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Knightian uncertainty and credit cycles

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Abstract

The Great Recession has been characterised by the two stylized facts: the build-up of leverage in the household sector in the period preceding the recession and a protracted economic recovery that followed. We attempt to explain these two facts as an information friction, whereby agents are uncertain about a new state of the economy following a financial innovation. To this end, we extend Boz and Mendoza (2014) by explicitly modelling the credit markets and by modifying the learning to an adaptive set-up. In our model the build-up of leverage and the collateral price cycles takes longer than in a stylized DSGE model with financial frictions. The boom-bust cycles occur as rare events, with two systemic crises per century. Financial stability is achieved with an LTV-cap regulation which smooths the leverage cycles through quantity (higher equity participation requirement) and price (lower collateral value) effects, as well as by providing an anchor in the learning process of agents.

Keywords: uncertainty, financial engineering, deregulation, leverage forecasting, macroprudential policy

JEL: G14, G17, G21, G32, E44, E58

Non-technical summary

In the run-up to the Great Recession the leverage in the household sector reached its unprecedented historically high levels. This was facilitated by financial sector deregulation, increased competition in the banking activities and the financial engineering which led to a creation of new securitisation products that acted as high quality private sector collateral. While the outcome has been a strong increase in the supply of loans and a more creative way of diversifying risk, it has also pushed the economy into a previously unexplored and unmapped state. The Great Recession was great because the GDP deviated from its long term trend for the US, UK and the euro area in a protracted manner.

We put forward a model that attempts to explain these two stylized facts as a combination of a financial friction and an information friction, whereby agents are uncertain about a new state of the economy following financial innovation. We begin with the framework outlined by Boz and Mendoza (2014) and extend it in three dimensions. First, we endogenise the credit market by introducing patient and impatient households a la Iacoveillo and Neri (2009). Second, we introduce financial intermediaries. Finally, we modify the learning mechanism using an adaptive set-up. In our model households are (intrinsically) rational but take economic decisions under incomplete information. The incompleteness is not caused by their cognitive limitations, as in rational inattention theory (Sims, 2003). Instead households 'learn by doing' and once a sufficient number of realizations of the state variable has materialized, the incomplete information set is completed. This learning set-up is incorporated into a New Keynesian model with credit market frictions, where a share of households needs external financing to consume. Because of limited enforceability of financial contracts, households are required to provide collateral for their loans, and so the relationship between the bank and household is tightened for many periods ahead.

We find that in such a set-up the build-up of risks and leverage, as well as the increases in consumption and the price of collateral take longer than in other DSGE models with standard financial friction. We also find that both the frequency and the amplitude of expansions and contractions are asymmetric - recessions are less frequent and deeper than expansions. Moreover, we find that boom-bust cycles are attenuated compared to equivalent financial friction models, and occur as rare events. Using the Cogley and Sargent's (2008) definition of a severe (or systemic) crisis, we find on average two such events per century. We also find that, different from standard boom-bust cycles, a systemic crisis can be followed by a sequence of subsequent contractions, as it makes the economy more unstable. This results in asymmetric distributions of key macroeconomic and financial variables, with high skewness and fat tails.

We also consider macroprudential policy in the form of an LTV-cap as a tool to smoothen the leverage spikes. In our framework it limits the borrowing capacity both via lower equity participation and by taming the price of collateral. We also find that, by reducing the amount of borrowing and leverage in upturns, the LTV-ratio regulation is effective in smoothing the leverage cycles and reducing the effects of a deep contraction on the real-financial variables. We also compare the simple LTV-cap to more elaborate versions of macroprudential rules, as in Lambertini et al (2013) or Angelini et al (2014). We find that both types of rules are equally efficient in smoothing the cycle. However, a simple LTV-cap will be preferred as it provides a clearer benchmark in the learning process of agents, generating a lower cost in relation to learning or information gathering.

1 Introduction

The Great Recession has been characterised by two features, which made it distinct from any other recession in the post-war era. First, it was preceded by a significant build-up of leverage, in particular in the household sector. Excessive borrowing in the mortgage market, was also facilitated by securitization, which reached its pick activity in 2007. Second, the depth and length of the recession resulted in a substantial deviation of GDP from its long term trend in the US, UK and in the euro area, which cannot be explained in a standard New Keynesian model set-up even after taking account of financial frictions.

We put forward a model that links these two stylized facts and attempts to explain the protracted recovery as a result of a combination of credit frictions and information friction in the environment of a rare systemic event. Financial innovation shocks push the economy into a previously unexplored and unmapped state. In this new state, agents do not know the true riskiness of new financial products and therefore optimize under incomplete information. The incompleteness is not caused by households' cognitive limitations, but because they need to learn the true riskiness of the financial products. This learning process requires sufficient number of realisation of the state variable in order for the information set to be complete. As learning takes time, the economy approaches the new steady state only sluggishly.

The core of the model follows Boz and Mendoza (2014). However, we introduce three important modifications. Following Iacoviello and Neri (2009) we first split households into patient, who save and produce land, and impatient, who borrow and consume land. In addition, we introduce a financial intermediary and explicitly model the credit market.

The most important friction in the model is uncertainty about the new state of the economy after the financial innovation shock. In this way, financial innovation interacting with credit/margin constraints can lead to underpricing of the risk associated with a new financial environment. This in turn can lead to the accumulation of leverage and surges in asset prices. Because of limited enforceability of financial contracts, households are required to provide collateral for their loans, and so the relationship between the bank and household is tightened for many periods ahead. Once the agents observe sufficient number of realisations of the new state of the economy and realise that they are overlevered, this can lead to a sudden stop a là Mendoza (2010). More formally, sudden stop is caused by the uncertainty regarding the transition probability of such events. Since systemic crises are rare events, agents inherently misprice the occurrence of such events (see for instance Zeira (1999), Caballero and Krishnamurthy (2008) or Boz and Mendoza (2014)). Uncertainty coupled with the Fisherian deflation mechanism leads to highly volatile and asymmetric distributions in asset prices, consumption, debt, loan and deposit rates. Our approach is loosely linked to the rational inattention theory (Sims, 2010), which recognises that people have finite information-processing capacity that explains well some of the frictions.

We find that early realizations of the new state result in a much higher (lower) debt, consumption, price of collateral and risk accumulation (de-leveraging) during upturns (downturns) compared to standard financial friction models. Moreover, the loan-to-deposit ratio of banks is rapidly increasing at the onset of the financial innovation phase, and remains very high until sudden stop has materialized for a few periods. We also demonstrate that sluggish learning can explain why the economy can diverge from its long term trend for an extended period of time. Next, we evaluate the efficacy of standard macroprudential tools, such as a cap on the loan-to-value (LTV) in reducing the leverage of the household sector.

The remainder of this paper is organised as follows. In Section 2 we introduce the model set-up. Section 3 is devoted to the main friction of the model - it discusses uncertainty and describes the mechanism of learning. Section 4 presents the results, and Section 5 concludes.

2 Model

2.1 Overview and the logic of the model

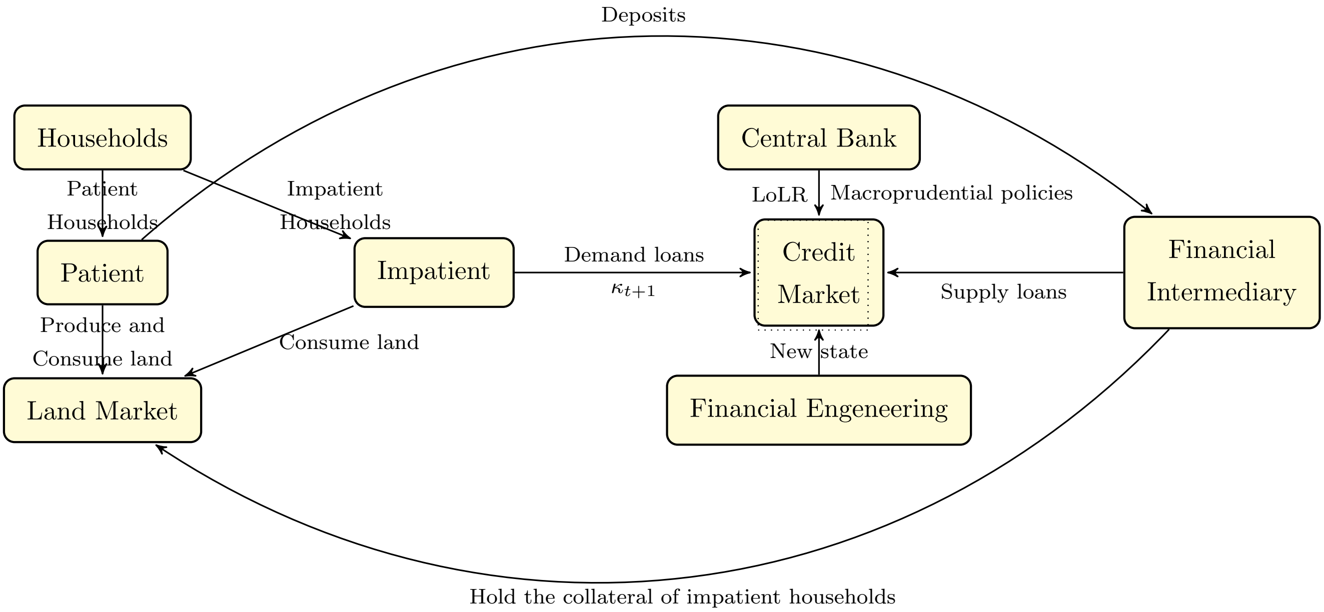
The backbone of the model is a standard New Keynesian set-up extended to include credit frictions, informational frictions (uncertainty) and heterogeneous expectations in learning. Credit frictions are caused by the limited borrowing capacity of impatient households. Informational frictions emerge because agents have limited capacity to estimate in real time the expected price of their collateral and the accumulated value of risk on their balance sheet as a result of their indebtedness. As a result, they face restrictions in determining precisely the expected loan-to-value (or leverage) ratio. Instead they are forced to simultaneously infer the (expected) value of the collateral and the (expected) LTV ratio, producing the informational friction in the model. Lastly, they infer these values using two alternative forecast rules. Agents evaluate the performance of each rule using an objective function and rationally choose the rule that performs best. In that sense, agents are intrinsically rational and gradually learn from the past. This layer in learning gives rise to heterogeneous expectations and allows for switching between these expectations in an endogenous and rational manner. In the analysis we will study the contribution of each layer of friction to model dynamics and for model's statistical performance, as well as the model's final achievements when we allow these frictions to interact. In the benchmark version, agents fully use rational expectations to infer the values of both land price and leverage, and so only credit frictions (and shocks) generate fluctuations. In the first extension, agents only forecast the expected value of leverage, while they precisely know the expected value of land price. In the second extension, i.e. in a complete model set-up, we demonstrate that the fluctuations are generated in the aftermath of shocks due to a mixture of credit and informational frictions and heterogeneous expectations.

Concerning the model set-up, we extend Boz and Mendoza (2014) by endogenizing credit production, allowing for heterogeneous expectations and modifying the learning mechanism to an adaptive set-up. In particular, we endogenize both the quantity and prices of deposits, loans, bank equity, allow agents to learn variables using time-consistent and adaptive heuristics and introduce two alternative forecast rules in learning. Moreover, we include a central bank who simultaneously sets a (time-varying) policy rate and a macroprudential rule.

Figure 1 provides a schematic overview of the model. Our economy is populated by three agents: households, financial intermediaries, and government. We divide households into two categories: the patient and the impatient types. What differentiates them is the degree of patience. The discount factor β of patient households is higher than those of the impatient. This forces the latter to complement their internal funds with loans from the credit market. While patient households both produce and consume land, impatient only consume it. Therefore, we explicitly model two markets: market for land and market for credit. More details will follow in the next section.

What differentiates this model from most other financial friction frameworks is that this one incorporates explicitly uncertainty. Financial sector developments prior to the Great Recession such as financial engineering, de-regulation of markets, and increased competition amongst financial intermediaries has meant that the new market structure is unknown and unexplored to the participants in financial transactions. As a result, agents do not know the true risks, leverage and price of collateral in the 'new' environment and therefore optimize under incomplete information. Our take on uncertainty is that agents are (intrinsically) rational insofar that they efficiently optimize over time, but do so under incomplete information regarding two variables in the model: the leverage ratio, and the price of collateral. The former is exogenous while the latter is endogenous, but dependent on the realization of the first. Agents engage in adaptive learning and learn about the 'true' values of leverage and asset price only after observing a sufficiently long set of realizations of both variables. Note that this learning is, however, slow since they only learn from their practical experiences.

With respect to the learning framework in Boz and Mendoza (2014), our agents are more active in their learning experience since one of the variables they forecast is endogenous. However, this variable is dependent on the exogenous (the shadow value of collateral constraint) variable which facilitates the tractability of the dynamic solution. Therefore, while agents can partially benefit from experimenting with the dynamic optimization to induce the endogenous land price, the exogenous component of this price will make such experimenting slow and costly. In other words, the values of the 'learning variables' cannot be directly deduced by recursively solving the remaining part of the model. Moreover, we will allow for heterogeneous expectations in heuristics, which will enrich the learning dynamics, but at the same time make it more tractable as switching between these expectations can explicitly be traced.



2.2 Benchmark model structure

We begin with the basic model structure that all three versions of the model include, and then sequentially add the different layers of friction until the complete model.

The consumption sector is populated by two types of infinitely lived households, each with a unit mass and they act atomistically in competitive markets.[1] Both types optimise under uncertainty. The key factor which differentiates them is the degree of impatience. The discount factor β of impatient households (I) is lower than the one of patient (P). This 'forces' the impatient households to engage in external credit market. For the sake of simplicity and tractability, we explicitly omit the labour supply decision of households which means that they only derive income from land and saving/borrowing.[2]

2.2.1 Patient Households

The representative patient risk-averse household chooses consumption c_t, land holdings l_{t+1}, and deposits d_t, taken as given the price of land q_t, the deposit rate R_t^d, and the gross real interest rate R_t so as to maximize a standard CRRA utility function:

[1] One could equivalently assume that in each period households die and are born with a constant probability so that on aggregate there is a unit mass of households.

[2] It would be straightforward to extend the model to include a labour market, as in for instance in Gerali et al (2010).

$$E_0[\sum_{t=0}^{\infty} \beta^t u(c_t)] \tag{1}$$

where $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, and σ is the parameter of relative risk aversion of households. Because of their relative patience, these households are natural lenders, and face the following budget constraint:

$$d_{t+1} + c_t^P \leq z_t g(l_t) + q_t l_t - q_t l_{t+1} + (1 + R_t^d) d_t + e_t \tag{2}$$

The share of patient households in the population is θ (and is time invariant). The production function $g(l_t) = l^\alpha$ is a standard neoclassical one and is subject to a stochastic productivity shock z_t, which is known to all agents.[3]Because in this paper we are interested in uncertainty regarding financial frictions, we omit from imperfect beliefs regarding the productivity shock. However, an immediate extension could be to also introduce macroeconomic uncertainty.

It is crucial to note that E_t^s in the utility function above represents expectations subject to agents' (subjective) beliefs using information available up to period t (inclusive). These beliefs will differ from the ones formulated under rational expectations.

2.2.2 Impatient Households

The impatient risk-neutral households (with the share of the total population equal to $1 - \theta$) maximize the same type of CRRA utility function:[4]

$$E_0[\sum_{t=0}^{\infty} \beta^t u(c_t)] \tag{3}$$

where $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, but face a different budget constraint due to their impatient nature:

[3]As is standard in this literature, we will assume that the TFP shock follows an autoregressive process. However, we could have equivalently assumed the TFP shock to follow a Markov process, without changing the core results.

[4]Note that we depart here from the representative agent assumption and make the impatient households heterogeneous by subjecting them to different initial land holding (or wealth). Aside from this initial wealth heterogeneity, which will generate a wealth distribution in period $t = 0$, the constrained optimization problem is equal for all agents within this category. We simply need this initial heterogeneity to motivate the endogenous learning dynamics within this group, and the (possible) reason for switches between one rule and the other. The learning dynamics will be explained in further detail at a later stage.

$$c_t^I \leq q_t l_t - q_t l_{t+1} - \frac{b_{t+1}}{R_t^b} + b_t \tag{4}$$

where b_t are the holdings of one-period discount loans (or bonds). Because of imperfections in the credit market (due to limited state-verification a la Towsend), impatient households face restrictions in the quantity of external financing obtained and must provide a collateral as a security.[5]Therefore, the LTV that the agent must satisfy limits the value of credit $\frac{b_{t+1}}{R_t^l}$ to a time-varying ratio of the market value of their land holdings, κ_t according to:

$$E_t[\kappa_{t+1}] q_t l_{t+1} \leq -\frac{b_{t+1}}{R_t^b} \tag{5}$$

From a microeconomic perspective, κ can be seen as the proportional cost of collateral repossession (or liquidation share) in case of default. Debt contract with margin clauses are also captured by this relation (Mendoza, 2010). A relaxation (tightening) of this constraint can either come from an increase (decrease) in the borrowing capacity κ_t or from an increase (decrease) in the value or quantity of the collateral $q_t l_{t+1}$. From a macroeconomic perspective, this relation can be interpreted as the LTV ratio (or leverage) set by the macroprudential authority. This interpretation will become evident later on when we study the impact of macroprudential policies on the model dynamics.

The random variable κ_t is continuous with an upper bound at 1 and a non-negative lower-bound. It is also time-varying. The framework is flexible enough to capture asymmetric regime-switching probabilities between high and low leverage capacities. In this benchmark version, agents have sufficient information to form rational expectations about the future value of κ. We will relax this assumption later.

2.2.3 Financial intermediary

The representative financial intermediary operates in a perfectly competitive market and uses deposits from patient households to give out loans to impatient households. As in Gerali et al. (2010) they are owned by patient households (captured by the

[5]See, for instance, Bernanke, Gertler and Gilchrist (1999), or Christiano, Motto and Rostagno (2013) for background information and microfoundations of the state-verification problem in external lending. We use the outcomes from that problem to motivate our collateral constraint, but because of the similarity with the aforementioned frameworks, we abstain from providing full microfoundations of that problem.

patients' discount factor $\beta_p^t \lambda_t^p$), and maximize the discounted sum of cash flows:

$$E_0 \sum_{t=0}^{\infty} \beta_p^t \lambda_t^p [(1+R^b)B_t - B_{t+1} + D_{t+1} - (1+R_t^d)D_t + (E_{t+1}^b - E_t^b) - \frac{\kappa_{E^b}}{2}[\frac{E_t^b}{B_t} - \nu^b]^2 E_t^b] \quad (6)$$

subject to the balance sheet constraint: $B_t = D_t + E_t^b$. B_t is the total amount of loans issued at time t, D_t the aggregate number of deposits received from patient households, E_t^b the bank capital, and ν^b is the long-term capital-to-asset ratio. The last term in the above maximization problem represents the cost of operating the financial intermediary. To motivate an undesirable social cost (externality) from excessive intermediary leverage from the point of view of the macroprudential policy maker, we impose a quadratic cost function whenever the intermediary's capital-to-asset ratio $\frac{E^b}{B_t}$ moves away from the target value ν^b. Because of the high number of competitors in the banking industry, the individual intermediary takes the deposit R_t^d and the loan rates R_t^b as given when maximizing its profits.[6]

The aggregate bank capital evolves according to:

$$E_{t+1}^b = (1-\delta^b)E_t^b + \pi_t^b \quad (7)$$

where δ^b measures the resources used in managing bank capital and π_t^b are overall real profits made by the financial intermediary at date t. These are described by the following relation:

$$\pi_t^b = R_t^b B_t - R^d D_t - \frac{\kappa_{E^b}}{2}[\frac{E_t^b}{B_t} - \nu^b]^2 E_t^b - Adj_t^b \quad (8)$$

with Adj_t^b denoting the adjustment costs for changing interest rates on deposits. This definition of profits is a narrow one as it coincides with the net interest rate margin. It does not include any other items from the income statement in order to maintain a closed-form solution for intermediary's optimization problem while keeping it simple.

[6]The intermediary also acts under incomplete information. That is why we have conditioned its expectations on the state s beliefs. However, their beliefs are of second order importance since they do not optimize with respect to κ_t nor do they engage in learning. κ_t is instead assumed to be out of direct control by either household or intermediary, and plays a key role only for the optimization of households. Therefore we will omit intermediary's subjective beliefs and in what follows, approximate its beliefs with the RE expectations operator.

2.2.4 Credit Market

Next we need to derive the lending and deposit rates that financial intermediaries charge. Iterating the balance-sheet constraint of financial intermediaries at date t and $t+1$ and inserting it into the cash-flow expression in equation 6, we get that the intermediary's objective is to maximize:

$$R_t^b B_t - R_t^d D_t - \frac{\kappa_{E^b}}{2}[\frac{E_t^b}{B_t} - \nu^b]^2 E_t^b \tag{9}$$

Taking first-order conditions with respect to B_t and D_t and combining them, we get that the spread charged on loans is equal to:

$$R_t^b = R_t^d - \kappa_{E^b}[\frac{E_t^b}{B_t} - \nu^b][\frac{E_t^b}{B_t}]^2 \tag{10}$$

Since patient households are risk-averse, they will ask for a safe rate on their deposits, that by no-arbitrage condition, will equal to the real rate R_t (see for instance Bernanke, Gertler and Gilchrist (1999) or Christiano, Motto and Rostagno (2013) for a microfoundation behind this result).[7] We can thus re-write the above expression as:

$$R_t^b - R_t = \kappa_{E^b}[\frac{E_t^b}{B_t} - \nu^b][\frac{E_t^b}{B_t}]^2 \tag{11}$$

This expression represents the trade-offs that the financial intermediary faces in setting the lending rate. The left-hand side represents the marginal benefit from increasing lending meanwhile the right-hand side represents the costs of increasing leverage (by deviating from the ν^b target). The final lending rate will be set where the two are equal.

2.2.5 Land Market

We can show that the effects of the collateral constraint on asset pricing can be derived by combining the Euler equations of land for the two households.[8] Solving the equations forward in which the future stream of land dividends is discounted at the stochastic discount factor and adjusted for the shadow value of the credit constraint:

[7]Following Gerali et al. (2010), we could equivalently assume that the financial intermediary has continuous and risk-free access to central bank liquidity at the safe rate R_t, which by arbitrage would make the deposit rate equal to the safe rate.

[8]We follow the method described in Mendoza (2010).

$$q_t = E_t \sum_{j=0}^{\infty} [\prod_{i=0}^{j} \frac{\beta u'(c_{t+1+i})}{u'(c_{t+i}) - \mu_{t+i}\kappa_{t+i}}] z_{t+1+j} g'(l_{t+1+j}) \quad (12)$$

This condition equalizes the equilibrium price of land with the marginal cost of investment. Looking at the denominator of 12, we see that the collateral constraint lowers land prices since it increases the rate of return at which future land dividends are discounted. It is forward-looking since not only will a binding constraint at t reduce the value of land, but also if agents expect that the constraint can bind at any future date $E_t[\mu_{t+i}\kappa_{t+i}]$ for any $i > 0$, the value of land will fall.

In our framework, because impatient agents borrow up to a maximum, the constraint will always be binding. This means that, under rational expectations, consumers know that future constraints will also be binding and that this will reduce the land price today. However, the amount of discount in land price will depend on the expected LTV of households. The more they are expected to leverage up, the higher the discount. *Ceteris paribus* compared to the occasionally binding constraint set-up of Mendoza (2010), our land price should be, on average, lower and thus the link between LTV and land price tighter and the credit friction stronger than in his framework.

We deviate from the original Mendoza framework in our paper for mainly two reasons. The first argument is an empirical observation. In the housing market of the US and many European countries, the vast majority of households are constrained when purchasing their residence. With the low and declining savings rate (and savings-to-income ratio) in the US and in the euro area over the past two decades, the number of households climbing up in the residential ladder who are not credit constrained and do not borrow up to a maximum of the purchase value has decreased. Therefore, making it always binding reflects better the housing market conditions of the previous decade than making it occasionally binding. The second argument is on theoretical basis. Our informational frictions and heterogeneous expectations framework makes the model non-linear. Including additional non-linearities, such as occasionally binding constraints would add additional layer of non-linearity with the risk of obstructing the true objective of the current paper, which is to study the role of the aforementioned informational and expectation frictions on household optimization and financial stability. Thus, in order to maintain the focus and effectively quantify the effect of these frictions on general equilibrium, we lean towards the always-binding constraint.[9]

If we further define the next period marginal utility of consumption as $\lambda_{t+1} \equiv \beta u'(c_{t+1})$ and return on land as:

$$R^q_{t+1} = \frac{z_{t+1+j} g'(l_{t+1+j}) + q_{t+1}}{q_t} \tag{13}$$

we can define the premium on land as (Mendoza, 2010):

$$E_t[R^q_{t+1} - R_t] = \frac{(1-\kappa_t)\mu_t - Cov^s_t(\lambda_{t+1}, R^q_{t+1})}{E_t(\lambda_{t+1})} \tag{14}$$

The land premium rises in every state in which the collateral constraint binds because of these three effects:

- The direct effect, $(1-\kappa_t)\mu_t$, is due to a rise in the shadow value of the collateral constraint (with an upper bound determined by κ_t, the amount of the collateral that can be turned into debt).
- The indirect effect, represented by a lower $Cov^s_t(\lambda_{t+1}, R^q_{t+1})$ and a higher $E^s_t(\lambda_{t+1})$.
- Because of the collateral constraint, the household's ability to smoothen her consumption is limited, leading her to transfer the consumption into the future.

To see the effects of this on the price of tangible, we can write the land price as a function of the return according to:

$$q_t = E_t \sum_{j=0}^{\infty} [\prod_{i=0}^{j} \frac{1}{E_t[R^q_{t+1}]}] z_{t+1+j} g'(l_{t+1+j}) \tag{15}$$

since the expected land return satisfies the condition $q_t E_t[R^q_{t+1}] = E_t[z_{t+1+j} g'(l_{t+1+j})]$. Then, as Aiyagari and Gertler (1999) show, an increase in expected return will lead to lower equity prices in the current period, since the discount rate of future dividends will increase due to the binding collateral constraint, in the current and/or next period. Thus, it is only sufficient for the collateral constraint to bind occasionally in the stochastic steady state for the entire equilibrium asset pricing function to

[9]As a side note, and remembering that the correlation between the number of times the constraint binds and the land price, the land price growth in a framework with the occasionally binding constraint will be faster. However, the growth in price in some states becomes unrealistically high, which is hard to sustain with any empirical or first-principle arguments.

be distorted by the constraint. Since it is always binding in our version, the effect on the price will be permanent.

Following Boz and Mendoza (2014), for simplicity we will assume that the aggregate land supply is fixed and equal to 1. Consequently, the market clearing condition in the land market:

$$1 = \theta l_t^P + (1-\theta) l_t^I \tag{16}$$

implies that the land holdings of the representative household must at each t satisfy $l_t = 1$, as well as the production function will be reduced to $z_t g(1)$.[10]

2.2.6 Central Bank

To close the model, we separately model the two policies of the central bank. Assuming that the variables without time subscripts denote their steady state values, we can characterize the monetary policy of the central bank with a standard Taylor-rule (expressed in deviations-from-the-target terms):

$$\frac{R_t}{R} = \frac{R_{t-1}}{R}^{\gamma_R} [\frac{\pi_t}{\pi}^{\gamma_\pi} \frac{y_t}{y}^{\gamma_y}]^{1-\gamma_R} e^{\epsilon_{R,t}} \tag{17}$$

where $\epsilon_{R,t}$ is a monetary policy shock.

On the other end, macroprudential policy is modeled as a set of *ex ante* rules that the intermediary sector must obey to. The first rule is a cap on the LTV ratio (independent of the state):

$$\kappa_t = \bar{\kappa} \tag{18}$$

Alternatively, we will test a more elaborate version of the above LTV-rule. Recently, several papers (Lambertini et al (2013), Angelini et al (2014)) have proposed Taylor-type macroprudential rules as a good approximation of the Basel II/III-style of regulatory requirements. We will therefore perform an alternative scenario where the central bank uses:

$$\kappa_t = \rho_\kappa * \kappa_{t-1} + (1-\rho_\kappa) * \kappa^* + (1-\rho_\kappa) * (b_t - b_{t-1}) \tag{19}$$

,where κ^* is the steady state value for the LTV-ratio. We calibrate it to 2 in line

[10]Hence all the variation in land will come in its value, which is a function of the intertemporal consumption smoothing of households, as well as the shadow value of collateral constraint.

with the above rule in order to facilitate the comparison between a static (state-independent) and a dynamic (state-dependent) version.

2.3 First extension

In the first extension, we go beyond the rational expectations assumption regarding future leverage, and allow agents to learn about its expected value using heuristics. To accommodate this, we will make a few modifications in the benchmark model above.

First, households maximise their utilities using their limited knowledge regarding the (future) LTV value. Thus, while rational optimizers, they optimize under uncertainty regarding one state variable. This uncertainty (or 'ignorance') regarding the *true* state applies to the entire population equally. Therefore, agents are rational in the sense that they use all available information (and models) at time t, but form subjective beliefs because they act under (evenly distributed) incomplete information.[11] We will proceed to modify their utility functions in 1 and 3 to:

$$E_0^s[\sum_{t=0}^{\infty} \beta^t u(c_t)] \tag{20}$$

,where E_t^s represents expectations subject to agents' (subjective) beliefs using information available up to period t (inclusive). Agents engage in (aggregate) learning and become fully aware of the *true* transition probabilities as they approach time $t = T$. We will describe the learning mechanisms in further detail once we have outlined the full model in the next section.[12]

In the same manner, we need to modify impatient household's collateral constraint in 5 to allow for uncertainty regarding expected LTV ratio:

$$E_t^s[\kappa_{t+1}]q_t l_{t+1} \leq -\frac{b_{t+1}}{R_t^b} \tag{21}$$

Since financial intermediaries use information regarding the expected leverage ratio of impatient households to define the level of lending and the expected dis-

[11]This is very different from model settings where one agent has more information than the other (asymmetric), or where agents use heterogeneous information and/or models (due to their cognitive restrictions) to infer the true states (irrationality).

[12]Preston (2005) pointed out that infinite horizon microfounded learning models fail to produce optimal dynamic consumption allocations while violating their intertemporal budget constraint, resulting in an inconsistency in the microfoundations. In defence, Hokapohja and Mittra (2011) showed that the intertemporal accounting consistency holds along the (infinite) sequence of temporary equilibria and that this model can be derived as a special case of Preston's framework.

counted cash flows, they maximize also using subjective beliefs. We thus modify equation 6 to:

$$E_0^s \sum_{t=0}^{\infty} \beta_p^t \lambda_t^p [(1+R^b)B_t - B_{t+1} + D_{t+1} - (1+R_t^d)D_t + (E_{t+1}^b - E_t^b) - \frac{\kappa_{E^b}}{2}[\frac{E_t^b}{B_t} - \nu^b]^2 E_t^b] \tag{22}$$

Lastly, note that the land price in this version is derived using rational expectations. The only information limitation is feeded from uncertainty regarding expected leverage in equation 12. However, the price itself is fully defined.

2.4 The complete model

Picking up on this last point, it is very difficult to sustain that the *true* expected leverage is unknown at time t, meanwhile land price is defined using rational expectations. Land price is an argument of the LTV-ratio of households, and thus must also be subject to uncertainty in order to maintain consistency of the model. Therefore, in the full model we proceed to modify the uncertainty framework to encompass both (expected) leverage and asset price, and modify the future stream of land dividends in 12 to:

$$q_t = E_t^s \sum_{j=0}^{\infty} [\prod_{i=0}^{j} \frac{\beta u'(c_{t+1+i})}{u'(c_{t+i}) - \mu_{t+i}\kappa_{t+i}}] z_{t+1+j} g'(l_{t+1+j}) \tag{23}$$

With this modification, the (subjective) premium on land in 14 is now defined as:

$$E_t^s[R_{t+1}^q - R_t] = \frac{(1-\kappa_t)\mu_t - Cov_t^s(\lambda_{t+1}, R_{t+1}^q)}{E_t^s(\lambda_{t+1})} \tag{24}$$

, with $E_t^s(\lambda_{t+1})$ as the next period (subjective) marginal utility of consumption. Now, with this information, we can finally redefine the price of land in 15 to:

$$q_t = E_t^s \sum_{j=0}^{\infty} [\prod_{i=0}^{j} \frac{1}{E_t^s[R_{t+1}^q]}] z_{t+1+j} g'(l_{t+1+j}) \tag{25}$$

The same effects from a binding collateral constraint described in the rational expectations version hold. However, if these strong effects were at work under rationally formed expectations (with the knowledge of the true state of κ), these effects are further accentuated if we in addition introduce learning into this framework.

To understand how, one needs to examine the interactions between the collateral constraint and learning regarding the κ_t variable. Suppose that the constraint was binding at t. In booms (or states with high leverage possibilities), the price of asset is higher, which will relax the LTV constraint. From equation 25 it implies that the land return is lower. So assuming that beliefs are optimistic (pessimistic) in a boom (bust), impatient households will assign a higher probability to lower (higher) future land returns than under RE. This will push land price further up (down), which via the LTV-constraint, will result in higher (lower) indebtedness.

Taking into account the tight and procyclical link between leverage and asset prices, and considering that the the value of κ_t (which is an argument of the land price q_t) is unknown and therefore forecasted, it is reasonable to also make the value q_t uncertain (and state contingent). Hence households will have to forecast the values of κ as well as q.

Apart from this extension, the remaining model structure is the same as in the first extension.

3 Uncertainty and Learning

Now that we described the model set-up, we need to devote some attention to the non-standard aspects of it. In particular, we wish to describe the environment and the processes that govern the learning of the agents. This learning framework is relevant for both the first extension and the complete model.

3.1 The general outline

We attempt to model a setting in which financial engineering and market deregulation has lead to a rapid increase in credit, leverage and risks. Agents know that the environment (and the value of all these variables) has changed, but they don't know exactly by how much. Thus, the uncertainty concerns the 'true' value of the LTV-ratio κ only (first extension), or the LTV-ratio and land price q together (complete model). [13] Therefore, in contrast to Boz and Mendoza (2011, 2014), we assume that there are more than two possible future regimes as the values of land and leverage can have many different realizations. Moreover, in our framework

[13] Equivalently, and using the approach by Boz and Mendoza (2014), one could say that the uncertainty is regarding the transition probability to a new state. This state is a subset to a bounded set between 0 and 1.

agents are adaptive learners and use heuristic rules to forecast the two variables. In the (very) long-run, their beliefs converge to rational expectations. In the short run, however, their beliefs will be different from the equilibrium with full information. They learn from past experiences and fully 'understand' the riskiness of the new financial environment only after they have observed a sufficiently large sample of data realizations. As a result, agents are slow learners and their learning process is strongly history dependent.[14]

Notice also that our learning framework allows for heterogeneous expectations regarding the future state variables and an endogenous switch between them. This adds on an additional friction in our model. A simpler setting would be to only accommodate one learning rule. While that would narrow the model, it generates three concerns. First, it is very restrictive to assume that all agents learn in the same way, and use the same models to forecast. Much of the dynamics in behavioural settings, such as that of Brock and Hommes (1998) occurs because of the heterogeneity in the cognitive capacities of agents, and their diverse use of information. Second, the learning process in the homogenous learning setting would be so slow that the convergence to rational expectations would occur in the proximity to *ad infinitum*. This might cause convergence problems for solving the entire model. One way to solve it is to restrict the number of (future) state variable realizations, as in Boz and Mendoza (2014). However, that is very abstract in particular since different agents might judge the *low* and *high* risk/leverage states differently, which would again pull us towards a heterogeneous expectations setting. Third, by allowing agents to objectively evaluate the performance of each rule in each period before they decide for which one to go in the next period, the model is able to accommodate some degree of rationality and consistency in the learning of agents (intrinsic rationality). By removing this assumption, like in Boz and Mendoza (2014), one actually distances the learning framework further away from rational expectations, and therefore somehow suggesting that agents' cognitive capacities are more limited or their level of ignorance more permanent than in our framework. Taking all these remarks into account, we therefore proceed with heterogeneous expectations. How-

[14]In contrast, agents are Bayesian learners in Boz and Mendoza (2014) and their learning space is constrained to only two realizations of the 'learning variable': High or Low leverage states. In addition, the uncertainty concerns leverage only (and not land prices, despite the fact that uncertainty will enter the land price function via the shadow value of collateral constraint.) Therefore, the speed of learning and convergence is expected to be higher in their model compared to ours once we acknowledge that the probability space of the (learning) variables in their model is much smaller.

ever, in order to disentangle the contribution of each friction to model dynamics (or the fat tails of the model variables), we first analyze the effects from heterogeneous expectations alone, and only after that introduce the multivariate learning process (or uncertainty).

Cecchetti et al (2000) and Cogley and Sargent (2008b) show that CRRA utility functions with Markov process for the consumption growth can generate asymmetric behaviour in consumption. High-growth states in consumption are persistent and common. However, once a low-growth state has been reached, the contractions are severe, with a mean decline of 6.785% p.a. Moreover, once the economy is in the low-growth state, there is a certain positive probability of running into a sequence of contractions, with a total decline in consumption amounting to 25% (assuming the contraction lasts for 4 years with a probability of 7.1%). We will use this threshold to identify *ex post* severe contractions (or systemic crises) in our model.

The current learning set-up means that agents learn quickly about the leverage/land price states that occur more frequently. Therefore, taking into account that severe contractions are rare, learning about them will also be slower and asymmetric with respect to expansions. Moreover, because the ergodic probability of a contraction is as small as 0.0434 (Cogley and Sargent, 2008b), the time elapsed before a sufficiently large sample of contractions has been observed is very large. This retards the learning of large contractions significantly.

3.2 Specification of the learning process

Let us now formalize the learning process. Our approach is similar to De Grauwe and Macchiarelli (2015) insofar that we use the same type of heuristics and updating of beliefs.

Under rational expectations, the forecasted variable will equal its realized value in the next period, i.e. $E_t X_{t+1} = X_{t+1}$, denoting generically by X_t any variable in the model. However, as anticipated above, we depart from this assumption in this framework by making the forecast contingent on imperfect information, but allowing the agents to learn. Expectations are replaced by a convex combination of heterogeneous expectation operators $E_t \kappa_{t+1} = E_t^s \kappa_{t+1}$ and $E_t q_{t+1} = E_t^s q_{t+1}$. In particular, agents forecast the LTV-ratio and the land price using two alternative forecasting rules: *fundamentalist* vs. *extrapolative rule*. Under the fundamentalist rule, agents are assumed to use the steady-state value of the LTV-ratio - κ^*, against a naive forecast based on the LTV's latest available observation (extrapolative rule). Equally for the value of land, *fundamentalist* agents are assumed to base their expectations on the steady-state value - q^* against the *extrapolatists* who naively base their forecast on the latest available observable.[15]Defining $i = (\kappa, q)$ we can formally express the fundamentalists as:

$$E_t^{s,f} i_{t+1} = i^* \tag{26}$$

and the extrapolative (or adaptive) rule as:

$$E_t^{s,e} i_{t+1} = \theta i_{t-1} \tag{27}$$

This particular form of adaptive expectations has previously been modelled by Pesaran (1987), Brock and Hommes (1997, 1998), and Branch and McGough (2009), amongst others, in the literature. Setting $\theta = 1$ captures the "naive" agents (as they have a strong belief in history dependence), while a $\theta < 1$ or $\theta > 1$ represents an "adaptive" or an "extrapolative" agent (Brock and Hommes, 1998). For reasons of tractability, we set $\theta = 1$ in this model, but the model dynamics would not be significantly altered with any of the other parameter values.

Note that for the sake of consistency with standard RE DSGE model, all variables here are expressed in gaps. Focusing on their cyclical component makes the model symmetric with respect to the steady state (see Harvey and Jaeger, 1993). Moreover, this facilitates the interpretation of the model as the fundamentalists can be seen as 'benchmarking' the variable values, meanwhile the problem of extrapolists is pinned down to guessing the deviation of these values from their benchmark (or steady state).

Next, agents' preference for one forecast over the other depends on the (historical) performance of the two rules given by a publicly available fitness measure, the mean square forecasting error (MSFE). After time $t + 1$ realization is revealed, the two predictors are evaluated *ex post* using MSFE and new fractions of agent types are determined. These updated fractions are used to determine next period (aggregate) forecasts of LTV-and land prices, and so on. Agents' rationality consists therefore in choosing the best-performing predictor using the updated fitness measure. There is a strong empirical motivation for inserting this type of switching mechanism amongst

[15]The latest available observation is the best forecast of the future, i.e. a random walk approach

different forecasting rules.[16]

The aggregate market forecasts of the LTV-ratio and land price are obtained as a weighted average of each rule ($i = \kappa, q$):

$$E_t^s i_{t+1} = \alpha_t^f E_t^{s,f} i_{t+1} + \alpha_t^e E_t^{s,e} i_{t+1} \tag{28}$$

where α_t^f is the weighted average of fundamentalists, and α_t^e that of the extrapolists. These shares are time-varying and based on the dynamic predictor selection. The mechanism allows to switch between the two forecasting rules based on MSFE / utility of the two rules, and increase (decrease) the weight of one rule over the other at each t. Assuming that the utilities of the two alternative rules have a deterministic and a random component (with a log-normal distribution as in Manski and McFadden (1981) or Anderson et al (1992)), the two weights can be defined based on each period utility $U_{i,t}^x$, $i = (\kappa, q)$, $x = (f, e)$ according to:

$$\alpha_{i,t}^f = \frac{exp(\gamma U_{i,t}^f)}{exp(\gamma U_{i,t}^f) + exp(\gamma U_{i,t}^e)} \tag{29}$$

$$\alpha_{i,t}^e \equiv 1 - \alpha_{i,t}^f = \frac{exp(\gamma U_{i,t}^e)}{exp(\gamma U_{i,t}^f) + exp(\gamma U_{i,t}^e)} \tag{30}$$

,where the utilities are defined as:

$$U_{i,t}^f = -\sum_{k=0}^{\infty} w_k [i_{t-k-1} - E_{t-k-2}^{s,f} i_{t-k-1}]^2 \tag{31}$$

$$U_{i,t}^e = -\sum_{k=0}^{\infty} w_k [i_{t-k-1} - E_{t-k-2}^{s,e} i_{t-k-1}]^2 \tag{32}$$

and $w_k = (\rho^k(1-\rho))$ (with $0 < \rho < 1$) are geometrically declining weights adapted to include the degree of forgetfulness in the model (De Grauwe, 2012). γ is a parameter measuring the extent to which the deterministic component of utility determines actual choice. A value of 0 implies a perfectly stochastic utility. In that case, each agent decides to be one type or the other simply by tossing a coin, implying a probability of each type equalizing to 0.5. On the other hand, $\gamma = \infty$ implies a fully deterministic utility, and the probability of using the fundamentalist

[16] See De Grauwe and Macchiarelli (2015) for a brief discussion of the empirical literature, Frankel and Froot (1990) for a discussion of *fundamentalist* behaviour, and Roos and Schmidt (2012), Cogley (2002), Cogley and Sargent (2007) and Cornea, Hommes and Massaro (2013) for evidence of *extrapolative* behaviour in the context of microeconomic and financial decision-making.

(extrapolative) rule is either 1 or 0. Another way of interpreting γ is in terms of learning from past performance: $\gamma = 0$ implies zero willingness to learn, while it increases with the size of the parameter, i.e. $0 < \gamma < \infty$.

As mentioned above, agents will subject the performance of rules to a goodness-of-fit measure and choose the one that generates least errors. In that sense, agents are 'boundedly' rational and learn from their mistakes. More importantly, this discrete choice mechanism allows to endogenize the distribution of heterogeneous agents over time with the proportion of each agent using a certain rule (parameter α). The approach is consistent with the empirical studies (Cornea et al, 2012) who show that the distribution of heterogeneous agents varies in reaction to economic or financial volatility (Carroll (2003), Mankiw et al (2004)).

3.3 The recursive solution method and the numerical set-up

We formulate the model in matrix format and solve it using recursive methods (see De Grauwe (2012) for further details). In essence, we solve the model in three steps. First, we initialise the learning framework for one or both state variables. In t=0, we assume a 50-50 share between the two rules, and allow the learning model to endogenously decide the subsequent shares as outlined in the previous section. Simultaneously, we define the core (reduced) system of the model, and solve it. Once the reduced system is solved, we recursively introduce those values into the rest of the model and solve for the entire model. In terms of ordering, we first initiate the learning process, solve for the reduced system, recursively solve the rest of the model, and then with that information at time t, we allow agents to form expectations regarding the state variables at time $t+1$ using the previously outlined utilities and switching mechanism.

This framework allows us to solve non-linear dynamics and is a more cost-effective alternative to the standard Bellman equation approach since we avoid using aggregate states and iterations to converge on the representative agent condition, matching individual and aggregate laws of motion for credit.

The model has eleven endogenous variables: land price, leverage, consumption, loans, interest rate on loans, deposits, interest rate on deposits, bank profits, bank equity, land, and the interest rate. The first four are obtained after solving the following reduced equilibrium system that iterates on the policy and pricing functions using households' FOCs and the forecasting rules:

$$\begin{bmatrix} 1 & \mu^{fx} & \bar{c} & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} q_t \\ \kappa_t \\ c_t \\ b_t \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha\nu & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_t^s[q_{t+1}] \\ E_t^s[\kappa_{t+1}] \\ E_t^s[c_{t+1}] \\ E_t^s[b_{t+1}] \end{bmatrix} +$$

$$+ \begin{bmatrix} 1-\nu & -\mu^{fx} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_{t-1} \\ \kappa_{t-1} \\ c_{t-1} \\ b_{t-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1+r_{t-1}^d \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{t-1}^b \\ l_{t-1} \\ d_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \epsilon_t^z \\ \epsilon_t^{Eb} \\ \psi_t \\ \epsilon_t^r \end{bmatrix}$$

Using matrix notation, we can write this as: $\boldsymbol{AZ_t = B\tilde{E}_t Z_{t+1} + CZ_{t-1} + DX_{t-1} + Ev_t}$. We can solve for $\boldsymbol{Z_t}$ by inverting: $\boldsymbol{Z_t = A^{-1}(B\tilde{E}_t Z_{t+1} + CZ_{t-1} + DX_{t-1} + Ev_t)}$ and assuring $\boldsymbol{A}$ to be non-singular.

Once these optimal values for the policy functions have been found, they are then inserted into the remaining general equilibrium system and the values of the remaining model variables are recursively solved. So, the solution for land, the interest rate on borrowing, deposits, bank profits, bank equity, and the interest rate are recursively obtained using the solutions obtained for land price, leverage, consumption and loans iterated above.

Expectation terms with an s E_t^s implies that we derive the optimal solution using the subjective beliefs governed by the learning process specified above.

Note that for the forecasts of land price and leverage, the expectation terms in equations 21, 23 and 25 are substituted by the discrete choice mechanism in 28.

The rational expectations model we solve by shutting off learning process (set it to 0), and set subjective expectations equal to rational expectations, $E_t^s = E_t$. In addition, we set the A matrix to diagonal $diag[1]$, C to 0, and B matrix to:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For the univariate learning model, the solution algorithm is almost the same except that we only insert learning on κ and set expected land price equal to its realised value.

We have four shocks in this model. ϵ_t^z is a standard TFP shock in the land production function. ϵ_t^{Eb} is a shock to bank capital (or equity), ψ_t denotes a shock to income (or collateral value), whereas ϵ_t^r is a standard monetary policy shock. Their parametrization will be discussed in the next subsection.

3.4 Calibration and simulations

We will divide the discussion in three parts. First, we will discuss the parameters related to the general equilibrium set-up. We will continue with the parameters related to the learning dynamics in the second part, followed by the calibration of the four shocks in the model. A full list of parameters and their values are reported in Table 1.

For the calibration of parameters related to the general equilibrium, we use the parameters calibrated or estimated in a number of closely related DSGE models. In particular, the (constant) risk aversion coefficient σ in households' utility function is, following Boz and Mendoza (2014), set to 2. We set the share of impatient households in the total economy to 0.61, as in Brzoza-Brzezina et al. (2014), in order to match the micro data on the share of liquidity constrained consumers reported in the Survey of Consumer Finances (SCF). This is also in line with the number reported in Justiniano et al. (2015). The discount factor β of patient households is higher and set to 0.9943 in order to obtain an annualized average real interest rate of slightly below 3%. This is in line with much of the literature, including Gerali et al. (2010) and Brzoza-Brzezina et al. (2014). The discount factor of the impatient types is lower and set to 0.975, as in Gerali et al. (2010).

For parameters related to financial intermediaries, we use the estimation results from Gerali et al. (2010) and De Grauwe and Macchiarelli (2015). In particular, we set the share of bank profits in bank equity equation ω^b to 1, the cost for managing banks' capital position δ^k to 0.1049, the adjustment costs of changing the interest rate on deposits Adj^b to 0 (since the unlimited access to liquidity from the central bank makes this process costless) and the target capital-to-loans ratio ν^b (or the inverse of the leverage target ratio) to 0.09. In order to make the deviation from this target value costly, we calibrate the cost parameter κ^{Eb} to 11.49, which is the value obtained from estimations in Gerali et al. (2010).

Turning to the land market, we use the values obtained in Boz and Mendoza (2014). In particular, we calibrate the factor share of land in the production α to 0.025, and we set the supply of land l fixed at 1. The (fixed) Lagrange multiplier μ^{fx} in the credit constraint, which is used to derive the shadow value of collateral in the land price function in equation 12, is set to 0.30.

Following Boz and Mendoza (2014), we set the consumption-GDP ratio in the aggregate resource constraint to 0.670, or two-thirds of the total output. Meanwhile, the remaining third is split between land and bank equity, where land-GDP ratio is set to 0.20 and bank equity-GDP to 0.13.

For the Taylor-rule parameters, we use the values estimated in Gerali et al. (2010). In particular, the interest rate smoothing (AR) coefficient is set to 0.77, the response to inflation in the Taylor rule to 2.01, meanwhile the response to output is set to 0.35. Equally, for macroprudential policy, we set the target (or cap) on household leverage $\bar{\kappa}$ to 2, and the response of LTV to credit growth ρ^{κ} in the Taylor-type macroprudential rule to 0.75, as in Lambertini et al (2013).

We turn to the parameters governing the learning process. The initial fraction of fundamentalists and extrapolists, α_0^f and α_0^e are each set to 0.5. The switching parameter, γ in equations 29 and 30 is set to 1, as in Brock and Hommes (1998). ρ, or the geometrically declining weight adapted to include a degree of forgetfulness in the learning dynamics in 31 and 32, is set to 0.5. For fundamentalists, we set the SS value of LTV, κ^* to 0.93 (as in Brzoza-Brzezina et al., 2014), and for the land price q^* simply to 1. To conclude this part, we make the land price highly contingent on its forecasted value by households, and therefore set the weight of the forecasted land price in the land price function ν equal to 0.7. That is in order to capture the uncertainty regarding its future value in the aggregate land dynamics.

We are considering four shocks in this model. A shock to TFP (or technology), (bank) capital quality, household income, and a monetary policy shock. The standard deviation of all shocks is normalized to 1 to facilitate the interpretation of the impulse responses. In line with the literature, the TFP and monetary policy shocks include an AR component equal to 0.90. (Bank) capital quality and income shocks, on the other hand, are each modelled as a white noise (with no AR component) since they lack a theoretical grounding for incorporating inertias into their process.

We simulate the model for 2000 periods, or 500 years.

Table 1: Parameters in the model and their descriptions

Parameter	Description	Value
Calibrated		Calibration
σ	Constant risk aversion parameter in CRRA utility function	2
ω	Share of impatient households int he economy	0.61
β^P	Discount factor of patient households	0.9943
β^I	Discount factor of impatient households	0.975
ω^b	Share of bank profits in bank equity accumulation	1
δ^k	Cost for managing banks' capital position	0.1049
Adj^b	Adjustment cost for changing the deposit rate	0
ν^b	Target capital-to-asset ratio	0.09
κ^{Eb}	Cost of deviating from target capital-to-asset ratio	11.49
α	Factor share of land in production	0.025
l	Aggregate land supply	1
μ^{fx}	(Fixed) Langrangian multiplier of the credit constraint	0.3
$\bar{c}$	Consumption-GDP ratio	0.67
$\bar{l}$	Land-GDP ratio	0.20
$\bar{c}$	Bank equity-GDP ratio	0.130
γ^r	Interest rate smoothing parameter	0.77
γ^π	Response to inflation in the Taylor rule	2.01
γ^y	Response to output in the Taylor rule	0.35
$\bar{\kappa}$	Cap on household LTV-ratio	2
ρ^κ	Response of LTV to credit growth	0.75
α_0^f	Initial fraction of fundamentalists	0.5
α_0^e	Initial fraction of extrapolators	0.5
γ	Switching parameter in MSFE	1
κ^*	SS LTV-ratio	0.93
q^*	SS land price	1
ν	Weight of forecasted land price in the land price function	0.7
$\tilde{c}$	SS consumption parameter in CRRA geometric series	0.125
ϵ^z	Standard deviation of the TFP shock	1
ϵ^{Eb}	Standard deviation of the capital quality shock	1
	Standard deviation of the income quality shock	1
ϵ^r	Standard deviation of the monetary policy shock	1
ρ_z	AR parameter in the TFP shock process	0.9
ρ_{Eb}	AR parameter in the capital quality shock process	0
ρ_ψ	AR parameter in the income shock process	0
ρ_r	AR parameter in the monetary policy shock process	0.9

For the RE and partial learning models, we remove the parameters which are not relevant for learning as described before. So, for instance, ν is removed in both the RE and partial learning versions, while $alpha_0^f$, $alpha_0^e$, γ are in addition removed in the RE version.

4 Quantitative results

We will simulate the three versions of the model and compare them in multiple dimensions. In the first part, we will depict and analyse the variables over the business cycle. In particular, we will run a long simulation of all three versions of the model for 2000 periods (or 500 years) and compare the evolution of the key variables over that period, focusing on particular on the probability and occupance of systemic events, and their macroeconomic effects. We will also compare the ergodic distributions in the three versions to better understand what frictions generate fat tails in our model. Lastly we will perform a statistical analysis and comparison using the moments generated by all model versions. To validate our conclusions, we will in addition run a number of statistical tests on the ergodic distributions of the variables. All these tests conducted on a the three versions of the model aim at identifying which frictions in our model gives rise to the systemic events and the asymmetries and fat tails that we the complete model generates.

In the second part, we will in particular focus on the learning dynamics in the model and examine leverage-and land price cycles that the complete model generates with respect to the version where learning in only one variable occurs. Here, particular attention will be paid to understanding the marginal benefit from deviating from rational expectations in terms of additional insights compared to standard credit friction models. In other words, are the benefits from deviating from rational expectations higher than the efficiency costs caused by a more complicated model structure and simulation algorithm? At the same time, we will inspect the role that market sentiment (optimism/pessimism) has in driving asset (land) prices. For robustness purposes, we will use multiple definitions of market sentiment. We will also analyse the role of heterogeneous expectations and multivariate informational frictions in generating systemic events and heavy consumption losses by comparing the evolution of consumption in the univariate and multivariate learning models.

In the third part, we will analyze (model consistent) impulse responses to the four shocks described above.

The last subsection will examine the effects of a macroprudential policy in terms of smoothing the business cycles, reducing the probability and frequency of systemic events, reducing the asymmetries and fat tails of model variables, and improving the overall welfare.

4.1 Forcing variables

The four shocks we will examine are:

- (Positive) TFP (or technology) shock, ϵ_{z_t}

$$y_t = z_t \epsilon_{z_t} g(l_t) \tag{33}$$

,where the TFP shock has an AR component, ρ^z calibrated to 0.9:

$$\epsilon_{z_t} = \rho^z + \eta \tag{34}$$

- (Negative) capital quality shock, ϵ_{Eb_t}

$$E^b_{t+1} = (1 - \delta^b) E^b_t + \pi^b_t + \epsilon_{Eb_t} \tag{35}$$

,where ϵ_{Eb_t} is a white noise shock to the evolution of bank equity stock.

- (Positive) income shock, ψ_t:

$$\kappa_t q_t l_{t+1} \psi_t \leq -\frac{b_{t+1}}{R^b_t} \tag{36}$$

,where ψ_t is a white noise shock to the collateral constraint of impatient households. And a

- Standard (negative) monetary policy shock (ϵ^r):

$$r_t = \gamma^r r_{t-1} + \gamma^\pi \pi_t + \gamma^y y_t + \epsilon^r \tag{37}$$

and ϵ^r is a white noise shock to monetary policy. In our simulations, we calibrate the interest rate smoothing parameter γ^r to 0.9. The standard deviation of all shocks is normalized to 1.

4.2 The nature of cycles in the model

We report three things in this section. The time-series evolution of model variables in the complete model are reported in Figure I.1. The comparison of three key model variables in the complete model versus the RE version are reported in Figure I.2. Table I.1 summarizes the correlations and Table I.2 — the statistical moments in the three model versions. For the ergodic distributions of the long-run simulation of model variables, we depict them in Figures I.4 and I.5, and the results from the statistical tests on the distribution of these variables are reported in Tables I.3 to I.5. The figures depict the distributions in the complete and RE versions, meanwhile the tables report the tests for all three model versions.

4.2.1 Macro-financial cycles

In all time series graphs, the zero-line represents the trend and the area above (below) it represents the positive (negative) cyclical deviations from the trend. The series should be interpreted as the filtered cyclical component of a time-series with an independent time-varying (or time-invariant) trend.

Starting with consumption cycles in Figure I.1, the first thing to note is the asymmetry in cycles that the complete model generates. While there are several episodes of strong consumption booms (around t=100,550,950,1100 or 1600), these are followed by even sharper contractions. So while the overall sharpest booms generate a rise in consumption of around 20% above the trend, the heaviest contractions lie at around 30% below the trend. Moreover, the persistence in booms is higher than the persistence in busts. Hence, both the frequency and the amplitude of expansions and contractions are different. This observation is confirmed by the statistically significant skewness in the ergodic distribution of consumption in the complete model (left) in Figure I.4 or Table I.2.

Next, the model is equally capable of generating diverse magnitudes of cycles. While the majority of the cycles are small, with some periodicity, large deviations from the trend also occur. Using Cogley and Sargent's (2008) definition that a severe (or systemic) crisis is one where consumption contracts by at least 25 %, we find 10 such contractions in 500 year. They are marked by grey arrows in Figure I.1. If we take an average over the entire sample, then two systemic crises occur every century. In most of these, the contraction in consumption is higher than 25%, which makes them clear candidates for a truly systemic crisis. Note, moreover that the contractions are proceeded by substantial consumption surges. This is in particular true for the second, seventh and tenth contraction, where consumption increases by 30-40% before it drastically reverses. Also, the build-up phase is much longer than the subsequent bust. Hence, this allows enough time for risks and leverage to build up before they cause a switch in the cycle. In terms of distributions, the periodic occurrence of large systemic crises should generate fat tails. Judging from consumption kurtosis (and that of other model variables such as bonds) in Table I.2, or the left-hand figure in Figure I.4, consumption has significantly fatter tails than a Gaussian distribution. The normality tests in Table I.5 formally confirm this as the null hypothesis of a Gaussian distribution for consumption, for instance, is rejected in all cases.

In the benchmark RE model, on the other hand, the cyclical swings are much smaller, and the distributions of the model variables are symmetric and/or thin tailed. Comparing the cyclical evolution of three key model variables in Figure I.2, from the y-axis of all graphs it is clear that the magnitude of the swings are four-to-five times smaller in the model version with RE compared to the complete model. At the same time, more of the mass lies in the neighbourhood of the zero-trend line (between -2 and 2) compared to the more extreme realizations in the complete model. Moreover, none of the recessions in this benchmark version can be considered *systemic* since the maximum fall in consumption at any point in time does not surpass 12%. Moreover, the ergodic distributions of the variables in the RE version are much closer to a (if not fully) Gaussian, which is further confirmed by the formal statistical tests of normality in Table I.2. Hence, we conclude that the asymmetry in the macro-financial cycles and the fat tails are a result of informational frictions and heterogenous expectations introduced in our model. We will discuss this in detail in the next section.

In addition, there are two key stylized facts that the complete model captures well. First, Cogley and Sargent (2008b) note that once an economy is in a low-growth state, there is a certain positive probability of running into a sequence of contractions. That is what we see after contraction 2. While the economy tries to recover from the first downfall, in ten years (on average) it runs into the next systemic crisis. As a matter of fact, five systemic contractions occur in less than 300 quarters (75 years). The second stylized fact that the full model captures is that a long build-up of debt, risks and liquidity in the (financial) system makes the entire economy much more unstable and prone to heavy reversals than an economy where the long build-up phase is controlled and shortened. In our model, that is exactly what occurs. Prior to t=700, the economy only experiences one systemic crisis. However, after the exceptionally long build-up phase in t=[180, 700], the economy suffers 9 crises in around 1000 quarters (or 6 crises in 700 quarters). That is a 6-fold increment during the same time interval. Hence this confirms the fact that a long period with high and sustained build-up of risks, credit, leverage (and speculation via asset prices) with only minor contractions changes the entire structure of the economy over the longer-run. This is because the heavy and sustained accumulation of debt and market optimism make the economy more susceptible to future shocks and significantly increases the probability of a sharp future downturn.

The same is observed for credit in Figure I.1, where 6 out of the 10 systemic crises result in the historically heaviest contraction in lending. Total lending to households decreases by between 15-20% during those systemic crises, which is highly significant. Moreover, the preceding build-up of credit before systemic crisis 2, and the following cycle of contractions are clearly visible also on the same graph.

A similar pattern is also found for land prices (last row on the left in Figure I.2). During the same episode of sharp consumption and lending contraction, land prices fell by between 30 and 45%. The causality in the model goes from land prices to consumption. The credit friction in the model means that a fall in the price of collateral will, through both the wealth and credit channels, reduce consumption. In addition, with the new informational friction, this effect is accentuated. This is because market sentiments, created by imperfect forecasts of future land prices can generate sustained (and long-lasting) increases in the perceived price evolution, just to be followed by a sharp reversal when the first signs of contraction appear. The more systemic crises there are, the more agents will remember those and include them in the calculation of the (subjective) probability of future price increases/drops. This is why both the share of pessimists *and* the number of times a price decrease are predominant in the forecast of land prices is significantly higher after the second systemic event, as shown by the last graph on the right in Figure I.1.

Turning to banks (bank equity and interest rates on loans), we also see that the systemic crises affect the profitability of the bank. Taking into account that bank equity and the interest rate on borrowing have the opposite signs in equation 11, once the price of household collateral (land) starts to contract, the financial intermediary is obliged to increase its borrowing rate, as the probability of default of impatient households has increased and so it is more risky to lend to them. However, that will reduce the amount of total borrowing, and thus the profits of the bank (since the fall in lending is higher than the rise in the interest rate margin). This will subsequently lead to a fall in bank equity, as governed by equation 7. On contrary, the higher the bank equity that a financial intermediary holds (a feature in upturns), the more leeway the bank has to extend its lending, and so it reduces its lending rate. This is why we see the opposite business cycle evolution for bank equity and the lending rate. Note also how sensitive the interest rate setting is to movements in bank equity. Roughly a 1% drop (rise) in bank equity from its long-term trend generates a 10% rise (drop) in the lending rate from its trend. This is due to the quadratic composite social cost imposed on excessive leverage in equation 11, which pushes the interest rate more than proportionally up. Due to this heavy (de)-leveraging (or rebalancing) over the cycle, the financial sector becomes a powerful propagator of shocks, originated within the financial sector as well as outside. Thus, a sufficiently high de-leveraging can, via the lending channel, cause a severe downturn in the real economy. We will analyze this mechanism in more depth in the section discussing impulse responses.

4.3 Distributions and statistical moments over the business cycle

In this second part, we wish to formally investigate the quantitative results from marginally introducing the different frictions by looking at the statistical moments and the ergodic distributions that each version of the model creates. Correlations are reported in Table I.1, moments in Table I.2, and the formal tests for distributions in Table I.3 (for the RE version), Table I.4 (for the univariate learning version), and Table I.5 (for the complete model).

We run 5 statistical tests to explore the underlying data-generating process of the simulated (or bootstrapped) data. The tests we employ are the Jarque-Bera test of normality, Lilliefors test, the Kolmogorov-Smirnov test, the chi-squared goodof-fit, and the test for t-distribution. The first three verify the null hypothesis that the underlying data-generating process is that of a normal distribution. We employ three different tests of normality in order to avoid the limitations of each individually and make a robust inference about our distributions. The other two tests include distributions which are symmetric but fat-tailed (t-distribution), and perfectly asymmetric (chi-squared).

4.3.1 Statistical performance of the models

The complete model is capable of generating high contemporaneous cross-correlations between key model variables, as well as high persistence in the time domain of key variables. From the last column in Table I.1, most of the (auto)correlations lie between 0.8 and 0.99. The mechanism responsible for these high correlations is the informational friction. By comparing the contemporaneous cross-correlations of the complete model versus the RE and univariate learning versions, it seems that introducing the informational friction leads to an increase in the correlations between key model variables. Introducing the informational friction in just *one* variable only is also not sufficient in generating that persistence, since the correlations of the former are significantly lower. Furthermore, some of the correlations have wrong signs in the *incomplete* model versions. Examples are consumption and leverage $[c_t, \kappa_t]$ in both the RE and univariate learning versions, loans and consumption $[b_t, c_t]$ in the univariate learning version, or land price and leverage $[q_t, \kappa_t]$ in the RE version. In the complete model, the first two have a positive and the last one a negative sign, but the opposite in the incomplete versions.[17] This is indicative of the fact that by omitting the full informational friction in this model, one (or several) of the model mechanisms might be misrepresented that results in erroneous variable interactions.

Note also that the autocorrelations are lower in the univariate learning version compared to either the RE or complete model. In other words, while the autocorrelations in the RE and complete models are very similar, they are somewhat lower in the one-variable informational friction version. We believe that the reason behind this result is the incomplete frictional interaction that is present in this version. As we mentioned earlier, expected land price is an argument of the leverage function. If we assume that the expected price is forecasted using rational expectations while leverage is exposed to informational friction, we are in fact reducing the transmission power of this friction. So, while in the RE model credit frictions and shocks are responsible for the persistence, in the univariate learning model the transmission capacity of credit friction is reduced by the noise coming from the incomplete learning friction. However, when the complete informational friction is incorporated, the model re-generates the previous persistence. Nevertheless, as persistence in the correlations from credit frictions and shocks was already very high, the additional informational friction does not substantially increase it. Introducing an incomplete friction, on the other hand, might create costs in terms of lower persistence.

Turning to the second moments in Table I.2, they are highest for most model variables in the RE version, followed by the complete model. The only exceptions are the learning variables themselves, who have the highest variation in the complete model, followed by the univariate-learning one. Hence, while the learning variables oscillate most when their is uncertainty attached to them, this does not necessarily

[17] Higher leverage allows households to consume more or inversely, impatient households can consume more either if land price goes up, leverage goes up, or both. Higher loans allow (impatient) households to consume more. Finally, a higher land price, *ceteris paribus* allows households to reduce their leverage, since land price is the denominator in the leverage expression.

mean that the oscillation in the remaining part of the model is also the highest. This is also consistent with the observation that even if the variance of the learning variable(s) is higher in the univariate learning version compared to RE, the variation in the remaining model variables is lower. This implies that information frictions, which result in higher variability in the learning variables might dampen the transmission of shocks to the remaining model, possibly because of the resulting slower adjustment/transition of the model to (known) shocks compared to the case when only credit frictions are present, where the transmission is quicker. Remember that in the complete model, we do not have uncertainty regarding the shock processes which means that the shock-generating process is known to agents at all times.

As a preliminary conclusion, the statistical comparison has shown that a combination and interaction of credit friction, informational friction and heterogeneous expectations generates the highest persistence in the model. Moreover, including all of these causes consistency in cross-correlations. At the same time, the oscillation of the model to shocks is lower compared to the case where informational frictions are absent despite the higher variability in the learning variables.[18] Omitting one friction can either cause consistency issues, lower persistence, or both. Equally, including incomplete informational friction (or learning) might instead deteriorate model performance compared to a fully modelled mechanism by reducing the persistence and cross-correlations in the model, and generate very small cyclical variability.

4.3.2 Ergodic distributions and frictions

In order to validate our statistical findings, we formalise it with a number of statistical tests on the resulting distributions of the model variables in each version of the model. We are in particular interested in two things. First, we wish to investigate the type of distributions that each friction generates. Second, and possibly more important, we wish to examine whether the extreme events (systemic crises) and fat tails we obtain in the full model are truly generated by the complete informational friction (or uncertainty) that we have claimed, or whether it is heterogeneous expectations, credit frictions or something else that generate those characteristics. To do so, we will holistically compare test results in the three versions.[19]

[18] This pattern has also been documented in other medium-size learning DSGE models, such as Slobodyan and Wouters (2012), where the learning improves the fit of the model to the data compared to the benchmark Smets and Wouters (2007) set-up.

[19] Note that in cases where test results imply that a variable has Gaussian as well as t- or chi-squared distribution, in the final decision for that variable we opt for the non-standard distribution

In the model with only credit frictions, the ergodic distributions of all variables are symmetric. The two core variables of the model, κ and q follow a Gaussian distribution, meanwhile the rest follow a t-distribution. This implies that if agents use rational expectations in forecasting leverage and land prices, then the model is reduced to a standard New Keynesian framework. Expansions and recessions occur periodically, but they are all moderate, equal and symmetric. Credit frictions do generate some outliers in the other variable that results in slightly fatter tails of the pdf. However, these outliers are symmetric which means that positive and negative realizations occur with equal and small probability. Hence, if an imperfect credit market is the only friction in a sudden-stop type of model with financial intermediaries, then the macro-financial cycles are moderate, symmetric with the majority of them occurring close to the mean. There are only a few symmetric outliers around the trend-line which means that heavier expansions/contractions happen seldom.[20]

Relaxing somewhat the rational expectations assumption and allowing for uncertainty regarding one variable changes only slightly the results. In fact, they become somewhat inconsistent. The tails of some distributions (consumption, credit, interest rate on credit, leverage and bank equity) get reduced, albeit they remain symmetric. The distribution of other variables, on the other hand, becomes fully asymmetric (chi-squared), such as land prices and return on land. At the same time, the null-hypothesis for Kolmogorov-Smirnov test is rejected in all cases. Hence, some data-generating processes seem to be borderline-Gaussian cases. This is in line with the inconsistency argument we proposed before for the partial learning model. On the one hand, we impose uncertainty on leverage, but allow for rational expectations in land price, which is an argument of the leverage function. From the RE model version and the literature, we know that variables under rational expectations have Gaussian (or t-) distributions. On the other hand, allowing for imperfect beliefs and learning generates non-standard distributions. Mixing the two might create contradictory results.

Allowing for the full learning mechanism, and allowing to interact with hetero-

since for some tests there is a tendency of over-fitting the Gaussian distribution. Only when two or three out of the three normality tests are positive do we take it that the data-generating process of that variable is truly Gaussian. Equally, if none of the tests show a positive result, we interpret it as none of the distributions can be fitted and the data-generating process of that variable is something different or more exotic.

[20] Remember from the business cycle analysis above that during the 2000 simulated periods in the RE version, not a single systemic event occurs.

geneous beliefs and credit frictions generates entirely different results. In the case of the full model, the null hypothesis of all tests and for almost all variables are rejected. The exceptions are nominal interest rate, interest rate on deposits and those on loans, which are fitted into a t-distribution and chi-squared distribution respectively. There are at least three implications from this. First, none of the variables follows a Gaussian distribution, which means that the full informational friction is indeed responsible for causing systemic events and asymmetric cycles in our model. Second, most variables do not follow any of the standard distributions tested here which means that their data-generating processes are more complex. In the full model, we have interactions between three frictions. Since these frictions also determine the macro-financial linkages, these links are also multi-layered and complex. Thus complex macro-financial linkages give rise to non-standard ergodic distributions. Third, and maybe most important, recognising that the agents' estimation of collateral price and risks is limited and highly susceptible to subjective beliefs is analytically very important.

4.3.3 Learning process and model dynamics

Apart from the business cycle properties of the model, we also wish to understand the contribution of learning (and heterogeneous expectations) specifically on the two uncertainty variables. In other words, how different are the leverage and land price cycles when we introduce this friction, compared to the case where they are absent? A direct comparison is provided in Figure I.6. The two cycles are generated using the 3 different versions of the model and directly contrasted. At the same time, we can use this information to deduct how much the subjective beliefs attenuate the fundamental land price cycle by looking at the additional swings generated by the market sentiment. Both versions will provide a good estimate of the significance of the optimistic/pessimistic market sentiment.

Nevertheless, to get a better understanding of the role of market sentiments in our model, we explicitly depict in Figure I.3 the evolution of those during the 2000 periods by explicitly tracking the fraction of agents that are optimistic about the future land price (fraction=1), or pessimistic (fraction=0). To complement this information, in the same graph we also depict the additional rise or fall in land prices on top of what would be the appropriate response to the structural shocks that hit the economy in the current period, i.e. the evolution of land prices above its fundamental value. To finalise, we also report the ergodic distributions of market sentiment in the complete model versus the model in which only fundamental shocks drive the cycles of land prices (RE version).

The graph on the top-left of Figure I.3 clearly demonstrates that the majority of land price fluctuation is due to the market sentiment. The black line, which is the fundamental movement in land prices, is only a small proportion of the total land price movement in the full model (blue dotted line). In addition, note that movement in the two prices is well-aligned (meaning that when the fundamental price increases, the full price does it, too).This signals the fact that agents' subjective beliefs are not irrational since they use information on fundamentals to forecast future price. This is due to the specific learning framework imposed on agents, which is intrinsically rational. Thus when we impose an inconsistent learning framework, such as the one where learning only occurs in leverage, the co-movement in total and fundamental land price should be less tight. Judging from the lower-left graph in Figure I.6, that is indeed the result. Movements in total land price (light-blue dotted lines) are of a higher magnitude than that of the fundamental (marine blue dotted line) since imperfect beliefs in leverage are having externalities on the price of collateral. However, since the direct impact from rational expectations are mixed with the indirect externality from leverage, the movement in price is not always consistent with the fundamentals. Moreover, in the complete model most of the probability mass of sentiment in the extremes (full pessimism and optimism) and, in relative terms, there is a higher probability mass in the pessimistic region (see the lower-left graph in Figure I.3). The top-right graph in the same figure, which depicts the proportion of agents forecasting a future land price increase (1) or decrease (0), goes in the same direction since the number of times the graph touches the minimum zero-line is higher than that of the maximum. Thus, while the land price is subject to sequential switch in the market sentiment, going from extreme optimism to extreme pessimism, the probability (or the number of times) the land price is in the pessimistic phase is slightly larger.[21]

For leverage (the upper graphs in Figure I.6), we observe a much tighter co-movement between leverage in the partial and the full model. Recall that in both versions this variable is subject to imperfect beliefs. However, the heavier swings in the complete model show that the interaction between beliefs in leverage and land price are indeed important since the contribution of this joint learning-dynamics

[21]Note from the lower-right graph in Figure I.3 that market sentiment does not play any role in driving land prices since the probability mass for any level of land price forecast is uniform.

in the leverage cycle is non-negligible. Thus, imposing only partial learning will indeed bias the leverage estimate. At the same time, we observe episodes of strong positive and strong negative correlation between the full and fundamental leverage. In our simulation, for instance, during the first 400 periods, while the fundamental leverage increases or is above the trend-line (and then drops), the full leverage drops or is below the trend-line (and then rises). Since this is at the start of the learning dynamics, agents lack a long history of data realizations to take into account in their forecasts, and therefore make more subjective conclusions about the future leverage evolution. As a results, as long as they don't observe a reversal in leverage, they over-estimate the boom in land price, credit and consumption, and thus under-estimate the leverage for a long period. They only switch their forecast once they observe a reversal, and remain pessimistic about future economic outlook for a long time, and thus over-estimate the true leverage evolution.

Our sequence of beliefs is in many ways similar to the ones obtained in Boz and Mendoza (2011, 2014), but with some important deviations. Comparing the third graph in their Figure 6 to our top-right graph in Figure I.3, in both frameworks the *optimistic* interval is initially more persistent. Moreover, as the number of low leverage regimes is observed, the number of switches to *pessimism* increases. Nevertheless since the state-space is dichotomous in their world, the reversals are also more abrupt. This should result in sharper turning-points over the business cycle, not because of the model dynamics but because of the constrained model learning construction. In our case, on the other hand, households 'guess' a full continuous state-space of values, and so the reversals are more gradual. Hence, if sharp declines are observed in the business cycles, they are entirely generated by the endogenous model dynamics (via the interaction between learning and financial frictions), and not by a demarcation of the state-space. Further to that, learning in their framework is significantly faster than in ours, which means that convergence to a RE model is achieved after a relatively short period of time. Looking at the first two graphs of Figure 6 in their paper, the subjective transition probability is very close to the actual probability already prior to 300 quarters (or 75 years). That is possible because the state-space is reduced and because agents engage in restricted Bayesian learning (which has been shown to converge faster). In our model, on the other hand, the environment is more uncertain and learning is slower. Because systemic crises are rare, learning about them is also slow, and that is why uncertainty regarding leverage and land prices remains in the model dynamics for a much longer period of time. Lastly, while in Boz and Mendoza (2014) agents know the land price and forecast only the transition probability of leverage, we extend it to include land prices, since it directly depends on the leverage (via the shadow value of collateral). We think that our approach is more realistic under the asset pricing of Mendoza (2010) since a complete knowledge of the land price would allow households to learn the 'true' value of leverage by solving the rest of the model and recursively extract the value of leverage.

4.4 Impulse response analysis

The last part of our model evaluation consists of studying the complete model responses to exogenous (stochastic) shocks. Figure I.7 depicts the impulse responses to a positive TFP shock and Figure I.8 to an expansionary monetary policy shock.

The numbers on the x-axis indicate number of quarters. All the shocks are introduced in t=100 and we observe the responses over a period of 50 quarters (or 12.5 years). Note that in these figures we depict the *median* impulse response in black amongst a distribution of impulse responses generated with different initializations of the learning parameters. The red lines in the Figures represent the 95% confidence intervals, or a full distribution of impulse responses. For the sake of clarity and focus in the discussions, we will only concentrate on the median impulse response, which is a good representation of the overall distribution. Moreover, we will only concentrate on the standard TFP and monetary policy shocks in the text since these are standard in the literature and are modelled with persistence. [22]

4.4.1 TFP shock

A 1% TFP shock improves the production of land, and therefore increases the land price by 1.5%. Because quantity of land is fixed, all of the efficiency improvement will go to land price, by improving the intertemporal consumption smoothing of households. Since value of household equity goes up, leverage of impatient households decreases by 1.6%, and their external financing possibilities improves. Via the collateral constraint, impatient households are able to borrow more for the same collateral, which initially pushes up the loans they obtain by 0.4%. For financial intermediaries, this leads to a higher bank equity value (0.04%), which gives them

[22] The results and the discussion of a negative bank capital quality shock and a positive (financial) wealth shock are available on request.

space to extend their credit line even further since their capital-to-asset ratio has increased. Via the interest rate margin equation, they reduce the interest rate on borrowing to households by 0.65%. This lower cost on loan repayment in turn allows impatient households to extend their borrowing even further in t=102, resulting in a peak increase in external financing at 0.65% above the pre-shock level. For financial intermediaries, this is an additional increase of bank equity by 0.02%, implying a total of 0.06% expansion in bank equity as a result of the TFP shock. Hence, the bank can extend its activity and size as a result of an improvement in the real (production) sector.

However, this extension in credit makes the households gradually more leveraged, and the opposite mechanism is then set in motion. The higher leverage raises the value of the left-hand side of equation 5, which reduces the amount of next-period borrowing (because of their negative relation), which in turn reduces their (future) consumption possibility, and therefore the price of land. This opposite mechanism continues until the economy returns to its pre-shock level.

4.4.2 Monetary policy shock

A reduction of 1% in the (risk-free) interest rate reduces the deposit rate by the same amount (since $R_t^d = r_t$). Since this reduces the financing cost for banks, they can therefore reduce their cost of lending in order to extend their asset side and increase their profitability. The resulting rise in bank lending increases the amount of credit that households get, and therefore their (expected) consumption possibility. Via the pricing function of land in equation 12, the land price also increases. This reduces the leverage of (impatient) households, and via the collateral constraint allows them to borrow more. The cost of borrowing therefore reduces even further, and the bank extends its credit even further. As a result, bank equity rises. The total effect of the expansionary monetary policy shock is that the interest rate on borrowings falls by 0.45%, the expansion in credit is 0.45%, the rise in land price is 0.9%, and the fall in leverage 1.55%. The resulting boom in consumption is first 0.8% followed by 1.1%, and the rise in profitability of intermediaries raises its total bank equity by 0.04%. Note that while the economy (including the financial sector) expands following both shocks, the expansion is quantitatively larger for the supply side (or TFP) shock.[23] That is not surprising since our framework lacks

[23]Remember that both shocks are calibrated int he same way. The standard deviation of the white noise component is standardized to 1% while AR component is calibrated to 0.9. That is

sticky prices or wages which would make the monetary policy transmission more persistent, as in standard NK-models. Therefore in our framework, policy makers should concentrate on supply-side policies to generate sustained booms rather than using (discretionary) monetary policy. Therefore, we also expect (relatively) a high efficiency of macroprudential policy in smoothing the cycles, since the policy can be viewed as a type of supply-side constraint on the "production" in the financial sector.

4.5 Macroprudential policy

We proceed by introducing a macroprudential policy in the complete model and quantify the (stabilizing) effects that a well-defined policy can have, in particular on reducing the number (and impact) of systemic crises. In what follows, we will evaluate one particular type of macroprudential policy. We focus on a cap on (household) LTV, where the central bank allows households to leverage up to a certain level (but not beyond), and therefore restricts intermediaries to extend their credit supply only up to a certain quantity.

Tables I.6 and I.7 summarize the statistical moments of the model variables with and without macroprudential policy. Figures I.9 and I.10 compare the ergodic distributions for a selection of key model variables in the the version with and without macroprudential policy. Lastly, the second row in Figure I.9 compares the number of systemic crises in the complete model with macroprudential policy compared to one without it. The last two in the same figure depicts the fraction of agents forecasting an optimistic versus pessimistic land price evolution for the same simulation as for consumption in the row above.[24] In our simulations, we set the LTV cap at 2. The effects are significant.

Starting with learning dynamics, because of a fixed point provided by the cap, the learning with respect to the leverage is rapid. It only takes a few periods for households to understand what their maximum limit is, and therefore their subjective expectations converge to this limit. Since impatient households wish to leverage up to a maximum, the entire probability mass will also lie at this limit, as the ergodic distribution of the lower-right graph in Figure I.9 shows.

For land price, the two graphs at the bottom of Figure I.10, the learning dynamics

why we can directly compare the two effects.

[24]For a list of figures for the remaining model variables, please do not hesitate to contact the authors.

also changes. A larger share of agents use the fundamentalist rule compared to the model without a macroprudential policy. We believe the reason is because agents now only need to learn about one variable meanwhile the other has a fixed point. Hence, benchmarking the land price q becomes more useful in forecasting since fluctuations in the model are reduced and therefore using a fixed reference point for forecasting becomes more effective.

In terms of the macroeconomic and financial model variables, we also see a significant change. Looking at the statistical moments and distributions, we see that most variables become more Gaussian. The distributions become more symmetric and the fat tails are reduced. In practical terms, it means that sharp rises or drops in these variables are reduced, as well as probabilities of systemic crises. Many (auto)-correlations are reduced, which implies less of the (market) sentiment driven cycles that we observed before. In addition, the volatilities and skewness of the variables are reduced by a factor of between 2 and 4. Meanwhile, the kurtosis increases slightly, which means that the distribution becomes more centred around its mean/median. That is clearly visible in the figures for consumption, land price and credit.

Also the number of systemic crises are reduced by 50 %. Instead of the original 10 crises in 500 years, we now get 5 crises over the same time period (graphs in the middle of the page in Figure I.10). In particular, the sequence of systemic crises that occurs after the second one in the model without a binding policy is almost eliminated. Moreover, the losses related to each of the systemic crises are also reduced. Noting that the scaling in the second graph is 3 to 4 times smaller, the losses in each of the crises is reduced by, on average, a factor of 3. Thus, an LTV-cap does not only reduce the probability of a systemic crises by 50%, but it also reduces the losses incurred by each. As a result, the business cycles become shorter and the amplitude of each smaller.

While it is clear that the policy smooths the cycles and reduces the systemic events, we would like to quantify these effects in terms of households' welfare. In standard RE DSGE models, one would value the welfare effects by calculating the (welfare) gains using a second order approximation of households' welfare. However, in our model, RE is substituted with subjective beliefs, which means that the policy maker does not know how to weight these beliefs into a general welfare function. Hence, imperfect information also concerns households' welfare.[25]

[25]Recently, Brunnermeier et al. (2014) are trying to define 'belief-neutral' welfare functions

To overcome this problem, we instead value the welfare using utility (or consumption equivalence) measure of an economy with and without the policy. Knowing the parameters in the utility function, and the median consumption of households in an economy with and without the LTV-cap, we can calculate the utility gains that households will get from imposing the rule.[26] Since household utility only depends on consumption, the gains will be expressed in consumption equivalence terms.

We find that the utility gains from using a cap on household LTV is 6.5%. It means that, on average, a household will consume 6.5% more when a central bank imposes a cap compared to an economy without it. Decomposing this gain, we find that 6% out of the 6.5% derives from an increase in the level of consumption, while 0.5% comes from a reduction in variability (or volatility) of consumption over the cycle. The reason for this heavy gain in level, we believe, comes from the reduction in the systemic crises. Systemic crises are events when most of the consumption level is reduced. In relative terms, this reduction in level is even higher than the reduction in volatility of consumption, since the *ex ante* probability mass of such event is not big. However, once that state becomes absorbing, the reduction in level is very high.

To conclude, we compare our results with a more elaborate version of the LTV-rule. Following the recent literature on macroprudential policy (see, for instance Lambertini et al. (2013) or Angelini et al. (2014)), we also try a Taylor-type (countercyclical) rule specified in equation 19. We find that a more complex LTV-rule generates very similar economic outcomes to the simpler rule we have used before. The only difference is that the Taylor-type rule smooths the fluctuations in the interest rates by more. We believe that the explanation for this similarity lies in the learning. While a Taylor-type rule increases the information content in the reaction function and allows the central bank to react countercyclically to a larger set of financial variables, it also delays learning since the fixed point is removed, and the leverage cap in itself will vary over the cycle. In other words, we introduce an additional layer of uncertainty in the learning of leverage. We see that switches in the use of extrapolative versus fundamentalist rule for leverage are higher under the Taylor-type rule, which generates additional dynamics in the learning framework. In

in models with distorted (or imperfect) beliefs. However, more work is necessary before a robust method can be obtained for loss function derivations.

[26]Note that since model variables have asymmetric distributions in the benchmark model, the median is more representative of the centre of the distribution, rather than the mean. That is why we use the median consumption in our calculations. We could, however, trivially re-run the same experiments using the mean consumption values.

terms of monitoring costs, the simple rule obviously requires much less monitoring of the set of financial variables, including their cyclical co-movements. On the other hand, the Taylor-type rule allows for flexible setting of the cap over the business cycle. Thus considering all costs and benefits, the simple LTV-rule will be preferred by passive policy-makers, who wish to provide binding constraints without incurring a lot of monitoring costs and noise. The other rule will be preferred by more active policy-makers who, ont he other hand, wish to retain the flexibility of gradually setting the caps and/or to make them cyclical.

5 Concluding remarks

Deregulation in the financial services industry since 1980's, the increased competition amongst financial intermediaries and the unprecedented expansion in financial engineering since mid-1990's has, in an exceptional manner, increased the size of the financial sector. Their credit lines to the real economy, and the consumption possibility of households has been historically the highest in the period prior to the Great Recession. The US (and to certain extent the EU) economy experienced one of its sharpest booms in early 2000's. On the other end, however, the pricing of risks and leverage became an increasingly difficult task as uncertainty regarding the true accumulation of risks on balance sheets and the true exposure of households increased. The mispricing of risks gave leeway to market speculation and market sentiment-driven cycles. We put forward a model that explains these two observations as information frictions following a financial innovation. We start from Boz and Mendoza (2014) and analyze the effects of dynamic optimization under uncertainty on the macro-financial cycles, and the probability of systemic crises. In particular, we are interested in understanding the role that macroprudential policy plays in reducing the probability of systemic events.

Including these facts into a general equilibrium model with credit frictions and an adaptive learning result in an increase in the amplitude and frequency of the cycles. The build-up phase of risks, credit, leverage and consumption is much longer and higher than in standard DSGE models. In the same way, once a reversal in lending occurs, the decline in all variables is also much sharper and lasts shorter. The probability of systemic crises is significant, and we find that, on average, 2 such crises occur every century. Moreover, we find that, different from standard boom-bust cycles, a systemic crisis can be followed by a sequence of subsequent contractions, as it makes the economy more unstable. The result is asymmetric distributions of key macroeconomic and financial variables, with high skewness and fat tails.

A simple cap on the LTV-ratio is effective in smoothing the cycles and reducing the effects of a deep contraction on the real-financial variables. The model distributions become much more symmetric and Gaussian. It also reduces the amount of borrowing and leverage in upturns. The number of systemic crises is halved, and the losses at each is reduced by, on average, a factor of 3. The consumption (utility) gains from such a policy are, on average 6.5% compared to an economy without a macroprudential rule. Also the stabilizing role of monetary policy is increased once a macroprudential rule is used. To conclude, a simple LTV-rule is preferred to a more elaborate Taylor-type version because it provides a strong 'benchmarking' to agents in their learning process, while generating same welfare (improving) effects at a lower information cost.

These are promising results in our understanding of the probability of systemic events, and their destabilizing macroeconomic impacts. While the road in reaching a full understanding of such events is long, these should hopefully be seen as a contribution in the right direction. Future research should therefore try to stretch the framework of this paper in multiple directions.

First, a robust comparison is necessary between the learning framework in this model and the Bayesian set-up in Boz and Mendoza (2014). Both are actively used in the literature, and a serious comparison in terms of long-term learning, memory and model dynamics should be welcomed.

Equally, the regime-switching in rules in this framework should be compared to homogenous learning set-ups. A lot of the dynamics in this model comes from the regime switching. It would therefore be interesting to see the type of macroeconomic dynamics we would get if agents use only one rule, possibly a more elaborate adaptive rule such as least-square learning.

It would also be interesting to conduct a robustness exercise to test the model performance for a larger parameter space of the learning variables. On the same lines, it would be highly relevant for policy purposes to find the optimal LTV-cap whereby gains from such a rule are maximized.

Lastly, systemic crises are rare and non-linear events. Therefore, it would be of high interest to zoom-in such periods and only study the dynamics once such event becomes absorbing. In particular, it would be interesting to examine the statistical moments, the distributions and the transmission channels under only such states. That would bring the model closer to the recent but blooming empirical literature on tail-events and hyper correlations.

References

[1] Aiyagari, S. R., and Gertler, M. (1999). Overreaction of asset prices in general equilibrium". *Review of Economic Dynamics*, 2(1): 3–35.

[2] Anderson S.P., De Palma A., and Thisse J.F. (1992), *Discrete Choice Theory of Product Differentiation*, MIT Press, pages 13–62.

[3] Angelini P., Neri S., and Panetta F. (2014), "The Interaction Between Capital Requirements and Monetary Policy", *Journal of money, credit and Banking*, 46(6): 1073-1112

[4] Bernanke B.S., Gertler M. and Gilchrist S. (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework", in J.B. Taylor and M. Woodford, eds, *Handbook of Macroeconomics*, Amsterdam:North-Holland.

[5] Bianchi, J., Boz, E., and Mendoza, E. G. (2012). "Macroprudential policy in a Fisherian model of financial innovation". *IMF Economic Review*, 60(2): 223–269.

[6] Boz, E., and Mendoza, E. G. (2011). "Financial innovation, the discovery of risk, and the US credit crisis" (No. w16020). National Bureau of Economic Research.

[7] Boz, Emine and Mendoza, Enrique G., (2014). "Financial innovation, the discovery of risk, and the U.S. credit crisis," *Journal of Monetary Economics*, Elsevier, vol. 62(C): 1–22.

[8] Branch W. A., and McGough B. (2009), "A New Keynesian Model with Heterogeneous Expectations", *Journal of Economic Dynamics and Control*, 34(5): pp. 1305–11

[9] Brock W.A., and Hommes C.H. (1997), "A Rational Route to Randomness", *Econometrica*, 65(5): pp. 1059–96

[10] Brock W.A., and Hommes C.H. (1998), "Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model", *Journal of Economic Dynamics and Control*, 22(8-9): pp. 1235–74

[11] Brunnermeier M.K., Simsek A., and Xiong W. (2014), "A Welfare Criterion for Models with Distorted Beliefs", National Bureau of Economic Research Working Paper Nr. 20691

[12] Brzoza-Brzezina, M., Gelain, P., and Kolasa, M. (2014), "Monetary and macroprudential policy with multi-period loans" (No. 2014/16). Norges Bank.

[13] Bullard, J. and Suda J. (2011), "The Stability of Macroeconomic Systems with Bayesian Learners", SSRN Working Paper 322.

[14] Camerer, C., and Ho, T. H. (1999), "Experienced-weighted attraction learning in normal form games" *Econometrica*: 827–874.

[15] Carroll C.D. (2003), "Macroeconomic Expectations of Households and Professional Forecasters", *The Quarterly Journal of Economics*, 118(1): pp. 269–98

[16] Christiano, L., Motto, R., and Rostagno, M. (2013). "Risk shocks" (No. w18682). National Bureau of Economic Research.

[17] Cogley T. (2002), "A Simple Adaptive Measure of Core Inflation", *Journal of Money, Credit and Banking*, 34(1): pp. 94–113

[18] Cogley T., and Sargent T. (2007), "Inflation-Gap Persistence in the US", mimeo, Northwestern University

[19] Cogley, T., and Sargent, T. J. (2008). "Anticipated Utility and Rational Expectations as Approximations of Bayesian Decision Making". *International Economic Review*, 49(1): 185–221.

[20] Cogley, T., and Sargent, T. J. (2008b). "The market price of risk and the equity premium: A legacy of the Great Depression?". *Journal of Monetary Economics*, 55(3): 454–476.

[21] Cornea A., Hommes C.H. and Massaro D. (2012), *Behavioral Heterogeneity in U.S. Inflation Dynamics*, CeNDEF Working Papers, 12–03.

[22] De Grauwe, P. (2012), *Lectures on Behavioural Macroeconomics*, Princetone University Press

[23] De Grauwe, P. and Macchiarelli C. (2015), "Animal Spritis and Credit Cycles", *Forthcoming in Journal of Economic Dynamics and Control*

[24] Evans, G. W., and Honkapohja, S. (2001), *Learning and expectations in macroeconomics* Princeton University Press.

[25] Frankel J., and Froot K., (1990), "Charists, Fundamentalists, and Trading in the Foreign Exchange Market", *American Economic Review*, 80(2), pp.181–85

[26] Gelman, A., J.B. Carlin, H.S. Stern, and D.B. Rubin (2000). "*Bayesian Data Analysis*", (London: Chapman and Hall, 2000).

[27] Gerali, A., Neri, S., Sessa, L., and Signoretti, F. M. (2010). "Credit and Banking in a DSGE Model of the Euro Area". *Journal of Money, Credit and Banking*, 42(s1): 107–141.

[28] Harvey A.C. and Jaeger A. (1993), "Detrending, Stylised Facts and the Business Cycle", *Journal of Applied Econometrics*, 8: pp.231–47

[29] Honkapohja, S., Mitra, K., and Evans, G. W. (2012). "Notes on Agents Behavioral Rules Under Adaptive Learning and Studies of Monetary Policy". Unpublished Memo

[30] Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2015), "Household leveraging and deleveraging", *Review of Economic Dynamics*, 18(1): 3–20.

[31] Lambertini, L., Mendicino, C., and Punzi, M. T. (2013). "Leaning against boom bust cycles in credit and housing prices", *Journal of Economic Dynamics and Control*, 37(8): 1500–1522.

[32] Mankiw N.G., Reis R. and Wolfers J. (2004), "Disagreement About Inflation Expectations", NBER Macroeconomics Annual 2003, 18: pp. 209–70

[33] Manski C.F., McFadden D.L., (1981), "Alternative Estimators and Sample Designs for Discrete Choice Analysis" in *Structural Analysis of Discrete Data and Econometric Applications*, Manski C.F. and McFadden D.L. (ed.), Cambridge: The MIT Press

[34] Mendoza, E. G. (2010). “Sudden stops, financial crises, and leverage”. *The American Economic Review*, 100(5): 1941–1966.

[35] Payzan-LeNestour, E., and Bossaerts, P. (2014), “Learning about unstable, publicly unobservable payoffs”, *Review of Financial Studies*, hhu069.

[36] Pesaran H.M., (1987), *The Limits of Rational Expectations*, Blackwell, Oxford

[37] Preston, Bruce (2005). “Learning about Monetary Policy Rules when Long-Horizon Expectations Matter” *International Journal of Central Banking* 2: 81–126.

[38] Roos M.W.M. and Schmidt U. (2012), ”The Importance of Time-Series Extrapolation for Macroeconomic Expectations” *German Economic Review*, 13(2): pp. 196–210

[39] Sims, Christopher A., (2003), ”Implications of rational inattention,” *Journal of Monetary Economics*, Elsevier, vol. 50(3), pages 665-690, April.

Appendices

I Tables and Figures

Table I.1: Model (auto)-correlations in the three model versions

Variables	(Auto)-correlations in RE model	(Auto)-correlations in kappa-only model	(Auto)-correlations in full model
$\rho(b_t, b_{t-1})$	0.995	0.90	0.988
$\rho(c_t, c_{t-1})$	0.994	0.76	0.972
$\rho(E_t^b, E_{t-1}^b)$	0.995	0.90	0.988
$\rho(q_t, q_{t-1})$	0.90	0.92	0.986
$\rho(r_t^b, r_{t-1}^b)$	-0.99	0.90	0.988
$\rho(y_t, y_{t-1})$	0.88	0.88	0.88
$\rho(\kappa_t, \kappa_{t-1})$	0.99	0.99	0.99
$\rho(b_t, \kappa_t)$	0.007	0.99	0.99
$\rho(c_t, ms_t)$	0	0.01	0.82
$\rho(b_t, c_t)$	0.998	-0.75	0.99
$\rho(c_t, \kappa_t)$	0.02	0.64	-0.98
$\rho(b_t, q_t)$	0.004	0.92	0.99
$\rho(q_t, \kappa_t)$	0.86	-0.89	-0.99
$\rho(r_t, y_t)$	0.05	0.07	0.36

Note: We compare the correlations in the rational expectations (RE) version, in the first extension, and the complete model. The variables are: b_t bonds, c_t consumption, E_t^b bank equity, q_t land price, r_t^b interest rate on loans, y_t output, κ_t leverage, ms_t market sentiment, r_t (nominal) interest rate.

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Table I.2: Statistical moments in the three model versions

Variables	St. dev. in RE model	St. dev. in kappa-only model	St. dev. in full model	Skewness in full model	Kurtosiss in full model
y_t	19.71	1.77	6.55	0.32	2.89
b_t	24.37	0.84	4.76	-0.09	2.65
c_t	24.38	2.62	9.55	-0.09	2.68
E_t^b	2.19	0.08	0.43	-0.09	2.65
κ_t	2.72	4.83	12.03	0.1	2.68
q_t	2.31	3.76	11.24	-0.06	2.64
ms_t	0	0.01	0.48	0.26	1.11
r_t	2.17	2.17	2.17	0.24	3.11
r_t^b	24.37	0.84	4.76	0.09	2.65

Note: We compare the standard deviations in the rational expectations (RE) version, in the first extension, and the complete model. We also examine the skewness and kurtosis in the complete model version. The variables are: b_t bonds, c_t consumption, E_t^b bank equity, q_t land price, r_t^b interest rate on loans, y_t output, κ_t leverage, ms_t market sentiment, r_t (nominal) interest rate.

Table I.3: Test statistics in the RE model

Variables	Test for Normality 1	Test for Normality 2	Test for Normality 3	Test for Chi-square	Test for t-distr.	Final outcome
c_t	YES	NO	NO	NO	YES	t-distr.
b_t	YES	NO	NO	NO	YES	t-distr.
r_t	NO	NO	NO	NO	YES	t-distr.
κ_t	YES	YES	YES	YES	NO	Gaussian
q_t	NO	YES	NO	NO	NO	Gaussian
r_t^b	YES	NO	NO	NO	YES	t-distr.
r_t^d	NO	NO	NO	NO	YES	t-distr.
r_t^q	NO	NO	NO	YES	YES	t-distr.
E_t^b	YES	NO	NO	NO	YES	t-distr.

Note: We run five statistical tests to deduce the distribution of variables in the model. To test the null-hypothesis that the variables are normally distributed (Gaussian), we run three distinct tests: Jarque-Bera, Lilliefors, and the Kolmogorov-Smirnov tests. In addition we individually test the null-hypothesis that the variables have a Chi-squared distribution, and a t-distribution respectively. NO means that the null hypothesis is rejected at 5 % level and YES that there is not sufficient evidence to reject the null at 5 % significance level. The last column in the table reports the final distribution of each variable after all five tests have been executed. All results hold at 1% significance level.

Table I.4: Test statistics in the kappa-only model

Variables	Test for Normality 1	Test for Normality 2	Test for Normality 3	Test for Chi-square	Test for t-distr.	Final outcome
c_t	YES	YES	NO	YES	NO	Gaussian
b_t	YES	YES	NO	YES	YES	Gaussian
r_t	NO	NO	NO	NO	YES	t-distr.
κ_t	YES	YES	NO	YES	NO	Gaussian
q_t	NO	YES	NO	YES	NO	Chi-squared
r_t^b	YES	YES	NO	YES	NO	Gaussian
r_t^d	NO	NO	NO	NO	YES	t-distr.
r_t^q	NO	NO	NO	YES	NO	Chi-squared
E_t^b	YES	YES	NO	YES	NO	Gaussian

Note: We run five statistical tests to deduce the distribution of variables in the model. To test the null-hypothesis that the variables are normally distributed (Gaussian), we run three distinct tests: Jarque-Bera, Lilliefors, and the Kolmogorov-Smirnov tests. In addition we individually test the null-hypothesis that the variables have a Chi-squared distribution, and a t-distribution respectively. NO means that the null hypothesis is rejected at 5 % level and YES that there is not sufficient evidence to reject the null at 5 % significance level. The last column in the table reports the final distribution of each variable after all five tests have been executed. All results hold at 1% significance level.

Table I.5: Test statistics in the full model

Variables	Test for Normality 1	Test for Normality 2	Test for Normality 3	Test for Chi-square	Test for t-distr.	Final outcome
c_t	NO	NO	NO	NO	NO	Neither
b_t	NO	NO	NO	NO	NO	Neither
r_t	NO	NO	NO	NO	YES	t-distr.
κ_t	NO	NO	NO	NO	NO	Neither
q_t	NO	NO	NO	NO	NO	Neither
r_t^b	NO	NO	NO	NO	NO	Neither
r_t^d	NO	NO	NO	NO	YES	t-distr.
r_t^q	NO	NO	NO	YES	NO	Chi-squared
E_t^b	NO	NO	NO	NO	NO	Neither

Note: We run five statistical tests to deduce the distribution of variables in the model. To test the null-hypothesis that the variables are normally distributed (Gaussian), we run three distinct tests: Jarque-Bera, Lilliefors, and the Kolmogorov-Smirnov tests. In addition we individually test the null-hypothesis that the variables have a Chi-squared distribution, and a t-distribution respectively. NO means that the null hypothesis is rejected at 5 % level and YES that there is not sufficient evidence to reject the null at 5 % significance level. The last column in the table reports the final distribution of each variable after all five tests have been executed. All results hold at 1% significance level.

Table I.6: Model (auto)-correlations comparison

Variables	(Auto)-correlations without macro-pru	(Auto)-correlations with macro-pru
$\rho(b_t, b_{t-1})$	0.988	0.954
$\rho(c_t, c_{t-1})$	0.972	0.836
$\rho(E_t^b, E_{t-1}^b)$	0.988	0.954
$\rho(q_t, q_{t-1})$	0.986	0.952
$\rho(r_t, r_{t-1})$	0.883	0.88
$\rho(\kappa_t, \kappa_{t-1})$	0.99	0.99
$\rho(r_t^b, r_{t-1}^b)$	0.988	0.954
$\rho(b_t, \kappa_t)$	0.99	0.01
$\rho(c_t, as_t)$	0.82	0.76
$\rho(b_t, c_t)$	0.989	0.888
$\rho(c_t, \kappa_t)$	-0.975	-0.01
$\rho(b_t, q_t)$	0.99	0.97
$\rho(q_t, \kappa_t)$	-0.985	-0.01
$\rho(r_t, y_t)$	0.17	0.30
$\rho(c_t, r_t^b)$	-0.99	-0.88
$\rho(q_t, r_t^b)$	-0.99	-0.97

Table I.7: Second and higher moments comparison

Variables	Standard deviations (pre/post)	Skewness (pre/post)	Kurtosis (pre/post)
b_t	4.76/1.46	-0.09/-0.04	2.65/2.72
c_t	9.55/3	-0.09/-0.03	2.68/2.87
E_t^b	0.43/0.13	-0.09/-0.04	2.65/2.72
κ_t	12.03/0.05	0.1/-44.69	2.68/2000
q_t	11.24/5.01	-0.06/0.03	2.64/2.7
as_t	0.48/0.43	0.26/0.60	1.11/1.55
r_t	2.17/2.17	0.24/0.24	3.11/3.11
r_t^b	4.76/1.46	0.09/0.04	2.65/2.72

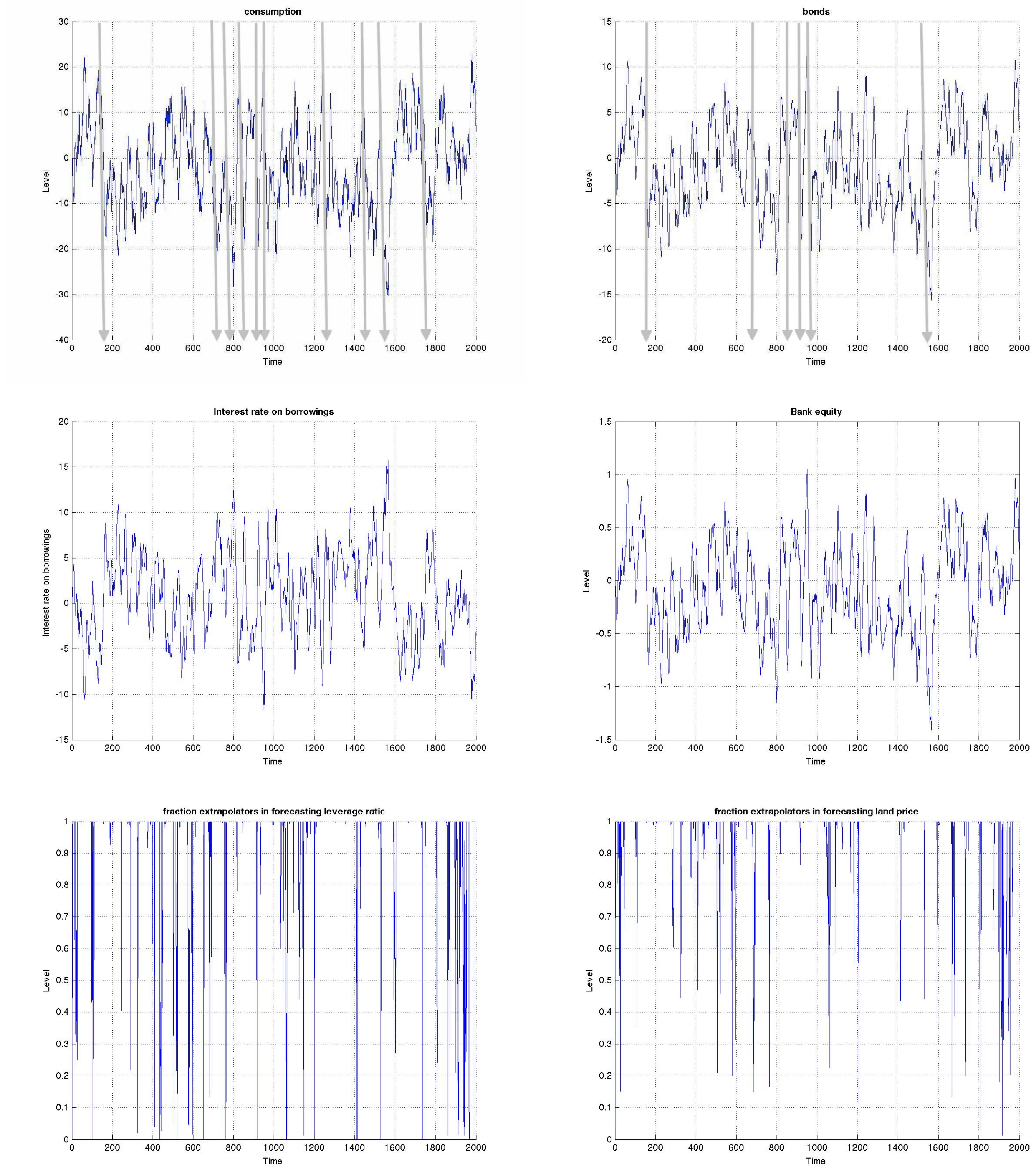

Figure I.1: The figures report the (business cycle) evolution of the model key variables after simulating the model for 2000 periods (or 500 years). From above and left the figures report the evolution of: consumption, bonds, interest rate on borrowings, bank equity, fraction of extrapolators in forecasting leverage, and fraction of extrapolators forecasting land prices. In addition the first two graphs have been marked with arrows that mark systemic crises according to the consumption loss criterion.

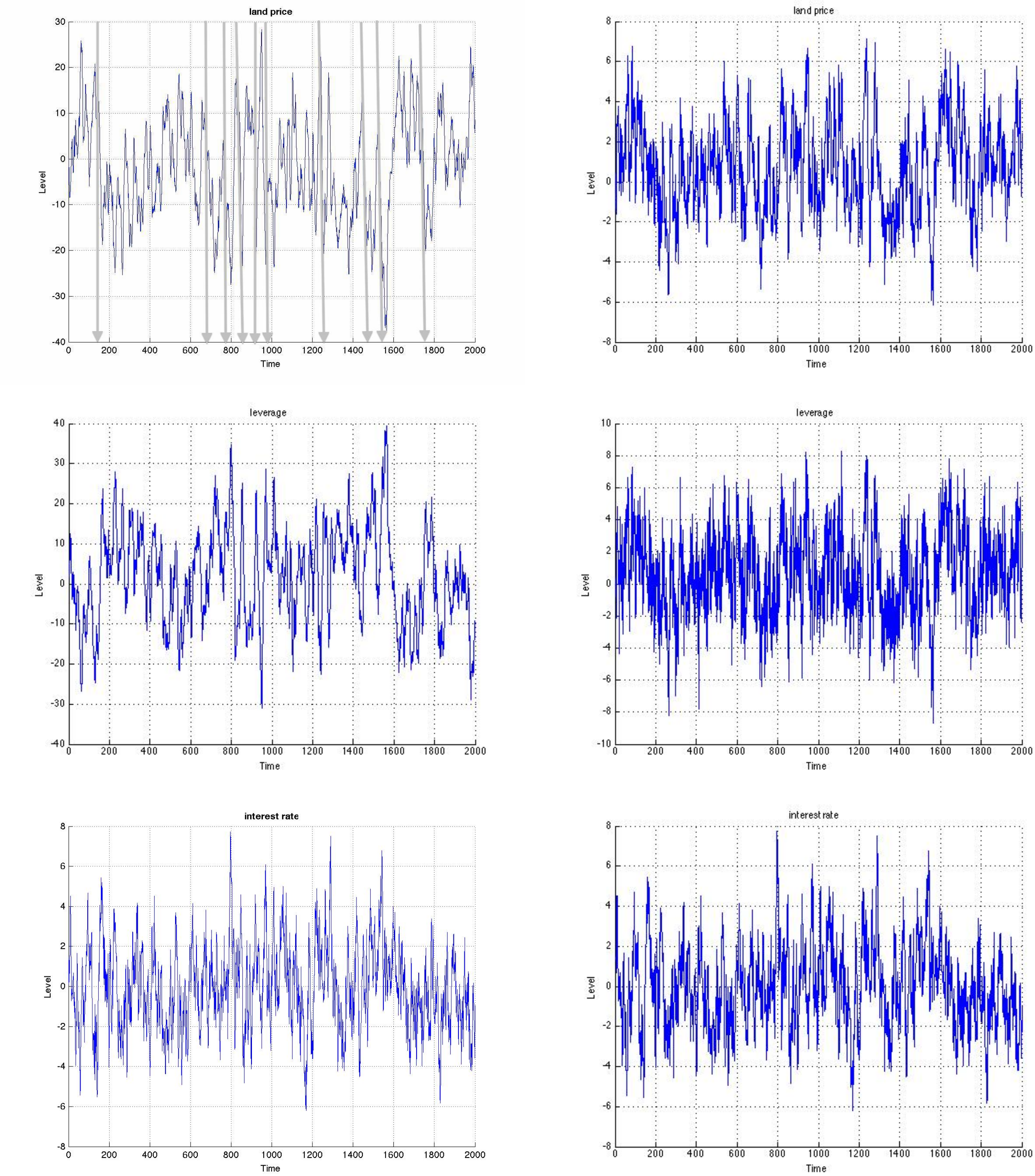

Figure I.2: The figures compare the evolution of three key model variables in the full model (left) and in the rational expectations version (right) after simulating both for 2000 periods (or 500 years). From above, the graphs report the evolutions of: land price, leverage and the interest rate. Note that the vertical scale of the figures on the left and on the right significantly differ.

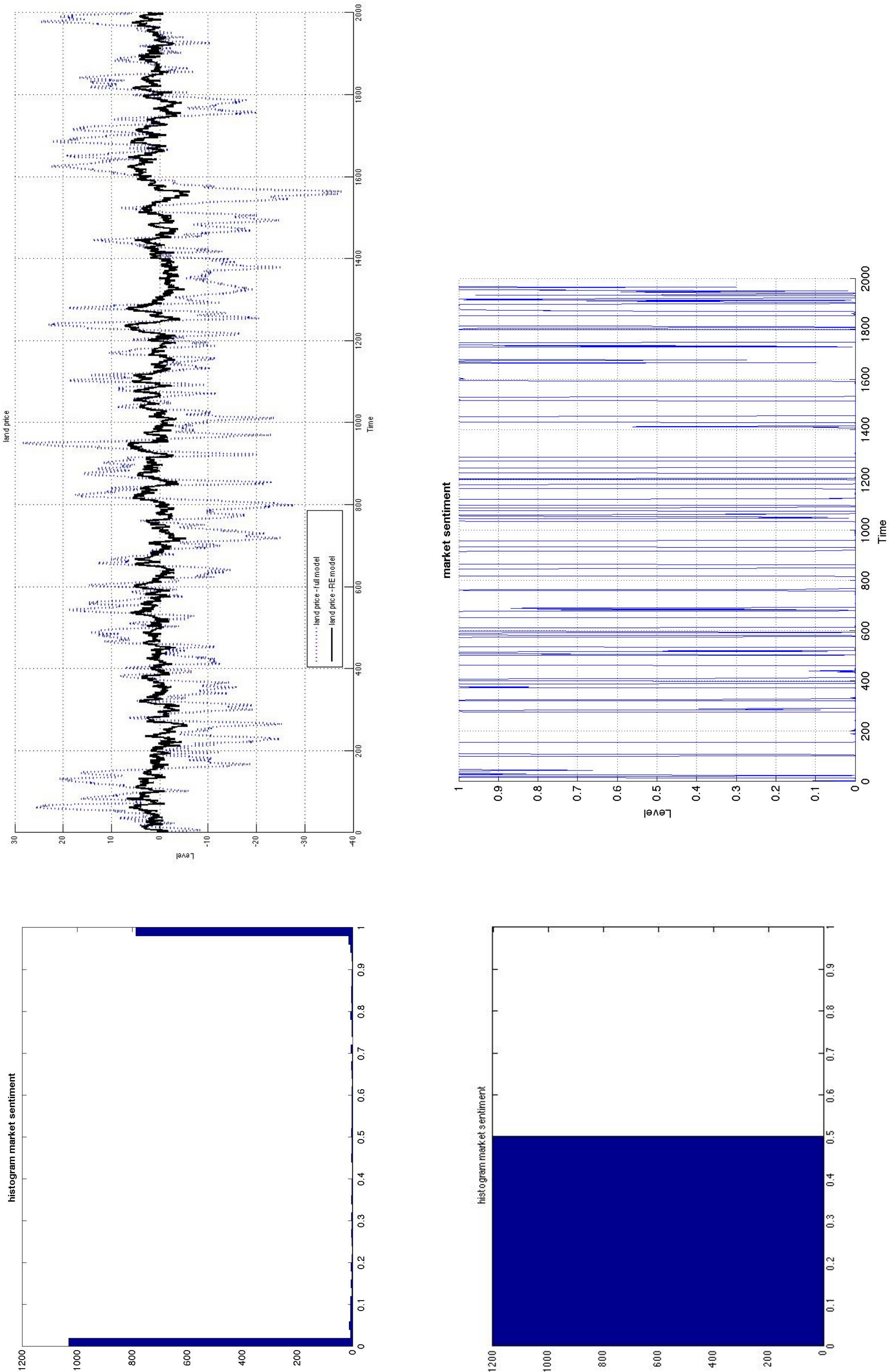

Figure I.3: Measures of market sentiment or optimism/pessimism in the full model. From above and right, the first graph measures the percentage share of agents that believe the price of land will increase in the next period (1), or decrease (0) over 2000 simulated periods. An alternative measure of optimism/pessimism is given in figure 1 on top and left. The dotted blue line is the evolution of land price in the full model, while the black is the evolution of land price in the rational expectations version. The difference between the two represents the expectations of a rise/fall above the fundamental value of land price, and thus optimism/pessimism. The lower graphs depict the distribution of market sentiment in the two models. The figure on the left is for the full model, and the figure ont he right is for the RE version.

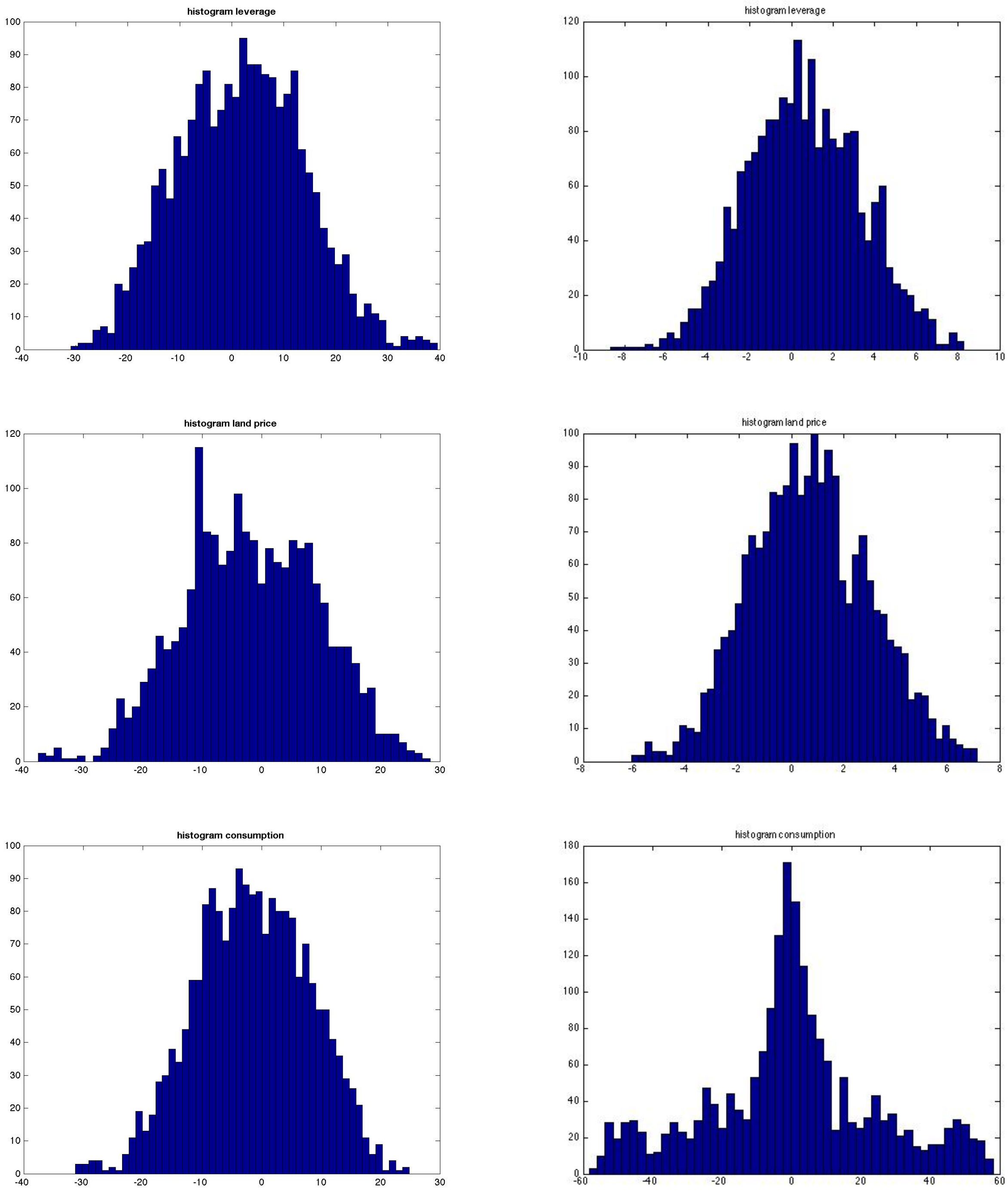

Figure I.4: Ergodic distributions of model variables after simulating the model for 2000 periods. The figures on the left are the distributions for the full model, and the graphs on the right for the rational expectations version. From top-down, we report leverage, land price, and consumption.

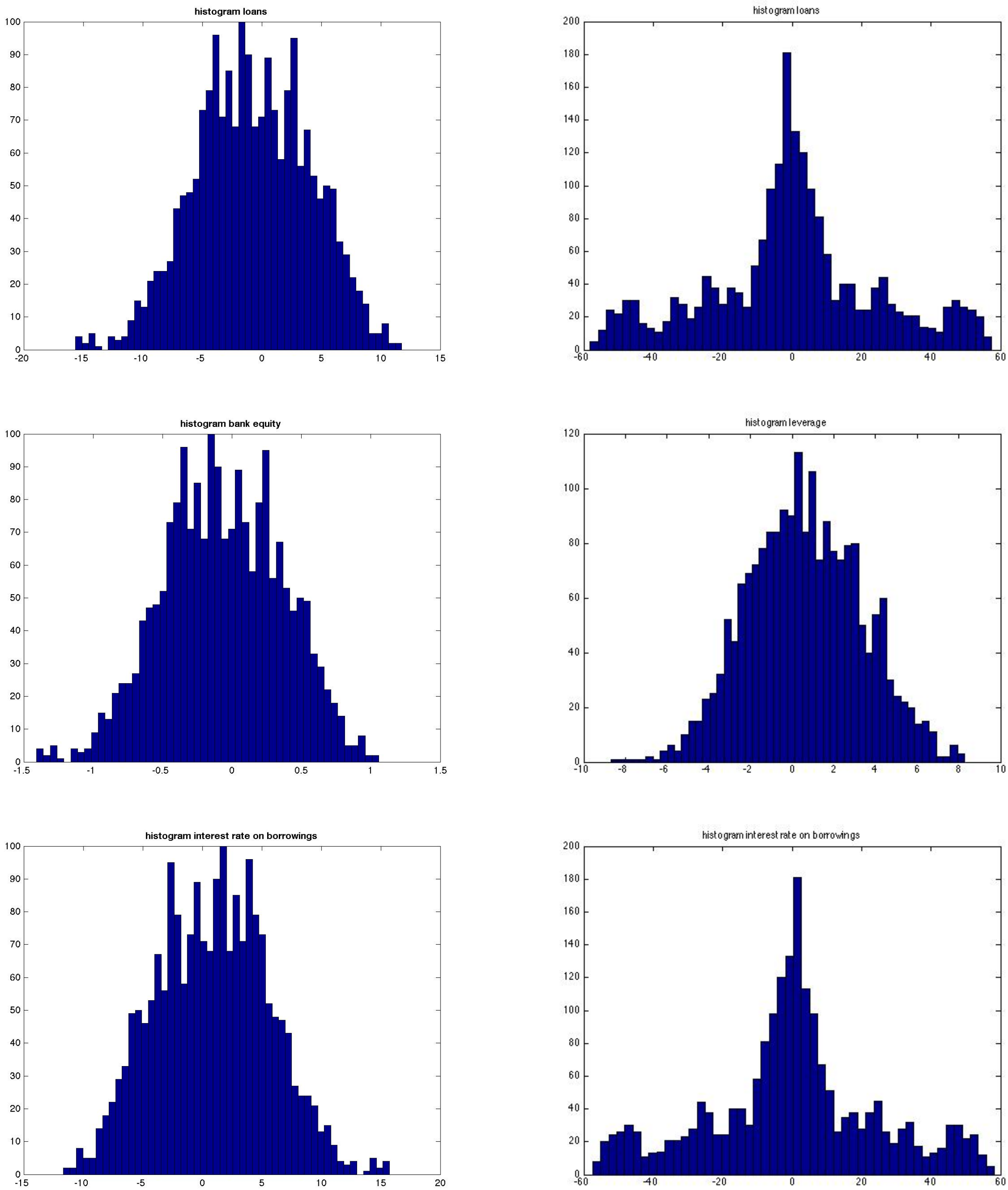

Figure I.5: Ergodic distributions of model variables after simulating the model for 2000 periods. The figures on the left are the distributions for the full model, and the graphs on the right for the rational expectations version. From top-down, we report loans, bank equity, and interest rate on loans.

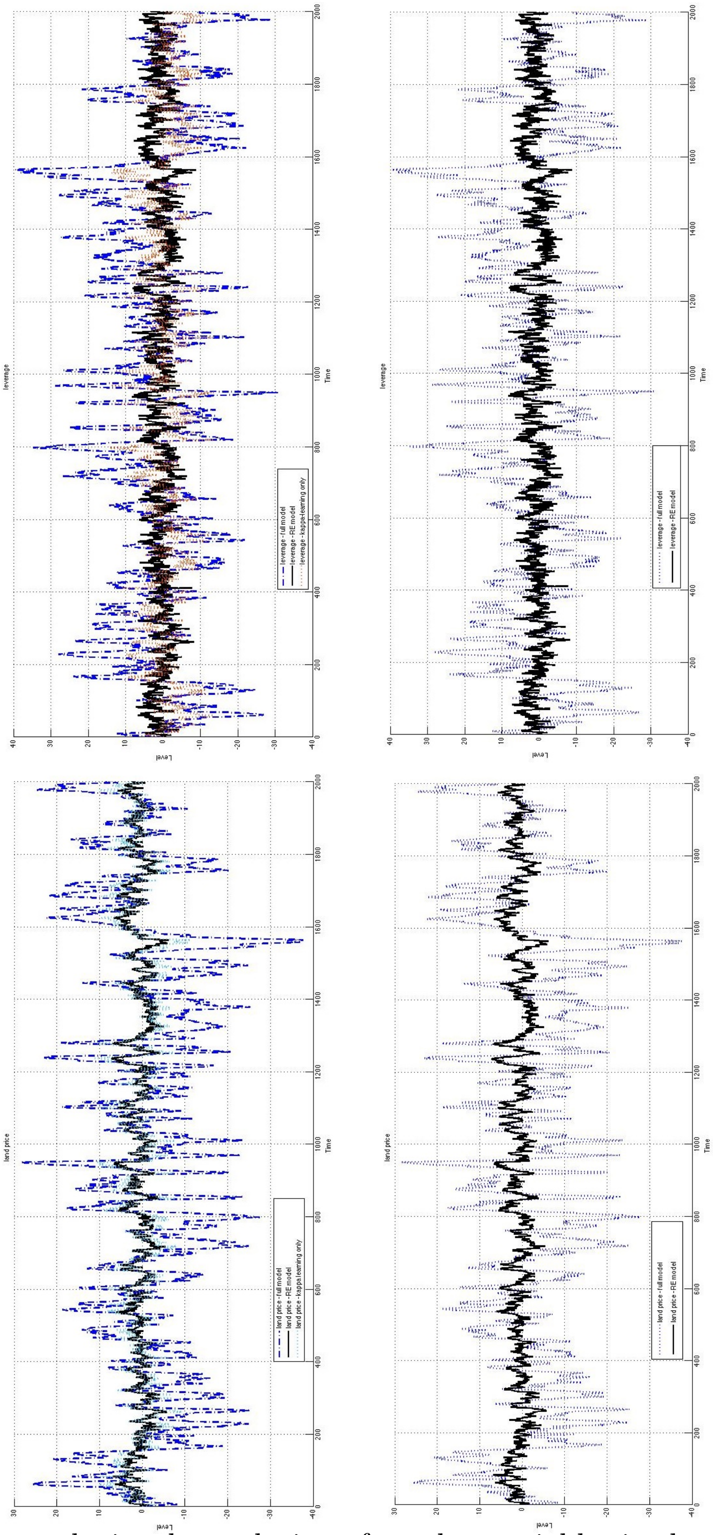

Figure I.6: Figures depict the evolution of two key variables in the distinct versions of the model. On the top, the figure on the left reports the evolution of leverage in the full model (blue dotted lines), model with only learning in LTV (brown dotted lines), and the rational expectations model (black solid lines). Figures on the right reports the same evolution, but excluding the version with only LTV learning. On the bottom of the page, the same information is provided for the land prices. The left figure reports the land price evolution in the full model (dark blue dotted lines), learning in only LTV (light-blue dotted lines), and the RE version (solid black lines).

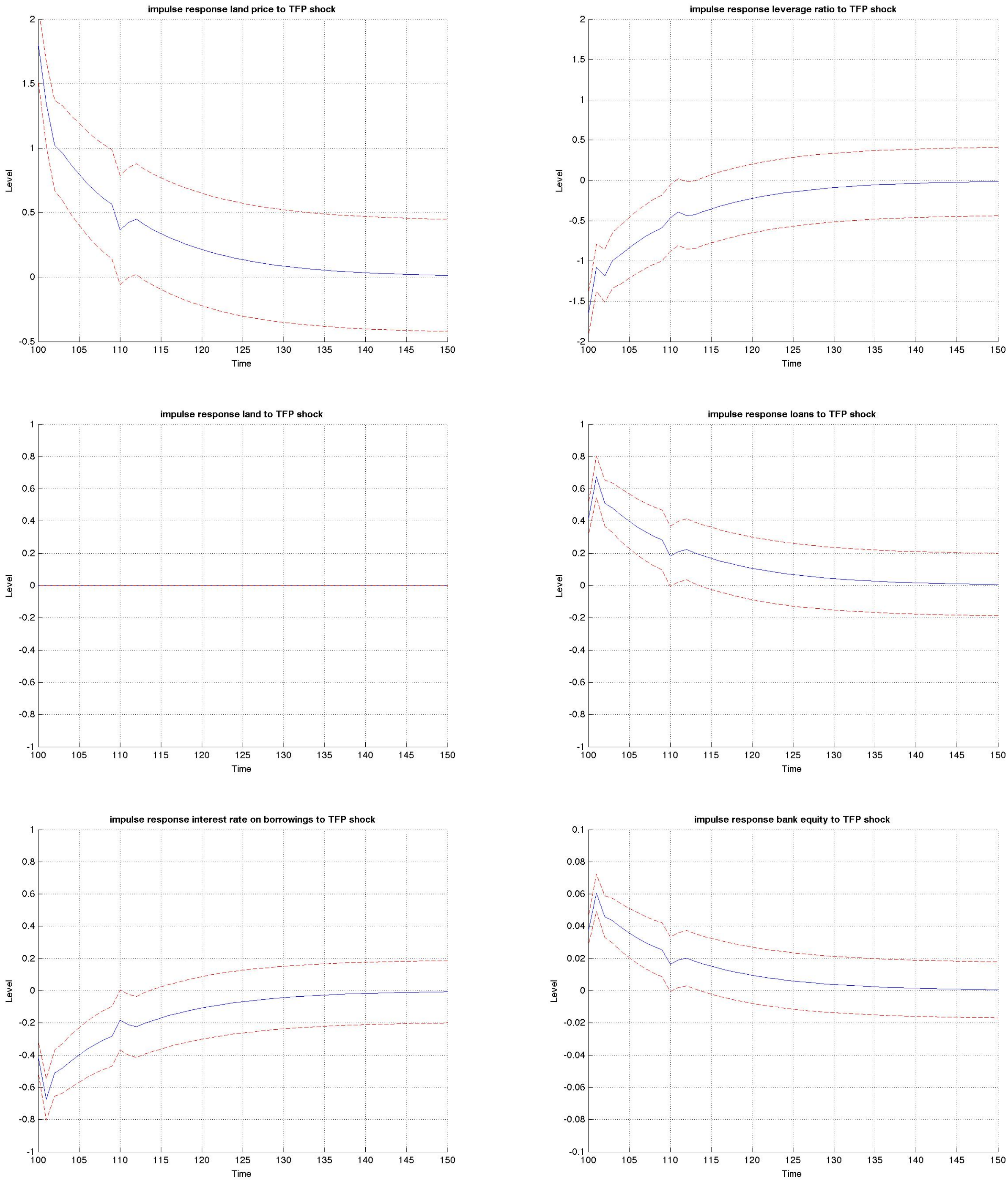

Figure I.7: Impulse responses to a positive TFP shock in t=100. From top-right, the figures report the responses of: land price, leverage, land, loans, interest rate on borrowings, and bank equity.

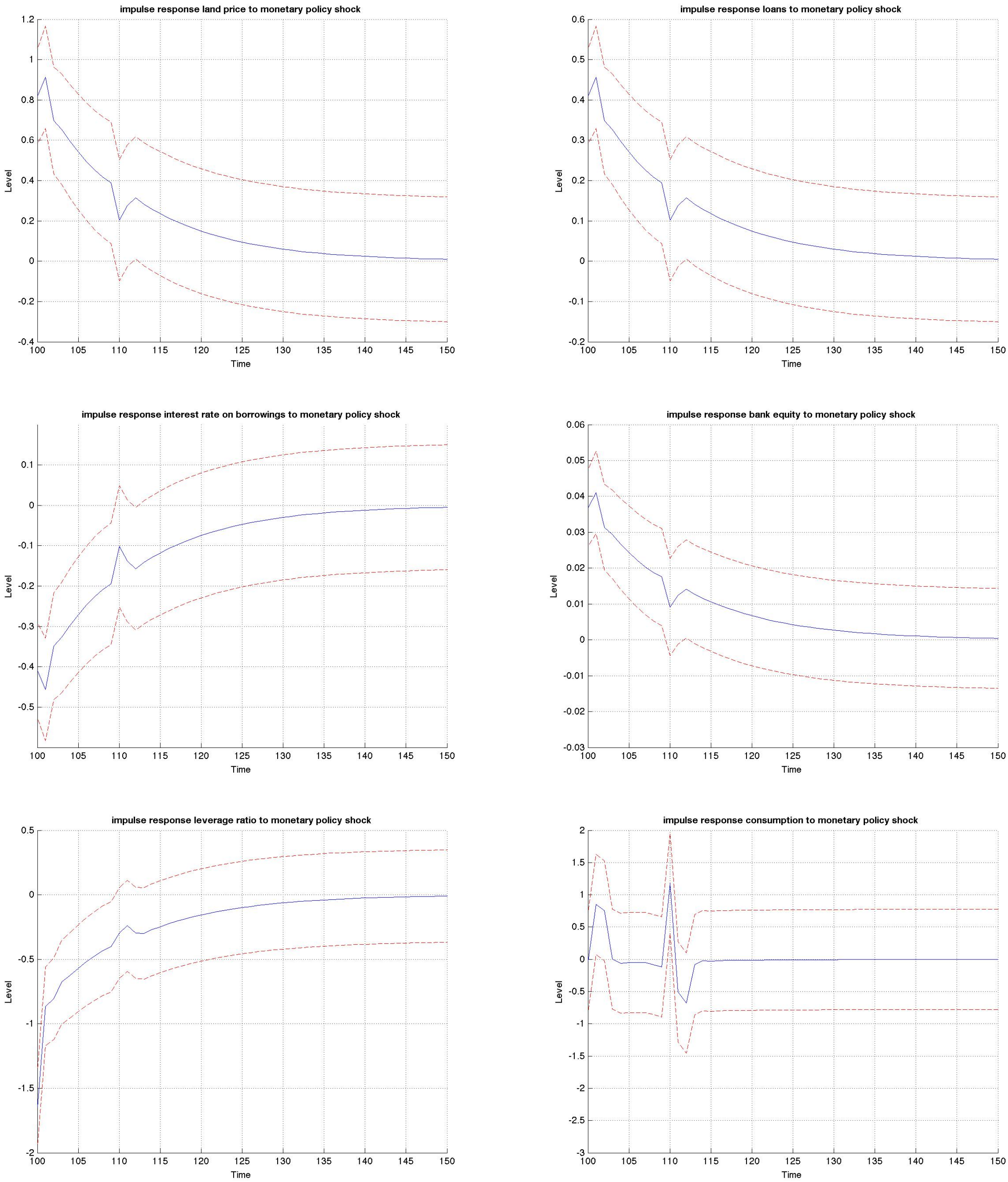

Figure I.8: Impulse responses to an expansionary monetary policy shock in t=100. From top-right, the figures report the responses of: land price, loans, interest rate on borrowings, bank equity, leverage, and consumption.

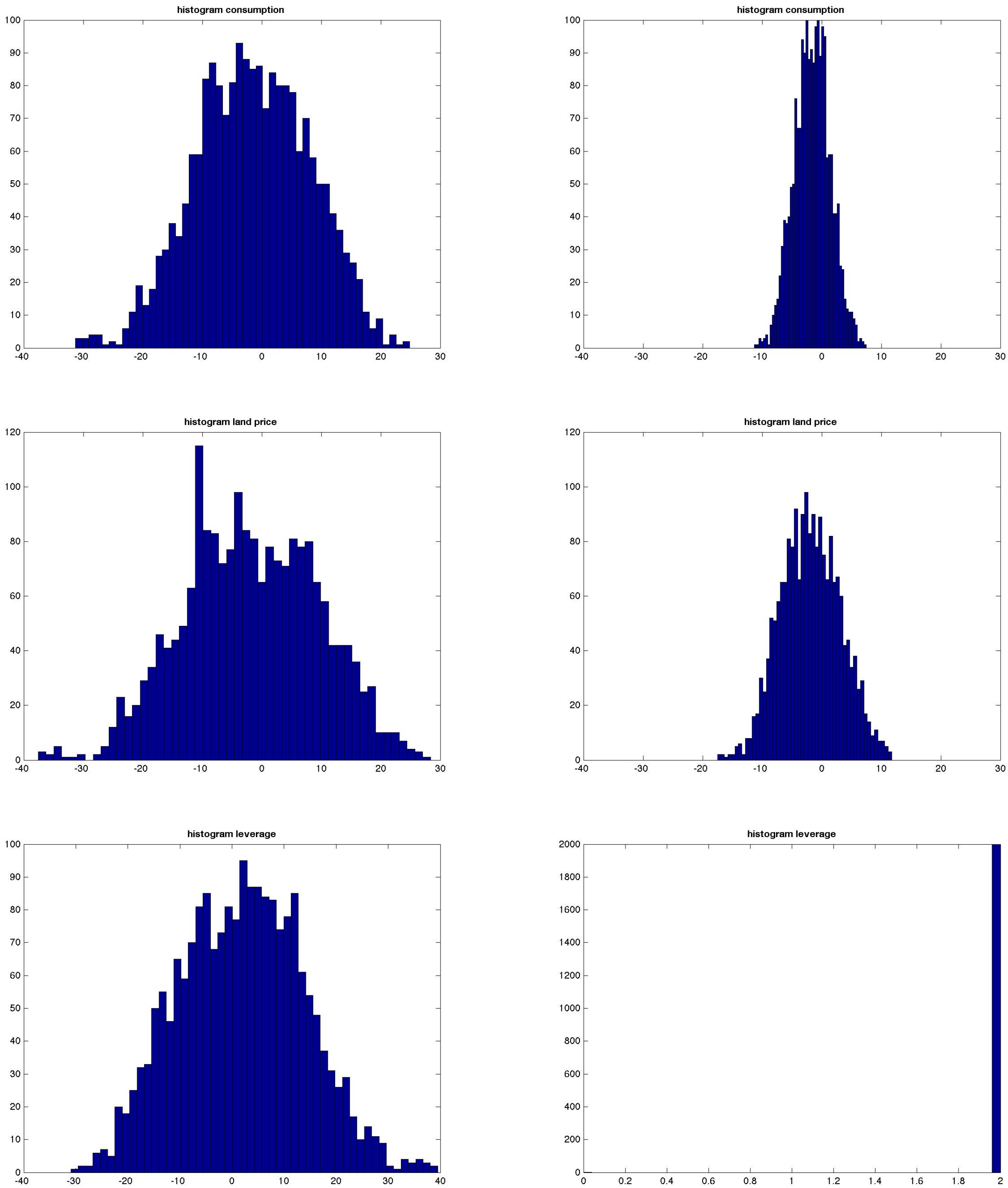

Figure I.9: Comparison of ergodic distributions in the model version without macroprudential policy (left) and with a binding macroprudential policy (right) after simulating both model versions for 2000 periods. From top-down, the figures report the distributions of: consumption, land price, and leverage.

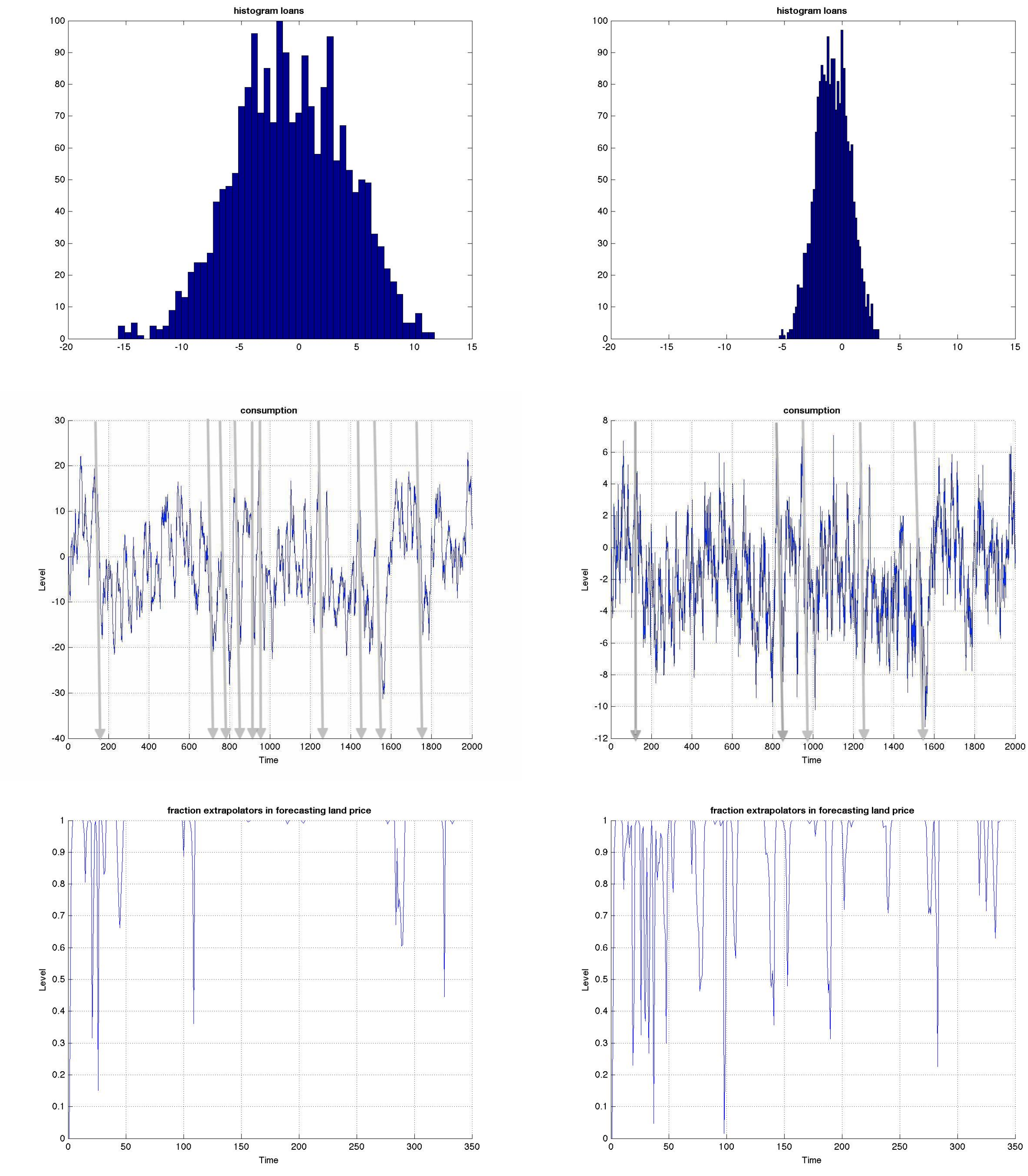

Figure I.10: Comparison of results between the model version without macroprudential policy (left) and with a binding policy (right). The results are obtained after simulating each model version for 2000 periods. From top-down, the figures report the ergodic distribution of consumption, the evolution of consumption over time, and the fraction of extrapolators forecasting land price. Note in the second row of figures that in addition, we mark systemic crises with arrows according to the consumption loss criterion in both versions of the model. Note that in the version with a binding macroprudential policy (left), the number of systemic crises is half compared to the version without the implementation of such policy.

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