prior to the financial shock. These courses of events completely turn opposite with the arrival of the financial shock, successfully replicating the current account reversal dynamics.

#### 4.2.3 Financial Heterogeneity and Predatory Price War

The financial heterogeneity plays an essential role in propagating the impact of the financial shock under the currency union because it is the financial strength of the foreign firms that allows them to engage in predatory price wars in order to drive out the competitors in both countries. If all countries are identical–in terms of technology, preferences, financial friction and composition of shocks hitting the economy, one may not generate the same degree of endogenous propagation. To show this, we consider an alternative calibration, in which all countries face the same magnitude of the fixed operation costs and are subject to the same financial shock: $\phi = \phi^* = 0.08$ and $\epsilon_{f,t} = \epsilon^*_{f,t} > 0$.

Figure 6 compares the two cases: the baseline calibration with asymmetric financial friction and financial shocks ($\phi = 0.08$, $\phi^* = 0.00$, $\epsilon_{f,t} > 0$, $\epsilon^*_{f,t} = 0$) and the alternative calibration with symmetric financial friction and financial shocks ($\phi = \phi^* = 0.08$ and $\epsilon_{f,t} = \epsilon^*_{f,t} > 0$). The results are striking in that the home country would prefer the alternative environment in which the foreign country were also subject to the same degree of financial market friction and underwent the same degree of financial shock. The dash-dotted lines of the figure suggest that the degree of the recession

Figure 7: Financial Heterogeneity, Relative Prices and Market Shares Dynamics




Note: Solid lines describe the baseline case with asymmetric financial condition and shocks and dash-dotted lines show the alternative case with symmetric financial condition and shocks. Blue color is for the home and and red color is for the foreign country.

is much smaller for the home country in the alternative environment. Since the foreign country now faces the same degree of liquidity crisis, the foreign firms cannot engage themselves in the predatory price war seen in the baseline case.

The foreign country now undergoes the same recession. This is in stark contrast to the baseline case where the foreign country experiences an export-driven boom. In response to its own liquidity shock, the foreign firms also raise their markup endogenously to secure cash flow and inflation rate follows the same route as the home counterpart (see panel (e)∼(f)). Figure 7 shows that with the nominal exchange rate held constant and the price indices of the two countries following the same path, the real exchange rate does not play any role in the macroeconomic dynamics, which is in stark contrast to the baseline case. Consequently, the burden of recession is evenly shared by the two countries and both countries lose equal magnitudes of market shares in their own territories and gain the same magnitudes of market shares in each other's economy.

#### 4.2.4 Is There Supporting Evidence?

Figure 4 suggests that the market shares of the foreign country expand in both home and foreign countries as a consequence of the financial crisis that the home country is undergoing because the foreign firms with ample liquid resources try to use the financial crisis of the home country to win market share competitions. Is there supporting evidence in the data?

Ideally, it is ideal to look at micro-level data in each member country, and this is what we have done for in our earlier work by matching COMPUSTAT firms and PPI correspondents (Gilchrist, Schoenle, Sim, and Zakrajsek (2013)). However, such good quality data are not available for the Euro-zone economy. Nevertheless, rough and crude measures of aggregate market shares can be constructed based on macroeconomic data. To that end, in figure 8, we compare the nominal value of country A's export to country B with the nominal GDP of country B. The idea is to think of the nominal GDP as the size of an economy and the relative export ratio as rough indicator of shares in relevant markets. In doing so, we delete energy, commodities and agricultural products, which do not fit the notion of customer markets. We normalize the ratios equal to one in 2010Q1, which we take as the onset of the European Balance-of-Payment crisis. Roughly around this period, Greece started finding it hard to finance its current account deficits from private sectors and the CDS spreads on Greek bonds started growing up to unsustainable levels.

Panel (a)∼(d) show the bilateral market shares of Portugal, Italy, Greece and Spain vis a vis Germany. Blue lines show the market shares of these countries in Germany and red lines the German market shares in these countries. Pictures show a remarkably similar pattern: since the beginning of the Balance-of-Payment crisis, German market shares have continued to grow in these countries; in contrast, the market shares of these countries in Germany have continued to shrink since the eruption of the crisis. The sizes of divergences and time series patterns are remarkably similar for all countries, and the time series patterns are consistent with what was shown for the model economy.

### 4.3 Welfare Consequence of Currency Union

Table 4 summarizes the welfare consequences of adopting a policy union when member countries face heterogeneous financial market friction. To evaluate the effect on welfare, we adopt the following, simple and stylized calibration strategy: we assume that the two countries are subject to aggregate technology shocks ($\epsilon_{A,t}$ and $\epsilon^*_{A,t}$) and financial shocks ($\epsilon_{f,t}$ and $\epsilon^*_{f,t}$) only; we calibrate the standard deviation of aggregate technology shocks as 1 percent each, and set the standard deviations of financial shocks such that they account for 50 percent of variance decomposition of home country output.[27] To evaluate the welfare, we define the value functions of the representative agents of the two countries as

$$W(\mathbf{s}) = U(x,h) + \beta\mathbb{E}[W(\mathbf{s}')|\mathbf{s}]$$
$$\text{and } W^*(\mathbf{s}) = U(x^*,h^*) + \beta\mathbb{E}[W^*(\mathbf{s}')|\mathbf{s}],$$

where $\mathbf{s}$ collects all the relevant state variables for the households in the two country. We approximate them using a second order approximation and report the analytical first moment in table 4.

[27] Obviously, more elaborate strategies can be adopted to provide more realistic representation of the macroeconomy. However, our main conclusions hold true even when a radically different strategy of calibrating the structure of shocks is employed as we have shown that the main problem associated with the currency union stays the same when the driving force of business cycle is entirely dominated by the aggregate technology shocks.

Figure 8: Euro-zone Market Share Dynamics


Note: Blue lines show the ratios of nominal values of export from Portugal, Italy, Greece and Spain to Germany relative to Germany's nominal GDP. Red lines show the ratios of nominal values of German exports to these countries relative to these countries' nominal GDPs. Export exclude energy, commodities and aggricultural products. The ratios are normalized to one in 2010Q1.

The first and second rows of table 4 show that the welfare levels of both home and foreign countries deteriorate by adopting a common currency. To put this result in perspective, we also report the consumption equivalent in the third column of the table, which is formally defined as the required increase in average consumption per period to make the agent living in an economy with the common currency indifferent with transitioning to an economy with the floating exchange rate. While the sign of the certainty equivalent change in consumption is intuitive, the degree of

Table 4: Welfare Consequence of Currency Union

| | Welfare | | Consumption Equivalent |
|---|---|---|---|
| | Currency Union (A) | Floating Ex. Rate (B) | Percent |
| Home country | -274.86 | -274.37 | 0.22 |
| Foreign country | -217.86 | -217.37 | 0.38 |
| Joint welfare | -492.82 | -491.48 | - |

Note: The consumption equivalent is the required minimum increase in average consumption per period holding labor hours constant to make the representative agent living in the economy under the floating exchange rate regime no worse off by transitioning to the currency union.

Table 5: Output and Consumption Volatility Under Alternative Environment

| | Output (GDP) volatility | | | Consumption volatility | | |
|---|---|---|---|---|---|---|
| | Union (A) | Floating (B) | B/A | Union (A) | Floating (B) | B/A |
| Home country | .0151 | .0108 | .72 | .0219 | .0099 | .45 |
| Foreign country | .0149 | .0087 | .58 | .0204 | .0093 | .46 |

Note: The consumption equivalent is the required minimum increase in average consumption per period holding labor hours constant to make the representative agent living in the economy under the floating exchange rate regime no worse off by transitioning to the currency union.

welfare deterioration caused by adopting a single currency appears to be small at least in terms of the certainty equivalent changes in consumption.

However, as is standard of welfare cost analysis of business cycle, this may be misleading because the representative agent is clearly a theoretical artifact we use for the sake of conveniency, and the aggregate uncertainty clearly and substantially understates the uncertainty facing individuals without perfect insurance. In this regard, somewhat more useful statistics can be found in standard deviations of output and consumption, which are reported in table 5.[28] The results shown in the table are striking: by agreeing to abolish the currency union and return to the floating exchange rate, home and foreign can reduce the output volatility as much as 28 and 42 percent, respectively. Even more strikingly, such a transition would reduce the consumption volatility of both countries more than 50 percent in the baseline calibration.

# 5 Two Fiscal Policy Arrangements

## 5.1 Complete Risk Sharing: Fiscal Union

Assuming that a return to the floating exchange rate is either infeasible or undesirable for whatever non-economic reasons, what could remedy the distortions without breaking the single currency

[28]Since about a half of output variation is due to the financial shock, one can easily map the changes in standard deviation of output into variation in unemployment rate using a variant of so called *Okun's law*.

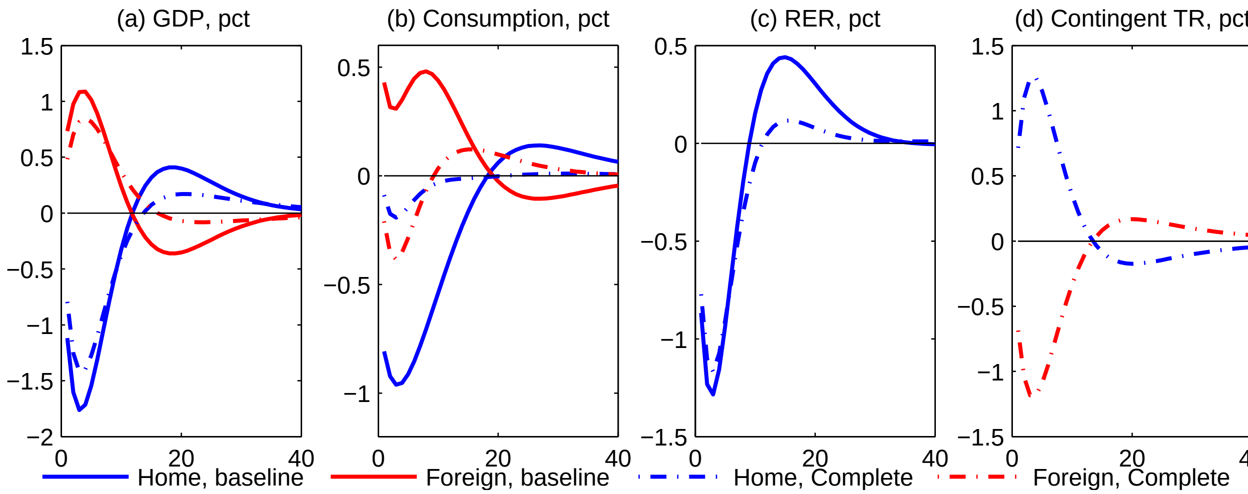
regime? To study this issue, we start by analyzing the nature of real allocation under a currency union with the two countries trading a complete set of state contingent bonds. This provides a natural benchmark against which the efficiency of other policy proposals can be gauged. For the sake of space, we mainly focus on the four aspects of the economy in this section: dynamics of GDP, consumption, real exchange and state-contingent cross-border transfer.

In the international macroeconomics literature, researchers often make the assumption of complete risk sharing. This is because it is often the case that the presence and absence of such risk sharing arrangement do not make substantial differences in the dynamics of endogenous quantities, including real exchange rate (see Steinsson (2008), for example). However, as shown by Figure 9, this is not the case in the current environment. In the Figure, solid lines represent the case of the baseline, that is, currency union without the complete risk sharing arrangement, with blue and red being home and foreign cases. Dash-dotted lines show the case of the currency union with the complete risk sharing with the same color convention.

A few conclusions can be easily made from the Figure. First, the state contingent bond trading evenly spreads out the cost of financial shock to the two countries, as shown by panel (b) of the Figure. In terms of the vertical distance from the baseline consumption paths, the two countries undergo the same degree of improvement/sacrifice depending on which country is hit by the shock. Second, the complete risk sharing arrangement works mainly through cross-border wealth transfer, rather than the changes in production share. Panel (a) shows that the production scales of the two allocations are not very different from each other, although the complete risk sharing does reduce the volatility of GDP for both countries. This is owing to the relative inefficiency of home country. To the contrary, the complete risk sharing arrangement organize the production such that the marginal costs are equalized across the countries and then redistribute the proceeds to achieve the risk sharing. Third, as shown by panel (d), this requires a substantial amount of international transfer of wealth, which, in this particular example, amounts to 1.2 percent of GDP. Finally, the complete risk sharing arrangement does not abolish the fluctuations in the real exchange rate, although the volatility is somewhat subdued in this environment. This is because, despite the cross-border transfer of wealth, the ratio of marginal utilities of consumption (more accurately, marginal utilities of consumption habit aggregator, $x$) declines.

Table 6 compares the welfare measures under the currency union with and without the fiscal union. A policy dilemma of risk sharing arrangement through the fiscal union between the two countries can be easily seen. While it substantially improves the welfare for the home country, it involves nontrivial, cross-border transfer of wealth. As a consequence, the welfare of foreign country deteriorates dramatically. The improvement in the joint welfare, shown in the last low of the table, implies that the gains for home dominates the loss for foreign country. The last column shows the certainty equivalent changes in consumptions relative to the baseline. Strikingly enough, the complete risk sharing arrangement increases the steady state consumption level 10 percent for home country, but decreases 9 percent for foreign country. The reason why the currency union cannot become a true union is that there simply is no reason for the residents of foreign country

Figure 9: Financial shock and Currency Union with Complete Risk Sharing




Note: Solid lines are the baseline case of the currency union with incomplete risk sharing arrangement with blue and red indicating the periphery and the core countries, respectively. Dashed-dotted lines depict the case of the currency union with complete risk sharing arrangement with the the same color convention.

Table 6: Effects on Welfare of Alternative Environments

| | Welfare | | Consumption Equiv |
|---|---|---|---|
| | MU (A) | Risk Sharing (B) | Percent |
| Home country | −274.86 | −253.21 | 10.28 |
| Foreign country | −217.86 | −236.96 | −9.13 |
| Joint welfare | −492.82 | −490.17 | – |

Note: The consumption equivalent is the required minimum increase in average consumption per period holding labor hours constant to make the representative agent living in the economy under the floating exchange rate regime no worse off by transitioning to the currency union.

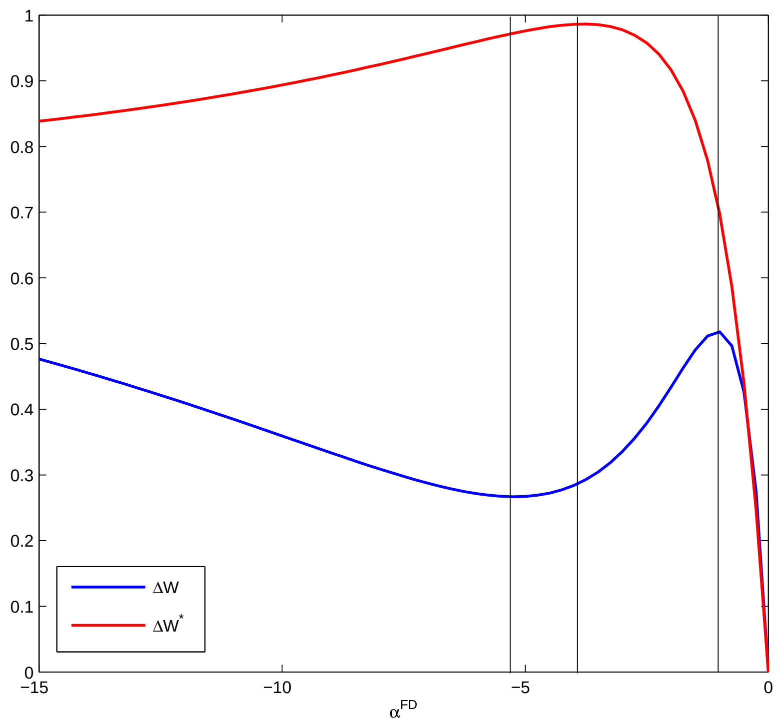
to agree with such transfers.

## 5.2 Fiscal Devaluations

In this section, we consider the effects of fiscal devaluation policy. The idea of fiscal devaluation policy is to replicate the effects of nominal devaluation by a mix of fiscal instruments (see Adao, Correia, and Teles (2009)). For instance, combining import tariff and export subsidy can replicate the effects of nominal devaluation on the competitiveness and terms of trade of a country implementing such a policy mix. Another example can be found in the mix of imposing value added tax (VAT) on all goods domestically sold and providing payroll subsidy to domestic firms. Both VAT and payroll subsidy are discriminatory fiscal tools because the exporters get reimbursement for VAT and foreign exporters are not eligible for payroll subsidy. Farhi, Gopinath, and Itskhoki (2014b) provide an in-depth analysis on a wide range of policy mixes that replicate the effects of a given degree of nominal devaluation under various asset market conditions.

There are at least two desirable aspects of fiscal devaluation policies. First, such policies can be

Figure 10: Welfare Differentials from Baseline without Fiscal Devaluation

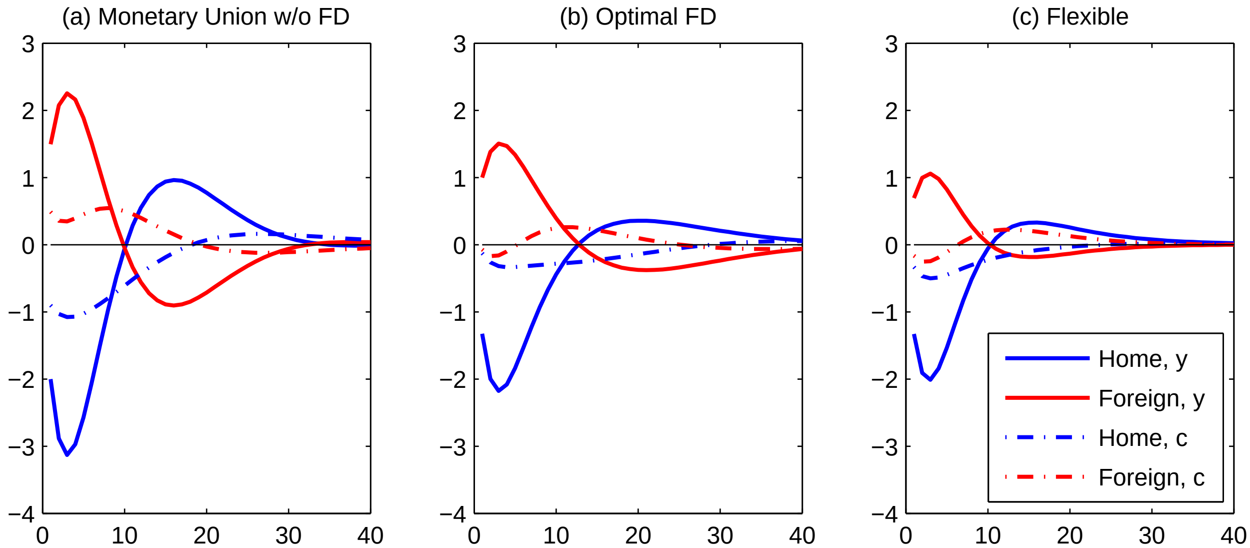



implemented unilaterally, and therefore can be tailored for the economic condition of the country implementing the policy. Second, fiscal evaluation policies can be designed is a way that the policies are revenue neutral. For instance import tariff can fund export subsidy and VAT can finance payroll subsidy. This second aspect makes fiscal devaluations particularly attractive for the periphery, which is undergoing fiscal crises.

However, despite the desirability of such policy options, the literature has yet to provide an extensive analysis on the impact of unilateral fiscal devaluations on the trading partners. In particular, if such policies achieve the effects of nominal devaluation by altering international relative prices and terms of trade, they may not be neutral to the trading partners in their effects. It then is unclear if such a unilateral policy move by the periphery will be welcomed by the core, especially if the core agreed to form a currency union because it wanted to avoid the manipulation of nominal exchanges rates by the central banks of the periphery. Can the peripheral countries carry out fiscal devaluations without the fear of retaliatory policy feedback from the core?

Since aforementioned literature has established equivalency of various policy mixes that generate the same effects of nominal devaluation, we focus on a simple mix of VAT ($\tau_t^v$) and payroll ($\varsigma_t^p$) subsidy carried out by the home country government. Under these policy, the marginal revenue of a home firm for selling its product in the home country is modified into $(1-\tau_t^\nu)p_{iht}p_{ht}$ where as its marginal cost of hiring becomes $(1-\varsigma_t^p)w_t$. It is assumed that the foreign country does not retaliate, and home firms are not subject to the same VAT in the foreign country. We assume that

Figure 11: Monetary Union w/ and w/o optimal FD vs Floating




Note: Solid lines are aggregate output and dash-dotted lines are consumption. Blue color is used for home country and red for foreign country.

home government operates the fiscal policies as stabilization tools and follow linear policy rules:

$$\tau_t^v = \varsigma_t^p = \frac{\delta_t}{1+\delta_t}$$
$$\delta_t = \alpha^{FD} \times \log\left(\frac{y_t}{\bar{y}}\right),$$

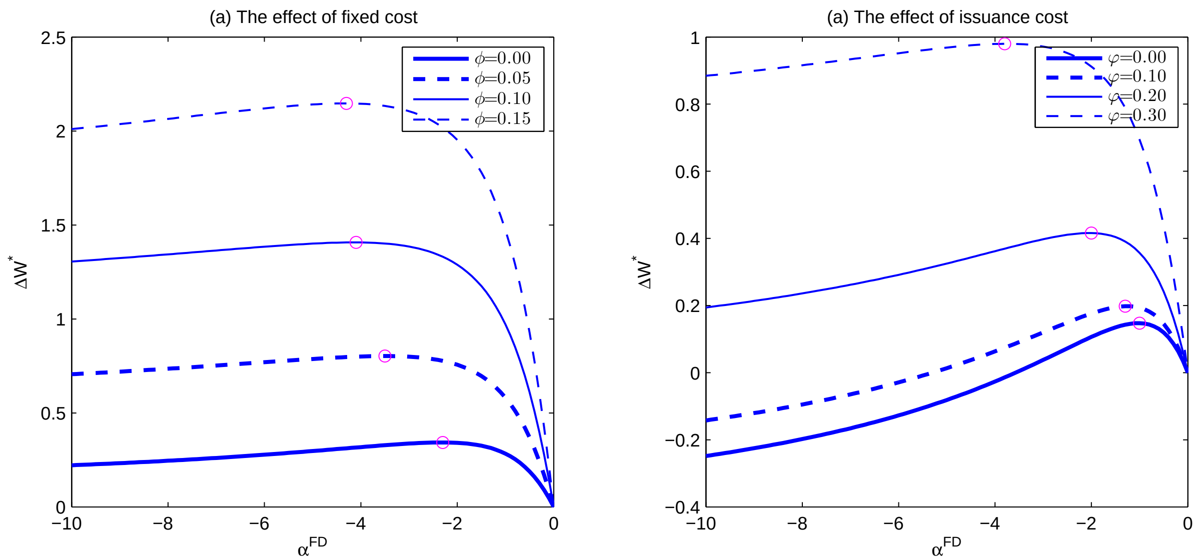
in which the magnitude of fiscal devaluation $\delta_t$ is linearly depends on output gap of the home country. To implement a countercyclical policy, $\alpha^{FD}$ should be nonpositive. We then perform an extensive grid search to calibrate $\alpha^{FD}$ that maximize the second-order approximate welfare of the home country and see what impact such a policy rule brings to the welfare of the foreign country.

Figure 10 shows our surprising finding: The welfare of the home country is maximized around $\alpha^{FD} = -1.0$, at which point the welfare is improved not only for home but also for the foreign country. In fact, the welfare maximizing point is in the welfare increasing region of the foreign country, which means that the foreign country has an incentive to provide a fiscal subsidy to make the home country adopt even stronger fiscal devaluation.

Figure 11 compares three allocations when the financial shock hits the home country: (a) under the currency union without fiscal devaluation; (b) under the currency union with fiscal devaluation optimized for home country welfare; (c) under the flexible exchange rate. Note that the magnitudes of the business cycles ensuing the financial are greatly reduced for both countries under the unilateral fiscal devaluation policy. The stabilization of consumption volatilities is remarkable again for both countries. The quality of resource allocation almost resembles the case of the flexible exchange rate.

What can explain the positive gains for the foreign country? To understand this, it is important

Figure 12: Financial Friction and Benefit of Fiscal Devaluation to the Core




to realize that there exists an important pecuniary externality working behind the monopolistic competition when pricing firms are facing financial friction. When the foreign firms take the predatory strategy of slashing the product prices in the middle of their trading partner's financial crisis, they take the general price levels as given, and as a result, they do not internalize the impact of their pricing strategy on the real exchange rate. This makes the foreign firms lower their prices to excessively low levels. Of course, their pricing strategies are individually rational, but they are not collectively in that they do not incorporate the impact on aggregate demand when driving out competitors. In general, to make private agents internalize externality, a distortionary taxation is required, and fiscal devaluation provides such a mechanism.

If our explanation is correct, one should expect that the potential welfare gains for the foreign country from the unilateral fiscal devaluation by the home country would increase as the degree of financial friction facing the home country, and therefore the strength of the pecuniary externality go up. Figure 12 shows this is indeed the case. In this figure, we show this in two different ways. In the left panel, holding the other parameters constant, we increase the size of the fixed operating costs for the home country. A greater fixed operating cost implies that the home firms have a greater probability of facing liquidity problem, which increases the effective shadow value of internal funds. In the right panel, we hold the size of the fixed operation cost. Instead, we increase the normal level of equity dilution cost $\bar{\varphi}$. Since the shock is given by $\varphi_t = \bar{\varphi} f_t$, this effectively increases the shock variance.

We then check what happens to the welfare differential to the foreign country as the home country policy coefficient for fiscal devaluation increases in absolute value. In the figure, red circles show the location of the policy coefficient that maximizes the welfare differential for the foreign country from the baseline with no fiscal devaluation for the home country. One can see that the welfare differential line generally moves up as the degree of financial market friction goes up and the welfare differential maximization for the foreign country generally calls for a greater policy reaction. This confirms our argument that if there is pecuniary externality working behind predatory price war and left unexploited by the Euro-zone governments, even the core will gain from the fiscal devaluation by the periphery.

## 6 Conclusion

We have analyzed the business cycle and welfare consequences of forming a currency union among countries facing heterogeneous financial market friction. We have shown that behind the sluggish adjustment of overvalued real exchange rates of the peripheral countries, there exist a mechanism that leads the firms in the core to lower their product prices to gain market shares and force the firms in the periphery to raise their prices to secure current cash flow to cope with liquidity problem. We have also shown that the common monetary policy may not be an effective tool to fight the current financial crisis and unilateral fiscal devaluation policy by the peripheral countries may be beneficial not only for themselves but for the core countries.

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# Appendices

## A Product Demand and Identities

### A.1 Derivation of Product Demands

In the symmetric equilibrium, all households choose the same levels of consumptions. Henceforth, we omit the household superscript. The cost minimization problem is then given by

$$\mathcal{L}_c = \sum_{k=h,f} \int_{N_k} P_{i,k,t} c_{i,k,t} di - \lambda_{c,t} \left[ \left\{ \sum_{k=h,f} \omega_k \left[ \int_{N_k} (c_{i,k,t}/s^{\theta}_{i,k,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}} \right\}^{1/(1-1/\varepsilon)} - x_t \right]$$

The efficiency condition for $c_{i,h,t}$ is given by

$$P_{i,h,t} = \omega_h \lambda_{c,t} \frac{(c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta}}{c_{i,h,t}} \left[ \int_{N_h} (c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}-1} x_t^{1/\varepsilon} \tag{A.1}$$

Similarly, the efficiency condition $c_{j,h,t}$ is given by

$$P_{j,h,t} = \omega_h \lambda_{c,t} \frac{(c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta}}{c_{j,h,t}} \left[ \int_{N_k} (c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}-1} x_t^{1/\varepsilon} \tag{A.2}$$

Taking the ratio of (A.1) and (A.2) yields

$$\frac{P_{i,h,t}}{P_{j,h,t}} = \frac{c_{j,h,t}}{c_{i,h,t}} \frac{(c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta}}{(c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta}}$$

or equivalently,

$$(c_{j,h,t}/s^{\theta}_{j,h,t-1})^{-1/\eta} = \frac{P_{j,h,t} s^{\theta}_{j,h,t-1}}{P_{i,h,t}} \frac{(c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta}}{c_{i,h,t}}$$

Rasing this expression to the power $1 - 1/\eta$, integrating the resulting expression with respect to $j$, and finally raising the resulting expression to the power $1/(1 - 1/\eta)$ yields

$$\left[ \int_{N_h} (c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta} dj \right]^{1/(1-1/\eta)} = \left[ \int_{N_h} (P_{j,h,t} s^{\theta}_{j,h,t-1})^{1-\eta} dj \right]^{1/(1-1/\eta)} c_{i,h,t} (s^{\theta}_{i,h,t-1})^{\eta-1} P^{\eta}_{i,h,t} \tag{A.3}$$

We define two aggregates

$$x_{h,t} \equiv \left[ \int_{N_h} (c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta} dj \right]^{1/(1-1/\eta)} \text{ and} \tag{A.4}$$

$$\tilde{P}_{h,t} \equiv \left[ \int_{N_h} (P_{j,h,t} s^{\theta}_{j,h,t-1})^{1-\eta} dj \right]^{1/(1-\eta)} \tag{A.5}$$

We can then rewrite (A.3) in terms of the two aggregates (A.4) and (A.5) as

$$\begin{aligned} c_{i,h,t} &= \left( \frac{P_{i,h,t}}{\tilde{P}_{h,t}} \right)^{-\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t} \\ &= \left( \frac{P_{i,h,t}}{P_{h,t}} \right)^{-\eta} \left( \frac{\tilde{P}_{h,t}}{P_{h,t}} \right)^{\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t}. \end{aligned} \tag{A.6}$$

Following the same steps, one can derive the home demand for foreign product as

$$\begin{aligned} c_{i,f,t} &= \left( \frac{P_{i,f,t}}{\tilde{P}_{f,t}} \right)^{-\eta} s^{\theta(1-\eta)}_{i,f,t-1} x_{f,t} \\ &= \left( \frac{P_{i,f,t}}{P_{f,t}} \right)^{-\eta} \left( \frac{\tilde{P}_{f,t}}{P_{f,t}} \right)^{\eta} s^{\theta(1-\eta)}_{i,f,t-1} x_{f,t}, \end{aligned} \tag{A.7}$$

where

$$x_{f,t} \equiv \left[ \int_{N_f} (c_{j,f,t}/s^{\theta}_{j,f,t-1})^{1-1/\eta} dj \right]^{1/(1-1/\eta)} \tag{A.8}$$

$$\text{and } \tilde{P}_{f,t} \equiv \left[ \int_{N_f} (P_{j,f,t} s^{\theta}_{j,f,t-1})^{1-\eta} dj \right]^{1/(1-\eta)} \tag{A.9}$$

Note that using (A.4) and (A.8), the consumption/habit aggregator $x_t$ can be written as

$$x_t = \left[ \sum_{k=h,f} \omega_k x_{k,t}^{1-1/\varepsilon} \right]^{1/(1-1/\varepsilon)} \tag{A.10}$$

We can then think of another cost minimization problem: minimizing the cost of obtaining $x_t$ by choosing $x_{k,t}$ when the unit price of $x_{k,t}$ is given by $P_{k,t}$, that is,

$$\mathcal{L}_x = \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} - \tilde{P}_t \left[ \left( \sum_{k=h,f} \omega_k x_{k,t}^{1-1/\varepsilon} \right)^{1/(1-1/\varepsilon)} - x_t \right]$$

where $\tilde{P}_t$ is the Lagrangian multiplier. The efficiency conditions for this program are given by

$$x_{h,t} = \omega_h^{\varepsilon} \left( \frac{\tilde{P}_{h,t}}{\tilde{P}_t} \right)^{-\varepsilon} x_t \tag{A.11}$$

$$\text{and } x_{f,t} = \omega_f^{\varepsilon} \left( \frac{\tilde{P}_{f,t}}{\tilde{P}_t} \right)^{-\varepsilon} x_t \tag{A.12}$$

Substituting these conditions in (A.10) yields the following condition.

$$1 = \left\{ \sum_{k=h,f} \omega_k \left[ \omega_k^{\varepsilon} \left( \frac{\tilde{P}_{k,t}}{\tilde{P}_t} \right)^{-\varepsilon} \right]^{1-1/\varepsilon} \right\}^{1/(1-1/\varepsilon)}$$

Solving this expression for $\tilde{P}_t$ results in an expression for a welfare based aggregate price index:

$$\tilde{P}_t = \left[ \sum_{k=h,f} \omega_k \tilde{P}_{k,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \tag{A.13}$$

### A.2 Accounting Identities

The following accounting identities are used in the main text:

$$\begin{aligned} \int_{N_k} P_{i,k,t} c_{i,k,t} di &= \int_{N_k} P_{i,k,t} \left( \frac{P_{i,k,t}}{\tilde{P}_{k,t}} \right)^{-\eta} s_{i,k,t-1}^{\theta(1-\eta)} x_{k,t} di && \text{(A.14)} \\ &= \tilde{P}_{k,t}^{\eta} x_{k,t} \int_{N_k} (P_{i,k,t} s_{i,k,t-1}^{\theta})^{1-\eta} di = \tilde{P}_{k,t} x_{k,t} \text{ for } k = h, f; \\ \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} = \sum_{k=h,f} \tilde{P}_{k,t} \omega_k & \left( \frac{\tilde{P}_{k,t}}{\tilde{P}_t} \right)^{-\varepsilon} x_t = \tilde{P}_t^{\varepsilon} x_t \sum_{k=h,f} \omega_k \tilde{P}_{k,t}^{1-\varepsilon} = \tilde{P}_t x_t; && \text{(A.15)} \end{aligned}$$

We define $p_{i,k,t} \equiv P_{i,k,t}/P_{k,t}$, $\tilde{p}_{k,t} \equiv \tilde{P}_{k,t}/P_{k,t}$ and $p_{k,t} \equiv P_{k,t}/P_t$ for $k = h, f$. Similarly, we define $p^*_{i,k,t} \equiv P^*_{i,k,t}/P^*_{k,t}$, $\tilde{p}^*_{k,t} \equiv \tilde{P}^*_{k,t}/P^*_{k,t}$ and $p^*_{k,t} \equiv P^*_{k,t}/P^*_t$ for $k = h, f$. Using these relative prices together with (8) and (11) under symmetric equilibrium, we can derive

$$\begin{aligned} \tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t} &= \left[ \sum_{k=h,f} \omega_k \tilde{p}_{k,t}^{1-\varepsilon} p_{k,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \\ &= \left[ \sum_{k=h,f} \omega_k p_{k,t}^{1-\varepsilon} s_{k,t-1}^{\theta(1-\varepsilon)} \right]^{1/(1-\varepsilon)}. \end{aligned} \tag{A.16}$$

# B Phillips Curve

Using the symmetric equilibrium condition and dividing the FOC for $c_{i,h,t}$ of the firm problem by $\mathbb{E}_t^a[\xi_{i,t}]$, one can express the ratio of the marginal sales to the marginal value of internal funds as

$$\begin{aligned} \frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} &= p_{h,t} - \frac{\mathbb{E}_t^a[\kappa_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} + (1-\rho) \frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} && \text{(B.1)} \\ &= p_{h,t} - \frac{\mathbb{E}_t^a[\xi_{i,t} a_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} \frac{w_t}{\alpha A_t} \left( \phi + c_{h,t} + c^*_{h,t} \right)^{\frac{1-\alpha}{\alpha}} + (1-\rho) \frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} \end{aligned}$$

Define aggregate (marginal) gross mark-up $\mu(s_t)$ as $\mu_t \equiv \dfrac{\alpha A_t}{w_t} \left( \phi + c_{h,t} + c^*_{h,t} \right)^{\frac{\alpha-1}{\alpha}}$. Define also financially adjusted markup $\tilde{\mu}_{h,t}$ as

$$\tilde{\mu}_t \equiv \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t} a_{i,t}]} \mu_t = \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t} a_{i,t}]} \frac{\alpha A_t}{w_t} \left( \phi + c_{h,t} + c^*_{h,t} \right)^{\frac{\alpha-1}{\alpha}}$$

We can then express (B.1) as

$$\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_{h,t}} + (1-\rho) \frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,h,t}]} \tag{B.2}$$

Dividing the FOC for $s_{i,h,t}$ through by $\mathbb{E}_t^a[\xi_{i,t}]$ and rearranging terms yields

$$\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = \rho\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{\lambda_{h,t+1}}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\right] + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{\nu_{h,t+1}}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\frac{c_{h,t+1}}{s_{h,t}}\right] \tag{B.3}$$

After substituting (B.2) in (B.3) and solving the expression forwardly, one can verify that

$$\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = \theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}_{t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s}-\frac{1}{\tilde{\mu}_s}\right)\right] \tag{B.4}$$

where $\tilde{\beta}_{t,s} \equiv m_{s,s+1}g_{h,s+1}\cdot\prod_{j=1}^{s-t}[\rho+\theta(1-\eta)(1-\rho)g_{h,t+j}]m_{t+j-1,t+j}$ with $g_{h,t} \equiv c_{h,t}/s_{h,t-1} = (s_{h,t}/s_{h,t-1}-\rho)/(1-\rho)$ denotes a growth-adjusted discount factor. Hence,

$$\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t}-\frac{1}{\tilde{\mu}_t}+(1-\rho)\theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}_{t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s}-\frac{1}{\tilde{\mu}_s}\right)\right] \tag{B.5}$$

# C Nonstochastic Steady State

To derive the steady state relationship, it is useful to state the problem of foreign firms and derive FOCs first. The firm problem can be expressed as the following Lagrangian:

$$\begin{aligned}
\mathcal{L}^* = \mathbb{E}_0\sum_{t=0}^{\infty}m_{0,t}^*\Bigg\{ & d_{i,t}^*+\kappa_{i,t}^*\left[\left(\frac{A_t^*}{a_{i,t}^*}h_{i,t}^*\right)^{\alpha}-\phi^*-(c_{i,f,t}^*+c_{i,f,t})\right] \\
& +\xi_{i,t}^*\Bigg[p_{i,f,t}^*p_{f,t}^*c_{i,f,t}^*+q_t^{-1}p_{i,f,t}p_{f,t}c_{i,f,t}-w_t^*h_{i,t}^*-d_{i,t}^*+\varphi^*\min\{0,d_{i,t}^*\} \\
& \qquad -\frac{\gamma}{2}\left(\frac{p_{i,f,t}^*}{p_{i,f,t-1}^*}\pi_{f,t}^*-\bar{\pi}^*\right)^2c_t^*-\frac{\gamma^*}{2}q_t^{-1}\left(\frac{p_{i,f,t}}{p_{i,f,t-1}}\pi_{f,t}-\bar{\pi}\right)^2c_t\Bigg] \\
& +\nu_{i,f,t}^*\left[(p_{i,f,t}^*)^{-\eta}s_{i,f,t-1}^{*\theta(1-\eta)}x_{f,t}^*-c_{i,f,t}^*\right] \\
& +\nu_{i,f,t}\left[(p_{i,f,t})^{-\eta}s_{i,f,t-1}^{\theta(1-\eta)}x_{f,t}-c_{i,f,t}\right] \\
& +\lambda_{i,f,t}^*\left[\rho s_{i,f,t-1}^*+(1-\rho)c_{i,f,t}^*-s_{i,f,t}^*\right] \\
& +\lambda_{i,f,t}\left[\rho s_{i,f,t-1}+(1-\rho)c_{i,f,t}-s_{i,f,t}\right]\Bigg\}
\end{aligned}$$

## C.1 Efficiency Conditions of Foreign Firms

The efficiency conditions for the firm problem in the foreign country are given by the followings:

$$d^*_{i,t} : \xi^*_{i,t} = \begin{cases} 1 & \text{if } d^*_{i,t} \geq 0 \\ 1/(1-\varphi^*) & \text{if } d^*_{i,t} < 0 \end{cases} \tag{C.1}$$

$$h^*_{i,t} : \xi^*_{i,t} w^*_t = \alpha \kappa^*_{i,t} \left( \frac{A^*_t}{a^*_{i,t}} h^*_{i,t} \right)^{\alpha-1} \tag{C.2}$$

$$\text{where } \; h^*_{i,t} \; = \; \frac{a^*_{i,t}}{A^*_t} (\phi^* + c^*_{i,f,t} + c_{i,f,t})^{1/\alpha}$$

$$c^*_{i,f,t} : \nu^*_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}] \; p^*_{i,f,t} p^*_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda^*_{i,f,t} \tag{C.3}$$

$$c_{i,f,t} : \nu_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}] \; q_t^{-1} p_{i,f,t} p_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda_{i,f,t} \tag{C.4}$$

$$s^*_{i,f,t} : \lambda^*_{i,f,t} = \rho \mathbb{E}_t[m^*_{t,t+1} \lambda^*_{i,f,t+1}] + \theta(1-\eta)\mathbb{E}_t \left\{ m^*_{t,t+1} \mathbb{E}^a_{t+1} \left[ \nu^*_{i,f,t+1} \frac{c^*_{i,f,t+1}}{s^*_{i,f,t}} \right] \right\} \tag{C.5}$$

$$s_{i,f,t} : \lambda_{i,f,t} = \rho \mathbb{E}_t[m^*_{t,t+1} \lambda_{i,f,t+1}] + \theta(1-\eta)\mathbb{E}_t \left\{ m^*_{t,t+1} \mathbb{E}^a_{t+1} \left[ \nu_{i,f,t+1} \frac{c_{i,f,t+1}}{s_{i,f,t}} \right] \right\} \tag{C.6}$$

$$p^*_{i,f,t} : 0 = \mathbb{E}^a_t[\xi^*_{i,t}] \left[ p^*_{f,t} c^*_{i,f,t} - \gamma \frac{\pi^*_{f,t}}{p^*_{i,f,t-1}} \left( \pi^*_{f,t} \frac{p^*_{i,f,t}}{p^*_{i,f,t-1}} - \bar{\pi}^* \right) c^*_t \right] - \eta \frac{\nu^*_{i,f,t}}{p^*_{i,f,t}} c^*_{i,f,t} + \gamma \mathbb{E}_t \left[ m^*_{t,t+1} \mathbb{E}^a_{t+1}[\xi^*_{i,t+1}] \pi^*_{f,t+1} \frac{p^*_{i,f,t+1}}{p^{*2}_{i,f,t}} \left( \pi^*_{f,t+1} \frac{p^*_{i,f,t+1}}{p^*_{i,f,t}} - \bar{\pi}^* \right) c^*_{t+1} \right] \tag{C.7}$$

$$p_{i,f,t} : 0 = \mathbb{E}^a_t[\xi^*_{i,t}] \left[ q_t^{-1} p_{f,t} c_{i,f,t} - \gamma \frac{q_t^{-1} \pi_{f,t}}{p_{i,f,t-1}} \left( \pi_{f,t} \frac{p_{i,f,t}}{p_{i,f,t-1}} - \bar{\pi} \right) c_t \right] - \eta \frac{\nu_{i,f,t}}{p_{i,f,t}} c_{i,f,t} + \gamma \mathbb{E}_t \left[ m^*_{t,t+1} \mathbb{E}^a_{t+1}[\xi^*_{i,t+1}] q^{-1}_{t+1} \pi_{f,t+1} \frac{p_{i,f,t+1}}{p^2_{i,f,t}} \left( \pi_{f,t+1} \frac{p_{i,f,t+1}}{p_{i,f,t}} - \bar{\pi} \right) c_{t+1} \right] \tag{C.8}$$

## C.2 Symmetric Equilibrium and Relative Prices

Before we move onto the model dynamics, it is useful to discuss how relative prices are determined and how they are related with each other in symmetric equilibrium. The risk neutrality, i.i.d. idiosyncratic shock and the timing convention aforementioned imply that all home country firms choose an identical price level for a given market, that is, $P_{i,h,t} = P_{h,t}$ and $P^*_{i,h,t} = P^*_{h,t}$. Similarly, $P_{i,f,t} = P_{f,t}$ and $P^*_{i,f,t} = P^*_{f,t}$. Due to the pricing to market mechanism, $P_{i,h,t} \neq S_t P^*_{i,h,t}$ and $P_{i,f,t} \neq S_t^{-1} P^*_{i,f,t}$ in general. However, the symmetric equilibrium implies $p_{i,h,t}(= P_{i,h,t}/P_{h,t}) = p^*_{i,h,t}(= P^*_{i,h,t}/P^*_{h,t}) = p_{i,f,t}(= P_{i,f,t}/P_{f,t}) = p^*_{i,f,t}(= P^*_{i,f,t}/P^*_{f,t}) = 1$ always.

In any path of symmetric equilibrium, the relative ratio of type specific, habit adjusted price index ($\tilde{P}_{k,t}$) and CPI index ($P_{k,t}$) satisfy the followings:

$$\tilde{p}_{h,t} = \tilde{P}_{h,t}/P_{h,t} = s^{\theta}_{h,t-1}$$
$$\tilde{p}^*_{h,t} = \tilde{P}^*_{h,t}/P^*_{h,t} = s^{*\theta}_{h,t-1}$$
$$\tilde{p}_{f,t} = \tilde{P}_{f,t}/P_{f,t} = s^{\theta}_{f,t-1}$$
$$\text{and } \tilde{p}^*_{f,t} = \tilde{P}^*_{f,t}/P^*_{f,t} = s^{*\theta}_{f,t-1}.$$

These relative prices can then be used to derive the demands for habit adjusted consumption baskets in the symmetric equilibrium: With a symmetric equilibrium condition $x^j_{h,t} = x_{h,t}$,

$$
\begin{aligned}
x_{h,t} &= \omega_h^{\varepsilon}\left(\frac{\tilde{P}_{h,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\
&= \omega_h^{\varepsilon}\left(\frac{\tilde{P}_{h,t}}{P_{h,t}}\cdot\frac{P_{h,t}}{P_t}\cdot\frac{P_t}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\
&= \omega_h^{\varepsilon} p_{h,t}^{-\varepsilon}\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{p}_t &= \left[\sum_{k=h,f} \omega_k \left(\frac{\tilde{P}_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\
&= \left[\sum_{k=h,f} \omega_k \left(\frac{\tilde{P}_{k,t}}{P_{k,t}}\cdot\frac{P_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\
&= \left[\sum_{k=h,f} \omega_k s_{k,t-1}^{\theta(1-\varepsilon)} p_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)}
\end{aligned}
$$

Similarly, it can be shown that

$$
\begin{aligned}
x_{f,t} &= \omega_f^{\varepsilon} p_{f,t}^{-\varepsilon}\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t, \\
x^*_{h,t} &= \omega_h^{*\varepsilon}(p^*_{h,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t, \\
x^*_{f,t} &= \omega_f^{*\varepsilon}(p^*_{f,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t,
\end{aligned}
$$

and

$$
\tilde{p}^*_t = \left[\sum_{k=h,f} \omega^*_k s_{k,t-1}^{*\theta(1-\varepsilon)} p_{k,t}^{*(1-\varepsilon)}\right]^{1/(1-\varepsilon)}.
$$

As is usual, only relative prices are determined in equilibrium: $p_{h,t}$, $p^*_{h,t}$, $p_{f,t}$, $p^*_{f,t}$, $\tilde{p}_{h,t}$, $\tilde{p}^*_{h,t}$, $\tilde{p}_{f,t}$, $\tilde{p}^*_{f,t}$, $\tilde{p}_t$, $\tilde{p}^*_t$ and $q_t$.

## C.3 Equilibrium Relative Prices and Quantities

The Phillips curves in the steady state is given by

$$p_h = \eta\frac{\nu_h}{\mathbb{E}^a_t[\xi_i]} \tag{C.9}$$

$$qp^*_h = \eta\frac{\nu^*_h}{\mathbb{E}^a_t[\xi_i]} \tag{C.10}$$

$$p^*_f = \eta\frac{\nu^*_f}{\mathbb{E}^a_t[\xi^*_i]} \tag{C.11}$$

$$\text{and } p_f q^{-1} = \eta\frac{\nu_f}{\mathbb{E}^a_t[\xi^*_i]}. \tag{C.12}$$

(C.9)∼(C.12) are the steady state Phillips curves of home good in the home country, home good in the foreign country, foreign good in the home country and foreign good in the foreign country, respectively. The notational convention is that $h$ and $f$ indicate the origin of the good, and asterisks and the absence thereof indicate the destination of the good with asterisks indicating the foreign country. For instance, $p_f^*$ is the (relative) price of good produced by and sold in the foreign country, whereas $p_f$ is the price of good produced by the foreign country, but sold in the home country in home currency unit, and hence $1/q$ attached to it to convert it to foreign currency unit.

The symmetric equilibrium and the law of motion for habit stock imply $c_h = s_h$, $c_h^* = s_h^*$, $c_f = s_f$ and $c_f^* = s_f^*$. Using these conditions together with the FOCs for habit stocks, one can derive

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_h}{\mathbb{E}^a[\xi_i]}, \tag{C.13}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]}, \tag{C.14}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]}, \tag{C.15}$$

$$\text{and} \quad \frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_f}{\mathbb{E}^a[\xi_i^*]}. \tag{C.16}$$

Combining (C.9)∼(C.12) and (C.13)∼(C.16) yields

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = p_h\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.17}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^*\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.18}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^*\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.19}$$

$$\text{and} \quad \frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q}\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.20}$$

which imply $qp_h^*/p_h = \lambda_h^*/\lambda_h$ and $qp_f^*/p_f = \lambda_f^*/\lambda_f$. Combining the FOCs (19), (21), (22), (C.2), (C.3) and (C.4), we have

$$\frac{\nu_h}{\mathbb{E}^a[\xi_i]} = p_h - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h}{\mathbb{E}^a[\xi_i]}, \tag{C.21}$$

$$\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^* - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]}, \tag{C.22}$$

$$\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^* - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]}, \tag{C.23}$$

$$\text{and} \quad \frac{\nu_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q} - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]}. \tag{C.24}$$

Substituting (C.9)∼(C.12) and(C.17)∼(C.20) in (C.21)∼(C.24) and solving for $p_k$ and $p_k^*$ yields

$$p_h = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}\frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}(\phi + c_h + c_h^*)^{\frac{1-\alpha}{\alpha}} \tag{C.25}$$

$$p_h^* = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}q^{-1}\frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}(\phi + c_h + c_h^*)^{\frac{1-\alpha}{\alpha}} \tag{C.26}$$

$$p_f^* = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}\frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}(\phi^* + c_f^* + c_f)^{\frac{1-\alpha}{\alpha}} \tag{C.27}$$

$$\text{and } p_f = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}q\frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}(\phi^* + c_f^* + c_f)^{\frac{1-\alpha}{\alpha}} \tag{C.28}$$

Note that the law of one price holds in the non-stochastic steady state: $p_h = qp_h^*$ and $p_f^* = p_f/q$, which also imply $\lambda_h^*/\lambda_h = 1$ and $\lambda_f^*/\lambda_f = 1$. This is simply because we assume the symmetry of the two markets in terms of elasticity of substitution, strength of customer relationship, etc. However, the law of one price is generally violated in stochastic simulation as two countries undergo different histories of asymmetric shocks, which affect the intensity of customer relationships, and hence the demand elasticities in the two countries, and different financing conditions. Firms in general exploit any discrepancies in customer relationship and discriminate prices across the border.

The external financing triggers in the steady state are given by

$$a^E = \frac{A}{w(\phi + c_h + c_h^*)^{1/\alpha}}(p_h c_h + qp_h^* c_h^*) \tag{C.29}$$

$$a^{E*} = \frac{A^*}{w^*(\phi^* + c_f^* + c_f)^{1/\alpha}}(p_f^* c_f^* + q^{-1}p_f c_f), \tag{C.30}$$

which can be used to compute $\mathbb{E}^a[\xi_i]$, $\mathbb{E}^a[\xi_i a_i]$, $\mathbb{E}^a[\xi_i^*]$ and $\mathbb{E}^a[\xi_i^* a_i^*]$:

$$\mathbb{E}^a[\xi_i] = 1 + \frac{\varphi}{1-\varphi}[1 - \Phi(z^E)] \tag{C.31}$$

$$\mathbb{E}^a[\xi_i a_i] = 1 + \frac{\varphi}{1-\varphi}[1 - \Phi(z^E - \sigma)] \tag{C.32}$$

$$\mathbb{E}^a[\xi_i^*] = 1 + \frac{\varphi^*}{1-\varphi^*}[1 - \Phi(z^{*E})] \tag{C.33}$$

$$\mathbb{E}^a[\xi_i^* a_i^*] = 1 + \frac{\varphi^*}{1-\varphi^*}[1 - \Phi(z^{*E} - \sigma)] \tag{C.34}$$

where

$$z^E \equiv \sigma^{-1}(\log a^E + 0.5\sigma^2) \tag{C.35}$$

$$\text{and } z^{*E} \equiv \sigma^{-1}(\log a^{*E} + 0.5\sigma^2). \tag{C.36}$$

(7) and (10) and their foreign counterparts imply that the following ratios should be satisfied in the steady state

$$\frac{c_{i,h}}{c_{i,f}} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta}s_{i,h}^{\theta(1-\eta)}x_h}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta}s_{i,f}^{\theta(1-\eta)}x_f} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta}s_{i,h}^{\theta(1-\eta)}\omega_h^{\varepsilon}\tilde{p}_h^{-\varepsilon}p_h^{-\varepsilon}\tilde{p}^{\varepsilon}x}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta}s_{i,f}^{\theta(1-\eta)}\omega_f^{\varepsilon}\tilde{p}_f^{-\varepsilon}p_f^{-\varepsilon}\tilde{p}^{\varepsilon}x}$$

$$\frac{c_{i,h}^*}{c_{i,f}^*} = \frac{p_{i,h}^{*-\eta}\tilde{p}_h^{*\eta}s_{i,h}^{*\theta(1-\eta)}x_h^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta}s_{i,f}^{*\theta(1-\eta)}x_f^*} = \frac{p_{i,h}^{*-\eta}\tilde{p}_h^{*\eta}s_{i,h}^{*\theta(1-\eta)}\omega_h^{\varepsilon}\tilde{p}_h^{*-\varepsilon}p_h^{*-\varepsilon}\tilde{p}^{*\varepsilon}x^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta}s_{i,f}^{*\theta(1-\eta)}\omega_f^{\varepsilon}\tilde{p}_f^{*-\varepsilon}p_f^{*-\varepsilon}\tilde{p}^{*\varepsilon}x^*}$$

Imposing the symmetric equilibrium conditions and using $\tilde{p}_k = s_k^{\theta}$, we have

$$\frac{c_h}{c_f} = \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_h}{p_f}\right)^{-\varepsilon}\left(\frac{s_h^{\theta}}{s_f^{\theta}}\right)^{1-\varepsilon} \tag{C.37}$$

$$\text{and } \frac{c_h^*}{c_f^*} = \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_h^*}{p_f^*}\right)^{-\varepsilon}\left(\frac{s_h^{*\theta}}{s_f^{*\theta}}\right)^{1-\varepsilon}. \tag{C.38}$$

Since $c_{i,k} = c_k = s_k = s_{i,k}$ and $c_{i,k} = c_k^* = s_k^* = s_{i,k}^*$, (9) and its foreign counterpart imply

$$x = \left[\sum_{k=h,f} \omega_k (c_k^{1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)} \tag{C.39}$$

$$x^* = \left[\sum_{k=h,f} \omega_k (c_k^{*1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}. \tag{C.40}$$

Aggregate (conditional) labor demand in home and foreign markets satisfy

$$h = \left[\frac{\phi + c_h + c_h^*}{A^{\alpha}\exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha}, \tag{C.41}$$

$$\text{and } h^* = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha}\exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha}, \tag{C.42}$$

which also implies goods market clearing conditions. The FOCs of households for labor hours can be expressed as

$$h = U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right], \tag{C.43}$$

$$\text{and } h^* = U_h^{-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right], \tag{C.44}$$

The labor market clearing conditions in home and abroad can then be given by

$$U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right] = \left[\frac{\phi + c_h + c_h^*}{A^{\alpha}\exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha} \tag{C.45}$$

$$\text{and } U_h^{-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right] = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha}\exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha}, \tag{C.46}$$

which characterize the labor market clearing conditions in home and abroad, and can be used to equilibrium wages in both markets. Finally, equilibrium consistency requires

$$1 = \left[\sum_{k=h,f} \omega_k p_k^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{C.47}$$

$$\text{and } 1 = \left[\sum_{k=h,f} \omega_k p_k^{*1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{C.48}$$

### C.4 Real Exchange Rate

In the case of complete risk sharing between the two countries, the real exchange rate at any point in time should satisfy (53).

$$q = \kappa \frac{U_x^*}{U_x} \left[ \frac{\sum_{k=h,f} \omega_k (p_k^* s_k^{*\theta})^{(1-\varepsilon)}}{\sum_{k=h,f} \omega_k (p_k s_k^{\theta})^{(1-\varepsilon)}} \right]^{-1/(1-\varepsilon)} \tag{C.49}$$

We assume that the equilibrium interest rates are determined by time preferences: $r = r^* = \beta^{-1} - 1$. This condition, in the case of incomplete risk sharing, pins down the equilibrium holdings of international bonds: $B_h = B_f = 0$, which, via the bond market clearing conditions, $B_h + B_h^* = 0$ and $B_f + B_f^* = 0$, pins down $B_h^* = B_f^* = 0$. In the case of incomplete risk sharing, the real exchange rate is determined such that $b_h = b_f = 0$, which, together with (69) implies

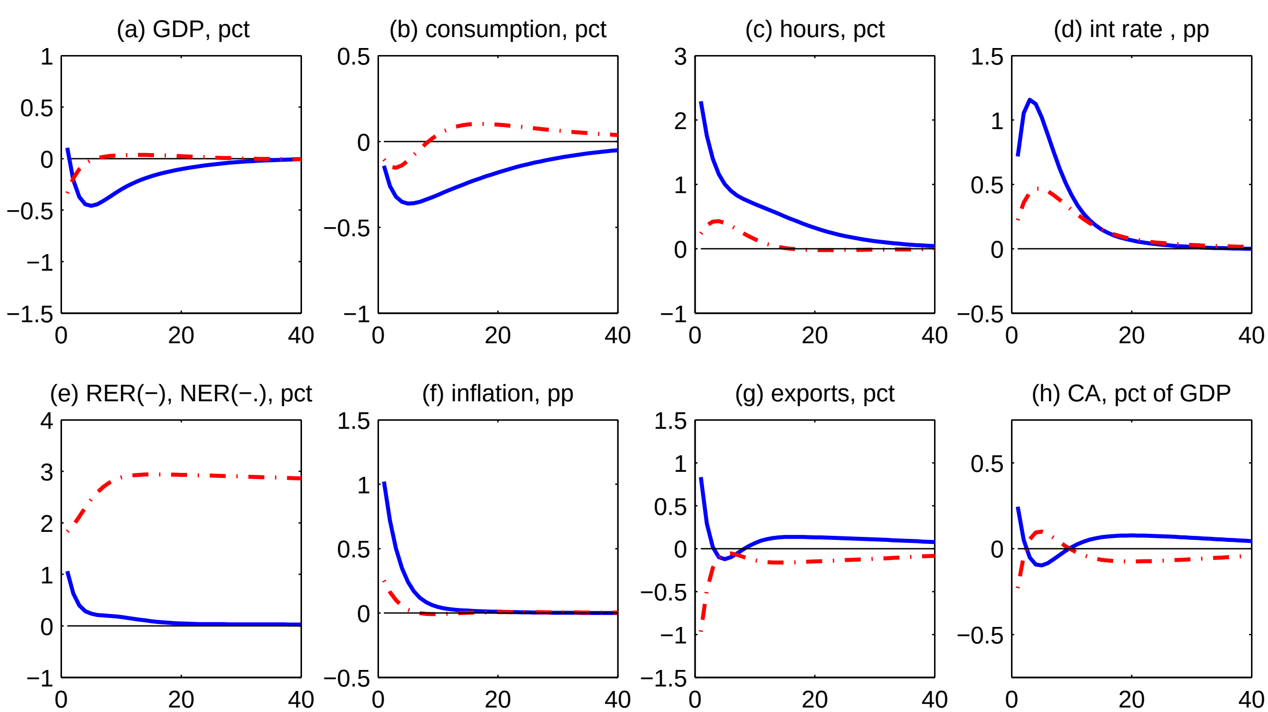
$$0 = wh - qw^*h^* + \tilde{d} - q\tilde{d}^* - (\tilde{p}x - q\tilde{p}^*x^*)$$

or equivalently,

$$q = \frac{wh + \tilde{d} - \tilde{p}x}{w^*h^* + \tilde{d}^* - \tilde{p}^*x^*}. \tag{C.50}$$

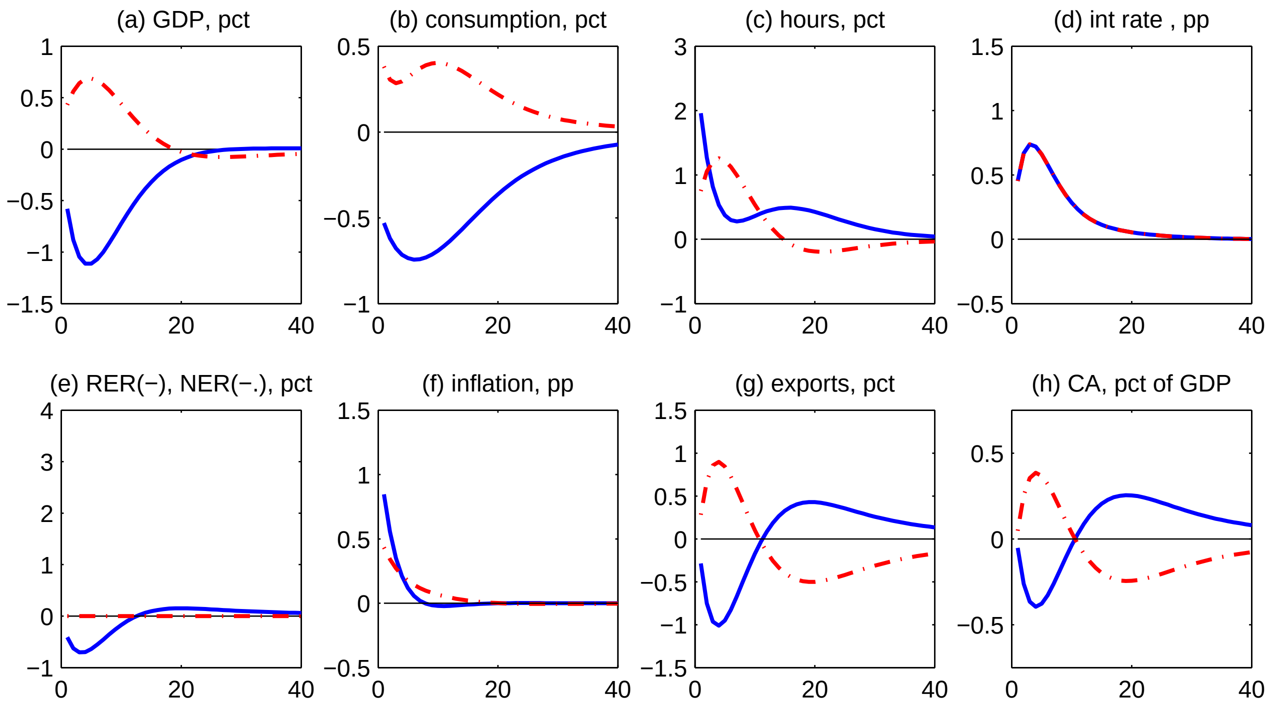
## D Impact of Technology Shocks

Figure 13: Technology Shock to Peripheral Country Under Floating




Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country.

Figure 14: Technology Shock to Peripheral Country Under Monetary Union




Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country.

# E System of Equations

There are total 71 equations for 71 endogenous variables in the system in the case with a floating exchange rate under the complete risk sharing arrangement. We provide these equations in the symmetric equilibrium forms:

$$0 = -\frac{h_t^{1/\zeta}/U_{x,t}}{w_t/\tilde{p}_t} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi_{w,t} - \pi_w)\pi_{w,t} \\ -\beta\frac{\gamma_w}{\eta_w}\mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}(\pi_{w,t+1} - \pi_w)\pi_{w,t+1}\frac{\pi_{w,t+1}}{\pi_{t+1}}\frac{h_{t+1}}{h_t}\right] \tag{E.1}$$

$$0 = -\frac{h_t^{*1/\zeta}/U_{x,t}}{w_t^*/\tilde{p}_t^*} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi_{w,t}^* - \pi_w)\pi_{w,t}^* \\ -\beta\frac{\gamma_w}{\eta_w}\mathbb{E}_t\left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t}^*/\tilde{p}_t^*}(\pi_{w,t+1}^* - \pi_w)\pi_{w,t+1}^*\frac{\pi_{w,t+1}^*}{\pi_{t+1}^*}\frac{h_{t+1}^*}{h_t^*}\right] \tag{E.2}$$

$$0 = -\frac{c_{h,t}}{c_{f,t}} + \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_{h,t}}{p_{f,t}}\right)^{-\varepsilon}\left(\frac{s_{h,t-1}^{\theta}}{s_{f,t-1}^{\theta}}\right)^{1-\varepsilon} \tag{E.3}$$

$$0 = -\frac{c_{h,t}^*}{c_{f,t}^*} + \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_{h,t}^*}{p_{f,t}^*}\right)^{-\varepsilon}\left(\frac{s_{h,t-1}^{*\theta}}{s_{f,t-1}^{*\theta}}\right)^{1-\varepsilon} \tag{E.4}$$

$$0 = -\tilde{p}_{h,t} + s_{h,t-1}^{\theta} \tag{E.5}$$

$$0 = -\tilde{p}_{f,t} + s^{\theta}_{f,t-1} \tag{E.6}$$

$$0 = -\tilde{p}^*_{h,t} + s^{*\theta}_{h,t-1} \tag{E.7}$$

$$0 = -\tilde{p}^*_{f,t} + s^{*\theta}_{f,t-1} \tag{E.8}$$

$$0 = -x_{h,t} + \omega_h^{\varepsilon} p_{h,t}^{-\varepsilon}\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t \tag{E.9}$$

$$0 = -x_{f,t} + \omega_f^{\varepsilon} p_{f,t}^{-\varepsilon}\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t \tag{E.10}$$

$$0 = -x^*_{h,t} + \omega_h^{*\varepsilon} (p^*_{h,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t \tag{E.11}$$

$$0 = -x^*_{f,t} + \omega_f^{*\varepsilon} (p^*_{f,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t \tag{E.12}$$

$$0 = -\tilde{p}_t + \left[\sum_{k=h,f} \omega_k s^{\theta(1-\varepsilon)}_{k,t-1} p^{1-\varepsilon}_{k,t}\right]^{1/(1-\varepsilon)} \tag{E.13}$$

$$0 = -\tilde{p}^*_t + \left[\sum_{k=h,f} \omega^*_k s^{*\theta(1-\varepsilon)}_{k,t-1} p^{*(1-\varepsilon)}_{k,t}\right]^{1/(1-\varepsilon)} \tag{E.14}$$

$$0 = -\pi_{h,t} + \frac{p_{h,t}}{p_{h,t-1}}\pi_t \tag{E.15}$$

$$0 = -\pi^*_{h,t} + \frac{p^*_{h,t}}{p^*_{h,t-1}}\pi^*_t \tag{E.16}$$

$$0 = -\pi_{f,t} + \frac{p_{f,t}}{p_{f,t-1}}\pi_t \tag{E.17}$$

$$0 = -\pi^*_{f,t} + \frac{p^*_{f,t}}{p^*_{f,t-1}}\pi^*_t \tag{E.18}$$

$$0 = -h^S_t + h^D_t \tag{E.19}$$

$$0 = -h^{*S}_t + h^{*D}_t \tag{E.20}$$

$$0 = -\mathbb{E}^a_t[\kappa_{i,t}] + \mathbb{E}^a_t[\xi_{i,t} a_{i,t}] \frac{w_t}{\alpha A_t}(\phi + c_{h,t} + c^*_{h,t})^{\frac{1-\alpha}{\alpha}} \tag{E.21}$$

$$0 = -\mathbb{E}^a_t[\kappa^*_{i,t}] + \mathbb{E}^a_t[\xi^*_{i,t} a^*_{i,t}] \frac{w^*_t}{\alpha A^*_t}(\phi + c_{f,t} + c^*_{f,t})^{\frac{1-\alpha}{\alpha}} \tag{E.22}$$

$$0 = -\nu_{h,t} + \mathbb{E}^a_t[\xi_{i,t}]\ p_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho)\lambda_{h,t} \tag{E.23}$$

$$0 = -\nu^*_{h,t} + \mathbb{E}^a_t[\xi_{i,t}]\ q_t p^*_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho)\lambda^*_{h,t} \tag{E.24}$$

$$0 = -\lambda_{h,t} + \rho\mathbb{E}_t[m_{t,t+1}\lambda_{h,t+1}] + \theta(1-\eta)\mathbb{E}_t\left\{m_{t,t+1}\mathbb{E}^a_{t+1}\left[\nu_{h,t+1}\frac{c_{h,t+1}}{s_{h,t}}\right]\right\} \tag{E.25}$$

$$0 = -\lambda^*_{h,t} + \rho\mathbb{E}_t[m_{t,t+1}\lambda^*_{h,t+1}] + \theta(1-\eta)\mathbb{E}_t\left\{m_{t,t+1}\mathbb{E}^a_{t+1}\left[\nu^*_{h,t+1}\frac{c^*_{h,t+1}}{s^*_{h,t}}\right]\right\} \tag{E.26}$$

$$0 = -\nu^*_{f,t} + \mathbb{E}^a_t[\xi^*_{i,t}]\ p^*_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda^*_{f,t} \tag{E.27}$$

$$0 = -\nu_{f,t} + \mathbb{E}^a_t[\xi^*_{i,t}]\ q_t^{-1} p_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda_{f,t} \tag{E.28}$$

$$0 = -\lambda^*_{f,t} + \rho\mathbb{E}_t[m^*_{t,t+1}\lambda^*_{f,t+1}] + \theta(1-\eta)\mathbb{E}_t\left\{m^*_{t,t+1}\mathbb{E}^a_{t+1}\left[\nu^*_{f,t+1}\frac{c^*_{f,t+1}}{s^*_{f,t}}\right]\right\} \tag{E.29}$$

$$0 = -\lambda_{f,t} + \rho\mathbb{E}_t[m^*_{t,t+1}\lambda_{f,t+1}] + \theta(1-\eta)\mathbb{E}_t\left\{m^*_{t,t+1}\mathbb{E}^a_{t+1}\left[\nu_{f,t+1}\frac{c_{f,t+1}}{s_{f,t}}\right]\right\} \tag{E.30}$$

$$\begin{aligned} 0 = &-p_{h,t}\frac{c_{h,t}}{c_t} + \gamma\pi_{h,t}(\pi_{h,t}-\bar{\pi}) + \eta\frac{\nu_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{c_{h,t}}{c_t} \\ &-\gamma\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\pi_{h,t+1}(\pi_{h,t+1}-\bar{\pi})\frac{c_{t+1}}{c_t}\right] \end{aligned} \tag{E.31}$$

$$\begin{aligned} 0 = &-q_t p^*_{h,t}\frac{c^*_{h,t}}{c^*_t} + \gamma q_t\pi^*_{h,t}(\pi^*_{h,t}-\bar{\pi}^*) + \eta\frac{\nu^*_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{c^*_{h,t}}{c^*_t} \\ &-\gamma^*\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}q_{t+1}\pi^*_{h,t+1}(\pi^*_{h,t+1}-\bar{\pi})\frac{c^*_{t+1}}{c^*_t}\right] \end{aligned} \tag{E.32}$$

$$\begin{aligned} 0 = &-p^*_{f,t}\frac{c^*_{f,t}}{c^*_t} + \gamma\pi^*_{f,t}(\pi^*_{f,t}-\bar{\pi}^*) + \eta\frac{\nu^*_{i,f,t}}{\mathbb{E}^a_t[\xi^*_{i,t}]}\frac{c^*_{f,t}}{c^*_t} \\ &-\gamma\mathbb{E}_t\left[m^*_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]}{\mathbb{E}^a_t[\xi^*_{i,t}]}\pi^*_{f,t+1}\left(\pi^*_{f,t+1}-\bar{\pi}^*\right)\frac{c^*_{t+1}}{c^*_t}\right] \end{aligned} \tag{E.33}$$

$$\begin{aligned} 0 = &-q_t^{-1}p_{f,t}\frac{c_{f,t}}{c_t} + \gamma q_t^{-1}\pi_{f,t}(\pi_{f,t}-\bar{\pi}) + \eta\frac{\nu_{i,f,t}}{\mathbb{E}^a_t[\xi^*_{i,t}]}\frac{c_{f,t}}{c_t} \\ &-\gamma\mathbb{E}_t\left[m^*_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]}{\mathbb{E}^a_t[\xi^*_{i,t}]}q^{-1}_{t+1}\pi_{f,t+1}\left(\pi_{f,t+1}-\bar{\pi}\right)\frac{c_{t+1}}{c_t}\right] \end{aligned} \tag{E.34}$$

$$0 = -\mu_t + \frac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}} \tag{E.35}$$

$$0 = -\mu^*_t + \frac{\alpha A^*_t}{w^*_t}(\phi^* + c_{f,t} + c^*_{f,t})^{\frac{\alpha-1}{\alpha}} \tag{E.36}$$

$$0 = -\tilde{\mu}_t + \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t} a_{i,t}]}\mu_t \tag{E.37}$$

$$0 = -\tilde{\mu}_t^* + \frac{\mathbb{E}_t^a[\xi_{i,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}^* a_{i,t}^*]}\mu_t^* \tag{E.38}$$

$$\begin{aligned} 0 = -a_t^E + &\frac{A_t}{w_t(\phi + c_{h,t} + c_{h,t}^*)^{1/\alpha}} \\ &\times\left\{ c_t\left[\frac{p_{h,t}c_{h,t}}{c_t} - \frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2\right] + q_t c_t^*\left[\frac{p_{h,t}^* c_{h,t}^*}{c_t^*} - \frac{\gamma^*}{2}(\pi_{h,t}^* - \bar{\pi}^*)^2\right]\right\} \end{aligned} \tag{E.39}$$

$$\begin{aligned} 0 = -a_t^{*E} + &\frac{A_t^*}{w_t^*(\phi^* + c_{f,t} + c_{f,t}^*)^{1/\alpha}} \\ &\times\left\{ c_t^*\left[\frac{p_{f,t}^* c_{f,t}^*}{c_t^*} - \frac{\gamma^*}{2}(\pi_{f,t}^* - \bar{\pi})^2\right] + q_t^{-1} c_t\left[\frac{p_{f,t}c_{f,t}}{c_t} - \frac{\gamma}{2}(\pi_{f,t} - \bar{\pi})^2\right]\right\} \end{aligned} \tag{E.40}$$

$$0 = -z_t^E + \sigma^{-1}(\log a_t^E + 0.5\sigma^2) \tag{E.41}$$

$$0 = -z_t^{*E} + \sigma^{-1}(\log a_t^{*E} + 0.5\sigma^2) \tag{E.42}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}[1 - \Phi(z_t^E)] \tag{E.43}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t} a_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}[1 - \Phi(z_t^E - \sigma)] \tag{E.44}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}[1 - \Phi(z_t^{*E})] \tag{E.45}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^* a_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}[1 - \Phi(z_t^{*E} - \sigma)] \tag{E.46}$$

$$0 = -h_t^D + \left[\frac{\phi + c_{h,t} + c_{h,t}^*}{A_t^\alpha \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha} \tag{E.47}$$

$$0 = -h_t^{*S} + \left[\frac{\phi^* + c_{f,t} + c_{f,t}^*}{A_t^{*\alpha} \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha} \tag{E.48}$$

$$0 = -U_{x,t} + (x_t - \delta_t)^{-\gamma_x} \tag{E.49}$$

$$0 = -U_{x,t}^* + (x_t^* - \delta_t^*)^{-\gamma_x} \tag{E.50}$$

$$0 = -y_t + \exp[0.5\alpha(1+\alpha)\sigma^2](A_t h_t)^\alpha - \phi \tag{E.51}$$

$$0 = -y_t^* + \exp[0.5\alpha(1+\alpha)\sigma^2](A_t^* h_t^*)^\alpha - \phi^* \tag{E.52}$$

$$0 = -1 + \mathbb{E}_t\Bigg[\beta\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}\frac{R_t}{\pi_{t+1}}\Bigg] \tag{E.53}$$

$$0 = -1 + \mathbb{E}_t\Bigg[\beta\frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t}\frac{R^*_t}{\pi^*_{t+1}}\Bigg] \tag{E.54}$$

$$0 = -R_t + R^{1-\rho_R}\Bigg[R_{t-1}\left(\frac{y_t}{y}\right)^{\rho_c}\left(\frac{\pi_t}{\bar{\pi}}\right)^{\rho_\pi}\Bigg]^{\rho_R} \tag{E.55}$$

$$0 = -R^*_t + R^{*1-\rho_R}\Bigg[R^*_{t-1}\left(\frac{y^*_t}{y^*}\right)^{\rho_c}\left(\frac{\pi^*_t}{\bar{\pi}^*}\right)^{\rho_\pi}\Bigg]^{\rho_R} \tag{E.56}$$

$$0 = -c_t + p_{h,t}c_{h,t} + p_{f,t}c_{f,t} \tag{E.57}$$

$$0 = -c^*_t + p^*_{h,t}c^*_{h,t} + p^*_{f,t}c^*_{f,t} \tag{E.58}$$

$$0 = -\pi_t + \Bigg[\sum_{k=h,f}\omega_k(p_{k,t-1}\pi_{k,t})^{1-\varepsilon}\Bigg]^{1/(1-\varepsilon)} \tag{E.59}$$

$$0 = -\pi^*_t + \Bigg[\sum_{k=h,f}\omega_k(p^*_{k,t-1}\pi^*_{k,t})^{1-\varepsilon}\Bigg]^{1/(1-\varepsilon)} \tag{E.60}$$

$$0 = -x_t + \Bigg[\sum_{k=h,f}\omega_k(c^{1-\theta}_{k,t})^{1-1/\varepsilon}\Bigg]^{1/(1-1/\varepsilon)} \tag{E.61}$$

$$0 = -x^* + \Bigg[\sum_{k=h,f}\omega_k(c^{*1-\theta}_{k,t})^{1-1/\varepsilon}\Bigg]^{1/(1-1/\varepsilon)} \tag{E.62}$$

$$0 = -s_{h,t} + \rho s_{h,t-1} + (1-\rho)c_{h,t} \tag{E.63}$$

$$0 = -s_{f,t} + \rho s_{f,t-1} + (1-\rho)c_{f,t} \tag{E.64}$$

$$0 = -s^*_{h,t} + \rho s^*_{h,t-1} + (1-\rho)c^*_{h,t} \tag{E.65}$$

$$0 = -s^*_{f,t} + \rho s^*_{f,t-1} + (1-\rho)c^*_{f,t} \tag{E.66}$$

## E.1 Complete Risk Sharing With Floating Exchange Rate

Under the complete risk sharing arrangement, the real exchange rate is determined by the risk-sharing condition:

$$0 = -q_t + \kappa\frac{U^*_{x,t}/\tilde{p}^*_t}{U_{x,t}/\tilde{p}_t} \tag{E.67}$$

## E.2 Incomplete Risk Sharing With Floating Exchange Rate

Under the incomplete risk sharing arrangement, the risk sharing condition (E.71), and the FOCs for risk-free bonds (E.57) and (E.58) should be replaced by the following equations:

$$0 = -(1+\tau b_{h,t+1}) + \beta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{R_t}{\pi_{t+1}} \right] \tag{E.68}$$

$$0 = -(1+\tau b_{f,t+1}) + \beta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{R^*_t}{\pi^*_{t+1}} \frac{q_{t+1}}{q_t} \right] \tag{E.69}$$

$$0 = -(1+\tau b^*_{h,t+1}) + \beta \mathbb{E}_t \left[ \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t} \frac{R_t}{\pi_{t+1}} \frac{q_t}{q_{t+1}} \right] \tag{E.70}$$

$$0 = -(1+\tau b^*_{f,t+1}) + \beta \mathbb{E}_t \left[ \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t} \frac{R^*_t}{\pi^*_{t+1}} \right] \tag{E.71}$$

$$0 = b_{h,t+1} + b^*_{h,t+1} \tag{E.72}$$

$$0 = b_{f,t+1} + b^*_{f,t+1} \tag{E.73}$$

$$0 = -(b_{h,t+1} + q_t b_{f,t+1}) + \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} q_t b_{f,t} + \frac{1}{2}(w_t h_t - q_t w^*_t h^*_t) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}^*_t) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}^*_t x^*_t) \tag{E.74}$$

where

$$0 = -\tilde{d}_t + \tilde{d}^+_t + (1-\varphi_t)\tilde{d}^-_t, \tag{E.75}$$

$$0 = -\tilde{d}^*_t + \tilde{d}^{*+}_t + (1-\varphi^*_t)\tilde{d}^{*-}_t, \tag{E.76}$$

$$0 = -\tilde{d}^+_t + \Phi(z^E_t) \Bigg[ p_{h,t} c_{h,t} + q_t p^*_{h,t} c^*_{h,t} - \frac{w_t}{A_t} \frac{\Phi(z^E_t - \sigma)}{\Phi(z^E_t)} (\phi + c_{h,t} + c^*_{h,t})^{1/\alpha} - \frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2} q_t (\pi^*_{h,t} - \bar{\pi}^*)^2 c^*_t \Bigg], \tag{E.77}$$

$$0 = -\tilde{d}^-_t + \frac{1-\Phi(z^E_t)}{1-\varphi_t} \Bigg[ p_{h,t} c_{h,t} + q_t p^*_{h,t} c^*_{h,t} - \frac{w_t}{A_t} \frac{1-\Phi(z^E_t - \sigma)}{1-\Phi(z^E_t)} (\phi + c_{h,t} + c^*_{h,t})^{1/\alpha} - \frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2} q_t (\pi^*_{h,t} - \bar{\pi}^*)^2 c^*_t \Bigg], \tag{E.78}$$

$$0 = -\tilde{d}^{*+}_t + \Phi(z^{*E}_t) \Bigg[ q_t^{-1} p_{f,t} c_{f,t} + p^*_{f,t} c^*_{f,t} - \frac{w^*_t}{A^*_t} \frac{\Phi(z^{*E}_t - \sigma)}{\Phi(z^{*E}_t)} (\phi^* + c_{f,t} + c^*_{f,t})^{1/\alpha} - \frac{\gamma}{2} q_t^{-1} (\pi_{f,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2} (\pi^*_{f,t} - \bar{\pi}^*)^2 c^*_t \Bigg], \tag{E.79}$$

and

$$0 = -\tilde{d}^{*-}_t + \frac{1-\Phi(z^{*E}_t)}{1-\varphi^*_t} \Bigg[ q_t^{-1} p_{f,t} c_{f,t} + p^*_{f,t} c^*_{f,t} - \frac{w^*_t}{A^*_t} \frac{1-\Phi(z^{*E}_t - \sigma)}{1-\Phi(z^{*E}_t)} (\phi^* + c_{f,t} + c^*_{f,t})^{1/\alpha} - \frac{\gamma}{2} q_t^{-1} (\pi_{f,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2} (\pi^*_{f,t} - \bar{\pi}^*)^2 c^*_t \Bigg]. \tag{E.80}$$

### E.3 Incomplete Risk Sharing With Monetary Union

Under the incomplete risk sharing with monetary union, (E.68)∼(E.71) should be replaced with

$$0 = -(1 + \tau b_{h,t+1}) + \beta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{R_t^U}{\pi_{t+1}} \right] \tag{E.81}$$

$$0 = -(1 + \tau b^*_{h,t+1}) + \beta \mathbb{E}_t \left[ \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t+1}/\tilde{p}^*_{t+1}} \frac{R_t^U}{\pi^*_{t+1}} \right] \tag{E.82}$$

and the bond market clearing condition $0 = b_{f,t+1} + b^*_{f,t+1}$ is deleted. The following identity is added to the system:

$$\frac{\mathbb{E}_t[q_{t+1}]}{q_t} = \frac{\mathbb{E}_t[S_{t+1}]}{S_t} \cdot \frac{\mathbb{E}_t[\pi^*_{t+1}]}{\mathbb{E}_t[\pi_{t+1}]}. \tag{E.83}$$

Note that $S_t$ is not a model variable as the level of nominal exchange rate cannot be determined in the steady state. However, $\pi^S_{t+1} \equiv S_{t+1}/S_t$ is well-defined as a model variable.

### E.4 Exogenous Variables

There are 6 exogenous variables:

$$0 = -\log A_t + \rho_A \log A_{t-1} + \epsilon_{A,t} \tag{E.84}$$

$$0 = -\log A^*_t + \rho_A \log A^*_{t-1} + \epsilon^*_{A,t} \tag{E.85}$$

$$0 = -\delta_t + \rho_\delta \delta_{t-1} + \epsilon_{\delta,t} \tag{E.86}$$

$$0 = -\delta^*_t + \rho_\delta \delta^*_{t-1} + \epsilon^*_{\delta,t} \tag{E.87}$$

$$0 = -\log \varphi_t + (1 - \rho_\varphi) \log \bar{\varphi} + \rho_\varphi \log \varphi_{t-1} + \epsilon_{\varphi,t} \tag{E.88}$$

$$0 = -\log \varphi^*_t + (1 - \rho_\varphi) \log \bar{\varphi}^* + \rho_\varphi \log \varphi^*_{t-1} + \epsilon^*_{\varphi,t} \tag{E.89}$$