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Contents

Abstract

This paper presents a *quarterly estimated structural macroeconomic model for the euro area*, denoted area-wide model (AWM). This model has been developed with four uses in mind: the assessment of economic conditions in the area, macroeconomic forecasting, policy analysis and deepening understanding of the functioning of euro area economy.

Five key features of the model are highlighted. First, it treats the euro area as a single economy. Second, it is a medium sized model which, while detailed enough for most purposes, is nonetheless sufficiently small to be manageable in the context of forecasting and simulation exercises. Third, the model is designed to have a long run equilibrium consistent with classical economic theory, while its short run dynamics are demand driven. Fourth, the current version of the AWM is mostly backward-looking, i.e. expectations are reflected via the inclusion of lagged variables. Finally, the AWM uses quarterly data, allowing for a richer treatment of the dynamics, and is mostly estimated on the basis of historical data (rather than calibrated).

The paper comprises the following elements. First, a general overview of the structure of the model and of its long-run and short-run properties is provided, with particular emphasis on how the model reaches its steady state. This is followed by a review of the key behavioural equations, showing e.g. the extent to which the standard behavioural equations are capable of fitting the historical euro area data which has been constructed. Finally results from two illustrative simulations are provided, i.e. a fiscal expenditure shock and a change in interest rates. Appended to the main text are the full list of econometric results, the detailed description of the database and the results of stochastic long run simulations. In addition, a companion file comprising all of the quarterly time series underlying the AWM is made available.

JEL classification system: C3, C5, E2
Keywords: European Monetary Union, Macroeconometric Modelling, Euro Area

Introduction

Prior to the move to monetary union it was widely recognised that "the ESCB will need to have at its disposal analysis capacities, including a broad range of econometric tools" (EMI, [1997]). It was envisaged that, as it is the case in most central banks, the econometric toolbox would include traditional estimated structural models, smaller scale reduced form models, calibrated theoretical models and various time-series tools such as VARs. In addition, given the specific circumstances of the euro area, the need for both an area-wide as well as cross-country approaches was also recognised.

The present paper presents one element in this toolbox, namely a quarterly structural macroeconomic model for the euro area. This model has been developed with four uses in mind. First, the model can assist in the *assessment of current economic and monetary conditions* in the euro area since it provides a means of assessing the impact of various ongoing developments on the economy. Second, by providing a coherent analytical framework which takes into account the behaviour of economic agents as estimated from historical data, the model is used for producing *forecasts* of future economic conditions in the euro area. Third, the model can be used to assess *effects of policy actions* on the economy ("the transmission mechanism"). Finally, by treating the euro area as a single economy, an area-wide model can help to develop an understanding of how the economy of the area as a whole functions and to focus attention on area-wide conditions. In this regard, given the absence of a well established body of empirical evidence regarding the behaviour of the euro area economy, the estimation of a range of key behavioural equations and the development of the necessary databases can provide a valuable starting point for further empirical analysis.

However, the development of an econometric model for the euro area poses formidable challenges. Even in 'normal' circumstances a number of difficulties arise, since there is, for example, no consensus on the theoretical framework, nor on the empirical methodology. These standard obstacles are supplemented with at least two major problems, which are euro-area specific in some sense. First, the monetary union (MU) area comprises a group of individual countries with – at least to some extent – different historical experiences, different economic structures, different institutional arrangements (e.g. financial systems, wage formation processes, role of governments, etc). Second, since econometric inference depends crucially on the estimation of parameters on the basis of historical data, specific difficulties arise in estimating an area-wide model, to the extent that the move to Stage Three consists of a major structural change in terms of monetary policy but may also lead to changes in other aspects of economic behaviour. There is, therefore, a risk that the equations could be subject to the Lucas [1976] critique. Moreover, there are significant problems in obtaining sufficient spans of historical data for the area. Despite these difficulties, the advantages of developing a tool such as an area-wide model are compelling, although the current version of the model should be seen as a first step in this direction and could be improved in a number of respects. In any case, the model has been found to be extremely useful in a number of practical contexts such as forecasting and simulation. In addition, the AWM is one model in a range of possible tools. Alternative approaches include multi-country models (see, for example, De Bondt et al. [1997] and Deutsche Bundesbank [2000]), very small scale models (such as Coenen and Wieland [2000]) as well as time-series approaches.

The remainder of this paper is structured as follows. Section 1 recalls both the rationale for developing such a model and some of the characteristics of the resulting tool. An overview of the structure of the model is provided in Section 2, which also reviews the specification and estimation results of the equations, which play a major role in the modelled economy. Section 3 focuses on the long run properties of the model and recalls the main adjustment mechanisms at work. In Section 4 the dynamic properties of the model as a whole are described and they are illustrated by variant simulation results. Section 5 concludes.

1 Key Features of the euro area-wide model

The model presented in this paper is characterised by a number of key features which should be highlighted from the start.

First, a unique feature of the model is that it *treats the euro area as a single economy*. Thus, all equations of the model relate to area-wide variables. Consumption in the area as a whole, for example, is expressed as a function of area-wide income and area-wide wealth. Trade, as a further specific feature, is defined in terms of gross flows, including intra-area trade. The model thus extends in a substantial way the tradition of area-wide econometric analysis within Europe, which up to now has been largely confined to studies of area-wide money demand.[1] The advantages of an area-wide model, compared to a multi-country approach, include: the fact that it requires considerably less resources to maintain and to carry out simulation exercises; the fact that it is relatively straightforward to handle and that it can be used to provide direct information concerning the impacts of various shocks on the area as a whole. Furthermore, an area-wide model can deal more appropriately with issues related to the growing integration of member countries. Moreover, treating the euro area as a single economy has the particular advantage in that it can help to underpin an area-wide focus in general economic analysis and policy discussion within the Eurosystem. However, these attractive features come at the cost of particular problems associated with the area-wide approach. These include difficulties regarding the construction of data, potential econometric problems arising from aggregation biases, potential lack of variability in data and the absence of a well established body of empirical results relating to the area per se. In addition, an area-wide model cannot take explicitly into account heterogeneity of behaviour across countries in the area resulting from institutional differences in, for example, housing and labour markets. [2]

Second, a key feature of the AWM is the *level of aggregation*. A decision has been made to opt for a relatively small-scale model which, while giving a reasonable degree of detail on the main components of aggregate demand and prices, is nonetheless sufficiently small to be manageable in the context of forecasting and simulation exercises. This is in line with current practice in academic macroeconomics[3] and increasingly in regard to the modelling practice among central banks in both Europe and in other industrialised countries. The current version of the model thus contains a total of 84 equations of which 15 are estimated behavioural equations. The rationale for opting for this degree of aggregation reflects both practical and conceptual considerations. Firstly, given the difficulties involved in constructing a dataset of area-wide variables, the small-scale approach is unavoidable in an area-wide context. Secondly, in view of the potential changes in economic behaviour that may occur following the move to Stage Three, it may be helpful to identify whether this will affect key parameters and to examine shifts in their estimated values following the regime shift associated with the introduction of the new currency. Such an approach is greatly facilitated by using small-scale models. Thirdly, small-scale models offer the potential advantage that a high degree of theoretical consistency across behavioural equations can be more easily ensured, which, in turn, enables a sharp focus on the issues of interest. It thus facilitates the process of ensuring that the model as a whole has desirable economic properties. As a result, the output of such models should be more readily interpretable in terms of theory. An additional implication of this approach is that it is relatively easy to assess and interpret the impact of choosing specific parameter values on the final outcomes of model simulations. Fourthly, forward-looking behaviour of agents can more readily be dealt with within the context of small-scale models, provided that backward-looking specifications are designed ex ante so as to easily permit forward-looking extensions of the model.

1 See, for example, Browne et al. [1997] for a comprehensive survey, Fagan and Henry [1998] and Coenen and Vega [1999] for recent contributions.

2 See Henry [1999] for a discussion of these issues.

3 Indeed by the standard of the current macroeconomic literature (e.g. Fuhrer and Moore [1995], McCallum and Nelson [1999]), the AWM could be considered as a large-scale model, although this not the case when specifically comparing with models used at central banks (e.g. at the US Fed FRB US, Brayton and Tinsley [1996], or at the Bank of Canada QPM, Coletti et al. [1996]).

A third key feature is the desired *economic properties* of the model. In line with most current mainstream macro models, the AWM has been specified to ensure that a set of structural economic relationships hold in the long run.These relationships are constrained to be consistent with a basic neo-classical steady state, in which in the long-run output is determined by technological progress and the available factors of production. Thus, the long-run of the model has been designed with a view to ensuring that money is both 'neutral' and 'superneutral' with respect to output. In the short-run, however, because of sluggish adjustment of prices and quantities, output is demand determined, but the model is designed to ensure a return to the neo-classical steady state. While the long-run properties are closely linked to the underlying theory, in the current version, the short-run dynamics are not explicitly derived from an optimisation framework, but are instead specified in a more traditional 'ad-hoc' form and estimated on the basis of historical data. The dynamics, however, are constrained by the need to fulfil long-run steady state properties via the use of ECM terms and appropriate homogeneity properties. Finally, another aspect of the current version is that it does not include any equations for 'rest of the world' variables, which are therefore treated as exogenous in simulations.[4]

Fourth, the current version is for the most part a traditional *backward-looking model* in which expectations are treated implicitly by the inclusion of lagged values of the variables in most equations (i.e. adaptive expectations). For the purpose of generating shorter-term forecasts – which are usually produced conditional on exogenous interest rates and exchange rates – such a tool is usually considered adequate. However, for other purposes, including simulation exercises, especially those involving policy changes, or the assessment of alternative policy rules, the backward-looking approach is clearly unsatisfactory and for many variables (especially financial variables such as long-term interest rates and exchange rates) is inherently implausible. In this reported version of the model, the forward-looking elements are limited to financial variables, specifically the exchange rate and the long term interest rate using respectively an Uncovered Interest Parity (UIP) condition and the expectations theory of the term structure. However, the framework employed is flexible enough to permit the introduction of forward-looking behaviour in other blocks of the model, in particular for wages and prices, in a straightforward manner.

Fifth, the model as it stands now does not comprise all of the elements that are necessary to comprehensively describe the *transmission mechanism of monetary policy*. The latter is simply summarised in this model by a direct influence of short-term interest rates on demand components. As a result, a number of standard channels are not explicitly modelled, such as the financial quantity and price channels. For instance there is no explicit role of credit variables in shaping liquidity constraints in the model, nor is there any description of the pass through of the short-term interest rates directly affected by monetary policy decisions to retail rates affecting households and corporate behaviour.[5]

The final feature worth pointing out relates to *the data and empirical* approach that has been followed in the development of the model. Regarding data, a decision has been made to develop a quarterly model. This has the advantage that it allows for a richer treatment of the short-run dynamics of the economy than would be allowed by, say, the use of lower-frequency annual data. This feature particularly enhances its usefulness for forecasting purposes. However, while the situation is improving continuously, severe data availability problems arise with respect to the euro

4 Given the share of the euro area in the global economy, it is likely that shocks to the euro area economy will have some impact on 'foreign variables' and these 'spillovers' are found in many multi-country models to be large in size (see Douven et al. [1997]). The spillovers in turn will imply further impacts on the euro area itself. By treating foreign variables as exogenous, these effects will not be taken into consideration in the AWM. However, it should be noted that the available evidence for the US (see, for example, Fair [1994]) suggests that these additional impacts are relatively small compared to the effect of the initial shock. These impacts could, in principle, be taken into account in simulations by supplementing the AWM with some equations for 'foreign variables'. Some experiments in this regard will be carried out in future work.

5 For an exhaustive description of the various mechanisms at play, see ECB [1999].

area, especially regarding longer spans of data which are necessary for estimation. There are currently no satisfactory databanks with long spans of area-wide time series which can be readily accessed. Thus, the model variables were created from scratch by the ECB staff using a range of national and international sources. The data extends back for most variables to the first quarter of 1970. In order to ensure maximum consistency in the data used across the ECB and within the Eurosystem, the older series have been linked to the series contained in the ECB Monthly Bulletin, where available (further details regarding the dataset are contained in Annex 2). As regards empirical methodology, the approach has, for the most part, been based on estimation of econometric equations on the basis of historical data. In developing econometric tools for a new economic entity such as the euro area, the need for striking the appropriate balance between 'fitting' the historical data, on the one hand, and ensuring that the model as a whole has appropriate economic properties, on the other, is especially acute. In particular, estimation is more delicate and questionable than when developing models for individual countries, so that calibration techniques could play a more prominent role. Calibration, as used e.g. extensively in Black et al. [1994], on the other hand, needs a very comprehensive understanding of the modelled economy, which is of course not yet available at the euro area level. Estimation has therefore been the preferred option, with a view to getting some benchmark initial estimates for key economic behaviour of the area as a whole, on the basis of specifications. The resulting equations are documented in the following section.

2 The estimated equations: summary view, key parameters and dynamic estimates

This section provides an overview of the equations of the model, starting with a summary of the whole model. It also provides information on the most relevant parameters of interest. This overview is followed by a formal presentation of the core dynamic equations. Moreover, a comprehensive listing of the equations is provided in Annex 1 (References to equations below, e.g. [B.2], refer to this annex).

2.1 A Bird's Eye View of the Model

The system reported in Box 1 provides a summary view of the whole model. Although comprising only 17 equations (one half accounting identities, the other half behavioural equations), this small-size system suffices to get a broad idea of the overall structure underlying the whole model.

Such a summary presentation implies a number of simplifications, such as using only two deflators – one domestic, one external – instead of a fully-fledged price system, omitting inventories, no explicit treatment of profits, no transfers, etc. In addition, some restrictions have to be imposed on the dynamics of the estimated equations, e.g. dynamic homogeneity[6] in the wage equation so as to ensure the existence of a vertical Phillips curve.

As shown in Box 1, the supply side of the model comprises a whole economy production function in which output depends on technical progress, the capital stock and the effective labour supply. Employment and investment are determined respectively by conditions derived from the inversion of the production function and profit maximisation, consistently with the assumed technology, under the assumption of competitive markets. The rate of structural unemployment – which together with the actual labour force determines effective labour supply – is an exogenous variable.

6 Dynamic homogeneity is a standard concept, the definition of which can be found e.g. in Jensen [1994].

Box 1

A summary view of the model

Supply side

$K \equiv (1-\delta)K_{-1} + I$	*K* CAPITAL stock accumulation
$I = I(Y, IR)$	*I* INVESTMENT – profit maximisation
$Ypot \equiv (1-\beta)K + \beta \bar{L}^{s} + Trend$	*Ypot* POTENTIAL OUTPUT – Cobb Douglas production function
$Ogap \equiv Y / Ypot$	*Ogap* OUTPUT GAP – goods market disequilibrium
$L = L(Y / Trend, K)$	*L* LABOUR – inverted production function
$W = W(P / Trend, U)$	*W* WAGES – real wage Phillips curve
$U \equiv (\bar{L}^{s} - L) / \bar{L}^{s}$	*U* UNEMPLOYMENT – labour market disequilibrium
$P \equiv P(W / Trend, Ogap)$	*P* PRICES – mark-up on unit labour costs

Demand components (other than investment)

$Y \equiv C + I + \bar{G} + X - M$	*Y* GDP – equal to demand components
$EX = X(\bar{Y}w, P / e\bar{P}_{w})$	*EX* EXPORTS – market shares function of competitiveness
$IM = M(Y, P / e\bar{P}w)$	*IM* IMPORTS – market shares function of competitiveness
$C = C(Y_d, A)$	*C* CONSUMPTION – saving ratio function of the wealth/ income ratio
$Y_d \equiv [W.L + \rho Y](1-t)$	Y_d INCOME – wages and non-wage net of direct taxes
$dY \equiv t.Y - \bar{G}$	*d* DEFICIT in GDP points – taxes minus public consumption
$A - A_{-1} \equiv X - M + dY + I - \delta K_{-1}$	*A* WEALTH – equal to households cumulated real savings

Monetary side

$M^{d} = M^{d}(P.Y, IN)$	M^{d} MONEY DEMAND – function of nominal GDP and nominal interest rate
$IR \equiv IN - \Delta P$	*IR* REAL INTEREST RATE – nominal interest rate minus inflation

Exogenous variables are denoted with a bar; endogenous variables are in capital letters.
$=$ are used for behavioural equations
$\equiv$ are used for accounting identities
Trend is the productivity trend

Prices are determined in the context of the wage–price block in which prices are a function of unit labour costs while wages are determined by a Phillips curve which is vertical in the long run.

Given the sluggishness of price adjustment, actual output is determined in the short-run by aggregate demand. The model contains fairly standard equations for the main components of demand – Private Consumption, Stockbuilding and Exports and Imports – while Government Consumption is exogenous and Investment is determined in the supply-side block.

Finally, the model contains equations for money demand, the exchange rate and long-term interest rates.

2.2 Key Empirical Features of the Estimated Equations

For ease of exposition, the equations listed in Box 1 correspond to the long-run specification that has been retained for the key equations. As to the dynamics of the latter, ECM specifications have been systematically estimated and generally found to fit the euro area data reasonably well over the last 25 years. Some summary features resulting from the estimation conducted, such as key long and short-run elasticities, are reported in Table 1 below, along with the corresponding *t*-ECM statistics. More details on equation dynamics are provided graphically in Annex 1. The reported *t-ECM*'s can be seen as a test for cointegration.[7] In view of the results, it appears that most of the long-run restrictions imposed are roughly consistent with the data used, although, in many cases, the speed of adjustment to equilibrium values is relatively low.

Table 1

Single-Equation Responses of key variables to 10% shocks on major determinants

	Year 1	Year 2	Year 5	Year 10	t-ECM
Employment					-4.7
Output	4.4	8.3	14.2	17.0	
Real wages	-1.8	-1.5	-0.6	-0.1	
Investment					-1.8
Output	10.0	9.9	9.0	6.3	
Real User Cost of Capital(*)	-0.5	-1.7	-5.3	-9.9	
Consumption deflator					-3.0
GDP deflator	6.4	8.7	9.4	9.4	
Import prices	0.8	1.0	0.8	0.6	
GDP deflator					-3.3
Unit Labour Costs	4.3	6.2	7.8	9.2	
Consumption					-3.3
Income	7.7	7.7	7.9	7.9	
Wealth	0.2	0.6	1.3	1.8	
Export Volume					-2.6
World demand	10.0	10.0	10.0	10.0	
Competitiveness	4.8	8.9	8.7	8.0	
Import Volume					-3.1
Domestic demand	19.8	16.8	11.6	10.4	
Competitiveness	-0.3	-1.1	-2.1	-2.6	
Export prices					-3.2
External prices	0.2	0.6	1.6	2.4	
Domestic prices	8.6	9.0	8.1	7.4	
Import prices					-2.0
External prices	2.7	2.6	1.8	1.2	
Domestic prices	4.4	5.3	5.8	6.2	

(*) 100 basis points to the Real interest rate

Bearing in mind the potential occurrence of structural breaks following the move to monetary union, some aspects of the euro area economy that appear in view of the econometric estimates are still worth highlighting. There is e.g. a significant short-run negative impact of real wages on employment, or a relatively high short-run elasticity of consumption with respect to income - which may reflect a still high proportion of liquidity-constrained households. It also appears that the price elasticity of exports is much higher than that of imports, presumably reflecting a quite

7 *As proposed in Banerjee et al. [1998].*

different product composition in both trade flows. Of course such observations should be taken with caution, to the extent that euro area econometric modelling is in its infancy and mostly relies de facto on data prior to monetary union. Given the risk that some of the equations might not be statistically stable, further attention should be paid in the future to the detecting and modelling of structural breaks.[8] This in turn would help to speed up convergence to the implied long-run equilibrium of the model, to the extent that appropriate modelling of the structural changes affecting long-run behaviour would increase the size of the coefficients on the ECM terms.

The actual model, albeit largely based on the above mentioned key mechanisms, comprises of course a much larger set of equations involving a number of specific aspects that appeared crucial in terms of either good estimation or simulation properties. Some further details on the equations entering the various blocks of the model are given below.

2.3 The main equations of the model

2.3.1 The production function and factor demand

The model includes a description of technology in which potential output is assumed to be given by a constant-returns-to-scale Cobb-Douglas production function with calibrated factor share parameters, see (2.1). The parameter β has been set as one minus the average wage income share in the sample, and is thus not estimated.

$$YPOT = TFT\ KSR^{\beta}\ LNN^{1-\beta} \tag{2.1}$$

Trend total factor productivity *TFT* has been estimated within-sample by applying the Hodrick-Prescott filter to the Solow Residual derived from this production function. This production function is used to derive theoretically consistent first-order conditions that enter other equations in the model, e.g. investment. It also provides the measure of potential output, which combined with actual output, determines the output gap.

The factor demand equations of the model – specifically for investment and employment – are specified in such a way as to be consistent in the long run with the underlying theoretical framework of the supply side. This is achieved by means of the inclusion of ECM terms which embody, respectively, the marginal productivity condition for capital and the consistency between employment and the production function (2.1). However, these relations only hold in the long run: in the short run, investment and employment are driven by short-run dynamic factors such as changes in demand.

INVESTMENT

$$\Delta LOG(ITR/YER)=0.18*\Delta LOG(ITR/YER)_{-1}$$
$$+0.53*(\beta*YER/KSR-(STRQ+\delta+0.01))_{-1}+dummies$$

ITR: Total investment
KSR: Total capital stock
STRQ: Real interest rate (quarterly)
YER: GDP real
β: Capital-share parameter in the Cobb-Douglas production function (= 0.41)
δ: depreciation rate of the capital stock (=0.01 per quarter)

8 *Once the functional form of any given behavioural equation is deemed robust enough on the basis of past observations, the model could be adjusted to accommodate for structural change. A number of methods could be used, such as time-varying parameters or non linear transition modelts (cf. Hall [1993] and Granger and Terasvirta [1993], respectively).*

In view of the well-known difficulties in estimating satisfactory aggregate investment equations (see e.g. Chirinko [1993]), the equation has been specified so that investment evolves around the theoretical steady state with the addition of some simple dynamic terms to capture observed short-term effects rather than putting the emphasis on statistical significance of parameters. Investment is consistent with the long-run capital stock determination (as described in the next section) supplemented with some accelerator effect in the short run, with unit elasticity imposed, i.e. a specification in terms of the ratio between investment and output.[9] It should be noted that this equation, via the cost of capital term, provides the main channel by which interest rates affect aggregate demand in the model.

Employment growth in the short run depends on real wages and output growth (both adjusted for trend productivity). In the longer term, in line with a number of models such as Bank of England [2000], employment adjusts to a level implied by the inversion of the production function (2.1).[10] The term DLNSS in the equation is a constant parameter which is set equal to steady state labour force growth. Together with the adjustments to the other variables in the dynamics, this implies a long-run solution of the equation in which employment growth equals labour force growth while the ECM term is zero. The dynamic response of the equation are presented in Annex 1 although, especially in the context of this equation, it is important to stress that these responses are of a partial equilibrium nature.[11]

EMPLOYMENT

$$\Delta LOG(\mathit{LNN})=0.69*DLNNSS$$
$$+0.18*\Delta LOG(\mathit{YEAR})-0.12*\Delta LOG(\mathit{WRNA})-0.13*\Delta LOG(\mathit{WRNA})_{-1}$$
$$-0.081*[LOG(\mathit{LNN})-(LOG(\mathit{YER})-ß*LOG(\mathit{KSR})-LOG(\mathit{TFT}))/(1-ß)]_{-1}+\mathit{dummies}$$

LNN: Total employment (including self-employed)
DLNNSS: a parameter set equal to trend labour force growth
ΔLOG(WRNA): real (product) wage growth minus trend productivity growth
KSR: Total capital stock
ΔLOG(YERA): Real GDP growth minus trend productivity growth
ß: Capital-share parameter in the Cobb-Douglas production function (= 0.41)

9 This restriction is not rejected by formal tests on the unrestricted version of the equation.

10 There are a number of possible ways in which the long-run condition for employment consistent with the theoretical framework of the model could be specified, apart from the inverted production function condition currently used. On the one hand, solving a profit maximisation problem would lead to an expression for the long-run level of employment as a function of output and the real wage. Alternatively, cost-minimisation subject to a given capital stock, would lead to an expression in which long-run employment would be a function of output, technical progress and relative factor prices. It can be easily shown, in the context of the current model as a whole, that each of these formulations leads in the long run to the same level of employment. The decision to adopt an inverse production function approach has been motivated by the superior empirical properties of this approach and by the fact that it is found to yield better simulation properties than the alternatives.

11 Specifically, the coefficient on output in the long run of the equation is 1/(1-*ß*) or approximately 1.7. Interpreted literally, this would imply that in response to a permanent rise in output of 1%, employment would rise by 1.7%. This feature, is more apparent than real, however. Since, in the long run, output is determined by the supply side, a permanent change in output cannot take place unless the other determinants of potential output (such as TFT) change by an appropriate amount. This of course would cancel the impact of YER in the ECM term. In addition, any rise in employment following an increase in output would lead, in the wage-price block, to higher real wages. This would tend to diminish the employment effect via the real wage term in the equation dynamics. That said, it may be the case that the formulation currently employed would lead to a sensitivity of employment to output in the short run which would be excessive compared to the 'stylised business cycle facts'. To assess the extent of this problem, impulse response functions for a bivariate VAR involving output and the estimated equation or alternatively an unrestricted employment equation were compared. It is found that the response of employment to output shocks is actually weaker for the first 6 quarters when the AWM employment equation is used. Thereafter, however, the return to baseline is notably less rapid than in the unrestricted VAR. However, at all horizons, the response of employment falls well within the confidence bounds of the unrestricted VAR impulse responses.

2.3.2 Components of Aggregate Demand

Regarding aggregate demand, expenditure on real GDP is split into six components which are modelled separately:

- private consumption *PCR* [D.3]
- government consumption *GCR* [Exogenous],
- investment *ITR* [B.8, see previous Section]
- inventories *SCR* [D.6]
- exports of goods and services *XTR* [E.3] and
- imports of goods and services *MTR* [E.4]

CONSUMPTION

$$\Delta LOG(PCR)=0.77*\Delta LOG(PYR)$$
$$-0.066*(LOG(PCR)+0.74-0.80*LOG(PYR)-0.199*LOG(WLN/PCD))_{-1}$$

PCR: Real private-sector consumption
YER: Real GDP
PYR: Real household' s disposable income (deflated by *PCD*, the consumption deflator)
WLN: Nominal wealth, defined as the sum of the capital stock, net foreign assets and public debt.

The model's consumption function is fairly standard (see e.g. Muellbauer [1994] for a survey of the currently used specifications and Church et al. [1994] for a review of estimates of such specification involving wealth and income for a number of models for the UK). Private Consumption is a function both of disposable income, comprising compensation, [12] transfers net of taxes and other income, and of wealth, defined as cumulated savings under the assumption that households own all of the assets in the economy (i.e. public debt, net foreign assets, and private capital stock).

INVENTORIES

$$\Delta LOG(LSR)=-0.0016*(LOG(LSR/YER)_{-1}-0.71)+0.0025$$

LSR: inventories
YER: Real GDP

Inventories are modelled in such a way that in the long run the ratio between cumulated inventories and GDP remains constant.

12 Nominal GDP is decomposed on the income side into total compensation *WIN*, indirect taxes *TIN* and Gross Operating Surplus *GON*, the latter being computed as a residual.

EXPORTS

$$\Delta LOG(XTR/YWRX)=0.22+0.16*\Delta LOG(XTR/YWRX)_{-7}$$
$$-0.38*\Delta LOG(XTD/YWDX))_{-1}-0.38*\Delta LOG(XTD/YWDX)_{-3}$$
$$-0.12*LOG(XTR/YWRX)_{-1}-0.098*LOG(XTD/YWDX)_{-1}+0.00099*TIME$$

XTR: Total exports (including both intra- and extra-area trade)
YWR: World GDP
YWD: World GDP Deflator
YWRX: World Demand, Composite indicator
YWDX: World Demand Deflator, Composite indicator
Where:
$LOG(YWRX)=0.4*LOG(YWR)+0.6*LOG(FDD-XTR)$
$LOG(YWDX)=0.4*LOG(YWD*EEN)+0.6*LOG(XTD)$
EEN being the nominal effective exchange rate

Exports and imports comprise both intra- and extra-area flows, thereby equations are not based on consolidated trade, i.e. taking into account only trade with the non-euro area countries. This reflects the current lack of sufficient spans of data on extra-area trade volumes and prices. Trade flows are otherwise modelled in a standard fashion, whereby market shares - in terms of world demand and domestic demand respectively - are a function of a competitiveness indicator involving trade prices, the competitors' index being computed as a weighted average of external and internal prices. In both cases deterministic trends were introduced to ensure cointegration between market shares and the corresponding relative prices. The approach to modelling trade is in line with e.g. Goldstein and Kahn [1985] or the updated review by Sawyer and Sprinkle [1996]. The external indicators for demand and prices as well as the effective exchange rate are based on weighted averages of indicators for the main trade partners of the euro area as a whole (i.e. only involving non-euro area countries).

IMPORTS

$$\Delta LOG(MTR)=-0.16+2.02*\Delta LOG(FDD)$$
$$-0.086*(LOG(MTR/FDD)\ +0.29*LOG(MTD/YED)-0.0034*TIME)_{-1}+dummies$$

MTR: Total imports (including both intra- and extra-area trade)
FDD: Domestic demand
YED: GDP deflator

2.3.3 Prices and Costs

On the price side, the current version of the model contains equations for a number of price and cost indicators. This system of prices has been estimated under the assumption that a form of the law of one price should hold, i.e. imposing static homogeneity on all price equations, which is equivalent to express the long-run ECM component of each of those equations in terms of relative prices only. Specifically, the main equations in the price/cost block are:

- GDP (factor cost) deflator YFD [C.6]
- GDP (Market Prices deflator, i.e. including indirect taxes and subsidies) YED [C.5]
- Average whole-economy earnings WRN [C.4]

- Consumer Expenditure Deflator PCD [C.7]
- HICP [C.9]
- Import and Export Deflators [F1 and F2]
- Investment deflator [C.8]

The key price index used in the model is the deflator for real GDP at factor costs *YFD* (excluding the effect of indirect taxes and subsidies). This key deflator is modelled as a function of trend unit labour costs. In the short-run, import prices also have some effects. The GDP deflator at market prices *YED* in turn is derived by using the accounting identity linking market prices to factor costs and indirect taxes net of subsidies, through an exogenous ratio in GDP terms. Dynamic homogeneity is strongly rejected by the data, which implies that in principle the mark-up in the long run depends on steady state inflation.[13] However, the constant term in the above equation ensures that the long-run real equilibrium of the model coincides with the theoretical steady state. In the short run, the mark-up also depends on the output gap, a feature which increases the response of the nominal side of the model to real shocks.

OUTPUT PRICE/GDP at F.C. DEFLATOR

$$\Delta LOG(YFD)=0.2*(INFT\text{-}DLOG(YFD)_{-1})+0.0039+0.03*LOG(YGAP)_{-1}$$
$$0.23*\Delta LOG(YFD)_{-1}+0.031*\Delta LOG(MTD)_{-1}$$
$$+0.25*\Delta LOG(ULT)+0.084*\Delta LOG(ULT)_{-1}+0.16*\Delta LOG(ULT)_{-2}$$
$$-0.045*LOG((1-\beta)*YFD/ULT)_{-1}$$

YFD: GDP deflator at Factor Cost
YGAP: Output gap
ULT: Trend Unit Labour Costs
TIN/YEN: Indirect Taxes/GDP
INFT: Inflation expectations

In addition, a term in inflation expectations is included in the short-run, the coefficient of which has been calibrated following simulation experiments. This expectational term may be viewed as a proxy for forward-looking behaviour (inflation expectations being set exogenously).[14] The specification employed resembles that used by Gerlach and Svensson [2000], whereby expected inflation is a weighted average of endogenous and exogenous elements. The equation for *YFD* can be rearranged as follows:

$INF = 0.2\ (INFT\text{-}INF_{-1})$ + lagged inflation terms + ECM term
$\Leftrightarrow INF = 0.2\ INFT - 0.2\ INF_{-1}$ + estimated INF
$\Leftrightarrow INF = 0.2\ INFT + 0.8\ INF_{-1}$ + (estimated INF - INF_{-1})

In a forward-looking setting the INFT term can be either the inflation objective of the monetary authority or future inflation, which at steady state should converge to the central bank's objective. In backward-looking mode, the latter is set equal to baseline historical inflation, so that the specification does not affect estimation results - the deviation from expectations term boiling down to zero - but would play a role in variant simulations.

13 See Price [1992] for a similar approach estimating forward-looking price ECM equations under the constraint of dynamic homogeneity, an hypothesis which cannot be rejected using the UK data, contrary to what our findings suggest for the euro area.

14 Accounting for such expectational components is clearly crucial for policy analysis (see e.g. Fuhrer and Moore [1995a] or Clarida et al. [1998]).

WAGE RATE

$$\Delta LOG(WRN/PCD/LPROD)=0.2*(INFT-\Delta LOG(PCD)_{-1})$$
$$+0.27*\Delta LOG(WRN/PCD/LPROD)_{-4}$$
$$-0.92*\Delta^2 LOG(PCD)-0.57*\Delta^2 LOG(PCD)_{-1}$$
$$-0.47*\Delta^2 LOG(PCD)_{-2}-0.33*\Delta^2 LOG(PCD)_{-3}$$
$$-0.56*\Delta^2 LOG(LPROD)-0.46*\Delta^2 LOG(LPROD)_{-1}$$
$$-0.40*\Delta^2 LOG(LPROD)_{-2}-0.26*\Delta^2 LOG(LPROD)_{-3}$$
$$-0.015*LOG(URX/URT)_{-1}+0.10*LOG((1-\beta)*YFD/ULT)_{-1}$$
$$+dummies$$

LPROD: Labour productivity
PCD: Consumption deflator
ULC: Unit Labour Costs
ULT: Trend Unit Labour Costs
URT: Trend unemployment rate
URX: Unemployment rate
WRN: Average compensation per head
WIN: Compensation to Employees
YER: GDP real
YFD: GDP deflator at factor cost
INFT: Inflation expectations

Wages are modelled as a Phillips curve in levels, with wage growth depending on productivity, current and lagged inflation – in terms of consumer prices – and the deviation of unemployment from its structural level (NAIRU). This latter variable is exogenous in the model, although it varies over time in sample, having been estimated using the Gordon [1997] approach. Since dynamic homogeneity holds, the long-run Phillips curve is vertical in the model. The short-run dynamics include a calibrated term in inflation as was the case for the price equation.[15]

The specification of the wage and the key price equations implies that two independent measures of demand can affect inflation.[16] The first factor is standard and appears in the wage equation, through the unemployment gap term. The second factor affects prices and has two aspects. The first is standard, namely the output gap term entering the price equation. The other one is less obvious, albeit present, coming from the fact that inflationary pressures are asymmetric because of the differing measures of productivity involved, namely trend productivity in prices and actual productivity in wages. In the reduced form of the price system this last feature would result in the inclusion of a productivity gap as an additional measure of inflationary pressure, besides the unemployment and output gaps. In addition, the long-run equilibrium for both wages and prices is pinned down by the pre-determined trend real unit labour costs or, equivalently, by the long-run labour share, in turn equal to the labour elasticity (1-ß) in the production function.

15 In case INFT would represent a one-year agead forecast, the calibration used would be consistent with empirical estimates for the US, as documented e.g. in Rudebusch [1999] where forward-looking price-price Phillips curves are estimated. In practice, having expectational terms in both equations is tantamount to having such a term in only one of them, albeit with a higher coefficient. However, in the absence of reliable estimates for such effects in the euro area, it has been deemed appropriate to treat potential effects of expectations on both wage and price formation symmetrically.

16 Both terms have been calibrated, so as to have tensions affecting both prices and wages in a symmetric and balanced manner. The output gap term was borderline-significant but kept in the equation, whereas the estimated Phillips curve impact has been rescaled to half of its point estimate. Without such a calibration, demand shocks would have led to some short-run overreaction of real wages.

There are two equations for consumption prices, one for the National Account deflator *PCD*, another one for the HICP (Harmonised Index of Consumer Prices). The roles played by the corresponding equations are quite different. While the consumption deflator is a key price indicator within the model's accounting framework and has a strong feedback on the model, the HICP is in contrast recursive to the rest of the model. The consumption deflator is a function of the GDP and import deflators supplemented with some transitory effect of commodity prices. The equation for HICP expresses the gap between this variable and the consumption deflator as a function of unit labour costs and the import deflator.

CONSUMPTION DEFLATOR

$$\Delta LOG(PCD)=0.0013+0.19*\Delta LOG(PCD)(-4)+0.45*\Delta LOG(YED)$$
$$+0.23*\Delta LOG(YED)_{-1}+0.07*\Delta LOG(MTD)+0.025*\Delta LOG(MTD)_{-1}$$
$$+0.0045*\Delta LOG(COMPR*EEN)-0.06*(LOG(PCD)-0.94*LOG(YED)$$
$$-\ 0.06*LOG(MTD))_{-1}\ +dummies$$

PCD: Consumption deflator
YED: GDP deflator
MTD: Import deflator
EEN: Nominal effective exchange rate
COMPR: Commodity prices

HICP

$$\Delta LOG(HICP/PCD)=0.36-0.047*LOG(HICP/PCD)_{-1}$$
$$-0.0053*LOG(HICP/MTD)_{-1}-0.027*LOG(HICP/ULC)_{-1}$$
$$+dummies$$

MTD: Imports of Goods and Services Deflator
ULC: Unit Labour costs

On the external side, import prices are a function of, on the one hand, export prices of the euro area itself to account for internal trade and, on the other, of foreign prices and commodity prices (measured by the HWWA index which is a weighted average of oil and non-oil commodity prices), both expressed in domestic currency. Export prices similarly have two components, internal and external, depending on the GDP deflator and foreign prices.

EXPORT DEFLATOR

$$\Delta LOG(XTD)=-0.0045+0.24*\Delta LOG(XTD)_{-1}$$
$$+0.72*\Delta LOG(YED)\ +0.12*\Delta LOG(EEN)+0.22*\Delta LOG(MTD)$$
$$-0.035*(LOG(XTD/YED)*0.7+LOG(XTD/(YWD*EEN))*0.3)_{-1}$$

XTD: Export deflator (total exports, both intra- and extra-area)
YED: GDP deflator
MTD: Import deflator
YWD: Foreign prices
EEN: Nominal effective exchange rate

IMPORT DEFLATOR

$$\Delta LOG(MTD)=-0.051+0.29*\Delta LOG(MTD)_{-1}$$
$$+0.57*\Delta LOG(YWDX)\ +0.099*\Delta LOG(COMPR*EEN)+0.037*\Delta LOG(COMPR*EEN)_{-1}$$
$$-0.044*(LOG(MTD/XTD)*0.65+LOG(MTD/(COMPR*EEN))*0.25\ +LOG(MTD/(EEN*YWD))*0.1)_{-1}$$

COMPR: Commodity prices
EEN: Nominal effective exchange rate
MTD: Import deflator (both extra and intra area)
XTD: Export deflator (id.)
YWD: Foreign deflator
YWDX: World Demand Deflator (both extra and intra area)

2.3.4 Fiscal and external accounts

The modelling of the fiscal block is very simplified, with a limited number of revenue and expenditure categories generally being exogenous - usually in ratios to GDP but real government consumption is exogenous in level terms. However, transfers to households (also in GDP percentage points) are modelled as a function of the unemployment rate on the basis of a calibrated equation, the proportionality coefficient between the two rates having been set to 0.2, which appeared consistent with country estimates. The version used for long-run simulation purposes also incorporates a calibrated fiscal rule in which the direct apparent tax rate – i.e. the ratio between direct taxes paid by households and GDP – is increased in response to the fiscal deficit relative to GDP observed the year before. The rule presumes apparent direct tax rates are changed with a view to reaching some given deficit ratio, namely 10% of the deviations of deficit from the target ratio are absorbed each period. This fiscal reaction is assumed to occur four quarters after the deviation has actually been observed, so as to allow for some inertia in the fiscal response.[17]

As regards the external accounts, the trade balance is given by the equations for trade volumes and prices discussed above [G.1 and G.2 in Annex]. Net factor income (including international transfers) is determined by a calibrated equation linking it to lagged values of the stock of net foreign assets [G.3]. The trade balance and net factor income equal the current account balance [G.4], which in turn is cumulated to give the stock of net foreign assets.

2.3.5 Monetary and financial sector

MONEY DEMAND

$$\Delta LOG(M3R)=-0.739+0.075*\Delta^2 LOG(YER)$$
$$+0.194*(\Delta STN+\Delta STN_{-1})/2-0.359*\Delta LTN_{-1}-0.526*(\Delta INF+\Delta INF_{-1})/2$$
$$-0.136*(LOG(M3R)-1.140*LOG(YER)+0.820*(LTN-STN)+1.462*INF)_{-2}+dummies$$

M3R: real M3
YER: real GDP
STN: short-term (3-month) interest rate
LTN: long-term (10-year) interest rate
INF: consumption deflator inflation, annualised

17 This fiscal rule is only one of the possible modelling approaches to such a necessary closure rule (see e.g. Mitchell et al. [1999] for a comparative analysis of alternative specifications).

Two equations are included in the financial sector: money demand and a yield curve. The money demand [I.1] equation is a fairly standard dynamic ECM equations for the new M3 aggregate[18], which expresses real money balances as a function of real income, short and long-term interest rates and inflation. The yield curve expresses the long-term interest rate in terms of the short-term rate. Two versions of the equation are currently available, a purely backward-looking and a purely forward-looking version.[19]

3 Long-Run Properties of the Model

Assessing the theoretical steady state

As noted above, the AWM is essentially a standard aggregate demand/aggregate supply model. Output is determined by aggregate demand in the short run, where the main components of demand (consumption, investment, net trade etc.) are separately modelled. In the long run, however, the supply curve is vertical with actual and potential output being determined by technology, the labour force and the natural rate of unemployment. In addition, the model has been specified with a view to ensuring that any deviation of output from potential due to either demand or supply shocks sets in train a process of price and wage adjustment eventually returning the economy to a long-run equilibrium which is determined by the model's supply side. In the long run, the price level and the level of nominal wages are determined by the particular nominal anchor used in simulating the model.

3.1 The long-run real equilibrium

The starting point in the specification of the model's supply side is the above mentioned two-factor Cobb-Douglas production function. It is assumed that factor markets are competitive and therefore the following marginal productivity conditions hold in the long run:

$$F'_{KSR}(KSR, LNN) = \beta YER / KSR = (r + \delta + \lambda) \quad (3.1)$$

$$F'_{LNN}(KSR, LNN) = (1 - \beta) YER / LNN = WRN / YFD \quad (3.2)$$

i.e. the marginal product of capital (*KSR*) equals the user cost (comprising the sum of the real interest rate and the depreciation rate plus a risk premium[20]) while the marginal product of labour (*LNN*) is equal to the real product wage, where *WRN* is the whole-economy nominal wage rate and *YFD* the output price in the form of the GDP-at-factor-cost deflator. Therefore (3.1) pins down the steady state capital-output ratio, while (3.2) can be expressed as a labour demand equation or, as done in the model, as an expression for the steady-state real wage consistent with maintaining labour's share in GDP. In addition, in steady state the level of output must be consistent with the production function, which can be re-arranged to yield an expression for employment:

$$LNN = \left(YER \cdot KSR^{\beta} \cdot TFT \right)^{\frac{1}{1-\beta}} \quad (3.3)$$

Last but not least, the model includes an explicit equilibrium unemployment rate to which the observed unemployment rate must converge. Specifically, in the current version of the model, the following assumptions are also made:

18 See Coenen and Vega [1999] for further details.
19 For the forward-looking equation the Fuhrer and Moore [1995] linear approximation has been used.
20 Consistent with the construction of the area-wide capital stock, the depreciation rate is a constant 4% per annum. The size of the risk premium is calibrated to ensure that the marginal productivity condition holds, on average, over the sample 1980-1997.

- The above mentioned natural rate of unemployment (*URT*) is an exogenous variable to the model;
- The labour force (*LFN*) is also exogenously determined;
- Following from the previous two assumptions, the effective labour supply is given by *LNT* = *LFN*(1-*URT*);
- Trend total factor productivity (*TFT*) is given exogenously.

These assumptions, together with equations (3.1) to (3.3), pin down – for a given steady-state real interest rate – the steady state level of output (*YER**), capital stock and the real wage. Substituting for the latter two variables yields an expression for YER* as a function of trend productivity, the effective labour supply, the steady-state real interest rate and the depreciation rate:

$$YER^* = TFT^{1/(1-\beta)}(\beta/(r^* + \delta + \lambda))^{\beta/(1-\beta)} LNT \tag{3.4}$$

In addition, in steady state, the unemployment rate is equal to the natural rate (*URT*), the real product wage is such that the steady state share of labour is consistent with the Cobb-Douglas production function parameter, and the capital output ratio is given by (3.1). Since capital stock adjusts sluggishly to its steady state level, the level of potential output (*YET*) – i.e. the level of output which can be produced at any given point in time *t* by the available factors – will be given by:

$$YET_t = TFT_t KSR_t^{\beta} LNT_t^{(1-\beta)} \tag{3.5}$$

As the capital stock adjusts gradually to its steady-state value, *YET* will converge to *YER**. One important aspect of the links between (3.4) and (3.5) is that they point to two different concepts of "potential output": one that could be termed as a medium-term or business cycle frequency measure of potential output, given by (3.5). The other is a longer horizon concept embodied in (3.4) when the capital stock has fully adjusted to its steady-state level.

The long-run system formed by equations (3.1)-(3.3) plus the condition that unemployment equals the natural rate is embodied in the model in the long-run solution of four equations. These together with (3.5), define a steady state. The marginal productivity condition for capital (3.1) enters the long-run version of the investment equation. The marginal productivity condition for labour (3.2) is incorporated in the wage equation and in the price equation. The production function (inverted as in (3.3)) enters the long run of the employment equation. Finally, the condition that *LNN=LNT* is incorporated in the wage equation. The long-run solution of these four equations are thus given by the theoretical steady state conditions, ensuring that output in the long run is given by the supply side of the model. It is obviously necessary to ensure that observed (i.e., actual as opposed to potential) output enters at least one of these conditions, to bridge the gap between the two measures. This is done by including it in the employment equation, thus ensuring that terminal labour productivity equals its theoretical counterpart. Since this is achieved for an employment level compatible with the NAIRU and labour force, it is logically necessary for terminal observed output and potential to coincide. The wage-price system ensures that terminal real wage is compatible with theoretical marginal productivity.[21] The investment equation ensures that the terminal capital stock will match its long-run counterpart, thus driving potential output to its terminal, interest rate-compatible level.[22]

21 In addition, the wage, output price and factor demand equations incorporate some dynamic homogeneity, to ensure that the resulting long run solution does not depend on arbitrary constants. Without dynamic homogeneity, the steady state of the model, while well defined, would not necessarily correspond to the conditions set out above. In particular, unemployment might not equal URT and steady state output could differ from that given by (3.4) by an arbitrary constant.

22 It does not matter whether observed output or potential enters the investment equation, as this equation sets the very-low-frequency steady state mentioned at the beginning of the section. At these low frequencies, using YER or YET is indifferent.

The precise steady state level of output will depend on the steady-state real interest rate which enters the user cost of capital in (3.1). The steady-state real interest rate is exogenous in the current model and has been calibrated on the basis of an historical average.

In order to complete the real long-run equilibrium it is necessary that the components of aggregate demand, in the long run, sum to *YER** as given in (3.3), which involves some additional hypotheses regarding, e.g. consumption and inventory accumulation behaviour and public finance:

$$YER^* = PCR + GCR + ITR + XTR - MTR + SCR \tag{3.6}$$

where:

- *PCR* real private consumption depends on real income and real wealth, the components of which are public debt, capital stock and net foreign assets *NFA*.
- *GCR* public consumption is exogenously given, assumed to represent a constant share of GDP.
- *ITR* is investment, the dynamics of which is consistent with that of the capital stock *KSR*.
- *XTR* and *MTR* real exports and imports depend on the real exchange rate and demand terms, world demand and GDP respectively.
- *SCR* change in inventories consistent with a constant stock to GDP ratio.

In the case at hand, the equality between demand and supply in (3.6) is achieved by means of a stock-flow interaction delivering an equilibrium value for the real effective exchange rate (*EER**). To see this, it is helpful to go through the various components in (3.6) one by one. The long-run investment to GDP ratio is already determined by the dynamics of the capital stock, i.e. by the supply side. In addition, inventories are simply proportional to GDP while Public Consumption is given exogenously. The two remaining components, namely private consumption and net trade (*XTR-MTR*), should then be consistent with each other, ensuring that (3.6) holds. Since private consumption in GDP terms is proportional to the wealth to GDP ratio, the adding-up constraint on demand components results in a relation linking wealth and net trade. The supply side ensures that the capital stock to GDP ratio reaches a given termnal value. A second additional assumption, i.e. the fiscal rule, implies that taxes levied by the public sector are endogenous so as to lead to a constant debt to GDP ratio. The only free component of wealth is therefore net foreign assets. Defining those as cumulated net trade, the adding-up condition boils down to a dynamic equation for the real exchange rate and, as indicated above, imposing equilibrium between supply and demand yields an equilibrium value for the real effective exchange rate.

Finally, in long-run simulations, care must be taken in specifying paths for exogenous 'rest of world' variables. For the time being, in simulation exercises it is assumed that steady-state real interest rates abroad are equal to that in the euro area and that steady-state output growth is equal in the two areas. This is consistent with a constant steady-state real exchange rate. These assumptions could be easily relaxed by a minor modification of the model.[23]

3.2 Determination of prices in the long run

As already explained, the current version of the model includes equations for a number of price indices. The long run of these equations determine *relative* prices but not the overall level of prices, which is determined by the nominal anchor of the model. Thus, for example, the long run of the equation for the GDP deflator sets the real wage consistent with a stable share of labour[24] while the consumption deflator equation specifies this deflator relative to the GDP deflator and import

23 E.g. by adding an endogenous risk premium term in the exchange rate equation.
24 The wage and price equations set jointly in the long run the real wage and the terminal level of employment.

prices. In order to pin down the long-run level and growth rate of the price system as a whole the model needs to be simulated using some nominal anchor. Technically, a number of possibilities could be employed for this purpose.

First, under strict monetary targeting the long-run price level would be given by the equilibrium condition for the demand for real money balances, given by a money demand function, along with an exogenously fixed nominal money supply. To take a simple example, assume that real money demand depends on real income (with a unit elasticity) and nominal short term interest rates (with a semi elasticity of ϕ), the long-run condition for the price level (*YED**) would be:

$$Ln(YED_t*) = M_t - YER_t * -\phi(r * + g_M - g_{YER})$$

Where the term in parentheses equals the steady-state nominal interest rate ($r*$ is the steady-state real interest rate and g_m and g_{yer} are the steady-state growth rates of the nominal money stock and output, respectively). Since the nominal money stock, which is controlled by monetary policy, pins down the domestic price level in this case, the nominal exchange rate would need to adjust – given exogenous foreign prices – to ensure equilibrium between real aggregate supply and demand.

Second, in case where short term interest rates were to depend on deviations of inflation or the price level from a given central bank's objective, [25] the price level would be pinned down in the long run by the price objective (*YEDT*) so that the following would hold:

$$YED_t* = YEDT_t$$

Again, since the domestic price level is pinned down, the nominal exchange rate would need to adjust – given exogenous foreign prices – to ensure the real equilibrium on the aggregate demand side. In case of an interest rate setting rule which would not explicitly take into account the price level, the terminal price level would depend not only on the inflation objective, but also on initial conditions.

Finally, if the model were run under fixed nominal exchange rates, the terminal price level would be pinned down by foreign prices. Given that the steady-state real exchange rate (*EER**) is pinned down by the real side of the economy as discussed above, the (fixed) exogenous nominal exchange rate (*EEN*) and exogenous foreign prices (*YWD*) would determine the domestic price level consistent with real equilibrium. For the GDP deflator (*YED*), therefore, the long-run price level would be given by:

$$Ln(YED_t) = Ln(EER*) + Ln(EEN) + Ln(YWD_t)$$

Such a configuration is recalled simply for illustration purposes to stress the generality of the approach. Obviously closing the model in this manner would not be appropriate for a large relatively closed economy such as the euro area.

3.3 Adjustment to equilibrium and short-run mechanisms

The theoretical equilibrium described above holds only for the long-run behaviour of the model (see Appendix 3 for some steady state simulations). As to its short-run behaviour, prices and wages do not adjust instantaneously to shocks. Due to this sluggishness in prices, the short-run equilibrium for output is demand-determined. As a result, transitory disequilibria appear in both

25 *See Bryant et al. [1993] for such policy modelling, and some empirical assessment of various types of rules.*

goods and labour markets, namely a deviation of output from potential level as well as a deviation of actual unemployment from the "natural rate".[26] In order to restore equilibrium, a number of mechanisms have to operate. These involve adjustments stemming from disequilibrium terms (from goods and labour markets) entering the price and wage equations as well as policy responses.

The story underlying the adjustment to equilibrium very much depends on the exchange regime and the type of interest rate setting and fiscal rules which are assumed. In the case at hand, the simulations reported below for illustrative purposes have been carried out in an environment where the exchange rate fulfils the UIP condition whereas short-term interest rates are determined by a standard Taylor [1993] rule. Tax rates are adjusted so as to ensure that a targeted deficit to GDP ratio is met. Obviously, because of the UIP condition, this setting is only compatible with forward-looking simulations and therefore the use of special solution techniques to solve the model is needed.[27] It is worth pointing out moreover that the plausibility or policy relevance of those otherwise relatively standard three relationships — i.e., the UIP condition, the calibrated Taylor rule , and the fiscal reaction — is not at stake as such. In fact, these supplementary equations are used primarily because they are necessary elements to close the model as a full system, which would otherwise not converge to some steady-state path.

In such a configuration the main adjustment mechanisms are as follows (taking the example of a positive aggregate demand shock):

- First, the shock mechanically increases output and employment, leading therefore to an increase in inflation via the Phillips curve. This triggers a rise in real short-term interest rates, since both arguments in the Taylor rule are deviating from their equilibrium values. This puts downward pressure on domestic demand, arising from weakening investment and therefore aggregate demand.
- Second, some external channel will operate too, although the impacts remain somewhat limited for a relatively closed economy such as the euro area. In line with the expected change in interest rates, the UIP condition would lead to an initial jump in the nominal exchange rate followed by a sustained but gradual depreciation. There would be ceteris paribus an initial appreciation of the real exchange rate, therefore exerting downward pressures on both prices (via diminished imported inflation) and demand (via lower net trade and also lower net foreign assets).
- Third, this initial nominal and real appreciation is reinforced by further "crowding-out" via an external channel. The additional inflation induces a real appreciation of the exchange rate, which would tend to weaken net trade and, in part, offset the initial increase in output. Moreover, increased demand would boost imports, leading to a further weakening of trade contribution to growth.
- Fourth, the 'automatic stabilisers' of fiscal policy imply in the case at hand that transfers to households should fall on foot of lower unemployment, helping to further dampen the growth of disposable income. In addition, in the case where the shock emanated from a fiscal expansion, the fiscal solvency rule gradually 'kicks-in' and the associated rise in direct taxes also dampens demand.

These adjustment processes would continue until output and inflation rates and growth rates had returned to their baseline values.

26 Obviously, short-term deviation of factor productivities from their steady state value can also occur.
27 The Troll software has been used, using the stacked-time algorithm in forward-looking mode (cf. Juilliard and Laxton [1996]).

4 Some Standard Simulation Results

To get some flavour of the model properties, this section presents two standard simulation exercises. The first one introduces an unexpected and permanent increase in real Government consumption by 1% of GDP, and the second, an unexpected and temporary 100 basis points increase in the short-term interest rate. The first simulation is run over a very long horizon since such a variant typically aims at assessing the extent to which a permanent shock would affect the model's long-run equilibrium. The second simulation, in turn, is analysed only over a shorter horizon, since the experiment conducted assumes that interest rates will remain exogenous, therefore not using the fully-fledged model. Of course a wide range of additional experiments have been conducted so as to assess further the model properties, the choice being made here however to only report in detail those simulations with significant illustrative elements underlying the dynamics of the model.[28]

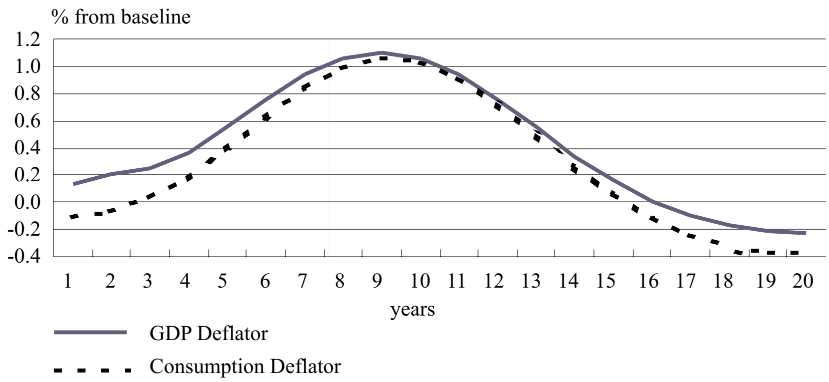
4.1 Shock to Government Consumption by 1% of GDP (ex-ante), permanent

The fiscal shock implemented is a permanent raise in real Government Consumption by 1% of GDP. The shock is a textbook-like test for any macroeconomic model. As documented above, on theoretical grounds, a return to the pre-shock level of activity is expected, to the extent that total supply should not be affected by this shock. An obvious further element worth analysing in the context of such a permanent shock is the speed at which the model goes back to a new equilibrium and the extent to which inflation rises above its steady state level before returning to base.

Prices respond to the expansionary shock quite progressively, see Chart 1. The deflationary impact of the initial appreciation of the exchange rate (3.4 %) counteracts the inflationary effect of additional activity. The increase in demand, however, pushes up both key deflators - consumption and GDP - inflation being higher than baseline for 9 years. After 20 years both inflation and price levels are close to baseline. The final equilibrium reached by the economy following this permanent demand shock implies a real appreciation of the euro of around 2.5%. The latter is needed to ensure a permanent reallocation of supply across demand components which is consistent with a permanently higher GDP share for Government consumption.

Chart 1

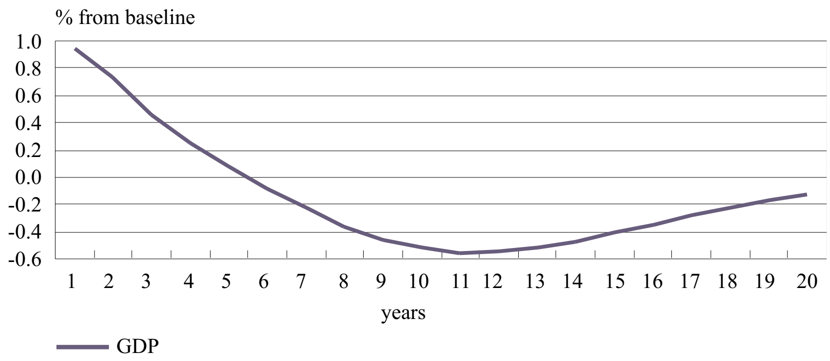
Fiscal shock: impact on the price levels



As to real activity (see Chart 2), the outcome is in line with expectations, the initial expansion is quickly crowded out with the result that the impact on GDP is less than one-to-one at all horizons. The initial expansion of exogenous output results in a rise in employment and lower unemployment, which in turn generates a pick-up in wage growth. This leads to an increase in consumption while accelerator effects boost investment. The deviation of GDP from baseline in the first year amounts however to only 0.9%, i.e. less than the shock itself, with Government consumption accounting for most of the initial reaction. After the initial expansion of output the above mentioned demand dampening mechanisms kick in, as expected. Real interest rates rise exert downward pressure on investment. In addition, the real exchange rate appreciation – induced initially by higher interest rates and later by higher domestic prices – leads to weaker net trade. After the second year, the deterioration of public finance moreover triggers a necessary adjustment of direct taxes to restore fiscal solvency, which in turn dampens consumption. The debt to GDP ratio is higher than baseline for about a dozen years, but this result is of course sensitive to the calibration chosen for the fiscal rule.

Chart 2

Fiscal shock: impact on the GDP level



The output reaction is remarkably smooth in terms of the return to steady state, which is reached after around 20 years. No hump shape is observed in annual terms as the highest impact on activity is reached the first year, a shallow monotonic decline following afterwards. As to the speed of the response, the initial impact takes only 3 years to be halved, and 3 further years to cross again the baseline.

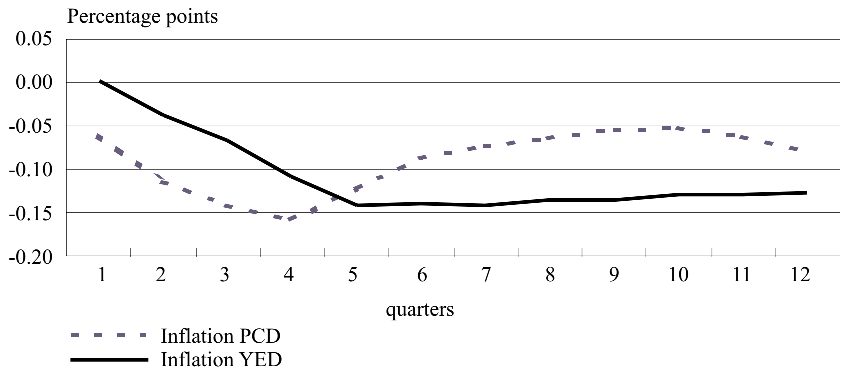
4.2 Interest rate increase of 100 basis point, sustained for two years

Interest rate shocks form also part of the basic tool-kit of the macro-modeller. The simulated shock in the case at hand is an increase of 100 basis points in the short-term interest rate, spanning two years, assuming moreover that the interest rate goes back to baseline after the shock. Such a variant is typically a short-run experiment, which can e.g. be conducted in the context of conditional forecasts, to the extent that one of the key equilibrating mechanisms – the interest rate setting rule – does not intervene over the simulation horizon.

All in all the results point to a somewhat stronger impact of interest rate changes on the euro area economy than what would be suggested by computing the average of the results reported by European central banks for the BIS [1995] exercise on the analysis of monetary policy transmission mechanisms. However, since a number of central banks have changed their models in the meantime, this reference is somewhat outdated. On the basis of the available evidence, it would appear that the AWM response could be more similar to what an updated exercise would presumably deliver.

Chart 3

Interest rate shock: impact on inflation rates

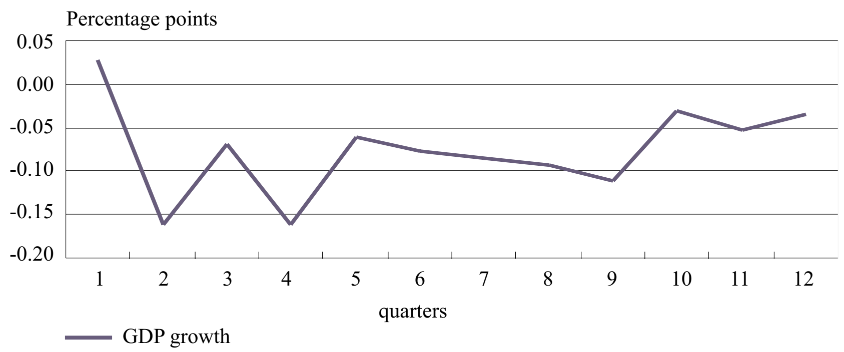


The inflation response (see Chart 3) is a mix of the depressing effect of lower activity and the initial appreciation of the exchange rate. Inflation, on the basis of the consumption deflator, drops immediately by around 0.2 percentage points, mainly because of lower imported inflation, whereas the output gap effect on the GDP deflator is felt only after one quarter. In the ensuing quarters, the initial exchange rate appreciation unwinds completely, as expected to the extent that interest rates are back to their baseline value, so that after a couple of years the gap between the two measures of inflation (consumption deflator and GDP deflator) tends to vanish. On the basis of additional simulations without an endogenous response of the exchange rate, it appears that the exchange rate contributes to two-thirds of the first year effect, this contribution dying out after three years.

As to activity, the outcome of higher interest rates is a lagged and gradual negative impact on GDP growth (see Chart 4), with a maximum deviation from baseline of around 0.15%. As a result, the level of GDP is below its steady state value by around 1 percentage point after 3 years. The main factor underlying these developments is the direct and strongly negative impact of higher interest rates on investment. This effect is supplemented with the consequences of the initial appreciation of the nominal exchange rate (of about 2%, reflecting the changes in short-term rates over the whole horizon). Both trade – via competitiveness – and consumption – through the foreign asset terms entering wealth – are negatively affected by the initial appreciation. The contribution of the exchange rate channel to this pattern for output is of the order of one third and remains more or less stable over the whole simulation horizon, contrary to what has just been mentioned for inflation.

Chart 4

Interest rate shock: impact on GDP growth



5 Conclusions

The work undertaken in the context of the AWM has been doubly fruitful: firstly, and obviously, because of the model that has resulted; but also because of the lessons learned as regards the features and behaviour of the euro area economy. The model and all the accompanying data and software infrastructure have been designed, tested and implemented. The resulting model has been found to be useful for practical purposes, in particular as a tool used in the context of forecasting and simulation exercises.

However, model development is a continuous process and no model can ever be considered to be 'final' in the sense that further improvements could and should not be envisaged. The AWM is no exception to that rule. In the context of the current version of the AWM, at least three further refinements can be identified.

First, with the situation regarding euro area data improving all the time, the model database needs to be continuously developed. A specific requirement will be to bring the database in line with the new ESA 95 data now coming on stream and to therefore re-estimate the model on the basis of this new data.

Second, regarding euro area trade variables, the model follows the Eurostat national accounts concepts in defining trade variables inclusive of intra-area trade. In terms of modelling, it may be preferable to model 'genuine' extra-area trade flows (i.e. excluding intra-area trade) although this involves formidable problems in terms of developing the necessary datasets and ensuring consistency between national accounts and balance of payments concepts.

Third, although the model includes forward-looking elements in a number of equations regarding financial variables, it is arguable that expectations should be incorporated in a wider range of the behavioural equations, particularly to allow for a specific role for expectations in price and wage formation. At present, the current version of the model is a backward-looking system in which activity and inflation follow lagged values of each other. In terms of the real side, a forward-looking investment equation and possibly the incorporation of forward-looking elements into the consumption function would be a useful enhancement of the model. More generally, expectations by themselves are a useful element in most models, although it is not necessarily the case that they should be model-consistent. In fact, survey-based, adaptive or model-consistent expectations can be used alternatively within the same framework, as long as this framework has been set up with explicit separation between expectations and genuine frictions. This would require to replace the particular expectation-formation mechanisms employed by a number of alternative specifications without altering the rest of the model, which is readily feasible within the context of the present model.

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ANNEX 1: Summary of the equations in the Area-Wide Model

This annex contains a detailed listing of the equations used in the model, providing also detailed estimation results. In addition, graphs are presented which show the impact over 50 quarters of a permanent 10% increase in the main explanatory variables for each of the relevant behavioural equations, with a view to providing a quick summary view of the economic properties of the individual equations.

1 Supply side

1.1 Production function [A]

The starting point in the specification of the supply side is an aggregate production function of the Cobb-Douglas functional form with constant returns to scale and neutral technical progress. The Potential Output equation is then defined as:

LOG(YET) = (1-BETA)*LOG(LNT)+BETA*LOG(KSR)(-1)+LOG(TFT) (A.1)

Where LNT is the level of employment consistent with unemployment being at the NAIRU;

LNT = LFN*(1-URT) (A.2)

and KSR is the capital stock;

KSR = (1-DELTA)*KSR(-1)+ITR (A.3)

and TFT is Trend total factor productivity which has been estimated using the Hodrick-Prescott filtered Solow residual.

The Output Gap is then defined as the ratio of Real GDP to Potential Output:

YGA = YER/YET (A.4)

YET: Potential output
BETA: Share of capital in value added, calibrated equal to one minus the average wage share (0.41)
LNT: Level of employment consistent with unemployment being at the NAIRU
KSR: Capital stock
TFT: Trend total factor productivity (Hodrick-Prescott filtered Solow residual)
LNT: Level of employment consistent with unemployment being at the NAIRU
LFN: Labour force
URT: NAIRU/trend unemployment rate (time-varying NAIRU)
DELTA: depreciation rate (1% per quarter)
ITR: Total investment
YGA: Output Gap
YER: Real GDP

1.2 Factor demands [B]

1.2.1 Employment

Employment is modelled conditional on output and the capital stock via an inversion of the production function. Equilibrium labour demand is therefore given by:

LOG(LSL) = (LOG(YER)-BETA*LOG(KSR)(-1)-LOG(TFT))/(1-BETA) (B.1)

The unemployment rate is defined as the number of unemployed divided by the labour force.

URX = UNN/LFN (B.2)

where the number of unemployed is defined as:

UNN = LFN-LNN (B.3)

Apparent productivity is also an important accounting identity, defined as follows:

LPROD = YER/LNN (B.4)

LSL: Long-run Employment (- inverted production function)
YER: real GDP
KSR: Capital Stock
TFT: Trend total factor productivity (Hodrick-Prescott filtered Solow residual)
URX: Unemployment rate
UNN: Number of Unemployed
LFN: Labour Force
LNN: Total employment
LPROD: Apparent labour productivity

The dynamic equation for employment reads:

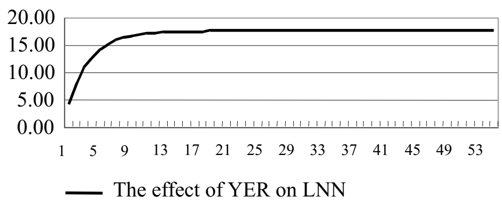
LNN equation: (B.5)

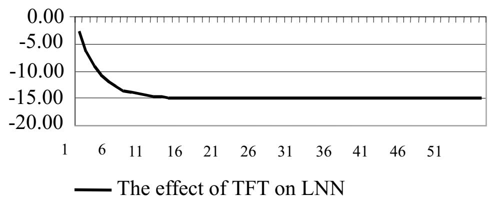
DEL(1: LOG(LNN)) = LNN.DLLNT*DEL(1: LOG(LNT))
+LNN.DLYERADJ*DEL(1: LOG(YER)-LOG(TFT)/(1-BETA))
+(1-LNN.DLLNT-LNN.DLYERADJ)*DEL(1: LOG(YER)-LOG(TFT)/(1-BETA))(-1)
+LNN.DLWRRADJ*DEL(1: LOG(WRN/YFD)-LOG(TFT)/(1-BETA))
+LNN.DLWRRADJ1*DEL(1: LOG(WRN/YFD)-LOG(TFT)/(1-BETA))(-1)
+LNN.D872*D872+LNN.D841*D841
+LNN.ECM*(LOG(LNN)-(-BETA*LOG(KSR(-1))
+LOG(YER)-LOG(TFT))/(1-BETA))(-1).

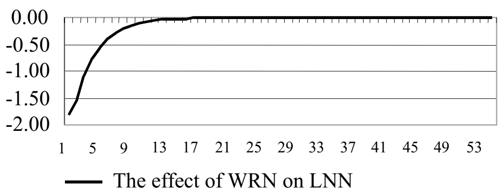
NOB = 76 NOVAR = 7 NCOEF = 7
RANGE: 1979Q1 to 1997Q4
RSQ = 0.92275 CRSQ = 0.916033
F(7/69) = NA PROB>F = NA
SER = 0.001524 SSR = 0.00016
DW(0) = 0.964136 COND = 3.727358
MAX:HAT = 1 RSTUDENT = NA
DFFITS = NA

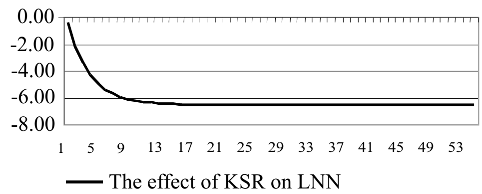
COEF	ESTIMATE	STER	TSTAT	PROB>|T|
LNN.DLLNT	0.692276	0.052972	13.068763	0
LNN.DLYERADJ	0.180177	0.039868	4.519293	2.49932901e-005
LNN.DLWRRADJ	-0.120938	0.036455	-3.317492	0.001452
LNN.DLWRRADJ1	-0.125018	0.038895	-3.214264	0.00199
LNN.D872	0.004483	0.001661	2.698309	0.008755
LNN.D841	-0.004429	0.001668	-2.655412	0.009829
LNN.ECM	-0.081495	0.017514	-4.653129	1.52989143e-005

LSL: Employment necessary to produce observed output, at normal levels of productivity
LNN: Total employment
LPROD: Productivity
YER: Actual output
YFD: GDP at Factor Costs deflator
WRN: Wage rate









1.2.2 Investment

The short-term real interest rate (at a quarterly rate) is defined as

STRQ = (1+STN/100)**0.25-1-INFQ (B.6)

where the nominal interest rate is deflated by the quarterly GDP deflator inflation rate, namely:

INFQ = DEL(4:LOG(YED))/4 (B.7)

Investment is derived from the first-order conditions of the optimisation programme of firms expressed directly in terms of investment's share in GDP, whereas an ECM term ensures that the long-run capital stock/output ratio depends on the real interest rate and the exogenous depreciation rate. The equation has been estimated on a shorter sample than others, since the real interest effect, measured by the ECM term, experiences a structural break in the late seventies.

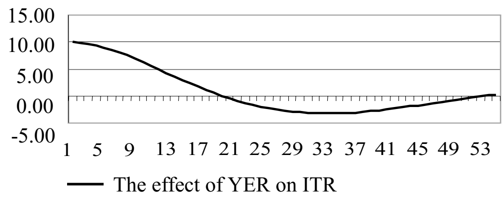
ITR Equation: (B.8)

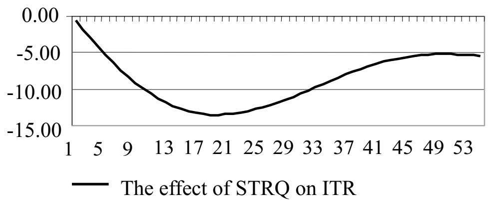
DEL(1: LOG(ITR/YER)) = 1*ITR.D894*D894+ITR.DITY1*DEL(1: LOG(ITR/YER))(-1)
+ITR.ECM*(BETA*YER/KSR-(STRQ+DELTA+ITR.ADJ))(-1)

NOB	= 76	NOVAR	= 3	NCOEF	= 3
RANGE:	1979Q1 to 1997Q4				
RSQ	= 0.111349	CRSQ	= 0.087003		
F(3/73)	= NA	PROB>F	= NA		
SER	= 0.010793	SSR	= 0.008504		
DW(0)	= 2.105432	COND	= 1.13729		
MAX:HAT	= 1	RSTUDENT	= NA		
DFFITS	= NA				

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
ITR.D894	0.020807	0.010856	1.916633	0.059199
ITR.DITY1	0.175554	0.110733	1.585376	0.117204
ITR.ECM	0.534642	0.308028	1.735691	0.086838

INFQ: Quarterly GDP deflator inflation rate
ITR: Total investment
KSR: Total capital stock
STN: Nominal interest rate
STRQ: Real interest rate (quarterly)
YER: GDP real





1.3 Price system [C]

1.3.1 Wages

ULC = WIN/YER (C.1)

ULT = WRN*LNT/YET (C.2)

WIN = WRN*LNN (C.3)

Wages are modelled as a Phillips Curve. The dependent variable is nominal wage growth deflated by consumption deflator minus productivity. The equation has been estimated after having imposed an ECM term similar to that used in the long-run price equation, reflecting wage bargaining. This also contributed to smooth the model response to expansionary shocks, by dampening the inflationary effects.

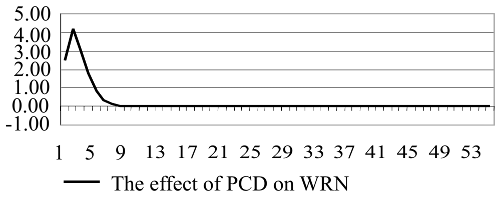
WRN Equation: (C.4)

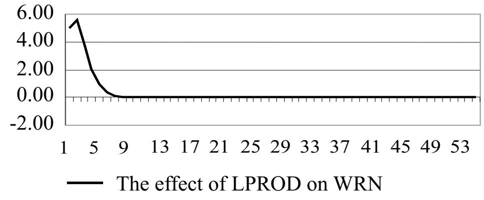
DEL(1: LOG(WRN/PCD/LPROD)) =WRN.DLWRCQ4*DEL(1: LOG(WRN/PCD/LPROD))(-4)
+WRN.DDLPCD*DEL(1: DEL(1: LOG(PCD)))
+WRN.DDLPCD1*DEL(1: DEL(1: LOG(PCD)))(-1)
+WRN.DDLPCD2*DEL(1: DEL(1: LOG(PCD)))(-2)
+WRN.DDLPCD3*DEL(1: DEL(1: LOG(PCD)))(-3)
+WRN.DDLPROD*DEL(1: DEL(1: LOG(LPROD)))
+WRN.DDLPROD1*DEL(1: DEL(1: LOG(LPROD)))(-1)
+WRN.DDLPROD2*DEL(1: DEL(1: LOG(LPROD)))(-2)
+WRN.DDLPROD3*DEL(1: DEL(1: LOG(LPROD)))(-3)
+WRN.DDLPROD4*DEL(1: DEL(1: LOG(LPROD)))(-4)
+WRN.LURX_GAP1*LOG(URX/URT)(-1)
+WRN.I81Q1*I81Q1+WRN.I84Q2*I84Q2+0*WRN.I98Q1*I98Q1
+(WRN.ECM)*LOG((1-BETA)*YFD/ULT)(-1)

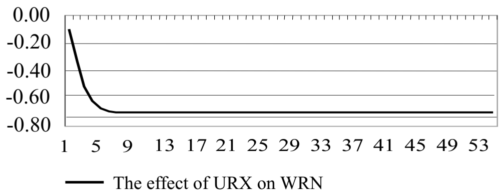
NOB	= 103	NOVAR	= 12	NCOEF	= 12
RANGE: 1972Q2 to 1997Q4					
RSQ	= 0.540956	CRSQ	= 0.485467		
F(12/91)	= NA	PROB>F	= NA		
SER	= 0.005209	SSR	= 0.002469		
DW(0)	= 1.402737	COND	= 4.286072		
MAX:HAT	= 1	RSTUDENT	= NA		
DFFITS	= NA				

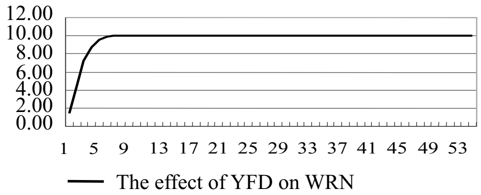
COEF	ESTIMATE	STER	TSTAT	PROB>|T|
WRN.DLWRCQ4	0.273892	0.100629	2.721797	0.007781
WRN.DDLPCD	-0.919736	0.19625	-4.6865599	.69063716e-006
WRN.DDLPCD1	-0.571055	0.201651	-2.831903	0.005696
WRN.DDLPCD2	-0.470158	0.201081	-2.338155	0.021571
WRN.DDLPCD3	-0.334405	0.190128	-1.758848	0.081965
WRN.DDLPROD	-0.562628	0.089468	-6.288594	0
WRN.DDLPROD1	- 0.456273	0.112985	-4.038351	0.000112
WRN.DDLPROD2	-0.399817	0.114714	-3.485339	0.000758
WRN.DDLPROD3	-0.261245	0.097938	-2.66745	0.009047
WRN.LURX_GAP1	-0.014744	0.002981	-4.945681	3.44000553e-006
WRN.I81Q1	-0.004171	0.005353	-0.779112	0.437935
WRN.I84Q2	-0.012449	0.005565	-2.237131	0.02772
WRN.ECM	0.100000			

LPROD: Labour productivity;
PCD: Consumption deflator;
ULC: Unit Labour Costs;
ULT: Trend Unit Labour Costs;
URT: Trend unemployment rate;
URX: Unemployment rate;
WRN: Average compensation per head;
WIN: Compensation to Employees;
YER: Observed output;
YFD: GDP deflator at factor cost.









1.3.2 GDP deflator

The key price index is the GDP at factor cost deflator (YFD). This deflator is the standard GDP deflator net of indirect taxes. The following two accounting identities bridge the gap between nominal GDP at factor costs and real GDP. YFD is multiplied by real GDP to get nominal GDP at factor cost. YFD is assumed to be the relevant price for firms.

YED = YFD/(1-TIN_YEN) (C.5)
YFN = YER*YFD (C.5a)

The estimated equation for the GDP at factor cost deflator involves trend unit labour costs and the import deflator. The model equation comprises moreover an output gap term the coefficient of which has been calibrated to the point estimate value, which is only marginally significant and therefore does not affect other coefficients' values.

YFD equation: (C.6)

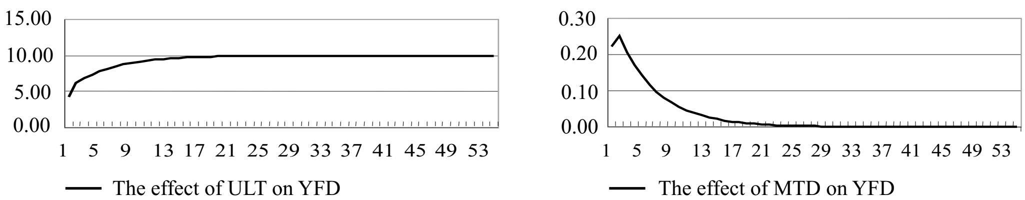
DEL(1: LOG(YFD)) = YFD.CST+YFD.DLYFD1*DEL(1: LOG(YFD))(-1)
+YFD.DLULT*DEL(1: LOG(ULT))+YFD.DLULT1*DEL(1: LOG(ULT))(-1)
+YFD.DLULT2*DEL(1: LOG(ULT))(-2)+YFD.DLMTD1*DEL(1: LOG(MTD))(-1)
+YFD.ECM*LOG((1-BETA)*YFD/ULT)(-1)

NOB = 102 NOVAR = 7 NCOEF = 7
RANGE: 1972Q3 to 1997Q4
RSQ = 0.928133 CRSQ = 0.923594
F(6/95) = 204.480934 PROB>F = 0
SER = 0.002424 SSR = 0.000558
DW(0) = 2.165686 COND = 17.753248
MAX:HAT = 0.366723 RSTUDENT = -3.068543
DFFITS = 0.904421

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
YFD.CST	0.00392	0.000784	5.001247	2.59097347e-006
YFD.DLYFD1	0.229684	0.088518	2.594784	0.010964
YFD.DLULT	0.246086	0.042084	5.847452	0

YFD.DLULT1	0.083541	0.047901	1.744033	0.084388
YFD.DLULT2	0.155765	0.043273	3.599625	0.000509
YFD.DLMTD1	0.031486	0.011367	2.769928	0.006744
YFD.ECM	-0.045006	0.013557	-3.31986	0.001278

YED: GDP deflator
YFD: GDP at Factor Costs deflator
ULT: Trend Unit Labour Costs
TIN_YEN: Indirect Taxes/GDP



1.3.3 Other deflators

The consumption deflator is in the long run as a weighted average of the GDP deflator and the import deflator (6% weight for the latter).

PCD equation, long run: (C.7a)

NOB = 115 NOVAR = 3 NCOEF = 3
RANGE: 1970Q1 to 1998Q3
RSQ = 0.999823 CRSQ = 0.99982
F(2/112) = 316254.142955 PROB>F = 0
SER = 0.007611 SSR = 0.006488
DW(0) = 0.102525 COND = 10.477097
MAX:HAT = 0.052008 RSTUDENT = 2.465406
DFFITS = 0.562798

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
LOG(YED)	0.939862	0.004753	197.759576	0
LOG(MTD)	0.059193	0.00533	11.104903	0
CONST	0.009146	0.000926	9.882009	0

The dynamic consumption deflator equation includes also commodity prices.

PCD dynamic equation (C.7b)

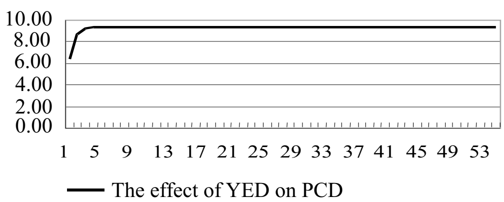
DEL(1: LOG(PCD)) = PCD.CST+PCD.DLPCD4*DEL(1: LOG(PCD))(-4)
+PCD.DLYED*DEL(1: LOG(YED))+PCD.DLYED1*DEL(1: LOG(YED))(-1)
+PCD.DLMTD*DEL(1: LOG(MTD))+PCD.DLMTD1*DEL(1: LOG(MTD))(-1)
+PCD.DLCOMPREEN*DEL(1: LOG(COMPR*EEN))
+PCD.ECM*(LOG(PCD)-PCD.ECM.LYED*LOG(YED)-PCD.ECM.LMTD*LOG(MTD))(-1)
+PCD.DI82Q1*DEL(1:I82Q1)+PCD.DI92Q4*DEL(1: I92Q4)+PCD.I77Q4I78Q1*I77Q4I78Q1

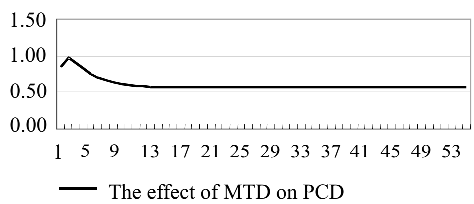
NOB = 110 NOVAR = 11 NCOEF = 11
RANGE: 1971Q2 to 1998Q3
RSQ = 0.976717 CRSQ = 0.974365
F(10/99) = 415.301031 PROB>F = 0
SER = 0.001399 SSR = 0.000194
DW(0) = 2.175761 COND = 17.849981
MAX:HAT = 0.546293 RSTUDENT = -3.62254
DFFITS = -2.914542

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
PCD.CST	0.001263	0.000393	3.212073	0.001778
PCD.DLPCD4	0.188246	0.033897	5.553486	0
PCD.DLYED	0.445458	0.044969	9.905897	0
PCD.DLYED1	0.226345	0.045123	5.016196	2.30840946e-006
PCD.DLMTD	0.071909	0.011663	6.165482	0
PCD.DLMTD1	0.02538	0.009588	2.64711	0.009446
PCD.DLCOMPREEN	0.004465	0.00223	2.002621	0.047953
PCD.ECM	-0.060559	0.020386	-2.970631	0.00373
PCD.DI82Q1	-0.003529	0.001004	-3.513849	0.000667
PCD.DI92Q4	-0.002979	0.000995	-2.992671	0.003492
PCD.I77Q4I78Q1	-0.003981	0.00102	-3.901539	0.000174

PCD: Consumption deflator
YED: GDP deflator
MTD: Import deflator
EEN: Nominal effective exchange rate
COMPR: Commodity prices (weighted average of oil and non-oil commodity prices)

The investment deflator is in the long-run a weighted average of the GDP deflator at factor costs and the import deflator (19% weight for the latter).





ITD equation: (C.8)

DEL(1:LOG(ITD/YFD)) = ITD.CST+ITD.DDLYFD*DEL(1:DEL(1: LOG(YFD)))
+ITD.DDLMTD*DEL(1:DEL(1:LOG(MTD)))+ITD.DLITDYFD1*DEL(1:LOG(ITD/YFD))(-1)
+ITD.DLITDYFD4*DEL(1: LOG(ITD/YFD))(-4)+ITD.DLITDYFD7*DEL(1:LOG(ITD/YFD))(-7)
+ITD.DLITDMTD1*DEL(1: LOG(ITD/MTD))(-1)+ITD.DLITDMTD5*DEL(1:LOG(ITD/MTD))(-5)
+ITD.DLITDMTD6*DEL(1:LOG(ITD/MTD))(-6)+ITD.ECM1*LOG(ITD/YFD)(-1)
+ITD.ECM2*LOG(ITD/MTD)(-1)

NOB	= 104	NOVAR	= 11	NCOEF	= 11	

RANGE: 1972Q1 to 1997Q4

RSQ	= 0.602798	CRSQ	= 0.560088
F(10/93)	= 14.113773	PROB>F	= 0
SER	= 0.002933	SSR	= 0.0008
DW(0)	= 2.03933	COND	= 20.169052
MAX:HAT	= 0.403859	RSTUDENT	= 3.596717
DFFITS	= 1.202549		

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
ITD.CST	0.004648	0.002238	2.07674	0.040584
ITD.DDLYFD	-0.368482	0.093599	-3.93683	0.000159
ITD.DDLMTD	0.111071	0.017597	6.311959	0
ITD.DLITDYFD1	0.313957	0.087306	3.596071	0.00052
ITD.DLITDYFD4	0.207904	0.080864	2.571021	0.011726
ITD.DLITDYFD7	0.16902	0.074912	2.256263	0.026397
ITD.DLITDMTD1	-0.116393	0.018354	-6.341712	0
ITD.DLITDMTD5	0.047888	0.020099	2.38267	0.019222
ITD.DLITDMTD6	0.037784	0.019592	1.928526	0.05684
ITD.ECM1	-0.052662	0.023138	-2.275945	0.025144
ITD.ECM2	-0.012261	0.0053	-2.313417	0.022904

ITD: Investment Deflator
YFD: GDP at Factor Cost deflator

The HICP is expressed as a ratio to the consumption deflator, which is a function of the import deflator and unit labour costs. In the long run, the HICP follows PCD with some differentiated impacts of domestic costs and foreign prices.

HICP equation: (C.9)

DEL(1: LOG(HICP/PCD)) = HICP.CST+HICP.D1*D1+HICP.D11*D1(-1)
+HICP.LHICPPCD*LOG(HICP/PCD)(-1)+HICP.LHICPMTD*LOG(HICP/MTD)(-1)
+HICP.LHICPULC*LOG(HICP/ULC)(-1)

NOB	= 72	NOVAR	= 6	NCOEF	= 6

RANGE: 1980Q1 to 1997Q4

RSQ	= 0.520376	CRSQ	= 0.484041
F(5/66)	= 14.32156	PROB>F	= 0
SER	= 0.001486	SSR	= 0.000146
DW(0)	= 2.206346	COND	= 2835.734012
MAX:HAT	= 0.235792	RSTUDENT	= 3.543209
DFFITS	= 1.440976		

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
HICP.CST	0.366631	0.19212	1.908349	0.060697
HICP.D1	0.003353	0.000432	7.760711	0
HICP.D11	0.000706	0.000435	1.624775	0.108977
HICP.LHICPPCD	-0.047578	0.027072	-1.757423	0.083483

HICP.LHICPMTD	-0.005347	0.004176	-1.28053	0.204841
HICP.LHICPULC	-0.026598	0.014201	-1.872952	0.065505

MTD: Imports of Goods and Services Deflator
ULC: Unit Labour costs

2 Domestic demand [D]

Demand is determined through equations for consumption, investment and stocks.

YER = PCR+GCR+ITR+SCR+XTR-MTR+YER_DIS (D.1)

FDD = PCR+GCR+ITR+SCR+XTR (D.2)

YER = Real GDP
PCR = Real Consumption
GCR = Real Government Consumption
ITR = Real Gross Investment
SCR = Change in Stocks
XTR = Real Exports of Goods and Services
MTR = Real Imports of Goods and Services
YER_DIS = GDP, Statistical Discrepancy
FDD = Final demand

2.1 Consumption

The level of consumption is determined in the long run by the household's disposable income and wealth.

PCR equation: (D.3)

DEL(1: LOG(PCR)) = PCR.DLPYR*DEL(1: LOG(PYR))
+PCR.ECM*(LOG(PCR)-PCR.CST- PCR.LPYR*LOG(PYR)-PCR.LWLR*LOG(WLN/PCD))(-1)

NOB = 72 NOVAR = 2 NCOEF = 2
RANGE: 1980Q1 to 1997Q4
RSQ = 0.729043 CRSQ = 0.725172
F(2/70) = NA PROB>F = NA
SER = 0.004007 SSR = 0.001124
DW(0) = 2.041378 COND = 1.403239
MAX:HAT = 0.135533 RSTUDENT = -3.091043
DFFITS = -0.758347

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
PCR.DLPYR	0.773468	0.067335	11.486938	0
PCR.ECM	-0.066069	0.019728	-3.349068	0.001309

PCR: Real private-sector consumption
YER: Real GDP
PYR: Real household' s disposable income

WLN: Nominal wealth (deflated with PCD, the consumption deflator), defined as the sum of capital stock, net foreign assets and public debt.

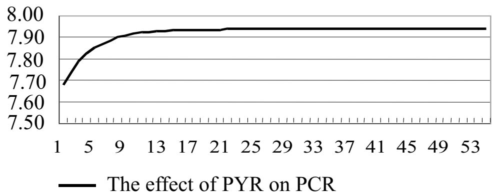
The following identity defines Household's disposable income.

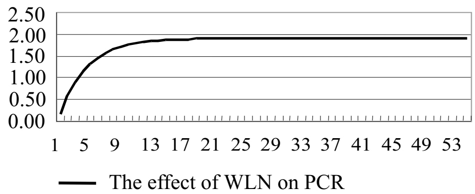
PYN = YFN-(GYN-TIN)+NFN+PYN_DIS (D.4)

Wealth is Net foreign assets plus capital stock plus public debt

WLN = NFA+KSR*ITD+4*GDN (D.5)

GYN: Government disposable income
GDN: Public Debt
TIN: Indirect Taxes
NFN: Net Factor Income from abroad
NFA: Net Foreign Assets
KSR: Whole-Economy Capital Stock
ITD: Gross Investment Deflator
WLN: Nominal wealth





2.2 Stocks

Inventories are primarily related to GDP, cumulated changes in inventories slowly adjusting to a constant ratio in terms of GDP.

LSR equation: (D.6)

LOG(LSR) = LOG(LSR(-1))+LSR.SGA*(LOG(LSR(-1)/YER(-1))-MEAN.LSX)+LSR.CST

NOB = 107 NOVAR = 2 NCOEF = 2
RANGE: 1971Q2 to 1997Q4
RSQ = 0.463321 CRSQ = 0.45821
F(1/105) = 90.647658 PROB>F = 0
SER = 0.022075 SSR = 0.051169
DW(0) = 0.464746 COND = 1
MAX:HAT = 0.096981 RSTUDENT = 3.269056
DFFITS = 0.63103

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
LSR.CST	0.023908	0.002134	11.202638	0
LSR.SGA	-0.044745	0.0047	-9.520906	0

LSR: inventories
YER: Real GDP

3 External side

3.1 Trade Flows [E]

3.1.1 Exports

The composite "world" demand indicator is defined as a weighted average of non-euro area GDP and domestic demand net of exports for the euro area:

LOG(YWRX) = 0.4*LOG(YWR)+0.6*LOG(FDD-XTR) (E.1)

The composite world deflator is in turn a weighted average of non-euro area GDP deflators expressed in euros and the euro area export deflator:

LOG(YWDX) = 0.4*LOG(YWD*EEN)+0.6*LOG(XTD) (E.2)

Such a weighting scheme has been employed, since trade variables that are readily available for the euro area include both intra – and extra-area trade.

The total exports for the euro area have been modelled as market share equations. The export equation is a function of a competitiveness indicator and a time trend.

XTR equation: (E.3)

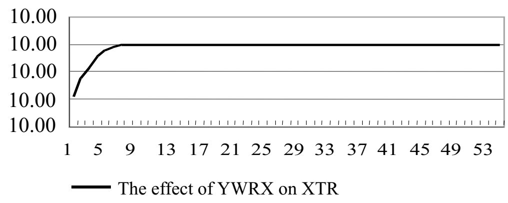
DEL(1:LOG(XTR/YWRX)) = XTR.CST + XTR.DLXTRYWRX7 * DEL(1: LOG(XTR /YWRX))(-7)
+ XTR.DLXTDYWDX1 * DEL(1: LOG(XTD / YWDX))(-1)
+ XTR.DLXTDYWDX1 * DEL(1: LOG(XTD / YWDX))(-3)
+ XTR.ECM * LOG(XTR / YWRX)(-1) + XTR.LXTDYWDX1 * LOG(XTD / YWDX)(-1)
+ XTR.TIME * TIME

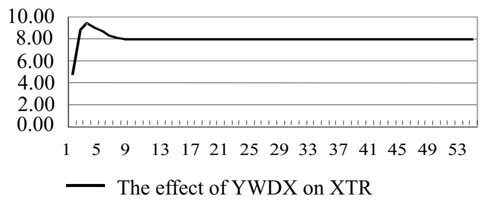
NOB	= 105	NOVAR	= 6	NCOEF	= 6
RANGE: 1972Q1 to 1998Q1					
RSQ	= 0.212883	CRSQ	= 0.173129		
F(5/99)	= 5.355081	PROB>F	= 0.000244		
SER	= 0.01602	SSR	= 0.025408		
DW(0)	= 2.062157	COND	= 172.611179		
MAX:HAT	= 0.196282	RSTUDENT	= 3.5386		
DFFITS	= -0.788606				

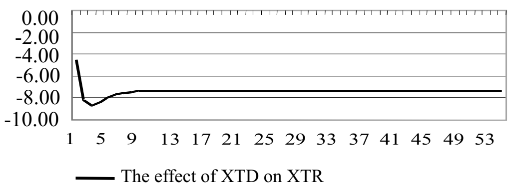
COEF	ESTIMATE	STER	TSTAT	PROB>|T|
XTR.CST	0.222518	0.086634	2.568484	0.011708
XTR.DLXTRYWRX7	0.15641	0.092638	1.688395	0.094483
XTR.DLXTDYWDX1	-0.377152	0.11565	-3.261156	0.001522
XTR.ECM	-0.121453	0.046452	-2.614577	0.010329
XTR.LXTDYWDX1	-0.097625	0.051067	-1.911678	0.058809
XTR.TIME	0.000989	0.000358	2.762421	0.006841

XTR: Total exports (including both intra- and extra-area trade)
YWR: World GDP
YWD: World GDP Deflator
YWRX: World Demand, Composite indicator
YWDX: World Demand Deflator, Composite indicator

EER: Real effective exchange rate (in terms of YED, YWD)
TBR: Real Trade Balance







3.1.2 Imports

The import equation is a function of domestic demand (intra-area) and a competitiveness term (MTD/YED), also expressed in market shares in the long run.

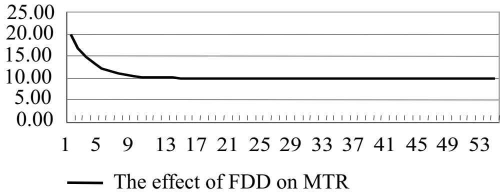
MTR equation: (E.4)

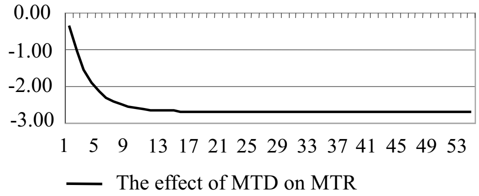
DEL(1: LOG(MTR)) = MTR.CST + MTR.DLFDD * DEL(1: LOG(FDD))
+ MTR.ECM *(LOG(MTR/FDD) - MTR.ECM.LMTDYED * LOG(MTD/YED) - MTR.ECM.TIME *TIME)(-1)
+ MTR.D743 * D743

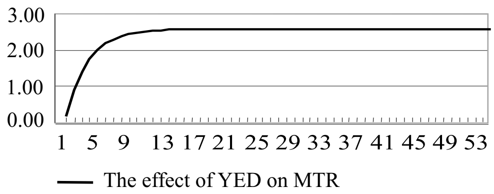
NOB = 110 NOVAR = 4 NCOEF = 4
RANGE: 1970Q3 to 1997Q4
RSQ = 0.854893 CRSQ = 0.850786
F(3/106) = 208.165421 PROB>F = 0
SER = 0.007488 SSR = 0.005943
DW(0) = 2.451185 COND = 158.574914
MAX:HAT = 1 RSTUDENT = NA
DFFITS = NA

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
MTR.CST	-0.158258	0.049683	-3.185365	0.001899
MTR.DLFDD	2.021422	0.092272	21.907282	0
MTR.ECM	-0.086131	0.027692	-3.110299	0.002401
MTR.D743	0.016717	0.007763	2.153454	0.03355

MTR: Total imports (including both intra- and extra-area trade)
FDD: Domestic, intra-area demand
YED: GDP deflator







3.2 Trade Deflators [F]

3.2.1 Export deflator

The export deflator depends in the long-run on the GDP deflator and on foreign prices (extra area). Static homogeneity has been imposed.

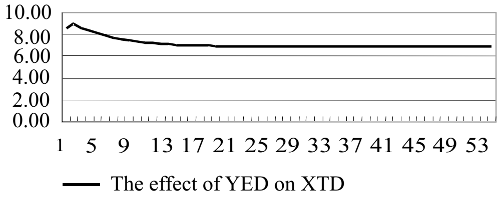
Equation XTD: (F.1)

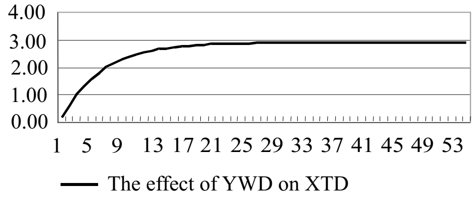
DEL(1:LOG(XTD))=XTD.DLXTD1*DEL(1: LOG(XTD))(-1)+XTD.DLYED*DEL(1:LOG(YED))
+XTD.DLEEN*DEL(1:LOG(EEN))+XTD.DLMTD*DEL(1: LOG(MTD))(-1)
+XTD.ECM*(LOG(XTD/YED)(-1)*0.7+LOG(XTD/(YWD*EEN))(-1)*0.3)
+XTD.CST

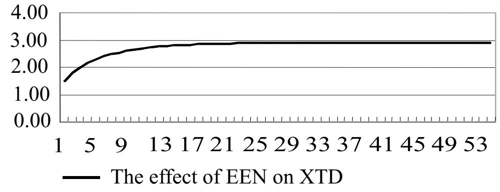
NOB = 110	NOVAR = 6	NCOEF = 6	
RANGE: 1970Q3 to 1997Q4			
RSQ = 0.791345	CRSQ = 0.781313		
F(5/104) = 78.886056	PROB>F = 0		
SER = 0.006958	SSR = 0.005036		
DW(0) = 1.646781	COND = 9.043161		
MAX:HAT = 0.345361	RSTUDENT = 3.537549		
DFFITS = 1.5291			

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
XTD.CST	-0.004465	0.001865	-2.394802	0.018419
XTD.DLXTD1	0.236281	0.122578	1.9276	0.056633
XTD.DLYED	0.720601	0.136161	5.292258	0
XTD.DLEEN	0.119249	0.026206	4.550374	1.45639525e-005
XTD.DLMTD	0.21966	0.060945	3.604217	0.000482
XTD.ECM	-0.034589	0.013469	-2.568083	0.011647

XTD: Export deflator (total exports, both intra- and extra-area)
YED: GDP deflator
MTD: Import deflator
YWD: Foreign prices
EEN: Nominal effective exchange rate







3.2.2 Import deflator

The import deflator depends in the long-run on the export deflator, commodity prices and foreign prices (extra area), with static homogeneity imposed.

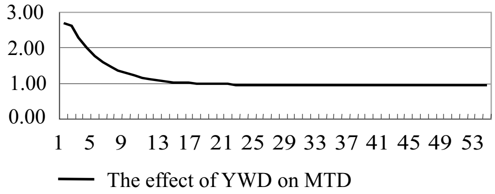
MTD Equation (F.2)

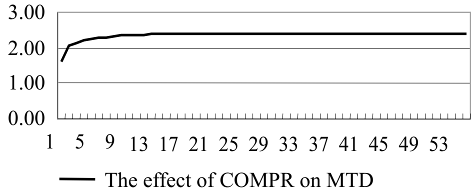
DEL(1: LOG(MTD)) = MTD.CST + MTD.DLMTD1 * DEL(1: LOG(MTD))(-1)
+ MTD.DLYWDX * DEL(1: LOG(YWDX))+ MTD.DLCOMPR * DEL(1:LOG(COMPR*EEN))
+ MTD.DLCOMPR1 * DEL(1: LOG(COMPR*EEN))(-1)
+ MTD.ECM *(LOG(MTD/XTD)(-1)* 0.65 + LOG(MTD/(COMPR*EEN))(-1) * 0.25
+ LOG(MTD/(EEN*YWD)) * 0.1)

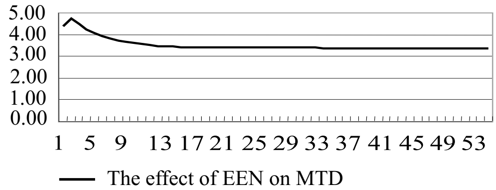
NOB	= 110	NOVAR	= 6	NCOEF	= 6
RANGE: 1970Q3 to 1997Q4					
RSQ	= 0.87139	CRSQ	= 0.865207		
F(5/104)	= 140.929787	PROB>F	= 0		
SER	= 0.009278	SSR	= 0.008952		
DW(0)	= 1.74746	COND	= 66.563925		
MAX:HAT	= 0.421287	RSTUDENT	= 4.183781		
DFFITS	= 2.994305				

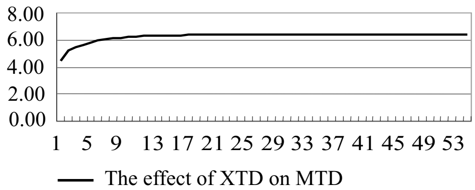
COEF	ESTIMATE	STER	TSTAT	PROB>|T|
MTD.CST	-0.050886	0.022765	-2.235232	0.02754
MTD.DLMTD1	0.288208	0.059129	4.874261	3.93313874e-006
MTD.DLYWDX	0.572207	0.076659	7.464298	0
MTD.DLCOMPR	0.099	0.011301	8.760518	0
MTD.DLCOMPR1	0.036614	0.013645	2.683334	0.008482
MTD.ECM	-0.044124	0.01958	-2.253504	0.026326

COMPR: Commodity prices
EEN: Nominal effective exchange rate
MTD: Import deflator (both extra and intra area)
XTD: Export deflator (id.)
YWD: Foreign deflator
YWDX: World Demand Deflator (both extra and intra area)









3.3 Trade and Current Account Balance [G]

TBR = XTR – MTR (G.1)
TBN = XTR*XTD – MTR*MTD (G.2)
NFN = Rho*NFA(-1) (G.3)
CAN = TBN+NFN (G.4)

TBR: Real Trade Balance
CAN: Current Account Balance
TBN: Nominal Trade Balance
NFN: Net Factor Income
NFA: Net foreign asset
Rho: Rate of return on foreign assets (calibrated)

4 Fiscal and Monetary Side

4.1 Fiscal Variables [H]

The Public Debt is calculated from Government Net Lending which is the difference between Government Savings and Public Investment. The fiscal block of the model just comprises a set of identities in expenditure and revenue categories (expressed as ratios to GDP).

GDN = GDN(-1) – GLN/4+GDN_DIS (H.1)

GLN = GSN-GIN+GLN_DIS (H.2)

GDN: Public Debt
GIN: Public Investment
GLN: Government Net Lending
GSN: Government savings

Transfers (as a percent of GDP) are modelled as a function of the unemployment rate.

TRN Equation (H.3)
TRN/YEN = 0.2*URX+TRN.CST

TRN.CST = 0.2
TRN: Transfers to households
URX: Unemployment rate
YEN: Nominal GDP

Transfers are counter-cyclical, increasing with the unemployment rate.

Government consumption is exogenous in real terms. The direct tax rate is determined by a fiscal solvency rule (see J.2).

4.2 Monetary and Financial Variables [I]

4.2.1 Money demand

M3 equation: (I.1)

The equation for M3 is taken from Coenen and Vega [1999].

$$\underset{}{\Delta(M3R)_t} = \underset{(11.07)}{-.739} + \underset{(1.86)}{.075}\ \Delta^2 YER_t + \underset{(2.63)}{.194}\ \overline{\Delta STN_t} - \underset{(4.43)}{.359}\ \Delta LTN_{t-1} - \underset{(10.73)}{.526}\ \overline{\Delta\pi_t}$$

$$- \underset{(11.17)}{.136}\ [(M3R) - 1.140 YER + .820(LTN - STN) + 1.462\pi]_{t-2} - \underset{(4.92)}{.009}\, DUM86_t + \hat{\omega}_{1t}$$

T=67 (1980:Q4-1997:Q2)	R^2=0.80	*s*=0.231%
	DW=2.17	
LM(1)= .478 [.49]	*LM(4)*=.137 [.71]	*LM(1,4)*=.787 [.13]
ARCH(4)=.286 [.89]	*HET*=.650 [.79]	*NORM*=.824 [.66]
RESET=.164 [.69]	*RED*=.456 [.81]	$HANS^1$=.060
$HANS^2$=.589	*FOR(6)*=6.28 [.39]	*CHOW(6)*=1.02 [.42]

where $\overline{\Delta STN_t} = \dfrac{\Delta STN_t + \Delta STN_{t-1}}{2}$ and $\overline{\Delta\pi_t} = \dfrac{\Delta\pi_t + \Delta\pi_{t-1}}{2}$,

4.2.2 Long term interest rates

LTN equation: (I.2)

Backward-Looking version:
DEL(1: LTN-STN) = LTN.DDSTN*DEL(1: DEL(1: STN))+LTN.DLTNSTN1*DEL(1: LTN-STN)(-1)
+LTN.LTNSTN1*(LTN-STN)(-1)+LTN.CST

NOB = 72 NOVAR = 4 NCOEF = 4
RANGE: 1980Q1 to 1997Q4

RSQ	=	0.462813	CRSQ	=	0.439114
F(3/68)	=	19.528461	PROB>F	=	0
SER	=	0.337964	SSR	=	7.766949
DW(0)	=	2.061098	COND	=	2.044877
MAX:HAT	=	0.153598	RSTUDENT	=	3.490815
DFFITS	=	0.7448			

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
LTN.CST	0.043275	0.045188	0.957664	0.341625
LTN.DDSTN	-0.511084	0.077191	-6.621061	0
LTN.DLTNSTN1	0.572504	0.101507	5.640032	0
LTN.LTNSTN1	-0.062762	0.038733	-1.620373	0.109781

Forward Looking Version:
LTN = (STN+DUR*LTN(+1))/(1+DUR)

DUR: the duration of the bond (calibrated at 32 quarters).

LTN: Long-Term Interest Rate
STN: Short-Term Interest Rate

4.2.3 Real exchange rate

The real exchange rate is defined as follows:

EER = YED/(EEN*YWD) (I.3)

EER: Effective Exchange Rate
YWD: World GDP Deflator

4.3 Tax Policy and short-term interest rate determination [J]

STN Equation (J.1)

STN = (1-STN.STN)*(150*(DEL(1: LOG(PCD))-INFT(-0))
+50*LOG(YGA))+STN.STN*STN(-1)

For illustration purposes, the interest setting equation in the form of a standard Taylor rule has been employed, with an immediate reaction (i.e. STN.STN=0) of interest rates to both current deviation of inflation from its target – that can be set equal to baseline inflation in simulation – and the output gap.

TDN Equation (J.2)

DEL(1:TDN/YEN) = -0.1*(GLN/YEN)(-4)

GLN: Public sector net lending
TDN: Direct taxes paid by households
YEN: Nominal GDP

The ratio of direct taxes to GDP reacts to the deficit observed one year before, so that the tax rate is raised from its previous level by a fraction of the excess deficit observed with respect to the historical average.

Area-Wide Model's Variables

CAN		*Current Account Balance*	**M3R**		*M3*
COMPR	X	*Commodity Prices (HWWA)*	**MTD**		*Imports of Goods and Services Deflator*
D1	X	*Dummy Variable*	**MTN**		*Imports of Goods and Services*
D743	X	*Dummy Variable*	**MTR**		*Imports of Goods and Services*
D841	X	*Dummy Variable*	**NFA**		*Net Foreign Assets*
D872	X	*Dummy Variable*	**NFN**		*Net Factor Income from Abroad*
D894	X	*Dummy Variable*	**NFN_YEN**	X	*Ratio, Net Factor Income from Abroad/GDP*
EEN		*Effective exchange rate*	**OGN**		*Other Net Government Income*
EEN_DIS		*Effective exchange rate Statistical Discrepancy*	**OGN_YEN**	X	*Ratio Other Net Government Income/GDP*
EER		*Effective exchange rate*	**PCD**		*Private Consumption Deflator*
FDD		*Total Demand*	**PCN**		*Private Consumption*
GCD		*Gov. Consumption Deflator*	**PCR**		*Private Consumption*
GCN		*Gov. Consumption*	**PYN**		*Household's Disposable Income*
GCN_DIS	X	*Gov. Consumption Statistical Discrepancy*	**PYN_DIS**		*Household's Disposable Income Statistical Discrepancy*
GCN_YEN		*Ratio Gov. Consumption/GDP*	**PYN_DIS_YEN**	X	*Ratio Household's Disposable Income Statistical Discrepancy/GDP*
GCR	X	*Gov. Consumption*	**PYR**		*Household's Disposable Income*
GCR_YEN		*Ratio Gov. Consumption/GDP*	**SCD**		*Variation of Stocks Deflator*
GDN		*Public Debt*	**SCN**		*Variation of Stocks*
GDN_DIS		*Public Debt Statistical Discrepancy*	**SCR**		*Variation of Stocks*
GDN_DIS_YEN	X	*Public Debt Statistical Discrepancy/GDP*	**SSN**		*Social Security Contributions Total*
GDN_YEN		*Ratio Public Debt/GDP*	**SSN_YEN**	X	*Ratio Social Security Contributions Total/GDP*
GEN		*Government Expenditure*	**STN**	X	*Short-Term Interest Rate*
GEN_YEN		*Government Expenditure/GDP*	**STRQ**		*Short-Term Quarterly Interest Rate*
GIN		*Public Investment*	**TBN**		*Trade Balance*
GIN_OTHER		*Public Investment other*	**TBR**		*Trade Balance*
GIN_OTHER_YEN	X	*Public Investment other/GDP*	**TDN**		*Direct taxes*
GIN_YEN	X	*Public Investment/GDP*	**TDN_YEN**	X	*Ratio Direct taxes/GDP*
GIX		*Implicit Public Debt Interest Rate*	**TDN_YEN_DIS**	X	*Ratio Direct taxes/GDP Statistical Discrepancy*
GLN		*Gov. Net Lending*	**TFT**	X	*Trend Total Factor Productivity*
GLN_DIS		*Net Lending Statistical Discrepancy*	**TIME**	X	*Time Trend*
GLN_DIS_YEN	X	*Net Lending Statistical Discrepancy/GDP*	**TIN**		*Indirect Taxes*
GLN_YEN		*Ratio Gov. Net Lending/GDP*	**TIN_YEN**	X	*Ratio, Indirect Taxes/GDP*
GLN_YEN_TARGET	X	*Target Ratio Gov. Net Lending/GDP*	**TRN**		*Transfers*
GON		*Gross Operating Surplus*	**TRN_FIRMS**		*Transfers to Firms*
GPN		*Gov. Primary Surplus*	**TRN_FIRMS_YEN**	X	*Ratio Transfers to Firms/GDP*
GPN_YEN		*Ratio Gov. Primary Surplus/GDP*	**TRN_OTHER**		*Other Transfers*
GRN		*Gov. Gross Revenue*	**TRN_OTHER_YEN**	X	*Ratio Other Transfers/GDP*
GRN_YEN		*Ratio Gov. Revenue/GDP*	**TRN_YEN**	X	*Ratio Other Transfers/GDP*
GSN		*Government savings*	**TRN_YEN_DIS**	X	*Ratio Other Transfers/GDP Statistical Discrepancy*
GSN_YEN		*Ratio Government savings/GDP*	**ULC**		*Unit Labour Costs*
GYN		*Government disposable income*	**ULT**		*Trend Unit Labour Costs*
GYN_YEN		*Ratio Government disposable income/GDP*	**UNN**		*Number of Unemployed*
HICP		*HICP*	**URT**	X	*Trend Unemployment*
I77Q4I78Q1	X	*Dummy Variable*	**URX**		*Unemployment*
I81Q1	X	*Dummy Variable*	**WIN**		*Compensation to Employees*
I82Q1	X	*Dummy Variable*	**WLN**		*Wealth*
I84Q2	X	*Dummy Variable*	**WRN**		*Wage Rate*
I92Q4	X	*Dummy Variable*	**XTD**		*Exports of Goods and Services Deflator*
I98Q1	X	*Dummy Variable*	**XTN**		*Exports of Goods and Services*
INFQ		*Quarterly GDP deflator inflation rate*	**XTR**		*Exports of Goods and Services*
INFT		*Growth of consumption deflator*	**YED**		*GDP*
INN		*Gov. Interest Payments*	**YEN**		*GDP Deflator*
INN_YEN	X	*Ratio Gov. Interest Payments/GDP*	**YEN_DIS**	X	*GDP Statistical Discrepancy*
ITD		*Gross Investment Deflator*	**YER**		*GDP*
ITN		*Gross Investment*	**YER_DIS**	X	*GDP, Statistical Discrepancy*
ITR		*Gross Investment*	**YET**		*Potential Output*
KSR		*Whole-Economy Capital Stock*	**YFD**		*GDP at Factor Costs Deflator*
LFN	X	*Labour Force*	**YFN**		*GDP at Factor Costs*
LNN		*Total Employment*	**YGA**		*Output Gap*
LNT		*Trend Employment*	**YIN**		*GDP, Income Side*
LPROD		*Labour Productivity*	**YIN_DIS**	X	*GDP at Factor Costs, Statistical Discrepancy*
LSL		*Long run Employment*	**YWD**	X	*World GDP Deflator*
LSR		*Stocks (level)*	**YWDX**		*World Demand Deflator, Composite Indicator*
LTN		*Long-Term Interest Rate*	**YWR**	X	*World GDP*
M3N	X	*M3*	**YWRX**		*World Demand, Composite Indicator*

N.B.: Exogenous variables are marked with an X.
Variables ending in N are nominal; R are real.

ANNEX 2: Overview of the area-wide model Database

The database used for the Area-wide (AWM) has evolved over the course of its development history. This annex explains the method and procedures used to create the Area-Wide Model (AWM) database. The database is quarterly with series from 1970q1 (where possible). The first section of this annex explains that the historical series for the AWM database are an aggregate of country data. The second section explains the method used to aggregate the historical series. The third section explains how the data for earlier periods are re-scaled to bring the data in line with the Monthly Bulletin published by the ECB.

1 Country data

The country series come from a variety of sources, but chiefly from the OECD National Accounts or Main Economic Indicators or the BIS database for back-dates and Eurostat data for recent periods. Recourse to data not in the public domain has been limited to the bare minimum. [1] Where one source does not provide data for a series from 1970q1 then two series are combined to create a longer historical series. In this case, the series are rebased to the same base year (i.e., 1990) and then joined.

1.1 Conversion technique

Where series were not originally available at the required frequency, an interpolation filter was employed to transform data into a quarterly frequency.

1.2 Seasonal adjustment and working day adjustment

The database of the AWM is seasonally adjusted data. The country data is seasonally adjusted before aggregating. Where only non-seasonally adjusted data is available for an original series then the series is seasonally adjusted using the SABL method (SABL = Seasonal Adjustment, Bell Labs).[2] All series are non working day adjusted.

1.3 Treatment of German Reunification

For the majority of variables, the "whole" Germany series starts in 1990 or 1991. In these cases, the West German series is used as the historical series. In order to remove the break in the joined series, the West German series has been re-scaled to the new whole German series, by the ratio of the two series on the start date of the whole German series.

1.4 Base years

Variables are rebased to the year 1990. The HICP base year is 1996.

1.5 Updating the databases

Each update of the database is frozen, and any improvements, changes or updates are encompassed in the next version of the database.

1 *Further detailed information can be provided upon request. The data set described in this annex can be downloaded from http://www.ecb.int/.*

2 *An overview of this method can be found in Cleveland et al. [1982].*

2 Aggregation method

The method of aggregation used for most variables is the so-called "Index method". Define the log-level index for any series X as follows:

$\ln X_z = \Sigma_z w_i . \ln X_i$

where w is the weighting vector. This is done for both nominal and real variables and then the deflators calculated. A full explanation of this method can be found in Fagan and Henry [1998]. For some variables, for example ratios (eg trade balance as a ratio of GDP), the aggregate is simply calculated as a weighted sum of the ratios (without expressing in logarithms). There are some series (eg employment) where the series are just summed.

The weights used in aggregating the individual country series are constant GDP at market prices (PPP) for the EU11 for 1995. If not all countries are available then the weights are re-scaled from the original EU11 weights.

Table 2.1

Weights used in aggregation

	EU11
Belgium	3.9
Germany	30.529
Spain	10.233
France	21.003
Ireland	1.128
Italy	20.333
Luxembourg	0.233
Netherlands	5.585
Austria	3.023
Portugal	2.363
Finland	1.669

3 Re-scaling of Area-wide data to monthly bulletin data

A general principle has been to bring the data as close as possible to the area-wide variables contained in the already mentioned Monthly Bulletin to ensure consistency between the data used in the model and the data used in the ongoing monitoring of the area-wide economy. This was done by either published series entirely replacing original series, or by linking them with the data contained in the original AWM database, i.e. the database including only information directly aggregated from country data. The linking was done, as a general rule, taking older data until 1995q4, and Monthly Bulletin data as of 1996q1. The break date used as a general rule (1996q1) was chosen in order to minimise the need to re-estimate the model. The variables have been re-scaled as follows.

- Real GDP and components are taken from Eurostat, the original source of the corresponding Monthly Bulletin data; they are backdated with rates of growth of the AWM's original series, the break date being 1996q1. Thus, from this point on, the two datasets are identical. For earlier years, the discrepancies are minor.
- GDP deflators are taken directly from the corresponding Monthly Bulletin series (which are calculated by ECB staff as a weighted average of the national deflators using PPP weights[4]) and are linked to the older AWM deflators, the break date being again 1996q1.
- Fiscal series, in the form of ratios over GDP, are taken from data distributed by the European Commission (their AMECO database), which has backdated them from 1970q1.

4 *This procedure is necessary since the published Eurostat figures for nominal GDP and its components are expressed in terms of the current exchange rate (in ECU terms), which implies that, for earlier years, the implicit deflators calculated from the Eurostat data would be distorted by exchange rate movements.*

These ratios are further treated to adapt them to the accounting framework of the model.

- The unemployment rate is taken from Eurostat, the same series as reported in the Monthly Bulletin, the data is linked from the first available data and then backdated with the original AWM's data. Backdating is done, as for the rest of the variables, in rates of growth.[5]
- Total employment has been taken, in rates of growth, from the Monthly Bulletin, making the link with the original series from the start of or the data.
- Total compensation to employees is still entirely taken from the original AWM database.

Finally, interest rate data are entirely taken from the Monthly Bulletin. They are backdated with the corresponding series contained in the original database (source: BIS and AMECO), with the particularity that aggregation weights were changed according to the number of countries available at each period.

Outside these broad categories, officially published area-wide series are still relatively scarce. As a result, it is necessary for the purpose of the AWM to use data which is only available in the original database of the model. For instance, the decomposition of the income side of GDP is derived through the aggregation of country data, as is household's disposable income. As regards the external side, adequate data of sufficiently long span on the value and volume of out-of-area trade in goods *and services* are not available. In the AWM (as in the Eurostat national accounts data contained in the Monthly Bulletin), exports and imports of goods and services are a gross concept (i.e. do not net out intra-area trade flows). While, in principle, this does not affect net trade and other 'balance' items of the current account, it does mean that both export and import figures overstate significantly the true trade of the area (since intra-area trade accounts for about half of gross exports).

5 Labour force re-scaling is less troublesome if the unemployment rate is linked in rates of growth.

ANNEX 3: Determining the model's steady state and long run convergence

The main theoretical features of the steady state of the model have been set out in Section 3. Simple as the outline may seem, it is essential to ensure that the steady state of the empirical model corresponds to the theoretical priors and also that the model actually converges over time to its steady state. As regards the first issue, two approaches are possible. The first is to re-express the equations of the model into long-run equations by substituting out lead and lag variables and solving the resulting 'long-run' version of the model moreover imposing the appropriate cross equation restrictions (see Deleau et al. [1990] for a general approach and Loufir and Malgrange [1995] for an application to Multimod). The second approach is to carry out very long run simulations of the actual model for a given (but consistent) path of the exogenous variable and checking the values generated by the model at the end of the horizon (as suggested by Ando and Loufir [1990]). This latter approach has the advantage that – in addition to quantifying the steady state itself – it yields information regarding the stability of the model and the speed with which it converges to the steady state. This approach is therefore adopted in this annex.

A number of factors have to be taken into account when drawing an explicit steady-state solution from the actual empirical model. The latter can be viewed as a complex dynamic system comprising a number of 'forcing' exogenous variables - such as world demand or factor productivity – the dynamics of which is unit rooted. The endogenous variables react to such underlying unit roots, and it is only under a number of conditions on the short-run or dynamic adjustment parameters that the whole empirical system does deliver a steady state. In the latter, endogenous variables reach terminal rates of growth (e.g. inflation) or can be expressed in terms of a fixed value (e.g. the saving ratio). These terminal conditions are a correct and sufficient characterisation of the steady state, and are usually easy to spell out and check. In some cases, such conditions can be directly plugged into the model, taking advantage of the ECM specifications employed (see e.g. Boutillier and Jacquinot [1996]).

The way in which the steady state of the AWM has been checked and assessed was to run a long simulation consistent with an artificial steady-state database with exogenous variables extended in an appropriate manner, which involved making hypotheses on a number of key parameters. Such an approach first requires the choice of the underlying growth parameters, namely an exogenously given trend growth in factor productivity and population. Also a terminal inflation rate (or price level) has to be determined, which applies to all exogenously given deflators. Nominal exogenous variables should of course follow a path consistent with both real variables and inflation at the steady state. Additional assumptions concern the evolution of fiscal variables and less important items such as risk premia or statistical discrepancies.

On the real side, the exogenous growth of population was chosen to match views on trend population growth for the zone, i.e. around 0.3% per year. Exogenous trend growth in total factor productivity was chosen to give a growth in output close to the 2.25% rate usually seen as sustainable growth for the zone.[6] The exogenous natural rate of unemployment contained in the model was extended using its last derived value. As to nominal variables, terminal inflation was set at 2% per year and relevant prices were extended from their last observed value using this rate of growth. Fiscal variables were extended using their ratio over GDP at their last observed value. As regard external variables, world growth and inflation were set equal to the corresponding values for the euro area.

6 *Potential grows in the model according to trend population growth plus trend total factor productivity, divided by the labour-share parameter of the production function.*

Starting from conditions observed in 1997Q4 and the exogenous information just described, a very long-horizon simulation can be performed, delivering a steady-state solution. In the context of this simulation, concluding that a steady state has been reached implies checking that a number of conditions have been met:

- GDP growth is stable at the hypothesised long-run value given above.
- GDP coincides with potential output, and unemployment with the (exogenously set) natural unemployment rate.
- Inflation is equal to the postulated rate set out above, and the exchange rate is neither appreciating nor depreciating.
- Real wages grow at the same pace as labour productivity, both trended or actual.
- The capital stock-GDP ratio has reached its theoretical level and the real interest rate equals the postulated steady state value.

The model met all these conditions after a number of years, confirming that the model settles down to a well-defined steady state consistent with the theoretical presentation outlined in Section 3. Table 2 collects some interesting ratios over GDP as a summary of the resulting steady state. The ratios presented are obviously a function of the assumptions made to extend the baseline, and should not be understood as reflecting steady-state conditions deeply rooted in the model; rather, they should point to the relative stability of the ratios when the model is run over a long period under 'reasonable' enough assumptions.

Table 2

Key ratios

	Steady State	1999Q4
Consumption to GDP ratio	64.4%	62.7%
Import penetration	33.5%	35.2%
Trade/GDP ratio	0.0	1.3
NFA/GDP ratio	-1.6	0.1
wealth/GDP ratio	15.5	14.1
mark-up on ULC	1.7	1.8

In practice, however, the speed of convergence to that steady state heavily depends on the initial conditions, i.e. the extent to which the starting point of the simulation corresponds to a configuration far away from the implicit long-run equilibrium of the model. This makes it difficult to give precise indications of the speed of adjustment to equilibrium only on the basis of such long-run simulations.

A clearer idea of the speed at which the model returns to equilibrium can be gauged by stochastic simulations of the model around the steady state. The model was shocked 200 times around the steady state reached in 2200Q1 (a date at which the model was undoubtedly at its steady state). Shocks affected all the stochastic equations for just one period. Shocks were drawn from a normal distribution, with variance equal to the historically observed ones, and were assumed to be uncorrelated in order to simplify the exercise. Such an experiment is meant at reproducing a credible 'real life' situation in which the initial conditions are not consistent with equilibrium, but are set for each econometric equation at values in line with the distribution of observed residuals - instead of corresponding to the historical residual at a given point in time.

Chart A.3.1. shows the 10%, 25%, 50%, 75% and 90% percentiles of the responses for each period, in the form of percentage point deviations from baseline for the output gap, and percentage points deviations from baseline for inflation. In both cases, the distribution of the trajectories seem to indicate that after around 25 years, the theoretical steady state is more or less reached. Even at the 90%/10% level, i.e. looking at less likely initial conditions, convergence seems to be relatively quick, although some cyclical effects seem to prevent the model from staying at steady state after 30 years or so. Focusing on the results appearing for the median trajectory, i.e. the 50% line (which would correspond to the no-shock situation in the case of a normal distribution for the residuals and a linear model), inflation converges back to baseline after 15 years whereas output gap closes after 20 years, always staying very close to the zero line.

One interesting feature of the graphs is the almost perfect symmetry in the responses, notwithstanding the non-linear nature of the model. This fact is evident when considering the distance from baseline for the line depicting the median shock (i.e., the 50% line), which is never significant. This means that a deterministic simulation performed with the model gives an accurate view of the mechanisms at work. Another interesting feature is the number of hump shapes appearing before full convergence back to steady state is achieved. The non-monotonic return to baseline is certainly a consequence of the diverse sources of sluggishness in the model. Thus, three broad waves can be detected for the output gap: one taking away most of the initial shock and lasting for around 5 years; a second one lasting for a further 10 years; and a third one leading to an almost perfect return to steady state after around 30-40 years. The inflation response is slightly more complex but follows broadly speaking the same pattern.

Chart A3.1

Shocks to the model around the steady state
Stochastic simulations

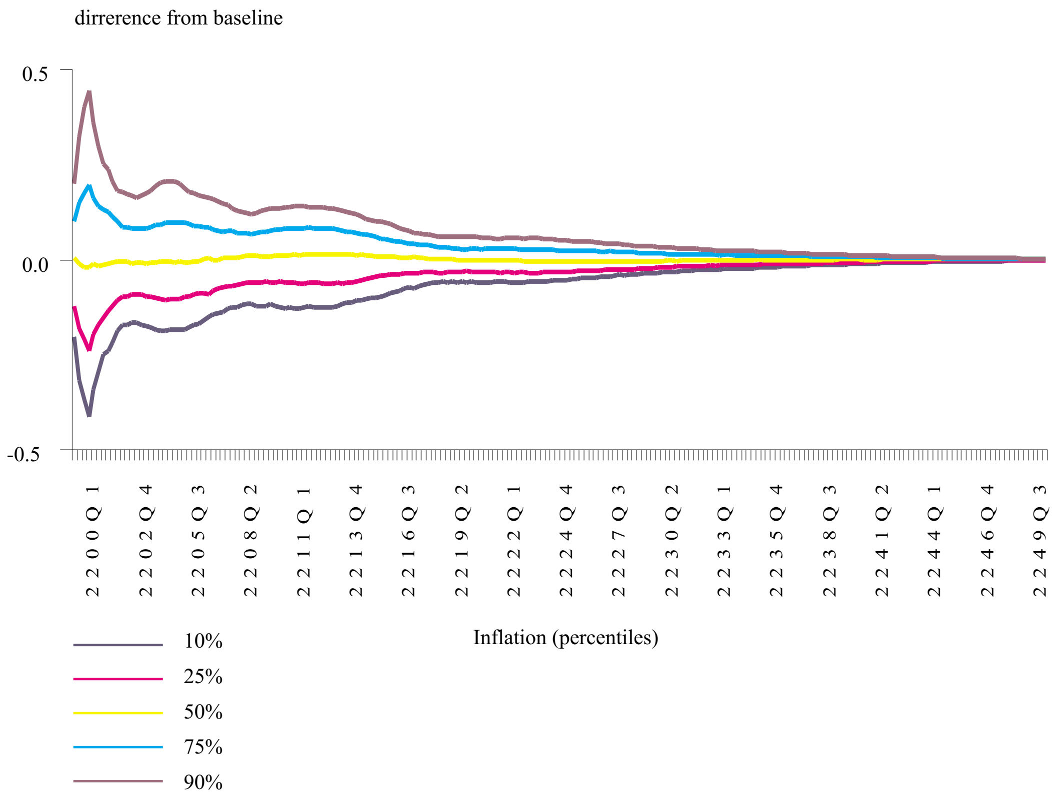
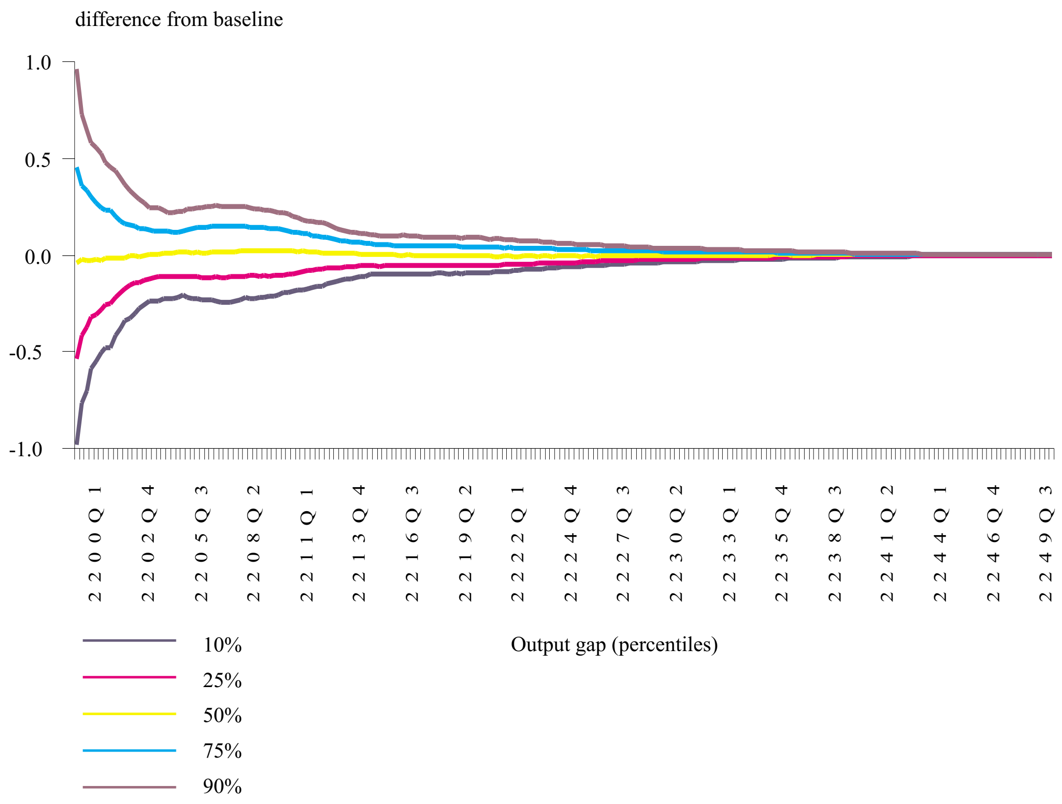


Chart A3.2

Shocks to the model around the steady state

Stochastic simulations



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