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MACROECONOMIC IMPLICATIONS OF TIME-VARYING RISK PREMIA

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Abstract

A large empirical literature suggests that risk premia on stocks or corporate bonds are large and countercyclical. This paper studies a simple real business cycle model with a small, exogenously time-varying risk of disaster, and shows that it can replicate several important facts documented in the literature. In the model, an increase in disaster risk leads to a decline of output, investment, stock prices, and interest rates, and an increase in the expected return on risky assets. The model matches well business cycle data and asset price data, and the countercyclicality of risk premia. I present an extension of the model with endogenous choice of leverage and endogenous default, and show that the model accounts well for the level and cyclicality of credit spreads, and in particular the relation between investment and credit spreads.

Keywords: business cycles, investment, credit spreads, risk premia, rare events.

JEL code: E32, E44, G12.

Non-technical summary

During the 2008-2009 financial crisis, the yield on risky securities (such as corporate bonds, or securities backed by real estate loans or consumer credit) rose, while the yield on safe assets such as short-term government bonds fell. Standard macroeconomic models, however, give no role to these spreads, and have typically only one "interest rate". Incorporating these spreads, which reflect both a borrower-specific default probability and an aggregate market price of risk, in a classical macroeconomic framework, in a rigorous fashion is useful both to understand macroeconomic movements and to design an optimal policy.

This paper proposes a simple framework to analyze this question, building on the literature on "rare disasters". This literature emphasizes that the existence of infrequent, but very deep recessions, can explain the observed equity premium. These "disasters" may be caused either by economic or financial crises (such as the Great Depression) or by wars and large natural disasters. Specifically, the paper studies the effect of a change in the risk of a disaster on economic decisions and prices. Technically, the model combines a real business cycle benchmark with a small, exogenously time-varying risk of disaster. An increase in disaster risk leads to a decline of output, investment, stock prices, and interest rates, and an increase in the expected return on risky assets and hence in credit spreads. This shock is equivalent, for quantities, to a preference shock, justifying the "equity premium shock" introduced by Smets and Wouters (2007). The model matches well business cycle data and asset price data, and the relations between prices and quantities, e.g. the countercyclicality of risk premia, the relation between the volatility index VIX (constructed by the Chicago Board of Trade using options prices) and GDP, or between the stock market and capital expenditures. Empirically, one can infer the probability of disaster from asset prices. Measured in this way, shocks to the probability of disaster appear to play an important role during the largest US recessions, including the current one.

The technical contribution of this paper is to show how a small, time-varying risk of an economic "disaster" can be incorporated in a tractable manner in a dynamic stochastic general equilibrium (DSGE) model. While the finance literature suggests that rare disaster models can explain a variety of asset pricing facts, this literature has so far been confined to endowment economies, and hence does not consider the feedback from time-varying risk premia to macroeconomic activity. In macroeconomics instead, the focus has been on linearized models where risk premia are tiny and nearly constant.

I also present an extension of the model where (nonfinancial) corporations choose their financial leverage optimally and consequently default when unable to service their debt. The motivation is that a large literature in empirical finance documents the "credit spread puzzle", i.e. corporate bond spreads are too high and too volatile given the standard measures of liquidity and credit risk. Traditional models of financial frictions do not reproduce the level, volatility and cyclicality of risk premia. The project is to connect these literatures by introducing a stylized model of capital structure in a neoclassical (real business cycle) model with disaster risk. The capital structure (bond vs. equity choice) is driven by the usual trade-off between bankruptcy costs and tax shield.

The model accounts well for the level and cyclicality of credit spreads, and in particular the relation between investment and credit spreads. The paper also shows that financial frictions considerably amplify the effect of disaster risk on quantities (by a factor of about three). In the model, the welfare cost of the advantageous tax treatment of debt is significant, because higher leverage leads to higher volatility as well as capital over-accumulation. Allowing firms to issue debt that is contingent on disaster has a substantial welfare benefit, as it curbs business cycle volatility. Last, the model is consistent with the idea that, as investors become more optimistic and believe that a disaster is less likely (e.g. because the economy is smoother, “the Great Moderation”), credit spreads fall, and firms decide to increase their leverage, making the economy more exposed to large negative shocks.

1 Introduction

The empirical finance literature has provided substantial evidence that risk premia vary over time, and that they are countercyclical.[1] Yet, standard business cycle models such as the real business cycle model, or the dynamic stochastic general equilibrium (DSGE) models used for monetary policy analysis, largely fail to replicate the level, the volatility, and the countercyclicality of risk premia. In these models, the variation in expected returns is entirely driven by variation in the risk-free interest rate. Is this a significant limitation of macroeconomic models? Do risk premia matter for macroeconomic dynamics?

A general answer to this question is difficult, because risk premia can arise through different mechanisms, ranging from preferences, to time-varying risk, to incomplete markets. However, a first step is to provide a framework with large risk premia to study the connection between risk premia and economic activity. The contribution of this paper is to introduce such a framework, by building a tractable real business cycle model with a small, stochastically time-varying risk of economic "disaster", following the work of Rietz (1988), Barro (2006), and Gabaix (2007). In my model, risk premia vary because the real quantity of risk varies, leading to a reaction of both asset prices and macroeconomic aggregates. Existing work has so far been confined to endowment economies, and hence does not consider the feedback from time-varying risk to macroeconomic aggregates. An increase in the probability of disaster creates a collapse of investment and a recession, as risk premia rise, increasing the cost of capital. Demand for precautionary savings increase, leading the yield on less risky assets to fall, while spreads on risky securities increase. These business cycle dynamics occur with no change in total factor productivity.[2]

Before turning to a quantitative analysis, I prove two theoretical results, which hold under the assumption that a disaster reduces total factor productivity (TFP) and the capital stock by the same amount. First, when the probability of disaster is constant, the path for macroeconomic quantities implied by the model is the same as that implied by a model with no disasters, but a different discount factor β. This "observational equivalence" (in a sample without disasters) is reminiscent of the numerical analysis of Tallarini (2000), who found that macroeconomic dynamics are essentially unaffected by the amount of risk or the degree of risk aversion. Second, when the probability of disaster is stochastic, an increase in probability of disaster is observationally

[1] See e.g. Campbell and Shiller (1988) and Fama and French (1989) for stocks, Cochrane and Piazzesi (2006) for Treasury bonds, Philippon (2008) and Gilchrist and Zakrajsek (2007) for corporate bonds, and Cochrane (2007) or Backus, Routledge and Zin (2008) for recent overviews.

[2] Because disasters are rare, the risk is usually not realized in sample. However, my results are not driven by sample selection (peso problem); see sections 4.3 and 5.5.

equivalent to a preference shock. This implies that these shocks have affect macroeconomic aggregates, and this provides an interpretation of the "equity premium shocks" introduced by Smets and Wouters (2003) and other authors in their estimations of DSGE models. Consistent with the literature, the paper argues that these shocks play a significant role in macroeconomic dynamics. However I arrive at this conclusion from a very different path, since these shocks are calibrated to replicate asset prices in my model.

Quantitatively, I find that this parsimonious model can match many asset pricing facts - the mean, volatility, and predictability of returns - while doing at least as well as the RBC model in accounting for quantities. This is important since many asset pricing models which are successful in endowment economies do not generalize well to production economies.[3] Most interestingly, the model matches well the relations between macroeconomic aggregates (such as investment or output) and asset prices (such as expected returns, the P-D ratio, or the VIX index). As is well known, this connection between prices and quantities is problematic for most macroeconomic models.

Empirical tests of the disaster model are notoriously difficult. Barro (2006) measured historical disasters in cross-country data. To measure the time-varying probability of disaster, I use the most natural restriction of the model - disaster risk affects powerfully asset prices. I infer the probability of disaster from the observed price-dividend ratio. I then feed into the model this estimated probability of disaster. The variation over time in this probability appears to account for a share of business cycle dynamics, and is especially important during the sharpest downturns such as the current recession.

This risk of an economic disaster may be a strictly rational expectation. For instance, during the recent financial crisis, many commentators, including well-known macroeconomists, have highlighted the possibility that the U.S. economy might fall into another Great Depression.[4] My results suggest that the probability of a disaster was indeed high in Fall 2008. More generally it could reflect a time-varying belief, which may differ from the objective probability - i.e., waves of optimism or pessimism (see e.g. Jouini and Napp (2008)). My model studies the macroeconomic

[3] As explained in Jermann (1998), Lettau and Uhlig (2000), Kaltenbrunner and Lochstoer (2008).

[4] Greg Mankiw (NYT, Oct 25, 2008): "*Looking back at [the great Depression], it's hard to avoid seeing parallels to the current situation. (...) Like Mr. Blanchard at the I.M.F., I am not predicting another Great Depression. But you should take that economic forecast, like all others, with more than a single grain of salt.*"

Robert Barro (WSJ, March 4, 2009): "*... there is ample reason to worry about slipping into a depression. There is a roughly one-in-five chance that U.S. GDP and consumption will fall by 10% or more, something not seen since the early 1930s.*"

Paul Krugman (NYT, Jan 4, 2009): "*This looks an awful lot like the beginning of a second Great Depression.*"

effects of such time-varying beliefs. (Of course in reality beliefs may are endogenous, but understanding the effects of a change in beliefs is important.) This simple modeling device captures the idea that aggregate uncertainty is sometimes high: people sometimes worry about the possibility of a deep recession. It also captures the idea that there are some asset price changes which are not obviously related to current or future TFP, i.e. "bubbles", "animal spirits", and which in turn affect the macroeconomy.

I then present an important extension of the benchmark model, where firms are financed not only with equity but also with defaultable bonds. This allows studying the default probability, and corporate credit spread. I show that this model replicates several features of credit spreads that have been emphasized in the recent empirical literature. First, in the data the probability of default of an investment grade bond is much smaller than its credit spread: the probability is about 0.4% per year (and there is substantial recovery upon default, around 50%), but the spreads are on average around 100bp.[5] Second, credit spreads are strongly correlated with investment, and the part of credit spreads that forecasts investment is not the expected default frequency, but rather the residual part of credit spreads, which Gilchrist and Zakrajsek dub the "excess bond premium". By their very nature, corporate bonds are safe in normal times, with limited default during ordinary recessions, but are exposed to the risk of a very large downturn, and hence disaster risk can replicate these features by generating a large, time-varying risk premium.

Introducing time-varying risk requires solving a model using nonlinear methods, i.e. going beyond the first-order approximation and considering higher order terms in the Taylor expansion. Researchers disagree on the importance of these higher order terms, and a fairly common view is that they are irrelevant for macroeconomic quantities. Lucas (2003), in his presidential address, summarizes: "*Tallarini uses preferences of the Epstein-Zin type, with an intertemporal substitution elasticity of one, to construct a real business cycle model of the U.S. economy. He finds an astonishing separation of quantity and asset price determination: The behavior of aggregate quantities depends hardly at all on attitudes toward risk, so the coefficient of risk aversion is left free to account for the equity premium perfectly.*"[6] My results show, however, that when risk varies over time, risk aversion affects macroeconomic dynamics in a significant way, and hence building a model to match the equity premium or other asset pricing facts leads to different quantity

[5] This is the spread of a BAA-rated corporate bond over a AAA-rated corporate bond (rather than a Treasury), so as to net out differences in liquidity.

[6] Note that Tallarini (2000) actually picks the risk aversion coefficient to match the Sharpe ratio of equity. Since return volatility is very low in his model (there are no capital adjustment costs), the equity premium is much smaller in his model than in the data.

implications.

Overall, the contribution of the paper is twofold. Substantively, the quantitative and empirical results of this paper suggest an important role for time-varying risk in accounting for business cycles and asset prices. This result obtains in the context of a model which matches well data on prices, quantities, and the relations between quantities and prices, which in itself is an important achievement. Besides this substantive contribution, the technical contribution of the paper is to provide a tractable framework which leads to volatile, countercyclical risk premia in a standard macroeconomic model. The tractability of the framework is such that extensions to include credit frictions, monetary policy, or several countries, are quite feasible.

The paper is organized as follows: the rest of the introduction reviews the literature. Section 2 studies a simple analytical example in an AK model which can be solved in closed form and yields the central intuition for the results. Section 3 gives the setup of the full model and presents the analytical results. Section 4 studies the quantitative implications of the model numerically. Section 5 considers some extensions of the baseline model. Section 6 presents an empirical evaluation of the model, backing out the probability of disaster from asset prices. Section 7 builds a substantial extension of the model, with endogenous choices of leverage (debt) and default.

Related Literature

Gabaix (2007, 2009) independently obtained propositions 1 and 2. On top of that, he develops a specific model where variation in the probability of disaster has no macroeconomic effect. In contrast, my paper uses the standard real business cycle model, and shows that a shock to the probability of disaster is equivalent to a preference shock (proposition 3) and hence has a macroeconomic effect. Unlike Gabaix then, my model generates an empirically compelling correlation between asset prices and macroeconomic quantities. Moreover, my paper is more quantitative and uses Epstein-Zin utility.

This paper is mostly related to four strands of literature. First, a large literature in finance builds and estimates models which attempt to match not only the equity premium and the risk-free rate, but also the variation of risk premia (i.e. the predictability of excess returns). Two prominent examples are Bansal and Yaron (2004) and Campbell and Cochrane (1999). However, this literature is limited to endowment economies, and hence is of limited use to analyze the business cycle or to study policy questions.

Second, my paper is closely related to a small literature which studies business cycle models (i.e. with endogenous consumption, investment and output), and attempts to match both business cycle statistics but also asset returns first and second moments.[7] Many of these studies consider only the implications of productivity shocks, and generally study only the mean and standard deviations of return and do not attempt to match the predictability of returns. My paper contributes to this literature by focusing on the variation of risk premia and the correlations between asset prices or returns, on the one hand, and macroeconomic quantities, on the other hand. In contrast to my paper, many of these studies also abstract from employment, which is a critical business cycle variable. Many of these studies have difficulty reconciling business cycle dynamics and asset returns, but my model does well in this dimension.

Third, the paper draws from the recent literature on "disasters" or rare events (Rietz (1988), Barro (2006), Barro and Ursua (2008), Gabaix (2007), Gourio (2008a,2008b), Julliard and Ghosh (2008), Martin (2008), Santa Clara and Yan (2008), Wachter (2008), Weitzmann (2007), Backus, Chernov and Martin (2009)). Disasters are a powerful way to generate large risk premia. Moreover, as we will see, disasters are relatively easy to embed into a standard macroeconomic model.

Last, my paper studies the real effects of a shock to uncertainty, a channel recently emphasized by Bloom (2009). Bloom (2009) considers a partial-equilibrium model with heterogeneous firms facing fixed and linear costs to adjusting capital or labor. In his model, the uncertainty shock is a temporary increase in the variance of aggregate and especially idiosyncratic productivity shocks. His model generates a recession and a decrease in endogenous aggregate TFP in response to an uncertainty shock. My model also generates a recession in response to higher uncertainty, but there are several differences: (1) risk is modeled differently since the higher uncertainty affects both productivity and the capital stock; (2) the mechanism is different since it relies on the general equilibrium feedback, i.e. risk-averse consumers are less willing to invest in risky capital when uncertainty is high; (3) the model does not generate any change in TFP. Most importantly, my model focuses on the relations between asset prices and the macroeconomy. For instance, my model can replicate the empirical finding that shocks to VIX affect output negatively.[8]

[7] A non-exhaustive list includes Jermann (1998), Tallarini (2000), Boldrin, Christiano and Fisher (2001), Lettau and Uhlig (2000), Kaltenbrunner and Lochstoer (2008), Campanele et al. (2008), Croce (2005), Papanikolaou (2008), Kuehn (2008), Uhlig (2006), Jaccard (2008), and Fernandez-Villaverde et al. (2008).

[8] Fernandez-Villaverde et al. (2009) also study the effect of shocks to risk, but they focus on a small open economy which faces exogenous time-varying interest rate risk.

2 A simple analytical example in an AK economy

To highlight the key mechanism of the paper, this section studies a streamlined model. Section 3 relaxes many of the simplifying assumptions, such as constant productivity, no adjustment costs, etc., which are made for clarity. Consider a simple economy with a representative consumer who has power utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

where C_t is consumption and γ is the risk aversion coefficient (and also the inverse of the the intertemporal elasticity of substitution of consumption). This consumer operates an AK technology:

$$Y_t = AK_t,$$

where Y_t is output, K_t is capital, and A is productivity, which is assumed to be constant. The resource constraint is:

$$C_t + I_t \leq AK_t.$$

The economy is randomly hit by disasters. A disaster destroys a share b_k of the capital stock.[9] This may be because of a war which physically destroys capital, but there are alternative interpretations. For instance, b_k could reflect expropriation of capital holders (if the capital is taken away and then not used as effectively), or it could be a "technological revolution" that makes a large share of the capital worthless. It could also be that even though physical capital is not literally destroyed, some intangible capital (such as matches between firms, employees, and customers) is lost. Finally, one can imagine a situation where the demand for some goods falls sharply, rendering worthless the factories which produce them.[10]

Throughout the paper I denote by x_{t+1} an indicator which is one if there is a disaster at time $t+1$, and 0 if not.

The probability of a disaster varies over time. To maintain tractability I assume in this section that it is *i.i.d.*: p_t, the probability of a disaster at time $t+1$, is drawn at the beginning of time t

[9] A disaster does not affect productivity A. I will relax this assumption in section 3. In an AK model, a permanent reduction in productivity would lead to a permanent reduction in the growth rate of the economy, since the *level* of A affect the *growth rate* of output.

[10] In a large downturn, the demand for some luxury goods such as boats, private airplanes, etc. would likely fall sharply. If this situation were to last, the boats-producing factories would never operate at capacity, and hence the value would fall to zero.

from a cumulative distribution function F. The law of accumulation for capital is thus:

$$K_{t+1} = ((1-\delta)K_t + I_t)(1 - x_{t+1}b_k).$$

Finally, I assume that the two random variables p_{t+1}, and x_{t+1} are independent. I also discuss this assumption in more detail in section 3.

This model has one endogenous state variable, the capital stock K and one exogenous state p, and there is one control variable C. There are two shocks: the realization of disaster $x' \in \{0,1\}$, and the draw of a new probability of disaster p'. The Bellman equation for the representative consumer is:

$$\begin{aligned} V(K,p) &= \max_{C,\, I} \left\{ \frac{C^{1-\gamma}}{1-\gamma} + \beta E_{p',x'}\left(V(K',p')\right) \right\} \\ s.t. &: \\ C + I &\leq AK, \\ K' &= ((1-\delta)K + I)(1 - x'b_k). \end{aligned}$$

The assumptions made ensure that V is homogeneous, i.e. V is of the form $V(K,p) = \frac{K^{1-\gamma}}{1-\gamma} g(p)$, where g satisfies the Bellman equation:

$$g(p) = \max_i \left\{ \frac{(A-i)^{1-\gamma}}{1-\gamma} + \beta \frac{(1-\delta+i)^{1-\gamma}\left(1-p+p(1-b_k)^{1-\gamma}\right)}{1-\gamma}\left(E_{p'}g(p')\right) \right\}, \tag{1}$$

and $i = \frac{I}{K}$ is the investment rate. This implies that consumption and investment are both proportional to the current stock of capital, but they typically depend on the probability of disaster as well:

$$\begin{aligned} C_t &= f(p_t)K_t, \\ I_t &= h(p_t)K_t. \end{aligned}$$

As a result, when a disaster occurs and the capital stock falls by a factor b_k, both consumption and investment also fall by a factor b_k. Given that there are no adjustment costs, the value of capital is equal to the quantity of capital, and hence it falls also by a factor b_k in a disaster.

Finally, the return on an all-equity financed firm is:

$$R^e_{t,t+1} = (1-\delta+A)\,(1-x_{t+1}b_k),$$

i.e. it is $1-\delta+A$ if there is no disaster, and $(1-\delta+A)\,(1-b_k)$ if there is a disaster. Clearly, the equity premium will be high, since the equity return and consumption are both very low during disasters. Moreover, the equity premium is larger when the probability of disaster p_t is higher.

Let us finally turn to the effect of p on the consumption-savings decision, i.e. the function $f(p)$. Using equation (1), the first-order condition with respect to i yields, after rearranging:

$$\left(\frac{A-i}{1-\delta+i}\right)^{-\gamma} = \beta\left(1-p+p(1-b_k)^{1-\gamma}\right)\left(E_{p'}g(p')\right). \tag{2}$$

Given that p is *i.i.d.*, the expectation of g on the right-hand side is independent of the current p. The term $(1-b_k)^{1-\gamma}$ is greater than unity if and only if $\gamma > 1$. Hence, the right-hand side is increasing in p if and only if $\gamma > 1$. Since the left-hand side is an increasing function of i, we obtain that i is increasing in p if $\gamma > 1$, it is decreasing in p if $\gamma < 1$, and it is independent of p if $\gamma = 1$.

The intuition for this result is as follows: if p goes up, investment in physical capital becomes more risky and hence less attractive, i.e. the risk-adjusted physical return on capital goes down.[11] The effect of a change in the return on the consumption-savings choice depends on the value of the intertemporal elasticity of substitution (IES), because of offsetting wealth and substitution effects. If the IES is unity (i.e. utility is log), savings are unchanged and thus the investment rate does not respond to a change in the probability of disaster. But if the IES is larger than unity, i.e. $\gamma < 1$, the substitution effect dominates, and i is decreasing in p. Hence, an increase in the probability of disaster leads initially, in this model, to a decrease in investment, and an increase in consumption, since output is unchanged on impact. Next period, the decrease in investment leads to a decrease in the capital stock and hence in output. This simple analytical example thus shows that a change in the perceived probability of disaster can lead to a decline in investment and output. The key mechanism is the effect of *rate-of-return uncertainty* on the optimal savings decision.[12]

[11] Following Weil (1989), I define the risk-adjusted return as $E(R^{1-\gamma})^{\frac{1}{1-\gamma}}$, where R is the physical return on capital.

[12] The effect of rate-of-return uncertainty differs from that of labor-income uncertainty, as is well known at least since Levhari and Srinivasan (1969) and Sandmo (1970). The example of this secton is related to work by Epaulard and Pommeret (2003).

Extension to Epstein-Zin preferences

To illuminate the respective role of risk aversion and the intertemporal elasticity of substitution, it is useful to extend the preceding example to the case of Epstein-Zin utility. Assume, then, that the utility V_t satisfies the recursion:

$$V_t = \left(C_t^{1-\gamma} + \beta E_t \left(V_{t+1}^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}}, \tag{3}$$

where θ measures risk aversion towards static gambles, γ is the inverse of the intertemporal elasticity of substitution (IES) and β reflects time preference.[13] It is straightforward to extend the results above; the first-order condition now reads

$$\left(\frac{A-i}{1-\delta+i} \right)^{-\gamma} = \beta \left(1-p+p(1-b_k)^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \left(E_{p'} g(p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}},$$

and we can apply the same argument as above, in the realistic case where risk aversion $\theta \geq 1$: the now *risk-adjusted* return on capital is $\left(1-p+p(1-b_k)^{1-\theta}\right)^{\frac{1}{1-\theta}}$; it falls as p rises; an increase in the probability of disaster will hence reduce investment if and only if the IES is larger than unity.[14] Hence, the parameter which determines the sign of the response is the IES, and the risk aversion coefficient (as long as it is greater than unity) determines the magnitude of the response only. While this example is revealing, it has a number of simplifying features, which lead us to turn now to a quantitative model.

3 A Real Business Cycle model with Time-Varying Risk of Disaster

This section introduces a real business cycle model with time-varying risk of disaster and study its implications analytically. The next section considers the quantitative implications of the model using numerical methods. The model extends the simple example of the previous section in the following dimensions: (a) the probability of disaster is persistent instead of *i.i.d.*; (b) the production function is neoclassical and affected by standard TFP shocks; (c) labor is elastically

[13] Note that it is commonplace to have a $(1-\beta)$ factor in front of $u(C,N)$ in equation (3), but this is merely a normalization, which it is useful to forgo in this case.

[14] The disaster reduces the mean return itself, but this is actually not important. We could assume that there is a small probability of a "capital windfall" so that a change in p does not affect the mean return on capital. Crucially, what matters here is the risk-adjusted return on capital, $E(R^{1-\theta})^{\frac{1}{1-\theta}}$, and a higher risk reduces this return. See section 5.6 for more details.

supplied; (d) disasters may affect total factor productivity as well as capital; (e) there are capital adjustment costs.

3.1 Model Setup

The representative consumer has preferences of the Epstein-Zin form, and the utility index incorporates hours worked N_t as well as consumption C_t:

$$V_t = \left(u(C_t, N_t)^{1-\gamma} + \beta E_t \left(V_{t+1}^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}}, \tag{4}$$

where the per period felicity function $u(C, N)$ is assumed to have the following form:

$$u(C, N) = C^{\upsilon}(1 - N)^{1-\upsilon}.$$

Note that u is homogeneous of degree one, hence γ is the inverse of the intertemporal elasticity of substitution (IES) over the consumption-leisure bundle, and θ measures risk aversion towards static gambles over the bundle.

There is a representative firm, which produces output using a standard Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} \left(z_t N_t \right)^{1-\alpha},$$

where z_t is total factor productivity (TFP), to be described below. The firm accumulates capital subject to adjustment costs:

$$K_{t+1} = \left((1-\delta)K_t + \phi \left(\frac{I_t}{K_t} \right) K_t \right) (1 - x_{t+1} b_k).$$

where ϕ is an increasing and concave function, which curvature captures adjustment costs, and x_{t+1} is 1 if there is a disaster at time $t+1$ (with probability p_t) and 0 otherwise (probability $1 - p_t$). At this stage b_k is a parameter, which may be zero - i.e., a disaster only affects TFP. We explore quantitatively the role of b_k in section 5.2.

The resource constraint is

$$C_t + I_t \leq Y_t.$$

Aggregate investment cannot be negative: $I_t \geq 0$. Depending on parameter values, this constraint may bind in the periods immediately following a disaster.

Finally, we describe the shock processes. Total factor productivity (TFP) is assumed to follow a unit root process, and is affected by standard normally distributed shocks ε_t as well as disasters. Mathematically,

$$\log z_{t+1} = \log z_t + \mu + \sigma\varepsilon_{t+1} + x_{t+1}\log(1 - b_{tfp}),$$

where μ is the drift of TFP, and σ is the standard deviation of normal shocks, and b_{tfp} is the reduction in TFP following a disaster. Here too, we will consider various values for b_{tfp}, including possibly zero - i.e., a disaster only destroys capital but does not actually affect TFP. Last, the probability of disaster p_t follows a stationary Markov process with transition function T. In the quantitative application, I will simply assume that the log of p_t follows an AR(1) process.

I assume that the three exogenous shocks $p_{t+1}, \varepsilon_{t+1}$, and x_{t+1} are all independent conditional on p_t. This assumption requires that the occurrence of a disaster today does not affect the probability of a disaster tomorrow. This assumption may be wrong either way: a disaster today may indicate that the economy is entering a phase of low growth or is less resilient than thought, leading agents to revise upward the probability of disaster, following the occurrence of a disaster. But on the other hand, if a disaster occurred today, and capital or TFP fell by a large amount, it is unlikely that they will fall again by a large amount next year. Rather, historical evidence suggests that the economy is likely to grow above trend for a while (Gourio (2008a), Barro et al. (2009)). In section 5.3, I extend the model to consider these different scenarios.

3.2 Bellman Equation

In this section I set up a recursive formulation of the problem, which is used to prove analytical results. The model has three state variables: capital K, technology z and probability of disaster p. There are two independent controls: consumption C and hours worked N; and three shocks: the realization of disaster $x' \in \{0,1\}$, the draw of the new probability of disaster p', and the normal shock ε'. The first welfare theorem holds, hence the competitive equilibrium is equivalent to a social planner problem, which is easier to solve. Denote $V(K,z,p)$ the value function, and define $W(K,z,p) = V(K,z,p)^{1-\gamma}$. The social planning problem can be formulated as:

$$\begin{aligned} W(K,z,p) &= \max_{C,I,N} \left\{ \left(C^{\upsilon}(1-N)^{1-\upsilon}\right)^{1-\gamma} + \beta \left(E_{p',\varepsilon',x'} W(K',z',p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \right\}, \qquad (5) \\ s.t. &: \\ C+I &\leq z^{1-\alpha} K^{\alpha} N^{1-\alpha}, \\ K' &= \left((1-\delta)K + \phi\left(\frac{I}{K}\right) K \right) (1 - x' b_k), \\ \log z' &= \log z + \mu + \sigma\varepsilon' + x' \log(1 - b_{tfp}). \end{aligned}$$

(Because we take a power $1-\gamma$ of the value function, if $\gamma > 1$, the max operator must be transformed into a min.) A standard homogeneity argument implies that we can write $W(K,z,p) = z^{\upsilon(1-\gamma)} g(k,p)$, where $k = K/z$, and g satisfies the associated Bellman equation:

$$\begin{aligned} g(k,p) &= \max_{c,i,N} \left\{ \begin{array}{c} c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} \\ +\beta e^{\mu\upsilon(1-\gamma)} \left(E_{p',\varepsilon',x'} e^{\sigma\varepsilon'\upsilon(1-\theta)} \left(1 - x' b_{tfp}\right)^{\upsilon(1-\theta)} g(k',p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\}, \qquad (6) \\ s.t. &: \\ c+i &= k^{\alpha} N^{1-\alpha}, \\ k' &= \frac{(1 - x' b_k)\left((1-\delta)k + \phi\left(\frac{i}{k}\right) k\right)}{e^{\mu + \sigma\varepsilon'}\left(1 - x' b_{tfp}\right)}. \end{aligned}$$

Here $c = C/z$ and $i = I/z$ are consumption and investment detrended by the stochastic technology level z. This homogeneity argument simplifies the problem substantially. It delivers some analytical results, and makes the numerical analysis simpler: first, k is stationary; second, the dimension of the state space is reduced.

3.3 Asset Prices

It is straightforward to compute asset prices in this economy. The stochastic discount factor is given by the formula

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\upsilon(1-\gamma)-1} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\upsilon)(1-\gamma)} \left(\frac{V_{t+1}}{E_t\left(V_{t+1}^{1-\theta}\right)^{\frac{1}{1-\theta}}} \right)^{\gamma-\theta}. \qquad (7)$$

The price of a one-period risk-free bond is $E_t\left(M_{t,t+1}\right)$, but this risk-free asset may not have an observable counterpart. Following Barro (2006), I will assume that government bonds are not risk-free but are subject to default risk during disasters.[15] More precisely, if there is a disaster, the recovery rate on government bonds is r, i.e. the loss is $1-r$. The T-Bill price can then be easily computed as $Q_{1,t} = E_t\left(M_{t,t+1}(1 + x_{t+1}(r-1))\right) \stackrel{def}{=} Q_1(k,p)$. The ex-dividend value of the firm assets P_t is defined through the value recursion:

$$P_t = E_t\left(M_{t,t+1}\left(D_{t+1} + P_{t+1}\right)\right),$$

where $D_t = F(K_t, z_t N_t) - w_t N_t - I_t$ is the payout of the representative firm, and w_t is the wage rate, given by the marginal rate of substitution of the representative consumer between consumption and leisure. The equity return is then $R_{t,t+1} = \frac{D_{t+1}+P_{t+1}}{P_t}$. If the positivity constraint on investment does not bind, the unlevered equity return can be rewritten, following a standard Q-theory argument (See Jermann (1998) or Kaltenbrunner and Lochstoer (2008)) as

$$R_{t,t+1} = \left(1 - x_{t+1} b_k\right) \phi'\left(\frac{I_t}{K_t}\right)\left(\frac{1-\delta+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)}{\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)} + \frac{\alpha K_{t+1}^{\alpha} z_{t+1}^{1-\alpha} N_{t+1}^{1-\alpha} - I_{t+1}}{K_{t+1}}\right), \tag{8}$$

where the first term emphasize that if $b_k > 0$, capital holders make a loss in the event of a disaster.

The empirical counterpart to this unlevered equity return is not stock returns, because in the real world, firms have financial leverage and operating leverage (e.g. fixed costs and labor contracts). This is a substantial source of profit and dividend volatility, which is not present in the model. Under the Modigliani and Miller theorem, in the absence of financial friction or taxes, the only effect of leverage is to modify the payout process and subsequently the asset prices. Rather than model the leverage explicitly, I follow the asset pricing literature (e.g. Abel (1999)) and compute the price of a claim to $D_t^{\text{lev}} = Y_t^{\lambda}$, where $\lambda > 1$ is the leverage parameter. This formulation implies that $\Delta \log D_t^{\text{lev}} = \lambda \Delta \log Y_t$, making dividends more volatile than output, as in the data. I will use the price of this levered claim to output as the model counterpart to stock prices. In section 5.1, I show that this formulation of leverage gives similar results to a formulation based on a constant debt-equity ratio.

3.4 Analytical results

This section proves some analytical results in the special case $b_k = b_{tfp}$, i.e. productivity and capital fall by the same amount if there is a disaster. Under this assumption, we first establish

[15] Empirically, default often takes the form of high rates of inflation which reduces the real value of nominal government debt.

the behavior of quantities and returns following a disaster. Then we establish the equivalence between disaster risk and a change in impatience (discount factor). All these results stem directly from equation (6).[16]

Proposition 1 *Assume that $b_k = b_{tfp}$. Then, a disaster leads consumption, investment, and output to drop by a factor $b_k = b_{tfp}$, while hours do not change. The return on capital is also reduced by the same factor, while the return on government bonds is reduced by a factor r. There is no further effect of the disaster on quantities or prices, i.e. all the effect is on impact.*

Proof. Equation (6) leads to policy functions $c(k,p), i(k,p), N(k,p)$ and $y(k,p) = k^{\alpha} N(k,p)^{1-\alpha}$ which express the solution as a function of the probability of disaster p (the exogenous state variable) and the detrended capital k (the endogenous state variable). The detrended capital evolves according to the shocks ε', x', p' through

$$k' = \frac{(1 - x' b_k)\left((1-\delta)k + \phi\left(\frac{i(k,p)}{k}\right)k\right)}{e^{\mu + \sigma\varepsilon'}\left(1 - x' b_{tfp}\right)}.$$

The key remark is that if $b_k = b_{tfp}$, then

$$k' = \frac{\left((1-\delta)k + \phi\left(\frac{i(k,p)}{k}\right)k\right)}{e^{\mu + \sigma\varepsilon'}}$$

is independent of the realization of disaster x'. As a result, the realization of a disaster does not affect c, i, N, y, since k is unchanged, and hence it leads consumption $C = cz$, investment $I = iz$, and output $Y = yz$ to drop by a factor $b_k = b_{tfp}$ on impact. Furthermore, once the disaster has hit, it has no further effect since all the endogenous dynamics are captured by k, which is unaffected. The statement regarding returns follows from the expression of the stock return (8): given that the investment-capital ratio and output-capital ratios are unaffected by the disaster, the only effect of the disaster is to multiply $R_{t,t+1}$ by the factor $(1 - b_k)$. ■

The intuition for proposition 1 stems directly from the condition for the steady-state of the neoclassical growth model, which is determined by the level of TFP according to the familiar formula $\frac{1}{\beta} - 1 + \delta = \alpha K^{\alpha-1}(Nz)^{1-\alpha}$. Given the preference specification, the steady-state hours are unaffected by the change in TFP. The decrease in z hence requires an equal decrease in K to reach a steady-state. When $b_k = b_{tfp}$, the amount of capital destruction is exactly what is

[16] An alternative derivation, using the Euler equation, is provided in the appendix.

required for the economy to jump from one steady-state to another steady-state, and there are no further transitional dynamics.

In contrast, when $b_k \neq b_{tfp}$, a disaster leads both to impact effects and to further transitional dynamics. For instance, a capital destruction without reduction in productivity leads to high investment and a recovery as the economy converges back to its initial steady-state. Inversely, a productivity decline without capital destruction leads to a persistently low level of investment as the economy adjusts gradually to reach its new steady-state.

We can now state the first main result.

Proposition 2 *Assume that the probability of disaster p is constant, and that $b_k = b_{tfp}$. Then the policy functions $c(k)$, $i(k)$, $N(k)$, and $y(k)$ are the same as in a model without disasters ($p = 0$), but with a different time discount factor $\beta^* = \beta(1 - p + p(1 - b_k)^{\upsilon(1-\theta)})^{\frac{1-\gamma}{1-\theta}}$. Assuming $\theta \geq 1$, we have $\beta^* \leq \beta$ if and only if $\gamma < 1$. Asset prices and expected returns, however, will be different under the two models.*

Proof. Following proposition 1, note that k' is independent of the realization of disaster x'. As a result, we can simplify the expectation in the Bellman equation (6):

$$g(k) = \max_{c,i,N} \left\{ \begin{array}{c} c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} \\ +\beta e^{\mu\upsilon(1-\gamma)} \left(E_{x'}\left(1 - x' b_{tfp}\right)^{\upsilon(1-\theta)} \times E_{\varepsilon'} e^{\sigma\varepsilon'\upsilon(1-\theta)} g\left(k'\right)^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\},$$

i.e.:

$$g(k) = \max_{c,i,N} \left\{ c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} + \beta^* e^{\mu\upsilon(1-\gamma)} \left(E_{\varepsilon'} e^{\sigma\varepsilon'\upsilon(1-\theta)} g\left(k'\right)^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \right\}.$$

We see that this is the same Bellman equation as the one in a standard neoclassical model, with discount rate β^*. As a result, the policy functions $c(k), N(k), i(k)$ and $y(k)$ are also the same as a standard neoclassical model.

Asset prices, on the other hand, are driven by the stochastic discount factor, which weights the possibility of disaster (see the expression of the SDF in the computational appendix). Both consumption and the return on capital are low in a disaster as show in Proposition 1, hence the equity premium will be larger than in a model without disaster risk. ■

This result has several implications. First, in a sample without disasters, the quantities implied by the model (consumption, investment, hours, output and capital) are exactly the same as those implied by the standard RBC model, provided that the discount factor is adjusted. In practice, this adjustment is small and hence has very little effect on quantity dynamics. For the benchmark calibration, we have $\beta = .994$, and $\beta^* \approx .9934$. As a particular implication, the response to a standard normal TFP shock ε will be exactly the same, hence the model will generate the standard patterns of higher investment, output, employment and consumption following an increase in TFP.

Second, this analytical result clarifies the numerical findings of Tallarini (2000). As discussed in the introduction, he found, in a model where the IES is unity, that higher risk aversion has little effect on business cycle quantity dynamics (a finding often interpreted as "fixing the asset pricing properties of a RBC model need not change the quantity dynamics"). In my model, if the IES is unity ($\gamma = 1$), β^* is exactly equal to β, hence no adjustment is required and the equivalence of dynamics is an exact result. The model nevertheless generates a large equity premium, since a disaster leads to a large decline in consumption and in the equity return. This proposition hence shows how to construct a model with large risk premia and reasonable business cycle dynamics, addressing the question studied by Jermann (1998) and Boldrin, Christiano and Fisher (2001).

Third, the result implies that the steady-state level of capital stock will be affected by the probability of disaster. If risk aversion θ is greater than unity, and the IES is above unity, then $\beta^* < \beta$, leading people to save less: the steady-state capital stock is lower than in a model without disasters. While higher risk to productivity leads to higher precautionary savings, rate-of-return risk can reduce savings.

While this first result is interesting, it is not fully satisfactory however, since the constant probability of disaster implies constant risk premia. As is well known, constant risk premia imply that price-dividend ratios and returns are not volatile enough. This motivates extending the result for a time-varying p.

Proposition 3 *Assume that $b_k = b_{tfp}$, and that p follows a stationary Markov process. Then the policy functions $c(k,p)$, $i(k,p)$, $N(k,p)$, and $y(k,p)$ are the same as in a model without disasters ($p = 0$), but with stochastic discounting (i.e. β follows a stationary Markov process). Assuming $\theta \geq 1$, β is inversely related to p if and only if $\gamma < 1$.*

Proof. The proof also uses the fact that k' is independent of x', to simplify the expectation inside the Bellman equation (6):

$$g(k,p) = \max_{c,i,N} \left\{ \begin{array}{c} c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} \\ +\beta e^{\mu\upsilon(1-\gamma)} \left(E_{x'|p} \left(1 - x' b_{tfp}\right)^{\upsilon(1-\theta)} E_{\varepsilon',p'} e^{\sigma\varepsilon'\upsilon(1-\theta)} g\left(k',p'\right)^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\}.$$

Define

$$\beta(p) = \beta\left(E_{x'|p}\left(1 - x'b_{tfp}\right)^{\upsilon(1-\theta)}\right)^{\frac{1-\gamma}{1-\theta}} = \beta\left(1 - p + p(1 - b_{tfp})^{\upsilon(1-\theta)}\right)^{\frac{1-\gamma}{1-\theta}}.$$

Since p is Markov, β is Markov too. Assuming $\theta \geq 1$, β is increasing in p if and only if $\gamma < 1$. We have:

$$g(k,p) = \max_{c,i,N}\left\{c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} + \beta(p)e^{\mu\upsilon(1-\gamma)}\left(E_{\varepsilon',p'}e^{\sigma\varepsilon'\upsilon(1-\theta)}g(k',p')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1-\gamma}{1-\theta}}\right\},$$

i.e. the Bellman equation of a model with time-varying β, but no disasters. ■

This result shows that the time-varying risk of disaster has the same implications for quantities as a preference shock. It is well known that these shocks have a significant effect on macroeconomic quantities (a point that we will quantify later). In a sense, this version of the model breaks the "separation theorem" of Tallarini (2000): when risk varies over time, risk aversion has an effect on the quantities. Asset prices will respond as well, generating correlations of risk premia and quantities.

This result is interesting in light of the empirical literature which suggests that "preference shocks" or "equity premium shocks" may be important (e.g., Smets and Wouters (2003)). Chari, Kehoe and McGrattan (2009) criticize these shocks which lack microfoundations. My model provides a simple microfoundation, which allows to tie these shocks to asset prices precisely, and justifies the label "equity premium shock". Of course, my model is significantly simpler than the medium-scale models of Smets and Wouters (2003), but I conjecture that this equivalence can be generalized to a large class of models.

Interestingly, this result also shows that it is technically feasible to solve DSGE models with time-varying risk premia. A full non-linear solution of a medium-scale DSGE model is daunting. But under this result, we can solve the quantities of the model by solving a model with shocks to β and no disasters, i.e. a fairly standard model which we can approximate well using log-linear methods. Once quantities are found, we can solve for asset prices using nonlinear methods. The computational appendix details this solution method.

The three propositions require that $b_k = b_{tfp}$; analytical results are impossible without this assumption. As discussed above, proposition 1 does not hold if $b_k \neq b_{tfp}$. On the other hand, numerical experiments suggest that proposition 2 is robust to this assumption, in that the dynamic response to a TFP shock is largely unaffected by the presence or type of disasters (i.e. b_k vs. b_{tfp}). Proposition 3 is somewhat more fragile. For instance, if disasters affect only TFP, and there are no adjustment costs, then an increase in p will lead people to want to hold more capital, for standard precautionary savings reasons. This is true regardless of the IES. We discuss this further and relax the assumption $b_k = b_{tfp}$ in Section 5.2.

4 Quantitative Results

This section first presents the calibration. I then successively study the implications of the model for business cycle quantities, for asset prices, and finally for the relations between asset prices and quantities. In general, the model cannot be solved analytically, leading me to resort to a numerical approximation. A nonlinear method is crucial to analyze time-varying risk premia. I use a standard policy function iteration algorithm, which is described in detail in the computational appendix.

4.1 Calibration

Parameters are listed in Table 1. The period is one quarter. Many parameters follow the business cycle literature (Cooley and Prescott (1995)). The risk aversion parameter is picked to replicate the mean equity premium, and it is set at 6. However, this is risk aversion over the consumption-hours bundle. Since the share of consumption in the utility index is .3, the effective risk aversion to a consumption gamble is 1.8 (Swanson (2010)).

The intertemporal elasticity of substitution of consumption (IES) is set at 2. There is a large debate regarding the value of the IES. Most direct estimates using aggregate data find low numbers (e.g. Hall (1988)), but this view has been challenged by several authors (see among others Bansal and Yaron (2004), Guvenen (2006), Mulligan (2004), Vissing-Jorgensen (2002)). As emphasized by Bansal and Yaron (2004), a low IES has the counterintuitive effects that higher expected growth lowers asset prices, and higher uncertainty increases asset prices. The IES plays a key role for only part of my results, namely the response of macroeconomic quantities to an increase in the probability of disaster.

The functional form for the adjustment cost function follows Jermann (1998): $\Phi(x) = a_1 \frac{x^{1-\eta}}{1-\eta} + a_2$, where a_1 and a_2 are set such that the steady-state is independent of η and marginal Q is one. The unique parameter η is set to match approximately the volatility of investment, relative to output, leading to $\eta = .15$, a value well in the range of empirical estimates.[17]

[17] The volatility of investment is limited by general equilibrium feedbacks, as in the RBC model, hence only moderate adjustment costs are required to lower further a bit the volatility of investment.

One crucial element of the calibration is the probability and size of disaster. I assume that $b_k = b_{tfp} = .43$ and the probability is .017 per year on average. These numbers are motivated by the evidence in Barro (2006) who reports this unconditional probability, and the risk-adjusted size of disaster is on average 43%. (Barro actually uses the historical distribution of sizes of disaster. In his model, this distribution is equivalent to a single disaster with size 43%.) In my model, with $b_k = b_{tfp} = .43$, both consumption and output fall by 43% if there is a disaster. Note that since the Solow residual is $z^{1-\alpha}$, the actual drop in productivity is 30.2%.

Whether one should model a disaster as a capital destruction or a reduction in TFP is an important question. Clearly some disasters, e.g. in South America since 1945, or Russia 1917, affected TFP, perhaps by introducing an inefficient government and poor policies. On the other hand, World War II led in many countries to massive physical destructions and losses of human capital. It would be interesting to gather further evidence on disasters, and measure b_k and b_{tfp} directly. This is beyond the scope of this paper. I concentrate on the parsimonious benchmark case $b_k = b_{tfp}$.This has the advantage of clarity, since the analytical results of section 3 apply, and generates the same consumption dynamics during disasters as assumed in the literature that uses endowment economies (Barro (2006), Gabaix (2007), Wachter (2008), Gourio (2008)). In section 5.2, I discuss an alternative calibration with $b_k = 0$, which generates many of the same results, provided that there are capital adjustment costs. Hence the capital destruction is not necessary for the model to match the data well.

The second crucial element is the persistence and volatility of movements in this probability of disaster. I assume that the log of the probability follows an AR(1) process:

$$\log p_{t+1} = \rho_p \log p_t + (1 - \rho_p) \log \overline{p} + \sigma_p \varepsilon_{p,t+1},$$

where $\varepsilon_{p,t+1}$ is *i.i.d.* $N(0,1)$.[18] The parameter $\overline{p}$ is picked so that the average probability is .017 per year, and I set $\rho_p = .92$ and the unconditional standard deviation $\frac{\sigma_p}{\sqrt{1-\rho_p^2}} = 1.85$, which allows the model to fit reasonably well the volatility and predictability of equity returns. Regarding the default of government bonds during disasters, I follow the work of Barro (2006): conditional on a disaster, government bonds default with probability .6, and the default rate is the size of the disaster. The leverage parameters λ is set to 2 (Abel (1999)).

On top of this benchmark calibration, I will also present results from different calibrations

[18]This equation allows the probability to be greater than one, however I will approximate this process with a finite Markov chain, which ensures that $0 < p_t < 1$ for all $t \geq 0$.

(no disasters, constant probability of disaster, and in section 5 more extensions) to illustrate the sensitivity of the results.

Some may argue that this calibration of disasters is extreme. A few remarks are in order. First, a long historical view makes this calibration sound more reasonable, as shown by Barro (2006) and Barro and Ursua (2008). An example is the U.K., which sounded very safe in 1900, but experienced a variety of very large negative shocks during the XXth century. Second, it is also possible to change the calibration, and increase risk aversion[19] while reducing the size or probability of disasters. One can also employ fairly standard devices to boost the equity premium, and reduce the probability of disaster further - e.g., the disasters may be concentrated on a limited set of agents, or some agents may have background risk (private businesses); or idiosyncratic risk might be countercyclical. These features could all be added to the model, at a cost in terms of complexity, and would likely reduce the magnitude of disasters required to make the model fit the data.

4.2 An increase in the probability of a disaster

We can now perform the key experiment of an increase in the probability of disaster, i.e. an increase in risk. Figure 1 plots the impulse response of quantities to a doubling of the probability of disaster at time $t = 6$, starting at its long-run average (.017% per year or 0.00425% per quarter).[20] Investment decreases, and consumption increases, as in the analytical example of section 2, since the elasticity of substitution is assumed to be greater than unity. Employment decreases too, through an intertemporal substitution effect: the return on savings is low and thus working today is less attractive. (This is in spite of a negative wealth effect which tends to push employment up; given the large IES the substitution effect overwhelms the wealth effect both for consumption and for leisure.) Hence, output decreases because both employment and the capital stock decrease, even though there is no change in current or future total factor productivity. This is one of the main result of the paper: this shock to risk leads to a recession. After impact, consumption starts falling. These results are robust to changes in parameter values, except for the IES which crucially determines the sign of the responses, and the assumption that $b_k = b_{tfp}$ (as we

[19] The risk aversion in my calibration less than two, and hence even lower than in Barro (2006), because the variation over time in the probability of disaster is an additional source of risk.

[20] For clarity, to compute this figure, I assume that there are no realized disaster. The simulation is started of after the economy has been at rest for a long time (i.e. no realized disasters, no normal shocks, and no change in the probability of disaster). I obtain this figure by averaging out over 100,000 simulations which start at $t = 6$ in the same position, but then have further shocks to ε or p.

discuss in section 5.2 below). The size of adjustment costs, and the level of risk aversion, affect only the magnitude of the response of investment and hours. This figure is consistent with proposition 3: the shock is equivalent, for quantities, to a preference shock to β. The model predicts some negative comovement between consumption and investment, which may seem undesirable.[21] I discuss this further in Section 5.4.

Regarding asset prices, figure 2 reveals that, following the shock, the risk premium on equity increases (the spread between the red–crosses line and the black-full line becomes larger), and the short rate decreases, as investors try to shift their portfolio towards safer assets - a "flight to quality". Hence, in the model, an increase in risk premia coincides with a recession. On impact (at $t = 6$), the increase in the risk premium lowers equity prices substantially, through a discount rate effect.

4.3 First and second moments of quantities and asset returns

Table 2 reports the standard business cycle moments obtained from model simulations. Results are reported both for a sample where no disaster actually takes place (i.e. agents fear a disaster but it does not occur in sample), and, in the starred rows, for a full sample that includes disaster realizations (i.e. population moments). The data row reports the standard U.S. post-WWII statistics. Given the lack of disasters in these data, one should compare the data to the model results in a sample without disasters.

Row 2 shows the results for the standard model (i.e. $b_k = b_{tfp} = 0$). The success of the basic RBC model is clear: consumption is less volatile than output, and investment is more volatile than output. The volatility of hours is on the low side, a standard defect of the basic RBC model driven by the specification of the utility function and adjustment costs.

Introducing a constant probability of disaster, in row 3, does not change the moments significantly. This is consistent with proposition 2. However, the presence of the risk shock - the change in the probability of disaster - leads to additional dynamics, which are visible in row 5. Specifically, the correlation of consumption with output is reduced. Total volatility increases, since there is an additional shock, but this is especially true for investment and employment. Overall the model gets closer to the data for most moments, except the relative volatility of investment which is slightly too high.

[21] Despite the fact that consumption rises on impact, states of nature with high probability of disaster are still "bad states", i.e. high marginal utility states. This is because the stochastic discount factor also includes current hours and future utility, and the higher uncertainty reduces the future value due to risk aversion (i.e. volatility is a priced factor; see e.g. Bansal and Yaron (2004) for a related analysis).

Turning to returns, table 3 shows that the benchmark model (row 4) can generate a large equity premium: about 6% (=4*(1.93-0.42)) per year for a levered equity (the model counterpart to real stocks). The unlevered equity also has a significant risk premium of 1.8% per year. These risk premia are computed over short-term government bonds, which are not riskless in the model; they would be larger if computed over the risk-free asset. Whether these risk premia are calculated in a sample with disasters or without disasters does not matter much quantitatively - the risk premia are reduced by 15–25 basis point per quarter or 0.6-1% per year. Hence, sample selection is not a critical issue.

Table 3 shows that the volatility of the levered equity approximately matches that of the data (7.14% per quarter vs. 8.14% in the data). This is in sharp contrast with the RBC model (1.59%) or the model with constant probability of disaster (1.53%). Importantly, the model matches the low volatility of short-term interest rates (0.85% vs. 0.81% in the data), an improvement over the studies of Jermann (1998) and Boldrin, Christiano and Fisher (2001) which implied highly volatile interest rates.

For completeness, it is important to note that an unlevered equity does not have volatile returns, however (0.40% per quarter). The intuition is that, without adjustment costs, Tobin q is unity, and the return on capital is simply $1 - \delta + \alpha K_{t+1}^{\alpha-1}(z_{t+1}N_{t+1})^{1-\alpha}$, which is very smooth. My calibration has only a small amount of adjustment costs, hence Tobin q varies little and the return on unlevered capital is smooth.[22]

4.4 Relations between asset prices and macroeconomic quantities

This section evaluates the ability of the model to reproduce some relations between asset prices and macroeconomic quantities.

[22] This foonote describes, for completeness, the implications of the model for the term structure of interest rates. Because the model does not incorporate inflation, it is difficult to estimate the extent to which the model fits the data. Moreover, these results for the yield curve are similar to those in the endowment economy model of Gabaix (2007). Assume that all bonds default by the same amount during disasters. The model then generates a negative term premium, consistent with the evidence for indexed bonds in the UK. This negative term premium is not due to what happens during disasters, since short-term bonds and long-term bonds are assumed to default by the same amount. As usual, TFP shocks generate very small risk premia. The term premium is thus driven by the third shock, i.e. the shock to the probability of disaster. An increase in the probability of disaster reduces interest rates, as the demand for precautionary savings rises. As a result, long-term bond prices rise. Hence, long term bonds hedge against the shock to the probability of disaster, they have lower average return than the short-term bonds, and the yield curve is on average downward sloping. Obviously, one possibility to make the yield curve upward-sloping is to assume that long-term bonds will default by a larger amount, should a disaster happen.

4.4.1 Countercyclicality of risk premia

An important feature of the data is that risk premia are countercyclical. This has been illustrated strikingly by the recent crisis, where the yield on risky assets such as corporate bonds went up while the yield on safe assets such as government bonds went down. This pattern is common to most U.S. recessions. To illustrate it simply, figure 3 reports the covariance between detrended output $\widetilde{y}_t$ and stock excess returns at different leads and lags, i.e. $Cov\left(\widetilde{y}_t, R^e_{t+k} - R^f_{t+k}\right)$,for $k = -12$ to $k = 12$ quarters. In the data (full line), this covariance is positive for $k < 0$, reflecting the well-known fact that excess returns lead GDP, but this covariance becomes negative for $k \geq 0$, implying that output negatively leads excess returns, i.e. *risk premia are lower when output is high.*[23] I concentrate on the covariance rather than the correlation because the size of the association is critical (correlations can look good even if there is only a tiny variation, provided it has the right sign). GDP is detrended using the one-sided version of the Baxter-King (1999) filter.

The fact that returns lead GDP, while interesting, might be rationalized by several models, e.g. a model of advance information and adjustment costs or time-to-build. More simply, as can be seen in the figure 3, even the basic RBC model generates this pattern, since high returns reflect positive TFP shocks, and positive TFP shocks lead to a period of above-trend output. More interesting, and more discriminating, is the right-side of this picture, i.e. high output is associated with low future excess returns. The model without shocks to p, i.e. the RBC model, does not generate any variation in risk premia, so the model-implied covariance is very close to zero.[24] In contrast, my model generates about the right comovement of output and risk premia. This is a validation of the model key mechanism: changes in risk drive both expected returns and output.

4.4.2 VIX and GDP

The VIX index is a measure of the implied volatility of the SP500, constructed by the CBOE from option prices with different strikes. Mathematically, it is defined as $\sqrt{4var^Q_t\left(r^m_{t+1}\right)}$, where the

[23] I use this particular statistic because it has a natural model counterpart. There are other, more powerful ways to show in the data that risk premia are countercyclical. First one can use additional variables, not present in the model: e.g. the unemployment rate forecasts excess stock returns negatively. Second, one can use a standard return forecasting regression, i.e. running future returns on the current dividend yield, the short rate and the term spread, and observe that the fitted values from this regression are significantly negatively correlated with detrended GDP.

[24] It is important to use a one-sided filter for this purpose, since with a two-sided filter output is low when future output is high, i.e. in the RBC model when future TFP is high, i.e. when future returns are high: hence, the RBC model generates a negative covariance between two-sided filtered output and future excess returns.

variance is taken under the risk-neutral measure, and the factor 4 annualizes the variance. In an influential study, Bloom (2009) shows using a reduced-form VAR that shocks to the VIX index have a significant negative effect on output. Figure 4 reproduces this results by depicting the impulse response of output to a shock to VIX in the data (full blue line). This impulse response function is computed using a Cholesky decomposition, under the orthogonalization assumption that a shock to VIX has no instantaneous impact on GDP.[25]

Running the same VAR on the model-generated data yields a response that is fairly similar to the data (red crosses). In the model, VIX is largely driven by the fear of a disaster, i.e. VIX is nearly one-to-one with the state variable p. Increases in p lead to an increase in VIX and a decline in output. Hence, the model generates an impulse response consistent with the data. In contrast, in a real business cycle model without disaster risk, VIX is small and nearly constant, and the VAR finds actually a positive effect of VIX on output.

4.4.3 Investment and Asset Prices

One enduring puzzle in macroeconomics and finance is the relation between investment and the stock market. While the Q-theory correctly predicts a positive correlation, the level of adjustment costs required to match the investment and the stock market is widely considered excessive (see e.g. Philippon (2009) for a recent discussion). In contrast, I show here (see also section 6) that my model captures well the magnitude of the relation between the stock market and investment, even with small adjustment costs.

One way to measure this association is to compute the covariance between investment and asset prices in the RBC model. Figure 5 presents the cross-covariogram $\gamma_k = Cov(i_{t+k}, \log(P_t/D_t))$,where i_{t+k} is HP-filtered log investment, for $k = -12$ to $k = 12$ quarters.

The black (diamonds) line shows the data, reflecting the well-known pattern that investment and the stock market are positively correlated, with the stock market leading investment. The blue line (crosses) presents the covariogram for the model with only TFP shock, i.e. the basic RBC model. The model generates actually a small negative covariance between the price-dividend ratio and investment, because TFP shocks have little effect on the stock market value - higher TFP increase future cash flows, but also increases interest rates, leading to offsetting effects on the levered equity.

[25] The orthogonalization assumption has little impact on these results. Following Bloom, both GDP and volatility are HP-filtered, but this is not critical either. Last, in the model, as in the data, it makes relatively little difference whether we use the implied volatility, based on the risk-neutral measure, or the physical volatility, $\sqrt{4var_t\left(r_{t+1}^m\right)}$.

The red line (diamonds) the covariogram for the benchmark model, i.e. including both TFP shocks and shocks to the probability of disaster. The key result is that the covariance is now of the right magnitude. Both models are, however, unable to replicate the exact timing of the association between the stock market and investment, i.e. to match the observed lag, but additional frictions such as time-to-plan may account for this.

4.4.4 IES estimation in the model

Similar to Bansal and Yaron (2004), my model requires an elasticity of intertemporal substitution (IES) larger than unity, which is at odds with traditional estimates such as Hall (1988). While it is impossible to do justice to the vast literature studying the IES, it is noteworthy that running the standard Hall regressions in my model would lead to small estimates, just like Hall found. Table 4 reports the slope and R2 from a univariate regression of consumption growth from t to t+1 on the time t short-term government bond rate (first two columns) or the pure risk-free rate (last two columns). The slope coefficient is the IES estimate. Each row corresponds to a variant of the model, which differ according to the utility function (power utility or Epstein-Zin), whether labor supply is fixed or labor is part of the utility function (which modifies the usual Euler equation), and whether the model has only shocks to TFP (no disaster risk) or the model has both TFP shocks and shocks to disaster risk.

In the simple RBC model (row 1), the Hall regression works well and suggests an IES of 1.76, which is close to the true value. If leisure is introduced in the utility function, the IES estimate recovered is already lower, around 1.00. Adding shocks to disaster risk disturbs the relationship between expected consumption growth and interest rates, because higher uncertainty leads (if the IES is greater than unity) to lower interest rates and higher expected consumption growth. As a result, the slope coefficient falls to 0.38 if one uses the short-term government bond rate, and 0.23 if one uses the pure risk-free asset. Moreover, using the equity return would lead to a slightly negative relationship. The results of this highly stylized model suggest that the IES used in this paper is not inconsistent with the empirical evidence.

4.4.5 Other asset pricing implications

The model has several other interesting implications for asset prices, which I describe briefly in this section. First, the model is consistent with the evidence that equity returns are predictable, but dividends are not. The standard regression

$$R^e_{t\to t+k} - R^f_{t\to t+k} = \alpha + \beta \frac{D_t}{P_t} + \varepsilon_{t+k},$$

yields a $R^2 = 25\%$ at the four-year horizon ($k = 4$) in the data (this figure, however, is sample-specific), and a $R^2 = 54\%$ in the model. In both data and model, the results are similar if the left-hand side variable is returns rather than excess returns. Second, in the model as in the data, dividends are much less forecastable than returns, i.e. in a regression

$$\frac{D_{t+k}}{D_t} = \alpha + \beta \frac{D_t}{P_t} + \varepsilon_{t+k},$$

the R^2 is 1% in the data at a four-year horizon, while it is 6% in the model. Hence, there is somewhat too much predictability of returns in the model, but the model is consistent with the finding that discount rate variation is the key driver of stock prices.

Third, the model generates an Euler equation error. A large literature has concentrated on the ability of models to generate a significant equity premium and volatile returns. Lettau and Ludvigson (2009) argue that a more challenging test is to generate a failure of the Euler equation, i.e. estimating the Euler equation with CRRA utility on data simulated from the model should lead, as in the data, to a rejection of the model. These author show that few models can pass this test, because in most models aggregate consumption is highly correlated with returns. In my model, the shock to the probability of disaster induces a negative comovement between asset returns and aggregate consumption, leading the CRRA model to be rejected.

Fourth, the model can generate qualitatively, but not quantitatively, results similar to those of Beaudry and Portier (2006, BP hereafter), which have stimulated a large recent literature on "news shocks". BP show empirically that shocks to the stock market lead to a gradual increase in TFP and GDP, suggestive of advance information about cash flows. An alternative interpretation of their findings is that the stock market movements are driven by changes in risk premia, which then feed back to GDP (and possibly in measured TFP through variation in utilization). To evaluate this possibility, I ran the same bivariate VAR with GDP growth and the stock market return in the data, in the RBC model, and in my model.[26] The impulse response are similar to BP, i.e. a "return shock" leads to a cumulative increase in GDP, however the magnitude of the

[26] Following BP, I use the orthogonalization assumption that the stock market shock does not affect GDP instantaneously. Obviously in my model this assumption is incorrect, hence the shocks picked by the VAR are combinations of the fundamental shocks, of opposite effect on output.

response is much smaller in my model than in the data.

5 Robustness and Extensions

In this section, I discuss several extensions of the baseline model, and check that the results are robust to various changes in the calibration.

5.1 A calibration with financial leverage

The benchmark model follows a formulation of leverage which is standard in the asset pricing literature, i.e. the dividend process is $D_t = Y_t^{\lambda}$ with $\lambda = 2$. One may worry that the nonlinearity is important. To check this, I computed the return on a levered equity, given an exogenous debt issuance policy.[27] Since the Modigliani-Miller theorem holds, the debt policy has no impact on the allocation. Assume that the firm each period adjusts its debt issues to keep the maturity equal to 5 years, and the book leverage ratio equal to 0.45 (Abel (1999), Barro (2006)). The expression for the levered firm return is

$$R_{t+1}^{\text{lev}} = \frac{P_{t+1} + D_{t+1} - \omega_t Q_{t+1}^{(n-1)}}{P_t - \omega_t Q_t^{(n)}},$$

where $Q_t^{(n)}$ is the price of a zero-coupon n-period bond, and ω_t is the number of bonds issued, e.g. for a constant book leverage policy $\omega_t Q_t^{(n)} = .45 K_t$. The mean return on the levered equity is then 2.25% per quarter, while the standard deviation of the return is 9.46%. This contrasts with 1.93% and 7.14% in my simple formulation of leverage. Moreover, in simulations, the two returns have a correlation above .95. Hence, the results are very similar if I use this formulation of leverage. Alternatively, one can assume that the firm keeps the market leverage ratio constant. In the benchmark model, the market value of the firm is somewhat more volatile than its capital stock, due to adjustment costs, but this effect is not very large, hence the results are similar (2.23% for mean return and 9.37% for volatility of return).

[27] I assume that the debt has the same default characteristics as government debt, i.e. it will default in a disaster, but by less than the capital stock. The results are stronger if the debt is truly risk-free. An interesting extension of the model is to make default endogenous.

5.2 A calibration without capital destruction

An interesting question is whether one should model a disaster as a reduction in TFP or a destruction of the existing capital stock.[28] Decreases in TFP arise for instance because of poor government policies or extreme misallocation, while destructions of the capital stock can be due to wars or expropriations (see the discussion in section 2). Tables 4 and 5 study the sensitivity of the key results to this assumption, and propose a different parametrization of the model without capital destruction which gives results similar to the benchmark. In tables 4 and 5, I keep the parameters as in the benchmark, except for those noted in the first column.

First, note that a calibration with only capital destruction and no TFP decline does not fit the data well (row 6). Business cycle statistics are, to a first order, similar to the benchmark model, but the equity premium is small and returns are not volatile. Intuitively, a disaster does not impact agents much in this economy, because capital share is only one-third, and hours increase following the disaster, thereby limiting the initial decline in output, and moreover the economy returns fairly quickly to its steady-state. Moreover, with recursive utility, agents take into account their future (high) consumption and hence do not mind disasters all that much.[29]

Second, when disasters affect solely TFP, and there are no adjustment costs (row 3), the model generates a sizeable equity premium, and volatile returns. The business cycle statistics, however, imply too much volatility of investment and the correlation of consumption and investment and output is negative, contrary to the data. There is also a qualitative change: an increase in the probability of disaster now leads to an increase in the capital stock for precautionary savings reasons. As a result, in this case, and regardless of the IES, an increase in the probability of disaster leads to a boom in investment and output, i.e. the sign of the impulse responses depicted in figure 1 are reversed.

Adding adjustment costs can however undo this effect. Intuitively, with adjustment costs, the price of capital will fall significantly if a disaster occurs. Hence investing in capital is now more risky when the disaster probability rises, generating rate of return uncertainty (as discussed in section 2). In row 4, I use the benchmark level of adjustment costs ($\eta = .15$), and in row 5 I use a higher value ($\eta = .5$). These calibrations now imply that a rise in the probability of disaster leads to a recession. The equity premium is high (about 4% per year) and returns are volatile

[28] In the benchmark, I assumed that a disaster affects TFP and the capital stock equally. This generates a responses of consumption following a disaster which is the same as the endowment economy literature (e.g. Rietz (1988), Barro (2006)).

[29] Following Gourio (2008), a low IES would make the equity premium larger, but this would also make increases in p lead to booms, generating unrealistic correlations between asset prices and quantities.

(over 6% per quarter). As the degree of adjustment costs is increased, the volatility of investment becomes close to the data, and the negative correlation of consumption with output or investment is overturned. Overall I conclude that a calibration without capital destruction can be successful, provided that adjustment costs are large enough.

5.3 Disaster Dynamics

As pointed out by Constantinides (2008) and Barro et al. (2009), disasters have more complicated dynamics in the real world than the pure jump typically assumed. First, disasters may last several years. Second, a recovery might then follow. This leads me to consider two variations on the model to study how these features affect my results. First, I consider disasters which last more than one period. Assume that a disaster leads only to a 20% drop in both productivity and the capital stock. However, a disaster also makes the probability of a disaster next period increase to 50%. Next period, either a disaster occurs, in which case the probability of a further disaster remains at 50%, or it doesn't, in which case this probability shifts back to a standard value. The last row of tables 4 and 5 shows the impact of this modification on the results. First, the business cycle dynamics are largely unaffected. Second, while the disaster is substantially smaller, the model still generates a high equity premium and volatile returns (though a bit less than in the data or benchmark). In this version of the model, a disaster initially leads to a large drop of investment, and a smaller drop in consumption, due to the very high risk of a further disaster which leads people to cut back on investment. Moreover, asset prices fall further during the disaster, since they are hit both by the realization of a disaster today and by the fear of another disaster tomorrow. The key lesson from this illustrative computation is that adding some fear of further disasters is a very powerful ingredient.

Second, I study how the results are affected by the presence of recoveries. More precisely, assume that following a disaster, there is a probability of recovery, i.e. an upward jump of capital and TFP which brings these quantities back to the initial trend. Following a disaster, one of three things can happen. Either there is a recovery right away (with probability 10%); there is no recovery (with probability 20%); or the recovery is uncertain (with probability 70%). When the recovery is uncertain, a new draw next period determines if there is a recovery, or not, or if it is still uncertain; and so on. Overall this leads to a recovery with probability 50%, with an uncertain timing. This is roughly in line with the estimates of Barro et al. (2009). The results of this model are shown in row 7. The business cycle statistics show somewhat less volatility than the benchmark. The equity premium and return volatility are also somewhat smaller than in the benchmark model, but they remain significant. Summarizing, the results of this section show that the model results are weakened, but not in a dramatic way,[30] when disasters are modeled in a more realistic way.

5.4 Comovement of consumption and investment

An implication of the model that may seem odd is that, when the probability of disaster rises, consumption initially increases, while output, employment and investment fall. Given the high IES, the wealth effect is overwhelmed by the substitution effect, hence hours go down and consumption goes up. More generally, given that productivity does not change, and the capital stock is predetermined, the labor demand schedule (marginal product of labor) is unchanged on impact, and, as explained by Barro and King (1984), this makes it impossible to generate on impact positive comovement between consumption and hours worked.[31]

It is not clear that this lack of impact comovement is necessarily a deficiency of the model. Consumption, investment, and hours are far from perfectly correlated in the data (see Table 2), which means that any model needs a shock which pushes consumption and investment in opposite directions sometimes. Indeed, we see in Table 2 that adding the shock to the probability of disaster brings the model closer to the data regarding the correlations. Moreover, while the impact response displays negative comovement, consumption eventually falls, although after a long delay.

Alternatively, there are some extensions of the model which may overturn this result. Here I discuss intuitively some possibilities. The most simple extension is simply to assume that the shocks to the probability of disaster and the TFP shocks are correlated. Indeed, we observe that during recessions, investors and workers seem to fear terrible outcomes. This can also be generated endogenously through a learning mechanism as follows. Disasters are modeled for convenience as jumps, but in reality the contraction is not instantaneous. As a result, it is sometimes difficult for consumers and firms to determine if a decrease in productivity and output is a standard recession

[30] In these experiments I have kept all parameters fixed as I changed the process for disasters, to illustrate the effect of disaster dynamics. Needless to say, to put the models on an equal footing, I could have recalibrated all the parameters, which would allow me to improve significantly the match of the data, e.g. by having slightly higher risk aversion.

[31] This issue is shared by many other papers which incorporate either a shock to preferences (e.g. Smets and Wouters 2008) a shock to investment good prices (e.g. Justiniano, Primiceri and Tambalotti 2010), a shock to micro-uncertainty (e.g. Gilchrist, Ortiz and Zakrajsek (2009), or a shock to financial frictions (e.g. Hall (2009)). These models adopt the last proposed solution to the comovement puzzle, namely sticky prices (except Hall (2009) who assumes that markups are countercyclical).

(a shock ε in our setup) or is the start of a large depression. A large decrease in productivity, due either to a disaster or to a large negative shock ε, will then lead agents to anticipate a further decrease next period. Because consumption reacts negatively to a standard TFP shock, this learning mechanism would attenuate the comovement puzzle.

Alternatively, it may be possible to employ non-standard preferences or adjustment cost formulations, as in Jaimovich and Rebelo (2009) for instance.

Finally and probably more interestingly, countercyclical markups may alter this result. Suppose one were to embed the model in a standard New Keynesian framework with sticky prices, which generates countercyclical markups endogenously. A perfect monetary policy could replicate the flexible price allocation, i.e. the results of this paper. In this case, an increase in the probability of disaster would require the central bank to decrease short-term interest rates. If, for some reason, monetary policy is not accommodative enough, or it is impossible to decrease interest rates because of the zero lower bound, then consumption would have to adjust. Since the real interest rate is too high, consumption would fall. This intuition suggests that this (very substantial) extension of the model may resolve the comovement puzzle.[32]

5.5 Government policy

In the model, the welfare theorems hold, implying no role for government policy. However, it is tempting to "offset" the time-varying wedge in the Euler equation created by the volatility in the probability of disaster. In the case where $b_k = b_{tfp}$, a simple policy can achieve this goal: the government commits to bail out capital holders. That is, the government provides a subsidy to capital holders, proportional to their holdings of capital, in the event of a disaster. If the government can finance this policy with lump-sum taxes, the agents' decisions are the same as in a model without disasters. The intuition for this result (proved in the appendix) can be easily seen in the model without adjustment costs and expected utility, where the return on capital is

$$R_{t+1}^K = (1 - x_{t+1} b_k) \left(1 - \delta + F_1 \left(K_{t+1}, z_{t+1} N_{t+1}\right)\right),$$

and the presence of disaster risk affects the equilibrium only through the Euler equation

$$U_1(C_t, N_t) = \beta E_t \left(R_{t+1}^K U_1(C_{t+1}, N_{t+1})\right).$$

[32] I thank Emmanuel Farhi for this suggestion.

Suppose now that the government engineers a state-contingent subsidy, so that the return to capital holders is $R_{t+1}^K (1 + x_{t+1}\zeta)$, where ζ is the amount of subsidy, to be determined. The equilibrium is then characterized by the equation

$$U_1(C_t, N_t) = \beta E_t ((1 - \delta + F_1 (K_{t+1}, z_{t+1}N_{t+1})) (1 - x_{t+1}b_k) (1 + x_{t+1}\zeta) U_1(C_{t+1}, N_{t+1})) . \quad (9)$$

Assuming that the utility function takes the form $U(C, N) = \frac{C^{1-\gamma}}{1-\gamma}v(N)$, it is easy to see that if $1+\zeta = (1 - b_k)^{\gamma-1}$, the time-varying wedge disappears: changes in p have no effect, since investors anticipate that they will be bailed out should a disaster happen. Hence, this policy reduces overall volatility. But of course this is sub-optimal since investment *should* vary in response to changes in the probability of disaster. This policy thus reduce welfare. This policy is attractive only if agents have incorrect beliefs, and the government wants to maximize the expected utility of the agents under the correct beliefs.

5.6 Time-varying volatility vs. time-varying jump probability

The model concentrates on a specific model of risk, i.e. a jump with a fixed size. However, the results of the paper immediately generalize to a larger class of shock processes. More precisely, let X_{t+1} be a random variable with strictly positive support, which affects both the capital stock and productivity, i.e.

$$K_{t+1} = ((1 - \delta)K_t + \Phi (I_t, K_t)) X_{t+1},$$

and

$$\log z_{t+1} = \log z_t + \mu + \sigma\varepsilon_{t+1} + \log (X_{t+1}),$$

then propositions 2 and 3 of section 2.4 can be restated as: the decision rules are the same as in a model where the agent's discount factor between time t and time $t + 1$, β_t is

$$\beta_t = E_t(X_{t+1}^{1-\theta})^{\frac{1-\gamma}{1-\theta}}, \quad (10)$$

with θ the risk aversion coefficient and γ the inverse of the IES. Hence, while I focused in the paper on the case where X_{t+1} is a binomial variable ($X_{t+1} = 1$ w/prob $1 - p_t$, and $X_{t+1} = 1 - b$ w/prob p_t), alternative stochastic processes generate similar effects. As an example, if X_{t+1} is log-normally distributed with unit mean, i.e. $\log(X_{t+1})$ is $N(-\sigma_t^2/2, \sigma_t^2)$, where σ_t follows an exogenous stochastic process, then

$$\beta_t = e^{-\theta(1-\gamma)\sigma_t^2},$$

and an increase in σ_t reduces β_t if and only if $\gamma < 1$ i.e. the IES is greater than unity, as in proposition 3. The key ingredient of my results is the fact that risk is time-varying, and not its specific distribution.[33] More generally, the expression (10) suggests that higher order moments may matter,[34] and the results of the paper hence readily apply to distributions with excess kurtosis, i.e. "fat tails".

6 The Empirical Importance of Time-Varying Risk

The previous sections show that a simple, parsimonious framework accounts for a variety of business cycle and asset pricing facts. However one may remain skeptical since the probability of disaster is difficult to measure.[35] This section tests the model directly by identifying the probability of disaster from asset prices – a natural approach, since asset prices are measured precisely and the probability of disaster affects them strongly. We then are able to back out the shocks to the probability of disasters, and to compare the data and the time series implied by two models: first, a model without disaster risk, where the only shock is a shock to TFP; second, a model with both TFP shocks and the shock to the probability of disaster.

Specifically, I pick the probability of disaster p_t to match the price-dividend ratio at each date. Following Campbell and Shiller (1988), we know that the stock market movements are largely due to variation in discount rates, which in my model come from variation in p. This motivates my choice of the P-D ratio as a reasonable source of information for p.[36] More precisely, in the model, given a vector of parameters Θ, the price-dividend ratio $\frac{P_t}{D_t}$ is a function of the two state variables, $k_t = \frac{K_t}{z_t}$ (where K_t is the capital stock and z_t is TFP) and of the probability of disaster

[33] The empirical implications are, however, somewhat different since normally distributed shocks occur every period. The calibration would then be different, with a significantly higher risk aversion and smaller shocks, but the effects would be similar in the end.

[34] For instance, Weitzman (2007) shows that this expression may not even be finite for some "natural" distributions. Martin (2008) provides a decomposition using cumulants, and shows in some examples that the higher cumulants (higher moments) can play an important role in this conditional expectation. For my calibration, the second moment (time-varying risk) is the most important. Calibrations which emphasize the risk of even more extreme disasters, however, may imply that the time-varying skewness or kurtosis matters.

[35] One indirect piece of evidence is that estimated DSGE models give a significant role to shocks to β, or to shocks to the relative price of investment goods, which have similar dynamic properties, in accounting for business cycle fluctuations. These estimation results are not based on asset prices data but on quantities alone.

[36] Alternatively, one could use option prices, e.g. the VIX index, to measure p. Since the model fits well the reduced-form relation between GDP and VIX (section 4.3), it is likely that if p is picked to match VIX in the data, the model would also match the GDP data well. Unfortunately, data for VIX are available only since 1986, limiting the power of this test.

p_t :

$$\frac{P_t}{D_t} = \psi\left(k_t, p_t; \Theta\right).$$

Standard data from the BEA lead to estimates of K_t and z_t, hence k_t, from 1948q1 to 2008q4. I then calculate, for each date, the value of the probability of disaster $\widehat{p_t}$ which allows to match exactly the observed price-dividend ratio in the data. Next, I feed this probability of disaster in the model, together with the measured TFP. For instance, the policy functions imply that aggregate investment is $I_t = z_t i(k_t, p_t)$. Finally, the implied series for investment and output are HP-filtered and compared to the data and to the baseline RBC model. The baseline RBC model is constructed using the same measurements of technology z_t and capital k_t, as $I_t = z_t i(k_t)$.[37]

Figure 6 depicts the probability of disaster obtained from this procedure. By construction, this time series is a nonlinear function of the price-dividend ratio $\frac{P_t}{D_t}$ and the detrended capital stock k_t. The short-run fluctuations hence mostly reflect changes in the stock market value. The probability of disaster - a measure of perceived risk - is highly volatile, consistent with the quantitative model. Interestingly, the highest probability of disaster is estimated to occur at the very end of the sample, in the last quarter of 2008.

Figures 7 and 8 present the quantity implications. The first figure presents the data and the RBC model, with the NBER recessions marked as shaded areas. As is well known, the RBC model does a reasonable job at matching macroeconomic aggregates, given the observed path for TFP, until 1985, even if it does not generate quite enough volatility, especially for investment. The second figure plots the data and the model with disaster risk. Comparing the two figures side by side, the difference between the two models is small in "normal times". During recessions however, my model generates a sharper drop in output and especially in investment. For instance, the effect of the 1975 or 1981 recessions on investment are dramatically underpredicted by the RBC model, while my model generates about the right decline in investment. Another episode of special interest is the current recession. Figure 9 "zooms in" on the most recent data. Little happens to TFP in 2008, hence the RBC model does not predict a sharp recession. My model, however, generates a large drop in investment and output through the shock to p_t, i.e. higher perceived risk.

Table 6 summarizes the fit of the models by computing several statistics: first, the correlation and covariance between the data and each model; second, the mean absolute error, i.e.

[37] It makes little difference if the capital stock is not measured, i.e. the time series for I_t are calculated using only an initial estimate of the capital stock K_{1948} and only TFP is fed into the model to construct the path.

$E\left|\text{data}_t - \text{model}_t\right|$.[38] A close look at figure 8 suggests that there is a slight lag between the model and the data, hence I also report these statistics when the model series are lagged by 2 quarters. (There may well be delays to decisions and various adjustment costs which create such a delay, not captured in the simple version of the model.) Finally, I report the same statistics for the subsample of recessions.

The statistics of table 6 are consistent with the discussion of the figures in the previous paragraph. First, without the lag, adding the probability of disaster does not improve much the fit of the RBC model, if at all. Second, taking into account the lag, the model with shocks to p now improves a bit on the RBC model, especially for investment, but also for employment and output: the correlation and covariance of model and data becomes higher (Table 6, columns 3 and 4), and there is a reduction in the mean absolute error. The model does more poorly for consumption, however. Last, the improvement in fit for investment, output and employment is quite significant if one looks at the subsample of recessions. For instance, the correlation between the model investment and the data investment goes from 44.9 to 61.8, and the mean absolute error goes down from 187 to 130 (Table 6 rows 2 and 10, last two columns). The correlation of model and data employment similarly goes from 34.0 to 43.3.

Table 7 shows a measure of volatility, $E\left|x_t\right|$, for $x =$data, RBC model, or RBC model with disaster risk. The table reveals that the model generates higher volatility: for instance, the investment statistic is only 3.74 in the RBC model vs. 9.55 in the data. The model with time-varying risk yields 6.51, a significant improvement. This is especially true in recessions: the volatility of investment or employment conditional on being in a recession almost doubles.

From an intuitive point of view, these result are not very surprising in light of the well-known empirical regularity that the stock market is correlated with GDP and investment. Section 4 shows that the model matches the relation between asset prices and investment. Hence, feeding in asset prices from the data (through p_t) allows the model to improve on quantities by using the empirical explanatory power of the stock market for investment or GDP. Overall, shocks to the probability of disaster, as *restricted* by asset prices data, appear to help the RBC model fit the data, especially during severe recessions, arguably the most interesting episodes.

[38] Here and table 7, I use the mean absolute error, because (unlike the variance) it is meaningful in a subsample with gaps, such as the subsample of recessions.

7 Extension: model with endogenous debt and default

In this long section, I extend the model to allow for endogenous debt (aka leverage) choice, with endogenous debt and equity financing This allows me analyzing the behavior of credit spreads, default probabilities, investment, and the "excess bond premium" as defined in Gilchrist and Zakrajsek (2011).

The capital structure choice modifies the standard RBC model equilibrium in two ways. First, the standard Euler equation is adjusted to reflect that investment is financed using both debt and equity, and the user cost of capital hence takes into account expected discounted bankruptcy costs as well as the tax savings generated by debt finance. Second, an additional equation determines the optimal leverage choice, by equating the marginal expected discounted (tax) benefits and (bankruptcy) costs of debt. The model remains highly tractable and intuitive, which allows to evaluate the role of defaultable debt and leverage choice on quantities and prices in a transparent fashion. In particular, the model encompasses the standard real business cycle model (the benchmark model) as a special (limiting) case.

The first result is that time-varying disaster risk generates large, volatile and countercyclical credit spreads, which are significantly larger than default probabilities. The second main result is that financial frictions amplify substantially – by a factor of about three – the response of the economy to a shock to the disaster probability. This amplification effect does not arise if the economy is subjected to TFP shocks.

The key mechanism is as follows. When the probability of economic disaster exogenously increases, the probability of default rises (holding leverage policy fixed). A higher probability of default directly raises expected discounted bankruptcy costs. However, expected discounted bankruptcy costs also rise through a second channel: agents anticipate that defaults are now more systematic, i.e. more likely to be triggered by a bad aggregate shock rather than a bad idiosyncratic shock. This higher systematic default risk increases the risk premium on corporate debt, making it more expensive to raise funds for investment. Overall, higher expected discounted bankruptcy costs increase the user cost of capital, leading to a reduction in investment. In equilibrium, firms also cut back on debt and substitute for equity, but since debt is cheaper due to the tax advantage, the user cost of capital has to rise. To summarize, higher disaster risk worsens financial frictions because debt is not efficient when disaster risk is high.

Before describing the model, we review briefly the literature on financial frictions and credit spreads.

7.0.1 Related Literature on credit spreads and financial frictions

This extension builds on the large macroeconomic literature studying general equilibrium business cycle models with financing constraints (Bernanke and Gertler (1989) and Kiyotaki and Moore (1997)). Some recent studies in this vein are Chugh (2010), Gomes and Schmid (2008), Jermann and Quadrini (2008), Mendoza (2010), Miao and Wang (2010), and Liu, Wang and Zha (2009). Amdur (2010), Covas and Den Haan (2009), and Hennessy and Levy (2007) study the business cycle behavior of capital structure. The paper is also closely related to Philippon (2009), who demonstrates how to link bond prices an real investment,[39] and to Gilchrist and Zakrajsek (2011), who construct an "excess bond premium" that contains significant macroeconomic information.

This extension studies the real effects of a shock to uncertainty, a channel recently emphasized by Bloom (2009), who emphasize the "wait-and-see" effect driven by lumpy hiring and investment behavior. My model focuses on aggregate uncertainty and lowers desired investment through general equilibrium effects (risk premia) and by exacerbating financial frictions. A related mechanism has recently been explored in the studies of Arellano, Bai and Kehoe (2010) and Gilchrist, Sim and Zakrajek (2010), who consider shocks to idiosyncratic uncertainty shocks as in Bloom (2009), but in a setup with credit frictions. I compare this mechanism and my mechanism in more detail below.

Finally, the extension relates to the vast literature on the "credit spread puzzle" (e.g. Leland (1994), Huang and Huang (2003), Hackbardt, Miao and Morellec (2006), Chen (2010), Chen, Collin Dufresne and Goldstein (2009), and Bhamra, Kuehn and Strebulaev (2009a, 2009b)). This literature documents that the prices of corporate bonds are too low to be accounted for in a risk-neutral model, and considers various risk adjustments, borrowed either from the long-run risk or the habits literature, to improve the fit of prices. Perhaps surprisingly, there is (to my knowledge) no model that studies the contribution of disaster risk to the credit spread puzzle. Moreover, the literature does not consider investment and is not set in general equilibrium, making it difficult to evaluate the macroeconomic impact of the financial frictions. On the other hand, this literature considers long-term debt and more detailed asset pricing implications.

For clarity, I restate the full model in this section, starting with the household problem, then the firm problem, and finally defining the equilibrium and asset prices.

[39] Philippon's results, which hold under the Modigliani and Miller theorem (given an exogenous leverage policy) do not require him to specify a full general equilibrium model.

7.1 Household

As in the main model, the representative household has recursive preferences over consumption and leisure, following Epstein and Zin (1989):

$$U_t = \left((1-\beta)(C_t^{\upsilon}(1-N_t)^{1-\upsilon})^{1-\psi} + \beta E_t \left(U_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \right)^{\frac{1}{1-\psi}}. \tag{11}$$

Here ψ is the inverse of the intertemporal elasticity of substitution (IES) over the consumption-leisure bundle, and γ measures risk aversion towards static gambles over the bundle. When $\psi = \gamma$, the model collapses to expected utility. While the additional flexibility of recursive utility is useful in calibrating the model, the key qualitative results can be obtained with standard CRRA preferences (See section 7.6.5).

The household supplies labor in a competitive market, and trades in stocks and bonds issued by the corporate sector.[40] The budget constraint reads

$$C_t + n_t^s P_t + q_t B_t \leq W_t N_t + \varrho_t B_{t-1} + n_{t-1}^s (P_t + D_t) - T_t, \tag{12}$$

where W_t is the real wage, B_{t-1} is the quantity of debt issued by the corporate sector in period $t-1$ at price q_{t-1}, each unit of which is redeemed in period t for ϱ_t, n_t^s is the quantity of equity shares, P_t is the price of equity, D_t is the dividend, and T_t is a lump-sum tax. We will normalize the number of equity shares n_t^s to one. In the absence of default, $\varrho_t = 1$, but $\varrho_t < 1$ if some bonds are not repaid in full. The household takes the process of ϱ_t as given, but it is determined in equilibrium by default decisions of firms, as we will see later.

Intertemporal choices are determined by the stochastic discount factor (a.k.a. marginal rate of substitution), which prices all assets:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\upsilon(1-\psi)-1} \left(\frac{1-N_{t+1}}{1-N_t} \right)^{(1-\upsilon)(1-\psi)} \frac{U_{t+1}^{\psi-\gamma}}{E_t \left(U_{t+1}^{1-\gamma} \right)^{\frac{\psi-\gamma}{1-\gamma}}}. \tag{13}$$

The labor supply decision is governed by the familiar condition:

$$W_t = \frac{1-\upsilon}{\upsilon} \frac{C_t}{1-N_t}. \tag{14}$$

[40] It is possible to introduce government bonds as well. If the government finances this debt using lump-sum taxes and transfers, Ricardian equivalence holds, and government policy does not affect the equilibrium allocation and prices.

7.2 Firms

We first describe the general structure of the firm problem, then we fill in the details.

7.2.1 Summary

There is a continuum of firms, which are all identical ex-ante and differ ex-post only in their realization of an idiosyncratic shock. For simplicity, we assume that firms live only for two periods. Firms purchase capital at the end of period t in a competitive market, for use in period $t+1$. This investment is financed through a mix of equity and debt. In period $t+1$, the aggregate shocks and the idiosyncratic shock are revealed, firms decide on employment and production, and then sell back their capital. Two cases arise at this point: (1) the firm value is larger than outstanding debt: the debt is then repaid in full and the residual value goes to shareholders as dividends; or (2) the firm value is smaller than outstanding debt: in this case the firm declares default, equityholders receive nothing, and bondholders capture the firm's value, net of some bankruptcy costs. In all cases, the firms disappear after production in period $t+1$ and new firms are created, which will raise funds and invest in period $t+1$, and operate in period $t+2$.[41]

The timing assumption clarifies the mechanism: it implies that a default *realization* does not affect employment, output and profits. Ex-ante however, default *risk* affects the cost of capital to the firm and hence its investment decision. This investment decision in turns affects employment and output, and in general equilibrium all quantities and prices. In section 7.6.1, we consider an extension where default affects production.

Since firms are ex-ante identical, they will all make the same choices. Because both production and financing technologies exhibit constant return to scales, the size distribution of firms is indeterminate, and has no effect on aggregate outcomes.

7.2.2 Production

All firms operate the same constant returns to scale Cobb-Douglas production function using capital and labor. The output of firm i is

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha},$$

[41] The assumption that firms live two periods, while obviously unrealistic, leads to substantial simplification of the analysis, which is useful to solve the model but also to clarify its implications. An important direction of future research is to incoporate long-lived firms and long-term debt in the model. Based on section II.A below, I conjecture that the model mechanism would still be relevant.

where z_t is aggregate total factor productivity (TFP), K_{it} is the individual firm capital stock, and N_{it} is labor. Both input and output markets are competitive and frictionless.

7.2.3 Productivity shocks

To model the possibility of large recessions, I assume that the aggregate TFP process in this economy is driven not only by the usual "small" normally distributed shocks standard in RBC theory, but also by rare large negative shocks.[42] Formally,

$$\log z_{t+1} = \log z_t + \mu + \sigma e_{t+1} + x_{t+1} \log(1 - b_{tfp}),$$

where $\{e_{t+1}\}$ is *i.i.d.* $N(0,1)$, and x_{t+1} is an indicator equal to 1 if a disaster happens, and 0 otherwise. The probability of a disaster at time $t+1$ is denoted p_t. I will also assume that the realization of disaster affects the capital stock (see the next paragraph). The probability of disaster p_t follows itself a Markov chain with transition matrix Q. The *three* aggregate shocks $\{e_{t+1}, x_{t+1}, p_{t+1}\}$ are assumed to be independent, conditional on p_t.

7.2.4 Depreciation shocks

Firms decide on investment at time t, but the actual quantity of capital that they will have to operate at time $t+1$ is random, and is affected both by realizations of aggregate disasters x_{t+1} as well as an idiosyncratic shock ε_{it+1}. Specifically, if a firm i picks $K^w_{i,t+1}$ at time t (w for wish), it actually has $K_{it+1} = K^w_{it+1}(1 - x_{t+1}b_k)\varepsilon_{it+1}$ to operate in period $t+1$, and $(1-\delta)K_{it+1}$ units of capital to resell. The idiosyncratic shock ε_{it+1} is *i.i.d.* across firms and across time, and drawn from a cumulative distribution function H, with mean unity.

7.2.5 Discussion of the assumptions regarding disasters

Barro (2006) and Barro and Ursua (2008) identify numerous large negative macroeconomic shocks in a cross-section of countries, which are usually caused by wars or economic depressions. In a standard neoclassical model there are two simple ways to model macroeconomic disasters – as destruction of the capital stock, or as a reduction in total factor productivity. My formulation allows for both.

[42] For parsimony and tractability, these rare disasters are modeled as one-time permanent jump in TFP; Gourio (2010, 2011) considers various extensions and shows that the key results are largely unaffected if disasters are modeled as smaller shocks that are persistent, and are followed by recoveries, provided that risk aversion is increased somewhat.

TFP appears to play an important role during economic depressions (Kehoe and Prescott, 2007). While economists do not understand well the sources of fluctuations in total factor productivity, large and persistent declines in TFP may be linked to poor government policies, such as expropriation or confiscatory taxes. They may also be caused by disruptions in financial intermediation, if these lead to inefficient capital allocation.

Capital destruction is clearly realistic for wars or natural disasters, but it can also be interpreted more broadly. Perhaps it is not the physical capital but the intangible capital (customer and employee value) that is destroyed during prolonged economic depressions.

In terms of economic mechanism, the model requires two ingredients: (1) that disasters are clearly bad events, with high marginal utility of consumption; (2) that the return on capital is low during disasters. These assumptions are certainly realistic. Introducing a large TFP shock is the simplest way to obtain (1) in a neoclassical model, and introducing a depreciation shock is the simplest way to obtain (2). An alternative to depreciation shocks is to introduce steep adjustment costs: since investment falls significantly during disasters, the price of capital would also fall, generating endogenously low returns on capital during disasters.

7.2.6 Capital structure choice

The choice of equity versus debt is driven by a standard trade-off between default (bankruptcy) costs and the tax advantage of debt. Specifically, I assume that bondholders recover a fraction θ of the firm value upon default, where $0 < \theta < 1$. Moreover, a firm which issues debt at a price q receives χq, where $\chi > 1$. That is, for each dollar that the firm raises in the bond market, the government gives a subsidy $\chi - 1$ dollar. For simplicity, I assume that the subsidy takes place at issuance.[43]

The price q is determined at time of issuance, taking into account default risk, and hence depends on the firm's choice of debt and capital as well as the economy's state variables. Equity issuance is assumed to be costless. When $\chi = \theta = 1$, the capital structure is indeterminate and the Modigliani-Miller theorem holds. When $\chi = 1$, the firm finances only through equity, since debt has no advantage. As a result, there is no default, and we obtain the standard RBC model. When $\theta = 1$, or more generally $\theta\chi \geq 1$, the firm finances only through debt, since default is not costly enough. We assume $\chi\theta < 1$, a necessary assumption to generate an interior choice for the capital structure.

[43] In reality, interest on corporate debt is deductible from the corporate income tax, hence the implicit subsidy takes place when firms' earnings are taxed.

7.2.7 Employment, Output, Profits, and Firm Value

To solve the optimal financing choice, we first need to determine the profits and the firm value. (The distribution of firm value determines the probability of default and hence the lending terms the firm can obtain ex-ante.) The labor choice is determined through the standard static profit maximization problem, given the realized values of both productivity and capital stock, and given the aggregate wage:

$$\pi\left(K_{it}, z_t; W_t\right) = \max_{N_{it} \geq 0} \left\{K_{it}^{\alpha}(z_t N_{it})^{1-\alpha} - W_t N_{it}\right\},$$

which leads to the labor demand

$$N_{it} = K_{it}\left(\frac{z_t^{1-\alpha}(1-\alpha)}{W_t}\right)^{\frac{1}{\alpha}}, \tag{15}$$

and the output supply

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha} = K_{it}\left(\frac{(1-\alpha)}{\frac{W_t}{z_t}}\right)^{\frac{1-\alpha}{\alpha}}.$$

These equations can then be aggregated. Define aggregates through $K_t = \int_0^1 K_{it}di$, $Y_t = \int_0^1 Y_{it}di$, etc., we obtain that $Y_t = K_t^{\alpha}(z_t N_t)^{1-\alpha}$,i.e. an aggregate production function exists, and it has exactly the same shape as the microeconomic production function. Aggregating 15 shows that the wage satisfies the usual condition $W_t = (1-\alpha)\frac{Y_t}{N_t}$.The law of motion for capital is obtained by summing over i the equation $K_{it+1} = K_{it+1}^{w}(1 - x_{t+1}b_k)\varepsilon_{it+1}$. As noted above, all firms are identical ex-ante, and they will make the same investment choice $K_{it+1}^{w} = K_{t+1}^{w}$. Since ε_{it+1} has mean unity, idiosyncratic shocks average out and the aggregate capital is

$$K_{t+1} = K_{t+1}^{w}(1 - x_{t+1}b_k).$$

Profits at time $t+1$ are given by

$$\pi_{it+1} = Y_{it+1} - W_{t+1}N_{it+1} = \alpha Y_{it+1} = \alpha K_{it+1}\left(\frac{(1-\alpha)}{\frac{W_{t+1}}{z_{t+1}}}\right)^{\frac{1-\alpha}{\alpha}} = K_{it+1}\alpha\frac{Y_{t+1}}{K_{t+1}},$$

i.e. each firm receives factor payments proportional to the quantity of capital it has, and to the aggregate marginal product of capital $\alpha\frac{Y_{t+1}}{K_{t+1}}$. The total firm value at the end of the period is

$$V_{it+1} = \pi_{it+1} + (1-\delta)K_{it+1} = K_{it+1}\left(1 - \delta + \alpha\frac{Y_{t+1}}{K_{t+1}}\right). \tag{16}$$

We define the aggregate return on capital as $R_{t+1}^K = (1 - x_{t+1}b_k)\left(1 - \delta + \alpha\frac{Y_{t+1}}{K_{t+1}}\right)$. The individual return on capital is $R_{it+1}^K = \varepsilon_{it+1} R_{t+1}^K$. The firm value is thus

$$V_{it+1} = R_{it+1}^K K_{t+1}^w = \varepsilon_{it+1} R_{t+1}^K K_{t+1}^w.$$

From ease of notation, I will from now on abstract from the firm subscript i, since all firms are identical and differ only ex-post in their realization of ε.

7.2.8 Investment and Financing Decisions

As noted above, all firms make the same choices for capital, debt, and hence equity issuance, which are linked through the budget constraint $\chi q_t B_{t+1} + S_t = K_{t+1}^w$. To find the optimal choice of investment and financing, we first need to find the likelihood of default, and the loss-upon-default, for any possible choice of investment and financing. This determines the price of corporate debt. Taking as given this bond price schedule, the firm can then decide on optimal investment and financing.

More precisely, the firm will default if its realized value V_{t+1}, which is the sum of profits and the proceeds from the sale of undepreciated capital, is too low to repay the debt B_{t+1}. This will occur if the firm's idiosyncratic shock ε is smaller than a cutoff value, which itself depends on the realization of aggregate states $(e_{t+1}, p_{t+1}, x_{t+1})$. Mathematically, at time $t+1$, the value of firms which finish operating is $V_{t+1} = \varepsilon_{t+1} R_{t+1}^K K_{t+1}^w$, hence default occurs if and only if

$$\varepsilon_{t+1} < \frac{B_{t+1}}{R_{t+1}^K K_{t+1}^w} \stackrel{def}{=} \varepsilon_{t+1}^*.$$

Given this default rule, the bond issue is priced ex-ante using the representative agent's stochastic discount factor:

$$q_t = E_t\left(M_{t+1}\left(\int_{\varepsilon_{t+1}^*}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}}\int_0^{\varepsilon_{t+1}^*} \varepsilon R_{t+1}^K K_{t+1}^w dH(\varepsilon)\right)\right).$$

In this equation, the first integral gives the value of the debt in the full repayment states. These states depend on the realization of shocks occurring at time $t+1$, notably disasters, through the threshold for default ε_{t+1}^*. The second term gives the average recovery in default states, divided among all the bondholders and net of bankruptcy costs. We can rewrite the bond price as

$$q_t = E_t\left(M_{t+1}\left(1 - H\left(\varepsilon_{t+1}^*\right) + \frac{\theta R_{t+1}^K K_{t+1}^w}{B_{t+1}}\Omega\left(\varepsilon_{t+1}^*\right)\right)\right), \tag{17}$$

where $\Omega(x) = \int_0^x sdH(s)$. Note the following properties of Ω, which follow from the fact that H is a c.d.f. with mean unity: (i) $\Omega(x) = 1 - \int_x^\infty sdH(s)$; (ii) $\lim_{x\to\infty}\Omega(x) = 1$; (iii) $\Omega'(x) = xh(x)$.

We can now set up the firm's problem at time t : it must decide how much to invest, how much debt to issue (and hence how much of the investment is financed through equity), so as to maximize the expected discounted equity value:

$$\max_{B_{t+1}, K_{t+1}^w, S_t} E_t\left(M_{t+1}\max\left(V_{t+1} - B_{t+1}, 0\right)\right) - S_t, \tag{18}$$

subject to:

$$\chi q_t B_{t+1} + S_t = K_{t+1}^w, \tag{19}$$

$$V_{t+1} = \varepsilon_{t+1} R_{t+1}^K K_{t+1}^w. \tag{20}$$

Equation (19) is the funding constraint: investment must come out of equity S_t, or the sale of bonds (including the subsidy) $\chi q_t B_{t+1}$. The objective function (18) takes into account the option of default for equityholders. Given that the firm defaults if $\varepsilon_{t+1} < \varepsilon_{t+1}^*$, we can rewrite this problem as:

$$\max_{B_{t+1}, K_{t+1}^w} E_t\left(M_{t+1}\left(\begin{array}{c} R_{t+1}^K K_{t+1}^w + \left(\chi\theta - 1\right) R_{t+1}^K K_{t+1}^w \Omega(\varepsilon_{t+1}^*) \\ + \left(\chi - 1\right) B_{t+1}\left(1 - H\left(\varepsilon_{t+1}^*\right)\right) \end{array}\right)\right) - K_{t+1}^w, \tag{21}$$

$$s.t. \quad : \qquad \varepsilon_{t+1}^* = \frac{B_{t+1}}{R_{t+1}^K K_{t+1}^w}.$$

In this expression, the first term is the expected discounted firm value, $E_t\left(M_{t+1}R_{t+1}^K K_{t+1}^w\right)$; the second term (which is negative since $\chi\theta < 1$) is expected discounted bankruptcy costs; and the third term is the expected discounted tax shield. The last term K_{t+1}^w is simply the cost of investment. By contrast, in a frictionless model, the firm would simply maximize $E_t\left(M_{t+1}R_{t+1}^K K_{t+1}^w\right) - K_{t+1}^w$. The difference is that the firm also takes into account the value of tax subsidies and default costs in making its decisions. Default costs are born by debt holders ex-post, but expected default costs are passed on into debt prices ex-ante, implying that equity holders actually bear the costs of default.

To solve this program, we simply take the first-order conditions with respect to K^w_{t+1} and B_{t+1}. The first-order condition with respect to K^w_{t+1} yields,

$$E_t\left(M_{t+1}R^K_{t+1}\left(1+(\chi\theta-1)\,\Omega(\varepsilon^*_{t+1})+(\chi-1)\,\varepsilon^*_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right)\right)=1. \tag{22}$$

Recall that $R^K_{t+1}=(1-x_{t+1}b_k)\left(1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\right)$ is the familiar expression for the unlevered physical return on capital, adjusted to reflect the possibility of disasters. In a model without financial frictions, the standard Euler equation implies $E_t\left(M_{t+1}R^K_{t+1}\right)=1$; here, equation (22) is modified to take into account the bankruptcy costs (the second term), which raise the cost of capital, and the tax shield (the third term), which reduces it. When $\chi=\theta=1$, we return to the standard equation, corresponding to the case of an unlevered firm. Overall the firm has always access to cheaper financing than in the frictionless (all-equity financed) model, since it always has the possibility to not take any debt. As a result, the steady-state capital stock is always higher when $\chi>1$ than in the frictionless version.

The first order condition with B_{t+1} is

$$(1-\theta)\,E_t\left(M_{t+1}\varepsilon^*_{t+1}h\left(\varepsilon^*_{t+1}\right)\right)=\left(1-\frac{1}{\chi}\right)E_t\left(M_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right). \tag{23}$$

This equation determines the optimal financing choice between debt and equity.[44] The left-hand side is the marginal cost of debt, i.e. an extra dollar of debt will increase the likelihood of default, and the associated bankruptcy costs. The right-hand side is the marginal benefit of debt, i.e. the higher tax shield in non-default states. Importantly, both the marginal cost and the marginal benefit are discounted using the stochastic discount factor M_{t+1}. The importance of this risk-adjustment is consistent with the empirical work by Almeida and Philippon (2007), who note that corporate defaults are more frequent in "bad times" and as a result the ex-ante marginal cost of debt is higher than a risk-neutral calculation would suggest. This risk-adjustment will play a substantial role in the analysis below: for a given debt level, an increase in the probability of disaster increases expected discounted default costs, not only because defaults become more likely, but also because they are more likely to occur during bad aggregate times.

We can define desired leverage $L_{t+1}=B_{t+1}/K^w_{t+1}$, which is decided at time t. The firm defaults

[44] For this equation to generate a unique threshold, some regularity condition must be imposed on the distribution H. The technical condition (which we assume from now on) is that the function $z\rightarrow\frac{zh(z)}{1-H(z)}$ is increasing. Bernanke, Gertler and Gilchrist (1999) make the same assumption in the context of a related model. Most distributions (such as the log-normal distribution) satisfy this assumption.

if $\varepsilon R^K_{t+1} < L_{t+1}$ i.e. if the return on capital is low relative to the leverage.

7.3 Equilibrium

The equilibrium definition is standard. First, the labor market clears:

$$(1-\alpha)\frac{Y_t}{N_t} = W_t = \frac{(1-\upsilon)C_t}{\upsilon\,(1-N_t)}. \tag{24}$$

Second, the goods market clears, i.e. total consumption plus investment plus bankruptcy costs equals output,

$$C_t + I_t + (1-\theta)R(\varepsilon^*_t)V_t = Y_t. \tag{25}$$

This equation implies that higher bankruptcy costs induce a negative wealth effect. In order to clarify the mechanism, I initially abstract from this effect, by assuming that the default cost is a tax, i.e. it is transferred to the government, which then rebates it to household using lump-sum transfers (T_t in equation 12). Then, the resource constraint is simply

$$C_t + I_t = Y_t. \tag{26}$$

Under this simplification, equations 22 and 23 are the only departures of our model from the standard real business cycle model: first, the Euler equation needs to be adjusted to reflect the tax shield and bankruptcy costs; second, the optimal leverage is determined by the trade-off between costs and benefits of debt finance. To summarize, the equilibrium is characterized by the equations (24), (26), as well as (22) and (23) and the definition of the stochastic discount factor (13) and (11).

7.3.1 Recursive Representation

It is useful, both for conceptual clarity and to implement a numerical algorithm, to present a recursive formulation of this equilibrium. This can be done in three steps. First, we make the simplifying assumption that the bankruptcy cost is a tax, instead a of a real resource cost. Second, we note that the equilibrium can be entirely characterized from time t onwards given the values of the realized aggregate capital stock K_t, the probability of disaster p_t, and the level of total factor productivity z_t, i.e. these are the three state variables.[45] Third, examination of

[45] The level of outstanding debt B_t at the beginning of period is not a state variable, since it does not affect production or investment possibilities. It does affect default, but because defaults do not affect production, and

the first-order conditions shows that they can be rewritten solely as a function of the detrended capital $k_t = K_t/z_t$ and p_t. This is a standard simplification in the stochastic growth model when technology follows a unit root, which also applies to our framework.

As a result the equilibrium policy functions can be expressed as functions of two state variables only, k and p. Hence, the model has the same states as the frictionless real business cycle (RBC) model. There is an additional equilibrium policy function to solve for, the desired leverage $L(k,p)$, and correspondingly, we have an additional first-order condition (equation (23)). Last, the first-order condition determining optimal investment, i.e. the standard Euler equation (equation 22)), is modified to take into account the marginal financing costs. The full list of equations of this recursive representation is in appendix.

7.3.2 Asset Prices

Any payoff can be priced using the stochastic discount factor, given by the representative agent's marginal rate of substitution. I focus here on four assets: a pure risk-free asset, a short-term government bond which may default during disasters, the corporate bond, and the equity. All these assets last only one period. The price of the risk-free asset can be calculated as the expectation of the stochastic discount factor, $P_t^{rf} = E_t\left(M_{t+1}\right)$. Following Barro (2006), the government bond is assumed to default by a factor Δ during disasters, and hence its price is $P_t^{gov} = E_t\left(M_{t+1}\left(1 - x_{t+1}\Delta\right)\right)$. The payoff to a diversified portfolio of corporate bonds, used in the household budget constraint (equation (12)), is $\varrho_{t+1} = 1 - H\left(\varepsilon_{t+1}^*\right) + \frac{\theta R_{t+1}^K K_{t+1}^w}{B_{t+1}}\Omega\left(\varepsilon_{t+1}^*\right)$, and the corporate bond price is $P_t^{corp} = q_t = E_t\left(M_{t+1}\varrho_{t+1}\right)$. Last, the equity value is

$$P_t^{eq} = E_t\left(M_{t+1}\left(R_{t+1}^K K_{t+1}^w\left(1 - \Omega\left(\varepsilon_{t+1}^*\right)\right) - B_{t+1}\left(1 - H\left(\varepsilon_{t+1}^*\right)\right)\right)\right).$$

Given constant return to scale and no equity issuance costs, the equity price satisfies a free entry condition: $P_t^{eq} = S_t$.

7.4 Quantitative results

This section studies the implications of the model presented in the previous section. First, I present a combination of analytical results and comparative statics to illustrate the workings of

bankruptcy costs are not in the resource constraint, the realization of default does not matter in itself – what matters is the possibility of default going forward. Here we rely on two assumptions: (1) the default cost is a tax; (2) default takes place after production.

the model. Then, a parametrized version of the model is solved numerically so as to delineate its predictions for business cycle quantities, for asset returns, an in particular for the level and volatility of credit spreads, and their relation with investment and GDP.[46]

7.4.1 Steady-state comparative statics

To better understand the model, it is useful to perform a "steady-state" analysis, as is commonly done in macroeconomics, but one that takes into account the risk of disaster. The first step is the following result.

Proposition 4 *Assume that $b_k = b_{tfp}$, i.e. capital and productivity fall by the same factor in a disaster Then, a disaster leads consumption, investment, output to also drop by the same factor $b_k = b_{tfp}$, while hours do not change. The return on physical capital is reduced by the same factor. There is no further effect of the disaster on quantities or prices, i.e. all the effect is on impact.*

Proof. The equilibrium is characterized by the policy functions $c(k,p), i(k,p), N(k,p), L(k,p)$ and $y(k,p) = k^{\alpha} N(k,p)^{1-\alpha}$ which express the solution as a function of the probability of disaster p (the exogenous state variable) and the detrended capital k (the endogenous state variable). The detrended capital evolves according to the shocks ε', x', p' through

$$k' = \frac{(1 - x' b_k)\left((1-\delta)k + i(k,p)\right)}{(1 - x' b_{tfp})\, e^{\mu + \sigma e'}}.$$

Since $b_k = b_{tfp}$,

$$k' = \frac{\left((1-\delta)k + i(k,p)\right)}{e^{\mu + \sigma e'}},$$

is independent of the realization of disaster x'. As a result, the realization of a disaster does not affect c, i, N, y, L since k is unchanged, and hence it leads consumption $C = cz$, investment $I = iz$, and output $Y = yz$ to drop by a factor $b_k = b_{tfp}$ on impact. Furthermore, once the disaster has hit, it has no further effect since all the endogenous dynamics are captured by k, which is unaffected. The statement regarding returns follows from the expression of the physical return, $R^K_{t+1} = (1 - x_{t+1} b_k)\left(1 - \delta + \alpha \frac{y_{t+1}}{k_{t+1}}\right)$. ■

To obtain further results, we consider a simplified version of the model, where we shut down the shocks to the probability of disaster and the TFP shocks e_{t+1}. As a result, the only source of shocks are disaster realizations, and can solve for the path of quantities and returns.

[46] Given the nonlinear form of the model, and the focus on risk premia, it is important to use a nonlinear solution method. The policy functions $c(k,p), N(k,p), g(k,p)$, and $L(k,p)$, are approximated using Chebychev polynomials and solved for using projection methods. The appendix details the computational method.

Proposition 5 *Assume that $b_k = b_{tfp}$, that $\sigma = 0$, and that $p_t = p$. The economy has a balanced growth path where $k_t, c_t, i_t, y_t, L_t, N_t$, the risk-free rate, the expected return on capital, and the probability of default, and the credit spread are constant, equal to k^*, c^*, i^*, etc. Along this balanced growth path, the level of capital, consumption, investment and output K_t, C_t, I_t, Y_t, are obtained by multiplying k^*, c^*, i^*, y^* by z_t, which is evolves as $z_{t+1} = z_t e^{\mu + x_{t+1}\log(1-b_z)}$.*

Proof. Given that $\sigma = 0$, and p is constant, the law of motion for capital further simplifies to $k' = \frac{((1-\delta)k+i(k))}{e^{\mu}}$. Call k^* the solution to this equation; the policy functions $c(k), i(k), N(k), L(k)$ then imply that these variables are also constant if $k = k$. Given this, consumption growth and other variables are iid, implying that expected returns and credit spreads are constant. ■

To visualize this result, note that macro quantities in this version of the model simply grow along constant trends, with no deviation except for occasional large downward jumps. During these jumps, realized returns on bonds and equity are low, but the dynamics of quantities are unaffected. The discount factor for this simplified version of the model depends only on the disaster realization:

$$M(x') = \frac{\beta e^{\mu((1-\psi)\upsilon-1)}(1-x' b_{tfp})^{(1-\gamma)\upsilon-1}}{(1-p+p(1-b_{tfp})^{(1-\gamma)\upsilon})^{\frac{\psi-\gamma}{1-\gamma}}},$$

and the economy's steady-state capital-labor ratio k/N and leverage $L = B/K^w$ are determined by the two equations:

$$\begin{aligned} & \frac{\beta e^{\mu((1-\psi)\upsilon-1)}}{(1-p+p(1-b_{tfp})^{(1-\gamma)\upsilon})^{\frac{\psi-\gamma}{1-\gamma}}}\left(1-\delta+\alpha\left(\frac{k}{N}\right)^{\alpha-1}\right) \\ = \; & (1-p)\left(1+(\chi\theta-1)\,\Omega\left(\varepsilon^*_{nd}\right)+(\chi-1)\,\varepsilon^*_{nd}\left(1-H\left(\varepsilon^*_{nd}\right)\right)\right) \\ & +p\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)-1}(1-b_k)\left(1+(\chi\theta-1)\,\Omega\left(\varepsilon^*_{d}\right)+(\chi-1)\,\varepsilon^*_{d}\left(1-H\left(\varepsilon^*_{d}\right)\right)\right). \end{aligned} \tag{27}$$

and

$$\begin{aligned} 0 \; = \; & (1-p)\left(\chi\left(\theta-1\right)\varepsilon^*_{nd} h\left(\varepsilon^*_{nd}\right)+(\chi-1)\left(1-H\left(\varepsilon^*_{nd}\right)\right)\right) \\ & +p\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)-1}\left(\chi\left(\theta-1\right)\varepsilon^*_{d} h\left(\varepsilon^*_{d}\right)+(\chi-1)\left(1-H\left(\varepsilon^*_{d}\right)\right)\right), \end{aligned} \tag{28}$$

with $\varepsilon^*_d = \frac{L}{(1-b_k)\phi}$ and $\varepsilon^*_{nd} = \frac{L}{\phi}$, and $\phi = 1-\delta+\alpha\left(\frac{k}{N}\right)^{\alpha-1}$ is the standard marginal product of capital.

While these expressions initially appear complicated, they provide significant intuition. First, note that they are recursive: equation (28) first determines the ratio of leverage to the marginal product $\frac{L}{\phi}$, and equation (27) then determines the marginal product of capital ϕ and hence $\frac{k}{N}$.[47] When there is neither disaster risk nor financial frictions, i.e. $p = 0$ and $\chi = \theta = 1$, the first equation collapses to the standard user cost equation,

$$\beta e^{\mu((1-\psi)\upsilon-1)} \left(1 - \delta + \alpha \left(\frac{k}{N}\right)^{\alpha-1}\right) = 1.$$

When there is disaster risk but no financial frictions (as in Gourio (2010)), the steady-state capital is determined as

$$\beta e^{\mu((1-\psi)\upsilon-1)} \left(1 - \delta + \alpha \left(\frac{k}{N}\right)^{\alpha-1}\right) \left(1 - p + p\left(1 - b_{tfp}\right)^{\upsilon(1-\gamma)}\right)^{\frac{1-\psi}{1-\gamma}} = 1.$$

Simple algebra shows that a higher probability of disaster p induces to a lower capital stock provided that the IES is greater than unity: agents are reluctant to invest in the more risky capital stock. Consider now the case of financial frictions but no disaster risk, equation (28) reflects simply the trade-off between the default costs and tax benefits of leverage:

$$\chi\left(1-\theta\right)\varepsilon^{*} h\left(\varepsilon^{*}\right) = \left(\chi - 1\right)\left(1 - H\left(\varepsilon^{*}\right)\right).$$

Last, in the full model, disaster risk affects the amount of desired leverage for two reasons. First, it changes the distribution of payoffs to the investment. Second, it changes the discount rates which multiply this distribution of payoffs (the term $\left(1 - b_{tfp}\right)^{\upsilon(1-\gamma)-1}$ in equation (28)).

7.4.2 The determinants of optimal leverage and investment

Figure 10 uses this simplified version of the model to illustrates the effect of several key parameters on the steady-state values of capital, leverage, default probability and credit spreads. Each column corresponds to one parameter. The first column shows the effect of idiosyncratic volatility σ_{ε}. Holding debt policy constant, higher idiosyncratic risk leads to more default and hence higher credit spreads, increasing the user cost of capital. This leads firms to reduce investment. In equilibrium, firms also endogenously reduce leverage, which mitigates the increase in default and

[47] Labor supply and the scale of the economy are then determined by preferences in the standard way. First, note that

$$c = k^{\alpha} N^{1-\alpha} - \delta k = N\left(\left(\frac{k}{N}\right)^{\alpha} - \delta\frac{k}{N}\right),$$

and second the MRS = MPL condition implies $\frac{1-\upsilon}{\upsilon}\frac{c}{1-N} = (1-\alpha)\left(\frac{k}{N}\right)^{\alpha}$. Since $\frac{k}{N}$ is known, this is one equation in one unknown N.

in credit spreads, but makes firms rely more heavily on equity issuance, which is more costly.

The second column shows the effect of the tax subsidy χ. A higher χ directly reduces the user cost of capital, since holding debt policy constant, the firm is able to raise more capital. Second, a higher χ makes debt relatively more attractive than equity, leading firms to take on more debt and increase leverage. This higher leverage leads to a higher probability of default and higher credit spreads.

Finally, the third column shows the effect of increasing the recovery rate parameter θ. Since the expected cost of bankruptcy falls, the user cost of investment falls and investment rises. Holding debt policy constant, a higher θ leads to a lower credit spread, since the recovery value is higher. However, since firms take on more debt, the probability of default and credit spreads go up.

7.4.3 User cost, financial frictions and probability of disaster

Turning now to the effect of the probability of disaster, figure 11 displays the effect of a rise in p on capital, leverage, credit spreads and the user cost $\alpha\left(\frac{k}{N}\right)^{\alpha-1}$, which is $r+\delta$ in the standard neoclassical model. Higher disaster risk leads to a reduction in leverage in equation (28), and hence an increase in the user cost (adjusted for the tax shield and bankruptcy costs) in equation (27) and a lower capital-labor ratio. The figure compares the frictionless model ($\chi = \theta = 1$, i.e. the firm is only equity-financed) and the model with the friction ($\chi > 1$). The percentage response of the steady-state capital stock to a change in the probability of disaster is substantially larger in the model with the financial friction, reflecting that the user cost is much more affected by an increase in disaster risk. An increase in disaster risk in itself increases the probability of default, but also makes the risk of default more likely to be driven by a bad *aggregate* realization, hence increases the cost of debt significantly, as reflected by the credit spread.[48] Overall, the probability of disaster p has an effect similar to that of σ_ε, which is the shock considered by Arellano, Bai and Kehoe (2010) or Gilchrist, Sim and Zakrajek (2010) in very recent studies. I return to this comparison in section 7.6.6.

7.4.4 Parametrization

Parameters are listed in Table 8. (I use a slightly different parametrization than in the benchmark model, though it has little effect.) The period is one year. Many parameters follow the business

[48] For high values of the probability of disaster p, the credit spread is decreasing in p. This counterintuitive result simply reflects that for very high p, firms reduce debt significantly to avoid bankruptcy and associated costs.

cycle literature (Cooley and Prescott (1995)). The risk aversion parameter is four, in order to get a reasonable level for the equity premium. Note that this is the risk aversion over the consumption-hours bundle. Since the share of consumption in the utility index is .3, the effective risk aversion to a consumption gamble is 1.33 (Swanson (2010)), a very low number by the standards of the asset pricing literature.

The intertemporal elasticity of substitution of consumption (IES) is set at 2. There is a large debate regarding the value of the IES. Most direct estimates using aggregate data find low numbers (e.g. Hall (1988)), but this view has been challenged by several authors (see among others Bansal and Yaron (2004), Gruber (2006), Mulligan (2004), Vissing-Jorgensen (2002)). As emphasized by Bansal and Yaron (2004), a low IES has the counterintuitive effects that higher expected growth lowers asset prices, and higher uncertainty increases asset prices. Section 7.6.5 analyzes how the results are affected by the intertemporal elasticity of substitution.

One crucial element of the calibration is the probability and size of disaster, which follow Barro (2006, 2009) and Barro and Ursua (2008) closely. The probability of a disaster is 1.7% per year on average. For computational simplicity, I summarize the historical distribution of disasters using a five-point distributions, with disaster sizes ranging from 15% to 57%.[49] While these disaster sizes may seem very large, they are the ones estimated by Barro and Barro and Ursua (2007) in a large international panel data set. The results are largely unchanged if the disaster size is set to be smaller – e.g., perhaps the US faces smaller disasters than most other countries – but risk aversion is correspondingly increased.

The second crucial element is the persistence and volatility of movements in this probability of disaster. I assume that the log of the probability follows an AR(1) process:

$$\log p_{t+1} = \rho_p \log p_t + (1 - \rho_p) \log \overline{p} + \sigma_p \varepsilon_{p,t+1},$$

where $\varepsilon_{p,t+1}$ is *i.i.d.* $N(0,1)$.[50] The parameter $\overline{p}$ is picked so that the average probability is .017 per year, and I set $\rho_p = .75$ and the unconditional standard deviation $\frac{\sigma_p}{\sqrt{1-\rho_p^2}} = 1.50$ in order to roughly match the volatility of credit spreads.

[49] The data from Barro and Ursua refers to consumption or output, but my model requires to parametrize the capital and TFP destruction. It would be interesting to gather further evidence on disasters, and measure b_k and b_{tfp} directly. This is beyond the scope of this paper. I concentrate on the parsimonious benchmark case $b_k = b_{tfp}$. Given this assumption, to match a drop of, say, 25% in consumption, requires exactly a drop of 25% of capital and z, hence the Barro and Ursua distribution of GDP losses leads directly to the distribution of capital and productivity losses. (Because TFP $= z^{1-\alpha}$, the drop in total factor productivity is smaller than 25%.)

[50] This equation allows the probability to be greater than one, however I will approximate this process with a finite Markov chain, which ensures that $0 < p_t < 1$.

As is standard, I use a log-normal distribution for H, the distribution of idiosyncratic shocks. The three remaining parameters determine the leverage choice: χ, θ and σ_ε, the variance of idiosyncratic shocks. Following the corporate finance literature, I set $\theta = 0.4$, consistent with estimates of recovery rates in "bad times". The parameters σ_ε and χ are then picked to match a target average probability of default and leverage. The target for the probability of default is 0.5% per year. I also set a target for leverage equal to 0.55. In the data leverage is somewhat smaller, perhaps 0.45. Targeting a leverage of 0.45 leads to an unrealistically large variance of idiosyncratic shocks σ_ε. This likely reflects that firm values are more volatile in the model than in the data. Higher volatility may be driven by fixed costs of production, which are equivalent to a higher target for leverage. Alternatively, the distribution of idiosyncratic shocks may exhibit skewness and/or kurtosis.[51]

7.4.5 Impulse response functions

I first illustrate the dynamics of the model in response to the three fundamental shocks: the standard TFP shock, the disaster realization, and a temporary shock to the probability of disaster. I next discuss how the model fits quantitatively both quantity and price data.

7.4.5.1 The effect of a TFP shock Figure 12 shows the response of quantities and returns to a one standard-deviation shock (i.e. 2%) to the level of total factor productivity. For clarity, this picture, as well as the ones following, assumes that no other shock is realized. The response of quantities is similar to that of the standard real business cycle model: investment rises as firms desire to accumulate more capital, employment rises because of the higher labor demand, and consumption adjusts gradually, leading to temporarily high interest rates. The equity return is high on impact, reflecting the sensitivity of firms' dividends to TFP shocks due to leverage, but corporate bonds are largely immune to small TFP shocks - the default and recovery rates are barely affected.[52] As a result, the path for the bond return mirrors that of the risk-free return. There is essentially no change in leverage or credit spreads, since the trade-off determining optimal leverage is hardly affected by the slightly higher TFP.

[51] The targets are not exactly matched in the full model because the calibration is done using the "steady-state" version of the model, studied in the previous section.

[52] I define the default rate as the share of firms in default. Because some of the capital is recovered in defaults, this is not the realized loss for debholders.

7.4.5.2 The effect of a disaster Figure 13 shows the response of quantities and returns to a disaster which hits at $t = 5$. The disaster realization leads capital and TFP to fall by the factors b_k and b_{tfp} respectively. The calibration assumes that these parameters are equal, and in this simulation, $b_k = b_{tfp} = 25\%$. As a result, the transitional dynamics are very simple, as seen in the figure, and as proved in the proposition 1: output, consumption and investment drop on impact by the same factor, and hours do not change.

The return on capital is also -25%, and is divided among equity and debt. But it is also further reduced by default, which leads to losses since $\theta < 1$. In this simulation, approximately 12% of firms are in default, the realized equity return is roughly -52% and the realized bond return is -4.5%. (The returns we compute are the average across all the firms, as defined in section 7.3: there are always some firms with very high idiosyncratic shocks which do not default.) Figure 14 illustrates that both equity and corporate debt are risky assets, since their returns are very low precisely in the states (disasters) when marginal utility is high (consumption growth is low). The figure confirms that a disaster does not generate any transitional dynamics in quantities, leverage, credit spreads, interest rates, or risk premia.

7.4.5.3 The effect of an increase in the probability of a disaster The important shock in this extension - as in the benchmark model - is the shock to the probability of disaster – i.e. an increase in perceived risk. Figure 14 presents the responses to an unexpected increase in the probability of disaster at time $t = 5$. The higher risk leads to a sharp reduction in investment. Simultaneously, the higher risk pushes down the risk-free interest rate, as demand for precautionary savings increases. This lower interest rate decreases employment through an intertemporal substitution effect. Hence, output decreases because employment decreases, even though there is no change in current or future total factor productivity, and even though the capital stock adjusts slowly. Intuitively, there is less demand for investment and this reduces the need for production.

Consumption increases on impact since households want to invest less in the now more risky capital. Consumption then falls over time. Qualitatively, these dynamics are similar to that in the frictionless version, but the quantitative results are quite different. To illustrate this clearly, figure 15 superimposes the responses to a shock to the probability of disaster for the frictionless model ($\chi = \theta = 1$) and for the current model. The response of macro quantities on impact is approximately three times larger in the model with financial frictions.

As argued in section 7.5.1, the mechanism through which disaster risk affects the economy is by changing the expected discounted bankruptcy costs. These become significantly higher, since default is (i) more likely and (ii) more likely to occur in "bad times". This increases the user cost for a given financial policy, leading firms to cut back on investment. Moreover, firms also adjust their financial policy, reducing debt and leverage.

Because risk increases, risk premia rise as the economy enters this recession: the difference between equity returns and risk-free returns becomes larger, and the spread of corporate bonds over risk-free bonds also rises (see the bottom panel of figure 14). This last result is not fully general, however. The equilibrium level of credit spreads depends on the *endogenous* quantity of debt, or leverage that firms decide to take on. For certain parameter values, the endogenous decrease in leverage leads, paradoxically, to lower credit spreads in response to a higher probability of disaster. However, for the parameter values that we use, firms do not decide to cut back on debt too much, and spreads rise with the probability of disaster. The model hence generates the required negative correlation between credit spreads and investment output. More generally, the model implies that risk premia are larger in recessions, consistent with the data.

7.4.6 Business cycle and financial statistics

Tables 10, 11 and 12 report standard business cycle and asset return statistics as well as default rates and leverage ratios.[53] To illustrate the role of disaster risk and time-varying disaster risk, I solve the model with the benchmark parameter values, under different assumptions regarding the structure of shocks: (i) only TFP shocks, (ii) TFP shocks and disasters, but a constant probability of disaster; (iii) TFP shocks and disasters, with a time-varying risk of disaster. I also consider three variant of the model: (a) with the financial friction, (b) with constant leverage, and (c) with no financial friction. The benchmark model results (a-iii) are indicated in bold in these tables. The variant with constant leverage adds the constraint that $B_{t+1} = \overline{L}K^w_{t+1}$, i.e. firms must pick debt and capital so that their ratio is constant (and equal to the average leverage in the benchmark model).

The models with only TFP shocks (rows 1 through 3) generate a decent match for quantity dynamics, as is well known from the business cycle literature. This model, however, generates rather small spreads for corporate bonds, and these spreads simply account for the average default of corporate bonds, because aggregate risk premia are very small. The spread is 51bp, twice below the data, whereas the probability of default is 79bp, larger than the data. Moreover, these spreads

[53]The leverage and default probability data are taken from Chen, Collin-Dufresne, and Goldstein (2009). The other data (GDP, consumption, investment, and credit spreads) are from FRED. I use BAA-AAA as the credit spread measure, and obtain similar results as Chen, Collin-Dufresne, and Goldstein. All series are annualized.

are essentially constant. The risk premium for equity is also very small and equity returns are not volatile. Note that except for investment, which is somewhat less volatile in the model with financial friction, the quantity moments are largely unchanged as we go from row 1 to row 3. Hence, financial frictions do not amplify the response to TFP shocks.[54] The smaller volatility of investment in the model with financial frictions may be explained by the higher steady-state capital stock (as in Santoro and Wei (2010)).

When constant disaster risk is added to the model (rows 4 through 6), the quantity dynamics are unaffected (table 9). Table 10 reveals that credit spreads are significantly larger however, because defaults are much more likely during disasters, when marginal utility is high. The model generates a higher equity risk premium and a plausible credit spread: the average spread is 129bp, and the probability of default is 50bp. However, the volatility of spreads is still close to zero. This motivates turning to the model with time-varying risk of disaster.

Rows 7 through 9 display the results for the models with time-varying disaster risk. The variation in the disaster risk does indeed lead to volatile credit spreads, roughly in line with the data. The equity premium is too low, but it is significant, and similar to that of the model with constant probability of disaster. Introducing the time-varying risk of disaster also generates new quantity dynamics: output and especially investment become more volatile. Moreover, credit spreads are countercyclical. Overall, the model fits well many stylized facts.

It is noteworthy that the model can generate volatile spreads only when disaster risk is time-varying. This suggests that variation in aggregate risk is important and plays a role in shaping business cycles.[55]

The amplification effect of disaster risk shock through financial frictions is visible in table 9: while the financial friction model exhibits less volatility than the RBC model when disaster risk is constant, it has more volatility than the RBC model when disaster risk is added. This is especially true for investment volatility, which nearly doubles as time-varying disaster risk is introduced.

The model with constant leverage generates even more volatility of quantities. Because firms cannot delever easily when the probability of disaster rises, the model generates more movements in spreads and investment. Finally, the model implies some volatility of leverage, but it falls

[54] The appendix presents a comparison of the impulse response functions to a TFP shock for the different models, which confirms this result.

[55] The key mechanism of the model is time-varying aggregate uncertainty. This time-varying aggregate uncertainty comes here from a time-varying probability of disaster, but the model implications are similar if the uncertainty takes the form of normally distributed shocks. If the shocks are small, risk aversion needs to be correspondingly higher.

somewhat short of the data. However, the one-period nature of firms in this model makes it difficult to interpret this statistic: the flow and stock of debt are equal in the model, while they behave differently in the data (Jermann and Quadrini (2009), Covas and Den Haan (2009)).

It is interesting to quantify the increase in *systematic* risk that occurs when the disaster probability rises. Figure 16 presents the correlation of defaults that is expected given the probability of disaster today, i.e. $Corr_t(def_{i,t+1}, def_{j,t+1})$ for any two firms i and j in the model economy. In normal times, the probability of disaster is low, and defaults are largely idiosyncratic since aggregate TFP shocks do not create much variation in default rates. Hence, this correlation is low. The correlation becomes much higher, however, when the probability of disaster rises. This is because defaults are now much more likely to be simultaneously triggered by the realization of a disaster. This higher correlation would show up in some asset prices such as CDO or CLO (collateralized debt or loan obligations). This higher correlation stems directly from the increase in aggregate uncertainty, holding idiosyncratic uncertainty constant. This correlation is affected by firms' choices, however, since they decide on how much debt to take which affects their default likelihood: for very large p, firms cut back on debt so much that this correlation may fall.

Overall, the model has two main deficiencies: first, the correlation of consumption and output is too low; second, the equity return is not volatile enough. The latter point is also driven by the fact that equities are only a one-period asset here, implying that the conditional volatility of equity returns equals the conditional volatility of dividends (i.e. there is only a cash flow effect and no discount rate effect).

7.5 Variations on the model

This section considers some implications and extensions of the baseline model, and the sensitivity of the quantitative results to parameter changes.

7.5.1 Default crises and time-varying resilience of the economy

For the purpose of analytical clarity, the benchmark model assumes that default does not affect output: (i) bankruptcy costs are a tax rather than a real resource cost, and (ii) a firm in default is as productive as a firm in good standing. This section relaxes these two assumptions: (i) in reality, bankruptcies are costly: costs include legal fees as well as the loss of intangible capital such as customer goodwill; (ii) firms in default are likely less productive as they need to reorganize and are constrained in their relations with suppliers and customers. Relaxing either of these assumptions implies that an economy with a high level of outstanding debt is prone to “default crises”: any negative shock may drive many firms into default, which further degrades the economy. The exact effect of (i) and (ii) is however different: (i) is a pure wealth effect, while (ii) reduces productivity and hence labor demand. Neither (i) nor (ii) affects the default decision ex-post, since the outside option of equity holders is zero.

An important implication of this extension of the model is that the economy's sensitivity to shocks (or resilience) is time-varying. For instance, as discussed in the previous section, a low probability of disaster leads firms to pick a high leverage. This makes the economy less resilient, i.e. its investment and output will fall more should a bad shock occur. This is consistent with a widely held view that during the 2000s, perception of risk fell, leading firms to increase leverage and making the 2008 recession worse.

Formally, we make the following two changes to the model. The first is to assume that a share ω of the bankruptcy costs is a real resource cost. The second is that firms in default have lower productivity, by a factor $(1-\zeta)^{\alpha}$. These two changes do not affect the expression for the default threshold $\varepsilon^*_{t+1} = \frac{B_{t+1}}{K^w_{t+1}R^K_{t+1}}$.Total output, taking into account the lower productivity of firms in default, is now

$$Y_t = (K_t)^{\alpha}(z_t N_t)^{1-\alpha}\left(\left(1-(1-\zeta)\Omega(\varepsilon^*_{t+1})\right)\right)^{\alpha}.$$

The resource constraint now reads

$$C_t + I_t + (1-\theta)\omega\Omega\left(\varepsilon^*_{t+1}\right)R^K_t K_t = Y_t.$$

We also need to modify consequently the firm value and bond price equations and the associated first order conditions; these equations are available in the appendix. As a result of this change, the quantity of debt B is now an additional state variable.

Figure 17 illustrates the negative effect of outstanding debt on the economy for the case $\zeta = 0.5$ and $\omega = 0$, i.e. firms in default are more productive. (The appendix presents an examples for the case of $\zeta = 0$ and $\omega = 0.5$, i.e. bankruptcies have real resource costs.) Ceteris paribus, a larger amount of debt increases default rates, and reduces output, employment, investment and consumption.

7.5.2 State-contingent debt

The defining characteristic of debt is that it is not state contingent. In the aftermath of the 2008 financial crisis, several economists have proposed that debt should be conditioned on large aggregate shocks. This section evaluates this proposal by allowing firms in the model to issue debt which repayments are contingent on the disaster realization x'.

The model is easily modified; first, the budget constraint now reads,

$$K_{t+1}^{w} = S_t + \chi q_t^{nd} B_{t+1}^{nd} + \chi q_t^{d} B_{t+1}^{d},$$

where B_{t+1}^{nd} (resp. B_{t+1}^{d}) is the face value of the debt to be repaid in non-disaster (resp. disaster) states, and q_t^{nd} (resp. q_t^{d}) the associated price:

$$q_t^{nd} = E_t\left((1-x_{t+1})M_{t+1}\left(\int_{\varepsilon_{t+1}^{*}}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}}\int_{0}^{\varepsilon_{t+1}^{*}} \varepsilon R_{t+1}^{K} K_{t+1}^{w} dH(\varepsilon)\right)\right),$$

where $(1 - x_{t+1})$ is a dummy equal to 1 if no disaster happens, and similarly for q_t^{d}.Taking first-order conditions leads to the following characterization of the equilibrium: first, the Euler equation is

$$E_t\left(M_{t+1}R_{t+1}^{K}\left(\begin{array}{c} 1+(\chi-1)\,L_{t+1}^{nd}(1-x_{t+1})\left(1-H\left(\varepsilon_{t+1}^{*}\right)\right) \\ +(\chi-1)\,L_{t+1}^{d}x_{t+1}\left(1-H\left(\varepsilon_{t+1}^{*}\right)\right) \\ +(\theta\chi-1)\,\Omega(\varepsilon_{t+1}^{*}) \end{array}\right)\right) = 1,$$

and second, optimal debt is determined through the two equations:

$$\begin{aligned} \frac{\chi-1}{\chi}E_t\left((1-x_{t+1})M_{t+1}\left(1-H\left(\varepsilon_{t+1}^{*}\right)\right)\right) &= (1-\theta)\,E_t\left(M_{t+1}\Omega'(\varepsilon_{t+1}^{*})\,(1-x_{t+1})\right), \quad (29)\\ \frac{\chi-1}{\chi}E_t\left(x_{t+1}M_{t+1}\left(1-H\left(\varepsilon_{t+1}^{*}\right)\right)\right) &= (1-\theta)\,E_t\left(M_{t+1}\Omega'(\varepsilon_{t+1}^{*})x_{t+1}\right). \end{aligned}$$

The Euler equation interpretation is similar to that of the benchmark model; the investor takes into account the total user cost of debt, which now must take into account the different leverage in disaster vs. non-disaster states. The optimal leverage condition simply says that, rather than equating expected discounted marginal costs and benefits of debt over all the states together, the firm can now equate these expected marginal costs and benefits *conditional on the disaster happening or not.* This added flexibility will lead the firm to issue little debt that is payable in disaster states, since bankruptcy is much more likely and costly in these states. As a useful special case, suppose that there are no TFP shocks or shocks to p, then the expectations are just expectations over the idiosyncratic shocks ε, and the first-order condition states, if we denote default cutoff in non-disaster states by ε_{t+1}^{nd*} and in disasters by ε_{t+1}^{d*}:

$$\frac{\chi - 1}{\chi}\left(1 - H\left(\varepsilon_{t+1}^{d*}\right)\right) = (\theta - 1)\,\Omega'(\varepsilon_{t+1}^{d*}),$$

$$\frac{\chi - 1}{\chi}\left(1 - H\left(\varepsilon_{t+1}^{nd*}\right)\right) = (\theta - 1)\,\Omega'(\varepsilon_{t+1}^{nd*}),$$

implying that $\varepsilon_{t+1}^{d*} = \varepsilon_{t+1}^{nd*}$, i.e. $\frac{B_{t+1}^{d}}{K_{t+1}^{w}(1-b_k)} = \frac{B_{t+1}^{nd}}{K_{t+1}^{w}}$ or $B_{t+1}^{d} = B_{t+1}^{nd}(1 - b_k)$. Hence, the firm targets the *same* default probability, conditional on a disaster happening, and conditional on no disaster happening. This implies a much lower face value of debt in disasters.

Figure 18 compares the response of the model with state-contingent debt to an increase in disaster risk, with the response of the benchmark model. The amplification effect largely disappears, and the model implies now no more volatility in investment than the frictionless RBC model. Hence, while the assumption that private contracts are not made contingent on aggregate realizations is made in many models (such as Bernanke, Gertler and Gilchrist (1999) or Kiyotaki and Moore (1997)), this result suggest that it is far from innocuous. Krishnamurthy (2003) similarly found that allowing for conditionality reduces or eliminate the amplification effect of financial frictions.

The benefits of debt conditionality in reducing volatility in response to shocks to disaster risk, comes on top of the obvious advantage that, should a disaster happen, there will be fewer defaults, which are likely to be costly (as in the previous section). This suggests that debt conditionality is likely valuable, provided that disasters can be well defined in a contract.

7.5.3 Welfare cost of the tax shield

Following a large literature in corporate finance, the model features as a prime determinant of capital structure the tax subsidy to debt, or tax shield. The tax shield is inefficient in the model for two reasons. First, the tax shield lowers the user cost of capital and hence encourages capital accumulation. However, the competitive equilibrium of the model without taxes is already Pareto optimal, hence the subsidy leads to *overaccumulation* of capital. Second, the tax shield also amplifies fluctuations in aggregate quantities, including consumption, and hence reduces welfare. Table 16 illustrates this effect by displaying the volatility of output, investment and employment, for various values of χ. Both in terms of steady-states and in terms of fluctuations then, the tax subsidy generates deadweight losses. A figure in appendix gives the welfare cost of the tax subsidy, as a function of χ. For our benchmark calibration of $\chi = 1.062$, removing the tax shield entirely would increase welfare substantially, equivalent to a permanent increase of consumption of approximately 3.52%. However, in the presence of a corporate tax, the tax shield may have some value as it brings the economy closer to the zero capital tax economy.

7.5.4 Capital adjustment costs

While the benchmark model abstracts from adjustment costs in the interest of simplicity, introducing them is useful to generate further volatility in the value of capital. In particular, the model implies that an increase in the probability of disaster has essentially no effect on realized equity returns or bond returns.[56] This implication is overturned if there are adjustment costs, because the price of capital then falls following an increase in the probability of disaster, since investment and marginal Q fall. It is simplest to consider an external adjustment cost formulation. Suppose that capital goods are produced by a competitive investment sector which takes I_t consumption goods at time t, and K_t capital goods at time t, and generates $K_{t+1} = (1-\delta)K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t$ capital goods next period. These capital goods are then sold in a competitive market to final goods producing firms at a price given by: $P_t^K = \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}$.The same formulas as in the model then apply, with the proviso that the return on capital R_{t+1}^K is now

$$R_{t+1}^K = \left(\frac{(1-\delta)\, P_{t+1}^K + \alpha \frac{Y_{t+1}}{K_{t+1}}}{P_t^K}\right)(1 - x_{t+1} b_k),$$

and $V_t = K_t R_{t+1}^K P_t^K$, and $\varepsilon_{t+1}^* = \frac{B_{t+1}}{R_{t+1}^K K_{t+1}^w P_t^K} = \frac{L_{t+1}}{R_{t+1}^K P_t^K}$. Following Jermann (1998), I set $\Phi(x) = a_0 + a_1 \frac{x^{1-\eta}}{1-\eta}$, where a_0 and a_1 are picked to make the steady-state investment rate and marginal Q independent of η. Tables 13 through 15 report model moments for two values of η, and a figure in appendix compares the impulse response function of the benchmark model (without adjustment costs) and the model with adjustment costs ($\eta = .1$), when the shock is an increase in the probability of disaster. As expected, adjustment costs smooth the response of investment and output. The qualitative dynamics, as well as the asset prices, remain similar. When the probability of disaster rises, the return on equity is now lower, and the return on the corporate bond is also slightly lower, reflecting the fall in the resale value of capital and the ensuing higher

[56] Technically, the only effect is through a decrease in the supply for labor which pushes the wage up, leading to slightly lower profits and hence slightly higher default rates.

default rate.

7.5.5 Role of the IES and risk aversion

While the households are assumed to have recursive utility, the model can also be solved in the special case of expected utility. When the elasticity of substitution is kept equal to 2, and the risk aversion is lowered to .5 to reach expected utility, the *qualitative* implications are largely unaffected. Tables 13 through 15 report the model moments with this specification. Because risk aversion is lower, all risk premia are lower, and the response of quantities to a probability of disaster shock is also smaller since agents care less about risk.[57]

In contrast, when the elasticity of substitution is small, a shock to the probability of disaster may lead to different *qualitative* effects. When the IES is low enough, investment, output and employment rise (rather than fall) as the probability of disaster rises. The intuition is that higher risk makes people save more, despite the fact that the capital is more risky. In the frictionless model, the threshold value for the IES is exactly unity. In the model of this paper, higher uncertainty has a more negative effect on investment demand, and hence the threshold value for the IES is lower than unity. Hence, for a certain range of values of IES below unity, the financial friction model implies that higher disaster risk lowers economic activity, while the frictionless model implies the opposite – an extreme example of the potential importance of financial frictions. Tables 13 through 15 report the model moments with a low IES (.25), which generates the opposite comovement. This specification is unattractive, since it implies that risk premia are procyclical, contrary to the data.

7.5.6 Comparison with idiosyncratic uncertainty shocks

Following Bloom (2009), several recent studies consider the effect of an increase in idiosyncratic uncertainty, σ_ε in our notation. While Bloom (2009) focused on the transmission of this shock through adjustment costs frictions, Arellano, Bai and Kehoe (2009), and Gilchrist, Sim and Zakrajek (2010) use default risk frictions, similar to my model. The shock to disaster risk is also an increase in uncertainty, and hence has a qualitatively similar effect. For instance, comparing figures 10 and 11 shows that the two parameters p and σ_ε have similar effects on steady-states.

[57] There is one qualitative change, but it is hard to discern in most statistics. A shock to the probability of disaster increases consumption, hence with expected utility it is a "good state", i.e. low marginal utility of consumption state. This is not the case with Epstein-Zin utility, since the future value is lower, making a high probability of disaster state a "bad state" (high marginal utility of consumption). This in turn implies that assets which pay off well in that state have higher risk premia rather than lower risk premia.

However, the channel through which the mechanism operates is somewhat different in my model, because an increase in aggregate uncertainty makes defaults more systematic and hence affects the bond risk premium.

To illustrate the differences in the mechanism, we can think of three experiments. First, the response of the economy to a shock to σ_ε is essentially unaffected by the coefficient of risk aversion. In contrast, as shown in section 7.5.1, the response to an increase in disaster risk in my model is stronger when risk aversion is larger. Second, in the frictionless version, an increase in disaster risk leads to a recession, whereas an increase in idiosyncratic risk has no effect on economic activity.[58] Finally, suppose that we consider a shock to disaster risk, such that high disaster risk states have low idiosyncratic volatility, making the total quantity of risk constant over time. In essence, we are changing only the relative importance of aggregate and idiosyncratic risk, and hence the *correlation* across firms. This shock reduces investment and output, if risk aversion is positive, even though total risk does not change at the microeconomic level. The appendix produces the impulse responses corresponding to these three experiments.

The aim of this discussion is not to argue that idiosyncratic uncertainty shocks are unimportant, but that the channel through which they operate is different than the channel through which aggregate uncertainty shock operate, at least in this model. The two approaches have different strengths: my model connects well with the evidence on the behavior of credit spreads, correlation risk and aggregate risk premia. In contrast, the studies of Arellano et al. and Gilchrist et al. focus on more realistic microeconomic heterogeneity, and take into account the effect of uncertainty on reallocation and on the labor wedge among other issues.

7.5.7 Samples with disasters

So far the results reported are calculated in samples which do not include disasters. Large excess returns arise for two reasons: first, a standard risk premium; second, a sample selection ("Peso problem") since the sample does not include the lowest possible return realizations. To quantify the importance of the second effect, tables 13 through 15 report the model moments in the benchmark model if the sample includes disasters. Quantities and returns are of course more volatile since they include some large realizations. The average excess returns on equities is 1.38% (vs. 2.30% in a sample without disasters). Similarly, the average return on corporate

[58] In some models, an increase in uncertainty would lead to a boom by leading to labor reallocation among firms with decreasing return to scale. But in the model of this paper, idiosyncratic shocks literally wash out because of the combined assumptions of constant return to scale and frictionless labor market.

bonds is 0.44% (vs. 0.60% in a sample without disasters (unreported in tables)). The dynamics of credit spreads and leverage are completely unaffected.

8 Conclusion

This work shows how introducing disaster risk into a standard RBC model improves its fit of asset return data, preserves its success for quantities in response to a TFP shock, and creates some interesting new macroeconomic dynamics. The model can replicate not only the second moments of quantities and asset returns, but also a variety of empirical relationships between macroeconomic quantities and asset prices, which have so far largely eluded researchers.

This parsimonious setup is fairly tractable, which allows to derive some analytical results and makes it easy to embed into richer models. One particular extension that I study in detail is when leverage and default are choice variables. I find that the model then reproduces well the level, volatility, and countercyclicality of credit spreads.

More broadly, the quantitative and empirical results of this paper suggest an important role for time-varying risk in macroeconomic models, and give some hope that we may be able to connect better asset prices and business cycles.

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Parameter	Symbol	Value	Target
Capital share	α	.34	Cooley and Prescott (1995)
Depreciation rate	δ	.02	Cooley and Prescott (1995)
Share of consumption in utility	υ	.3	Cooley and Prescott (1995)
Discount factor	β	.994	Mean short-term interest rate
Adjustment cost curvature	η	0.15	Investment volatility
Trend growth of TFP	μ	.0025	Measured TFP
Standard deviation of ordinary TFP shock	σ	.01	Measured TFP
IES	$1/\gamma$	2	A priori (see text)
Risk aversion over the consumption-leisure bundle	θ	6	Average equity premium
Size of disaster in TFP	b_{tfp}	.43	Barro (2006)
Size of disaster for capital	b_k	.43	Barro (2006)
Persistence of log(p)	ρ_p	.92	Predictability regressions
Unconditional std. dev. of log(p)	$\frac{\sigma_p}{\sqrt{1-\rho_p^2}}$	1.85	Volatility of returns
Leverage	λ	2	Abel (1999), Barro (2006)
Recovery rate for bonds during disasters	$1-r(1-b)$	.828	Barro (2006)

Table 1: **Parameter values for the benchmark model.** The time period is one quarter.

	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\sigma(\Delta \log Y)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$	$\rho_{I,C}$
Data	0.57	2.68	0.92	0.98	0.45	0.68	0.71	0.49
No disaster	0.66	1.86	0.24	0.78	1.00	1.00	0.99	0.99
Constant p	0.67	1.87	0.24	0.78	1.00	1.00	0.99	0.99
Constant p*	0.96	1.12	0.06	3.10	1.00	1.00	0.52	0.99
Benchmark	0.73	3.03	0.54	0.83	0.66	0.85	0.72	0.21
Benchmark*	0.96	1.35	0.15	2.88	0.87	0.90	0.42	0.60

Table 2: **Business cycle statistics.** Second moments implied by the model, for different calibrations. Quarterly data. The statistics are computed in a sample without disasters, except for the rows marked with a star, which are computed in a full sample. rho(A,B) is the correlation of the growth rate of time series A and B. Data sources in appendix.

	$E(R_f)$	$E(R_b)$	$E(R_e)$	$E(R_{e,lev})$	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e)$	$\sigma(R_{e,lev})$
Data	—	0.21	—	1.91	—	0.81	—	8.14
No disaster	0.71	0.71	0.71	0.74	0.04	0.04	0.24	1.59
Constant p	0.02	0.32	0.77	1.22	0.04	0.04	0.25	1.53
Constant p*	0.02	0.24	0.58	0.92	0.04	0.85	2.20	4.07
Benchmark	0.15	0.42	0.88	1.93	1.37	0.85	0.40	7.14
Benchmark*	0.17	0.36	0.69	1.60	1.29	1.28	2.06	7.94

Table 3: **Financial Statistics.** Mean and standard deviation of returns implied by the model for (a) a pure risk-free asset, (b) a one-quarter government bond, (c) a claim to dividends, (d) a claim on levered output. Quarterly data. The statistics are computed in a sample without disasters, except for the rows marked with a star, which are computed in a full sample. Data sources in appendix.

	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\sigma(\Delta \log Y)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$	$\rho_{I,C}$
Data	0.57	2.68	0.92	0.98	0.45	0.68	0.71	0.49
Benchmark	0.73	3.03	0.54	0.83	0.66	0.85	0.72	0.21
$b_k = 0, \eta = 0$	0.77	9.45	1.07	1.34	-0.13	0.89	0.89	-0.41
$b_k = 0$	0.80	5.55	0.85	0.99	0.24	0.85	0.78	-0.21
$b_k = 0, \eta = .5$	0.90	3.71	0.65	0.80	0.56	0.77	0.57	-0.03
$b_{tfp} = 0, \eta = .15$	0.77	3.67	0.66	0.87	0.50	0.83	0.71	-0.03
Recoveries	0.71	2.56	0.44	0.81	0.79	0.88	0.74	0.43
Multiperiod disasters	0.73	2.90	0.52	0.82	0.69	0.86	0.73	0.26

Table 4: **Robustness and Extensions. Business cycle statistics.** Second moments implied by the model, for different calibrations. Quarterly data. The statistics are computed in a sample without disasters.

	$E(R_f)$	$E(R_b)$	$E(R_e)$	$E(R_{e,lev})$	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e)$	$\sigma(R_{e,lev})$
Data	—	0.21	—	1.91	—	0.81	—	8.14
Benchmark	0.15	0.42	0.88	1.93	1.37	0.85	0.40	7.14
$b_k = 0, \eta = 0$	0.57	0.77	0.60	1.71	0.72	0.38	2.07	6.15
$b_k = 0$	0.51	0.72	0.64	1.71	0.69	0.29	1.68	6.25
$b_k = 0, \eta = .5$	0.48	0.68	0.72	1.71	0.74	0.33	1.11	6.22
$b_{tfp} = 0$	0.70	0.80	0.91	0.86	0.32	0.12	0.51	1.60
Recoveries	0.38	0.56	0.86	1.42	0.94	0.57	0.34	4.68
Multiperiod disasters	0.49	0.62	0.86	1.52	1.04	0.64	0.38	5.52

Table 5: **Robustness and Extensions. Financial statistics.** Mean and standard deviation of returns implied by the model for (a) a pure risk-free asset, (b) a one-quarter government bond, (c) a claim to dividends, (d) a claim on levered output. Quarterly data. The statistics are computed in a sample without disasters.

			b	R^2	b	R^2
CRRA	cons.	TFP shocks only	1.75	0.20	1.75	0.20
CRRA	cons. + leisure	TFP shocks only	1.00	0.05	1.00	0.05
Epstein-Zin	cons.	TFP shocks only	1.76	0.22	1.76	0.22
Epstein-Zin	cons. + leisure	TFP shocks only	0.92	0.04	0.92	0.04
CRRA	cons.	TFP shocks + p shocks	2.26	0.15	1.48	0.15
CRRA	cons. + leisure	TFP shocks + p shocks	0.60	0.10	0.38	0.09
Epstein-Zin	cons.	TFP shocks + p shocks	0.51	0.14	0.40	0.13
Epstein-Zin	cons. + leisure	TFP shocks + p shocks	0.38	0.12	0.23	0.12

Table 6: **Estimation of the IES in model-generated data.** This table reports the slope and R2 from a univariate regression of consumption growth from t to t+1 on the time t short-term government bond rate (first two columns) or the pure risk-free rate (last two columns). Each row corresponds to a variant of the model, which differ according to the utility function (power utility or Epstein-Zin), whether labor supply is fixed or labor is part of the utility function, and whether the model has only shocks to TFP (no disaster risk) or the model has both TFP shocks and shocks to disaster risk.

		Full sample		With two-quarter lag		Recessions	
		RBC	RBC+p	RBC	RBC+p	RBC	RBC+p
$Corr$(model,data)	C	43.26	24.96	31.20	11.74	-16.29	-31.72
	I	50.69	44.18	54.61	60.81	44.92	61.83
	N	16.07	11.02	48.90	48.90	33.96	43.36
	Y	67.71	64.33	56.56	60.80	28.88	39.27
Cov(model,data)	C	0.31	0.17	0.23	0.08	-0.10	-0.19
	I	5.15	8.00	5.56	10.85	3.08	8.79
	N	0.08	0.14	0.26	0.60	0.10	0.39
	Y	1.11	1.24	0.92	1.16	0.29	0.48
$E\left\|\text{data-model}\right\|$	C	194.51	210.35	206.65	227.82	43.51	50.99
	I	848.01	917.13	831.41	768.78	187.55	129.70
	N	369.59	384.13	345.39	326.76	90.44	72.00
	Y	243.94	253.55	255.29	247.30	65.31	56.20

Table 7: **Fit of the model.** The table reports three statistics of fit, for each time series (C,I,N,Y), and for the full sample, with a two-quarter lag, and for the subsample of recessions. The statistics are the correlation between model and data, the covariance between model and data, and the mean absolute error. See section 6 for the construction of the series.

		Full sample			Recessions		
	$x_t =$	Data	RBC	RBC+p	Data	RBC	RBC+p
$E\left\|x_t\right\|$	C	2.11	1.18	1.18	0.52	0.29	0.22
	I	9.55	3.74	6.51	2.72	1.00	1.92
	N	3.72	0.53	1.23	1.03	0.14	0.35
	Y	3.09	1.82	2.11	1.05	0.48	0.59

Table 8: **Volatility and conditional volatilities.** The table reports the mean absolute value of each time series (C,I,N,Y), for the data and for each model; results are reported for the full sample and for the subsample of recessions. This is a measure of the volatility implied by each model. See section 6 for the construction of the series.

Parameter	Symbol	Value
Capital share	α	.3
Depreciation rate	δ	.08
Share of consumption in utility	υ	.3
Discount factor	β	.98
Trend growth of TFP	μ	.01
Standard deviation of TFP shock	σ	.02
Intertemporal elasticity of substitution	$1/\psi$	2
Risk aversion	γ	4
Mean probability of disaster		.017
Distribution of $b_{tfp} = b_k$: values		(.15,.25,.35,.45,.57)
Distribution of $b_{tfp} = b_k$: probabilities		(.333,.267,233,.033,.133)
Persistence of $\log(p)$	ρ_p	.75
Unconditional std. dev. of $\log(p)$	$\frac{\sigma_p}{\sqrt{1-\rho_p^2}}$	1.5
Idiosyncratic shock volatility	σ_ε	0.2267
Tax subsidy	$\chi - 1$	0.0616
Recovery rate	θ	0.4

Table 9: **Parameter values for the extension with endogenous leverage.** The time period is one year.

		$\sigma(\Delta \log Y)$	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\rho_{C,Y}$	$\rho_{I,Y}$
	Data	2.78	0.65	2.52	0.96	0.61	0.80
No disaster risk	Endog. leverage	1.80	0.55	1.88	0.33	0.97	0.99
	Constant leverage	1.80	0.55	1.89	0.33	0.97	0.99
	RBC	1.82	0.56	2.47	0.35	0.96	0.98
Constant Disaster risk	Endog. leverage	1.81	0.55	1.97	0.34	0.96	0.99
	Constant leverage	1.81	0.55	1.97	0.34	0.96	0.99
	RBC	1.83	0.56	2.50	0.35	0.96	0.98
Time-varying Disaster risk	**Endog. leverage**	**2.11**	**0.77**	**3.38**	**0.83**	**0.12**	**0.86**
	Constant leverage	2.46	0.89	4.62	1.05	-0.23	0.86
	RBC	1.86	0.60	2.89	0.46	0.79	0.91

Table 10: **Business cycle statistics (annual).** Second moments implied by the model with endogenous leverage, for different versions of the model. The statistics are computed in a sample without disasters. rho(A,B) is the correlation of the growth rate of time series A and B. The endogenous leverage model is in bold.

		$E(R_f)$	$E(R_e)$	E(y)	σ(y)	ρ(y,gdp)	$\sigma(R_f)$	$\sigma(R_e)$
	Data	0.80	7.60	0.94	0.41	-0.37	2.50	16.20
No disaster risk	Endog. leverage	2.46	2.44	0.51	0.00	-0.56	0.23	0.39
	Cst leverage	2.46	2.39	0.55	0.01	0.64	0.23	0.39
	RBC	2.59	2.54	-0.01	0.00	0.11	0.27	0.33
Constant	Endog. leverage	1.21	3.84	1.29	0.00	-0.65	0.22	0.41
Disaster risk	Cst leverage	1.21	3.81	1.32	0.01	0.65	0.22	0.41
	RBC	1.32	2.65	-0.01	0.00	-0.03	0.26	0.33
Time-varying	**Endog. leverage**	**1.31**	**3.60**	**0.98**	**0.46**	**-0.53**	**2.33**	**1.18**
Disaster risk	Cst leverage	1.31	3.85	1.21	1.40	-0.80	2.58	1.88
	RBC	1.41	2.65	-0.01	0.00	-0.07	2.05	0.34

Table 11: **Financial Statistics, 1.** Mean and standard deviation of the risk-free return, the equity return, and the spread between the corporate bonds and the risk-free bond (denoted y). The statistics are calculated in a sample without disasters. The correlation is the correlation between the spread BAA-AAA and HP-filtered GDP.

		E(Lev)	Std(Lev)	E(ProbDef)	Std(ProbDef)
	Data	0.45	0.09	0.39	NA
No disaster risk	Endog. leverage	0.56	0.00	0.79	0.01
	Constant leverage	0.57	0.00	0.86	0.03
	RBC	0.00	0.00	0.00	0.00
Constant	Endog. leverage	0.54	0.00	0.50	0.01
Disaster risk	Constant leverage	0.55	0.00	0.53	0.02
	RBC	0.00	0.00	0.00	0.00
Time-varying	**Endog. leverage**	**0.54**	**0.04**	**0.58**	**0.23**
Disaster risk	Constant leverage	0.54	0.00	0.49	0.02
	RBC	0.00	0.00	0.00	0.00

Table 12: **Financial Statistics, 2.** Mean and volatility of leverage and of probability of default. The statistics are calculated in a sample without disasters. Data from Chen, Collin-Dufrense and Goldstein (2009).

	$\sigma(\Delta \log Y)$	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\rho_{C,Y}$	$\rho_{I,Y}$
Data	2.78	0.65	2.52	0.96	0.61	0.61
Benchmark financial friction	2.11	0.77	3.38	0.83	0.12	0.86
Samples with disasters	5.47	1.00	1.58	0.32	0.81	0.80
Adjustment costs ($\eta = .1$)	1.75	0.83	2.60	0.62	0.54	0.83
Adjustment costs ($\eta = .2$)	1.58	0.90	2.11	0.48	0.73	0.80
IES = .5	1.59	0.78	1.58	0.23	0.96	0.96
IES = .25	1.57	0.94	2.11	0.51	0.70	0.80
Risk aversion = .5	1.97	0.69	2.83	0.67	0.39	0.88

Table 13: **Extensions of the model: business cycle statistics (annual).**

	$E(R_f)$	$E(R_e)$	E(y)	σ(y)	ρ(y,gdp)	$\sigma(R_f)$	$\sigma(R_e)$
Data	0.80	7.60	0.94	0.41	-0.37	2.50	16.20
Benchmark financial friction	1.31	3.60	0.98	0.46	-0.53	2.33	1.18
Samples with disasters	1.30	2.68	0.98	0.46	-0.52	2.34	7.05
Adjustment costs ($\eta = .1$)	1.31	3.62	0.98	0.46	-0.46	2.17	1.75
Adjustment costs ($\eta = .2$)	1.32	3.62	0.98	0.46	-0.37	2.09	2.07
IES = .5	1.61	3.90	0.98	0.47	-0.10	2.36	1.11
IES = .25	1.97	4.27	0.98	0.46	0.21	2.40	1.11
Risk aversion = .5	2.28	3.68	0.77	0.28	-0.63	1.50	0.91

Table 14: **Extensions of the model: Financial Statistics, 1.**

	E(Lev)	Std(Lev)	E(ProbDef)	Std(ProbDef)
Data	0.45	0.09	0.39	0.51
Benchmark financial friction	0.54	0.04	0.58	0.23
Samples with disasters	0.54	0.04	0.81	2.55
Adjustment costs ($\eta = .1$)	0.54	0.04	0.58	0.24
Adjustment costs ($\eta = .2$)	0.54	0.05	0.58	0.24
IES = .5	0.54	0.04	0.58	0.23
IES = .25	0.54	0.04	0.58	0.23
Risk aversion = .5	0.55	0.03	0.65	0.18

Table 15: **Extensions of the model: Financial Statistics, 2.**

	$\sigma(\Delta \log Y)$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	E(Lev)
$\chi = 1.062$ (Benchmark)	2.11	3.38	0.83	0.54
$\chi = 1.06$	2.11	3.41	0.82	0.54
$\chi = 1.05$	2.05	3.41	0.77	0.53
$\chi = 1.04$	2.00	3.37	0.71	0.51
$\chi = 1.03$	1.96	3.30	0.64	0.50
$\chi = 1.02$	1.91	3.14	0.57	0.47
$\chi = 1.01$	1.89	3.00	0.51	0.43
$\chi = 1.005$	1.87	2.94	0.48	0.39
$\chi = 1.002$	1.86	2.92	0.47	0.34
$\chi = 1.001$	1.86	2.91	0.46	0.30
$\chi = 1$ (RBC)	1.86	2.89	0.43	0.00

Table 16: **Effect of tax shield parameter on mean leverage and volatilities of quantities.**

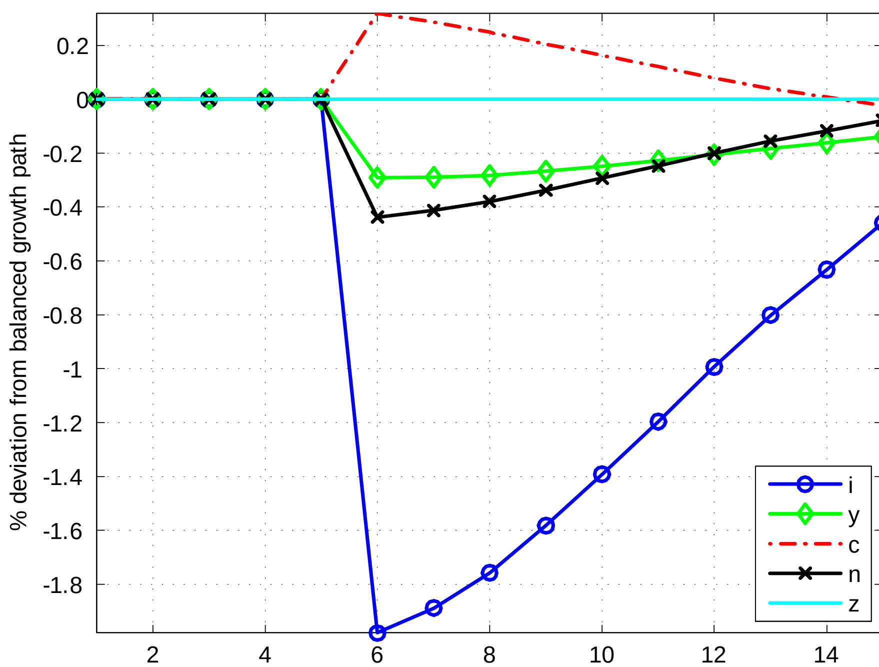

Figure 1: **The effect of an increase in the probability of disaster on macroeconomic quantities**. Impulse response of (I,Y,C,N,Z) to a shock to the probability of disaster at $t = 6$. Time (x-axis) is in quarters. The probability of disaster goes from its long-run average (0.425% per quarter) to twice its long-run average then mean-reverts according to its AR(1) law of motion. For clarity, this figure assumes that there is no shock to TFP, and no disaster realized.

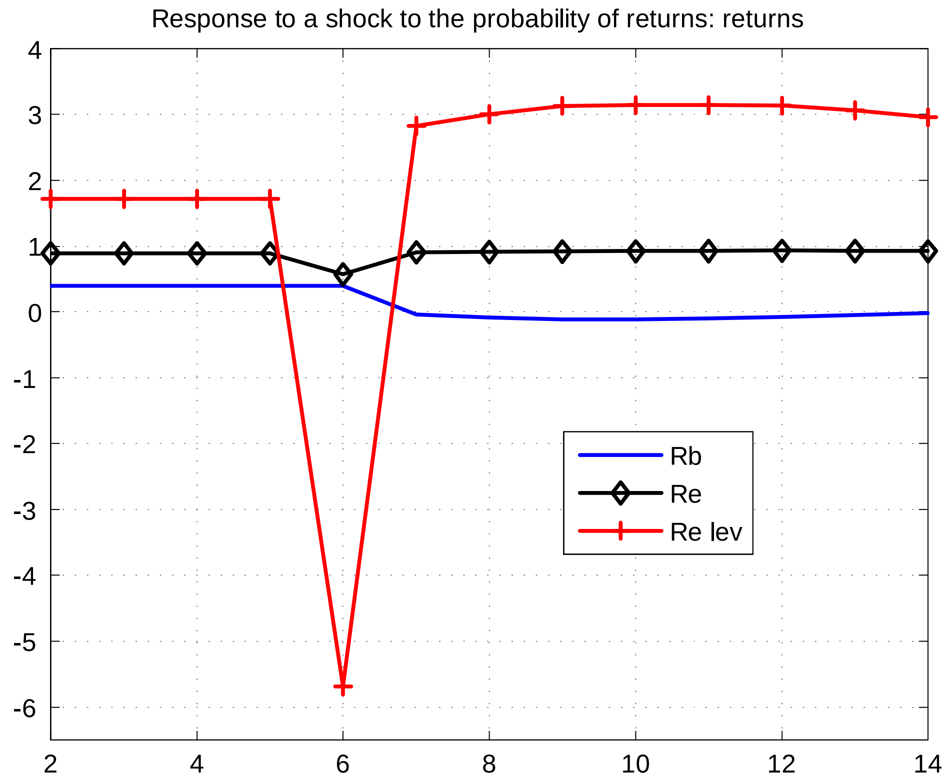

Figure 2: **The effect of an increase in the probability of disaster on asset returns and spreads**. Impulse response of asset returns to a shock to the probability of disaster at $t = 6$. Time (x-axis) is in quarters. The probability of disaster doubles at $t = 6$, starting from its long-run average. The figure plots the short-term government bond return, the equity return, and the levered equity return.

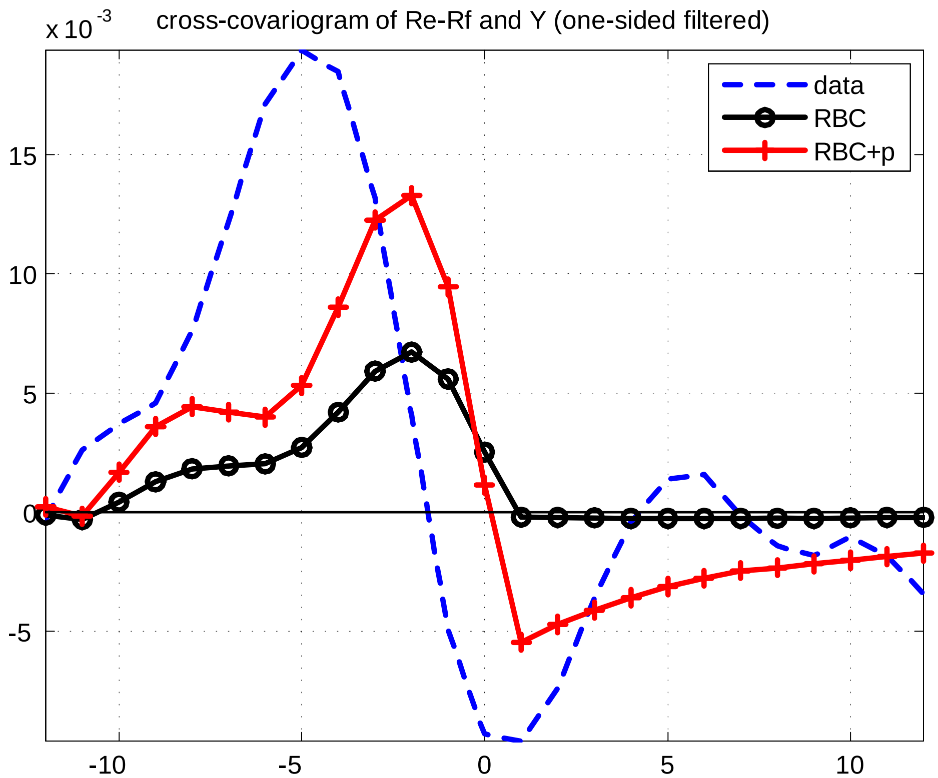

Figure 3: **Cross-covariogram of GDP and excess stock returns in the model and in the data.** Cross-covariogram of the (one-sided Baxter-King filtered) log GDP, and excess stock returns, in the data (blue dashed line), the RBC model, i.e. the model with only TFP shocks, (black circles) and the benchmark model with both p-shocks and TFP shocks (red crosses). The lag/lead (x-axis) is in quarters. The model covariograms are obtained by running 1000 simulations of length 200 each, and averaging.

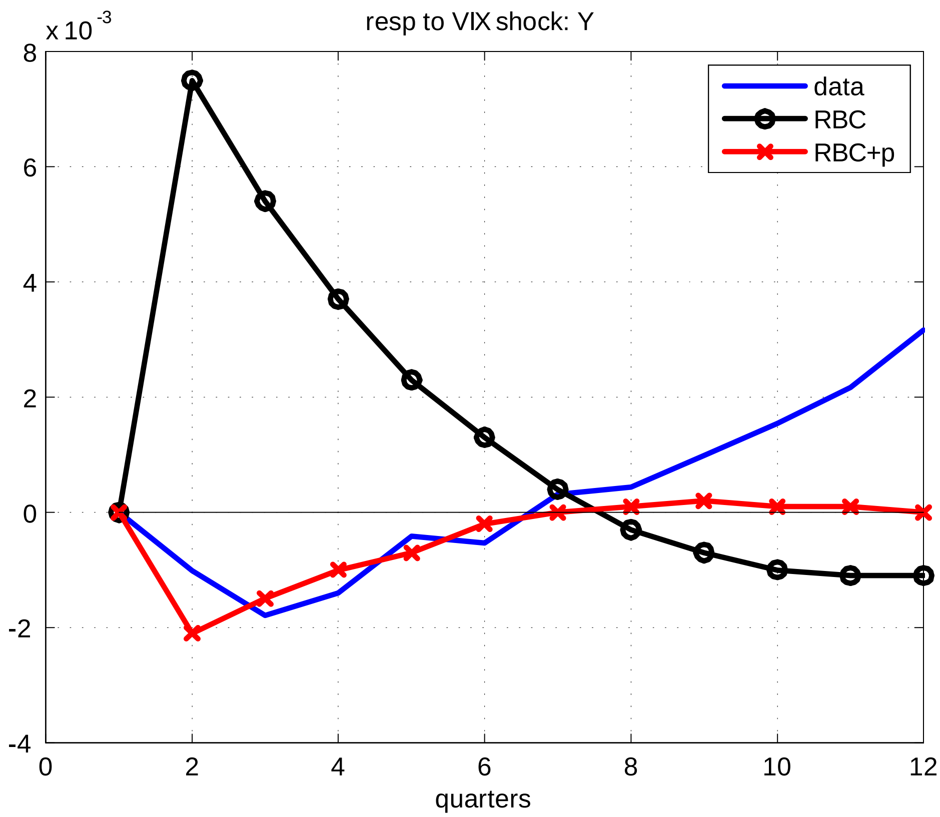

Figure 4: **Impulse response to a shock to VIX in a bivariate (VIX,GDP) VAR, in the model and in the data.** This figure gives the IRF to a one-standard deviation shock to VIX, in a bivariate VAR of HP-filtered log GDP, and HP-filtered VIX, in the data (blue full line), the RBC model, i.e. the model with only TFP shocks (black circles), and the benchmark model with both p-shocks and TFP shocks (red crosses). The model IRFs are obtained by running 1000 simulations of length 200 each, running the VAR on each simulation, and averaging. Orthogonalization assumption: GDP doesn't react to a VIX shock at $t = 0$.

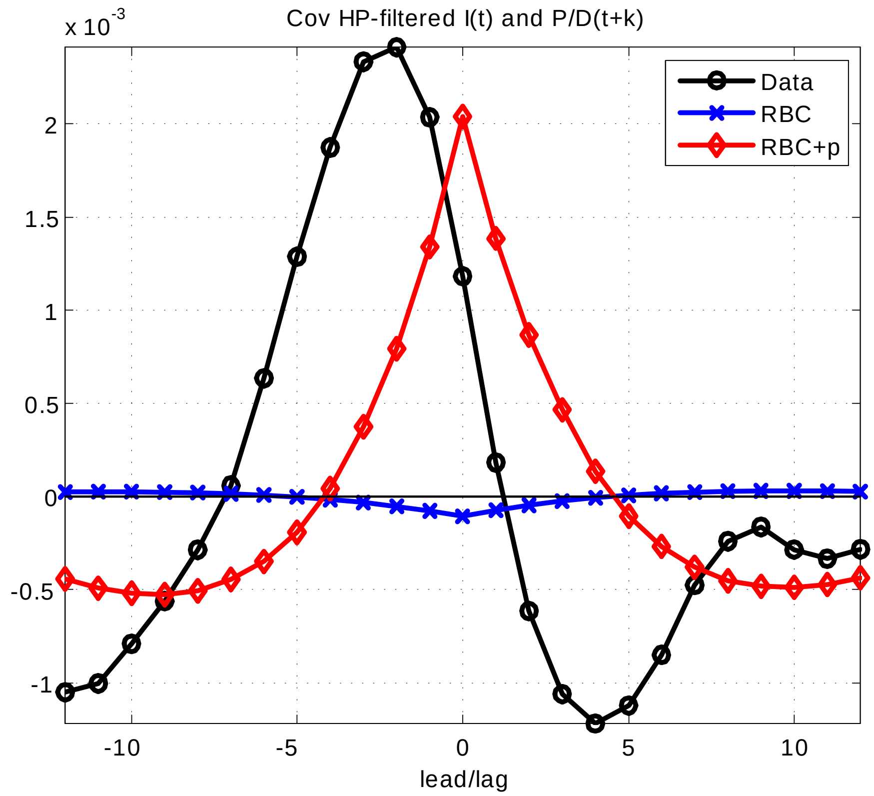

Figure 5: **Cross-covariogram of investment and P-D ratio in the model and in the data.** Cross-covariogram of the log HP filtered investment, and the HP filtered P-D ratio, in the data (black diamond line), in the RBC model, i.e. the model with only TFP shocks (blue crosses), and the benchmark model with both p-shocks and TFP shocks (red diamonds). The model covariograms are obtained by running 1000 simulations of length 200 each, and averaging.

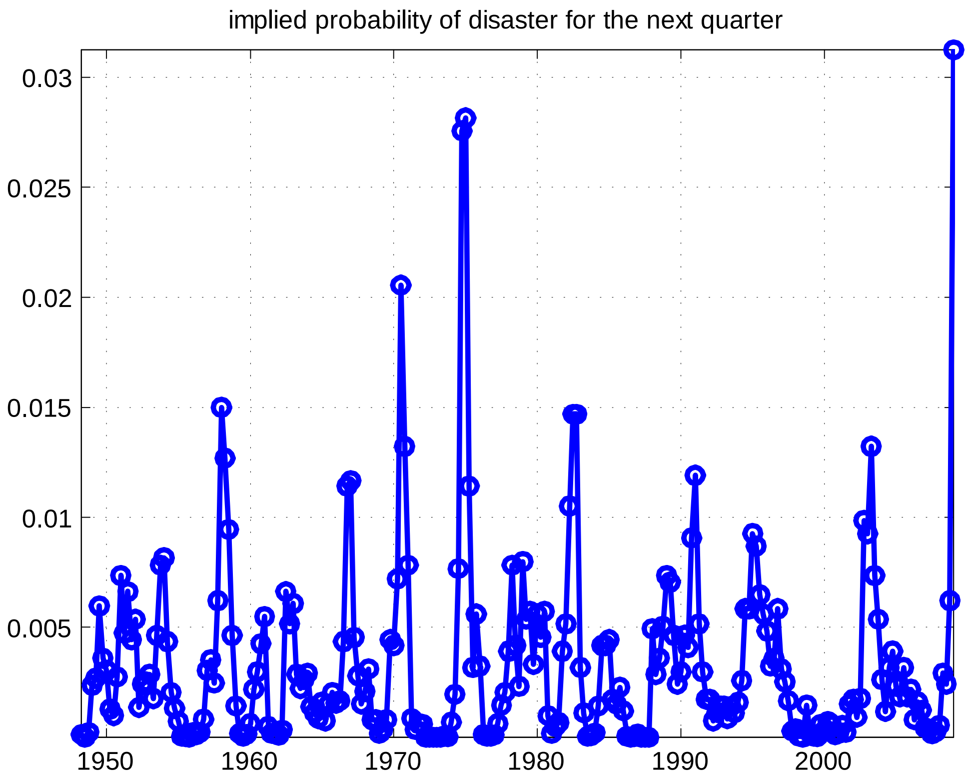

Figure 6: **Time-series for the quarterly probability of disaster (1948q1 to 2008q4).** This picture plots p_t, as implied by the model given the observed price-dividend ratio from CRSP and the measured capital stock and TFP (see section 6).

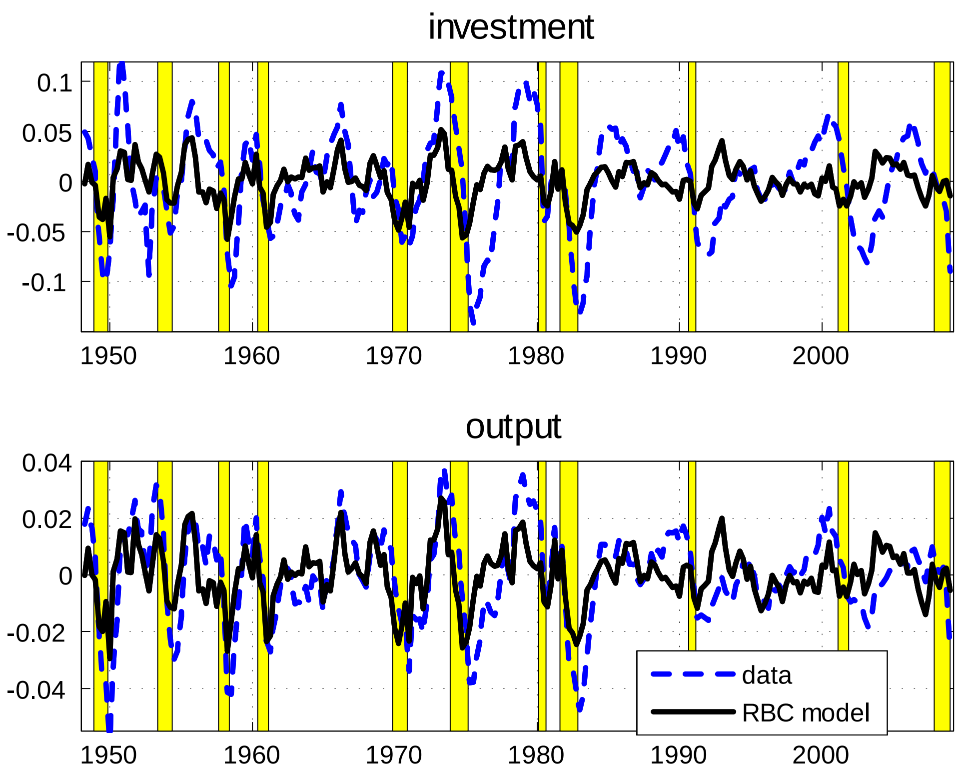

Figure 7: **Time-series of investment and output, in the data and in the RBC model.** This picture plots the data and the model-implied time series for macro aggregates for the RBC model (when TFP is fed into the model). All series are logged and HP-filtered, 1947q1-2008q4.

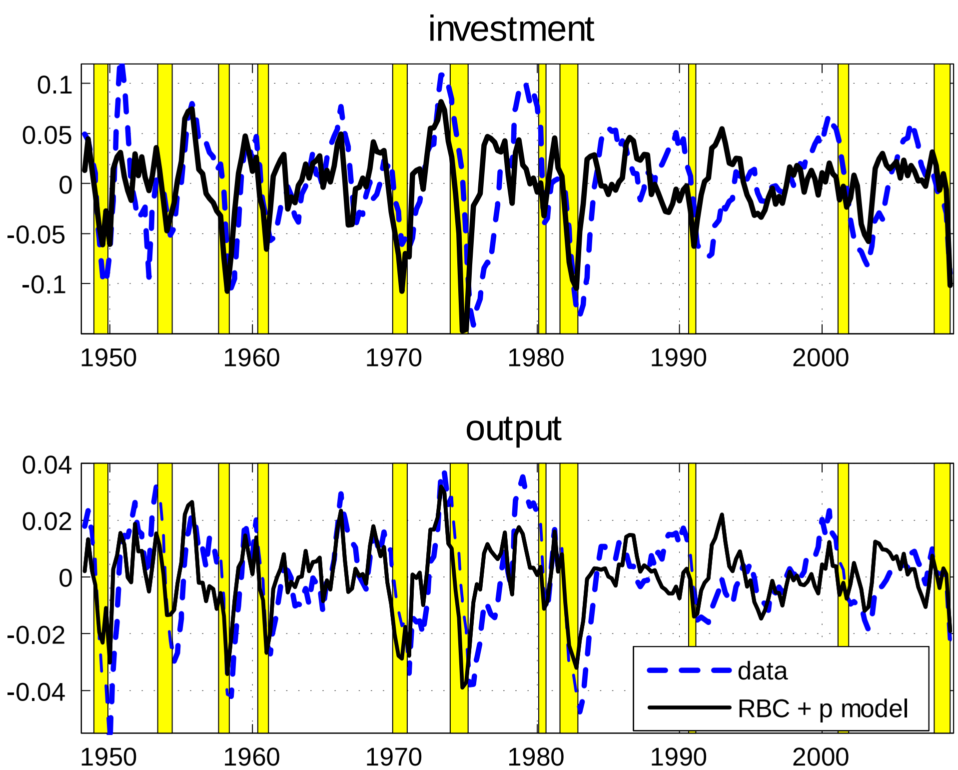

Figure 8: **Time-series of investment and output, in the data and in the benchmark model.** This picture plots the data and the model-implied time series for investment and output for the benchmark model (when both TFP and the disaster probability is fed into the model). All series are logged and HP-filtered, 1947q1-2008q4.

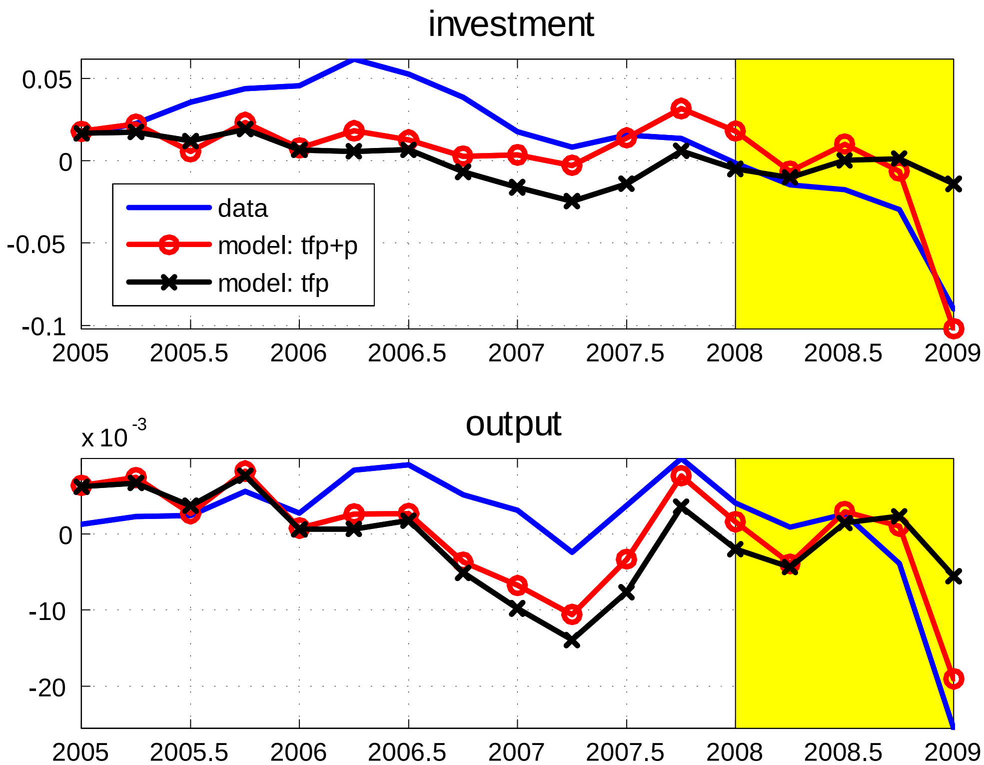

Figure 9: **Time-series of investment and output, in the data, in the RBC model, and in the benchmark model, from 2005q1 to 2008q4.** This picture is a zoomed-in version of the previous two figures, and displays the data, the model-implied time series for macro aggregates for the RBC model (when TFP is fed into the model) and for the benchmark model (when both TFP and the probability of disaster are fed into the model). See section 6 for details. All series are logged and HP-filtered, over 1947q1-2008q4, then cut from 2005q1 onwards.

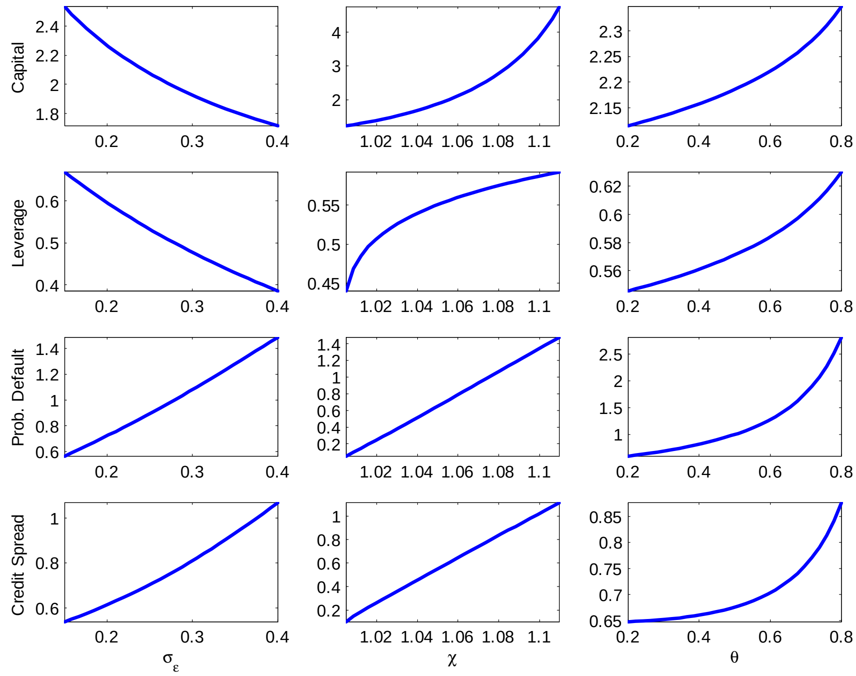

Figure 10: **Comparative statics on steady-state.** Effect of idiosyncratic volatility σ_ε, tax subsidy χ, and recovery rate θ, on capital, leverage, probability of default (in %), and credit spread (in %).

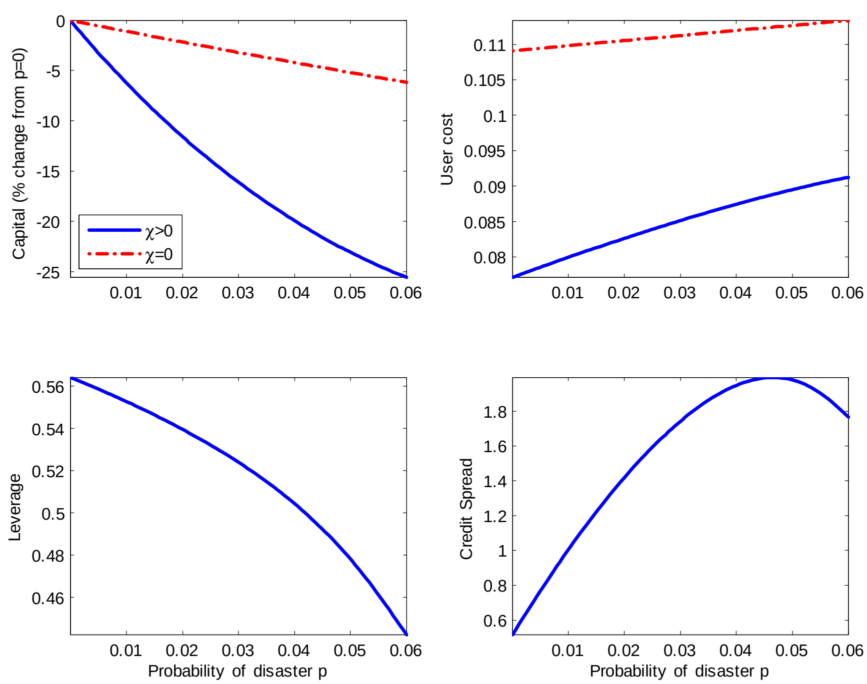

Figure 11: **Comparative statics on steady-state.** Effect of an increase in the probability of disaster on capital, leverage, credit spreads (in %), and the user cost of capital, for the frictionless model ($\chi = 0$, red dot-dashed line) and the benchmark model ($\chi > 0$, blue full line).

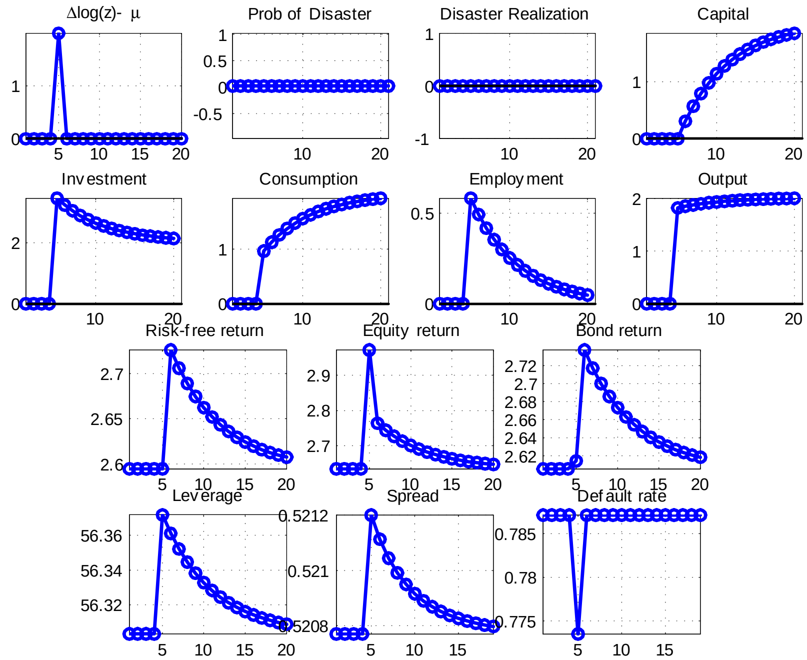

Figure 12: **Impulse response function of model quantities and returns to a one standard deviation shock to total factor productivity**. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

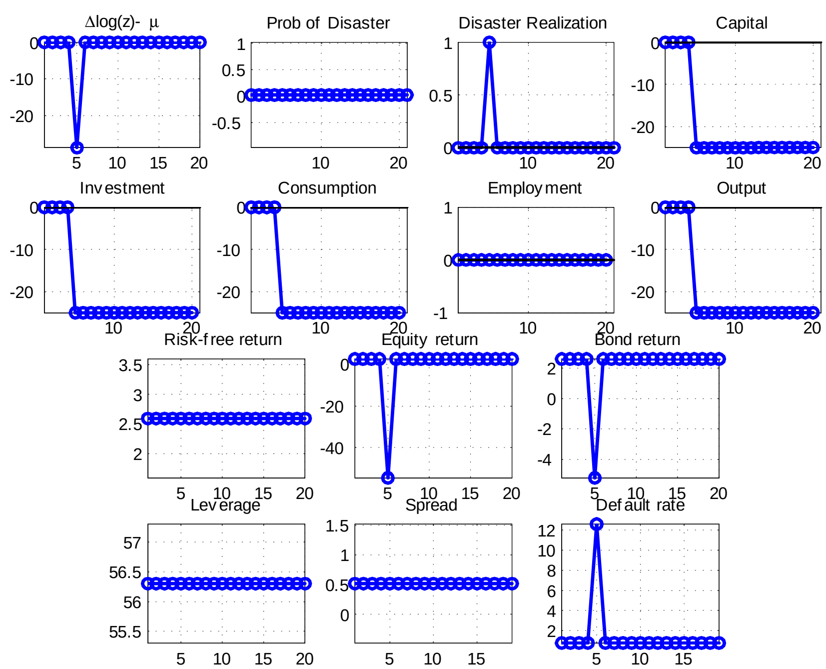

Figure 13: **Impulse response function of model quantities and returns to a disaster realization**. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

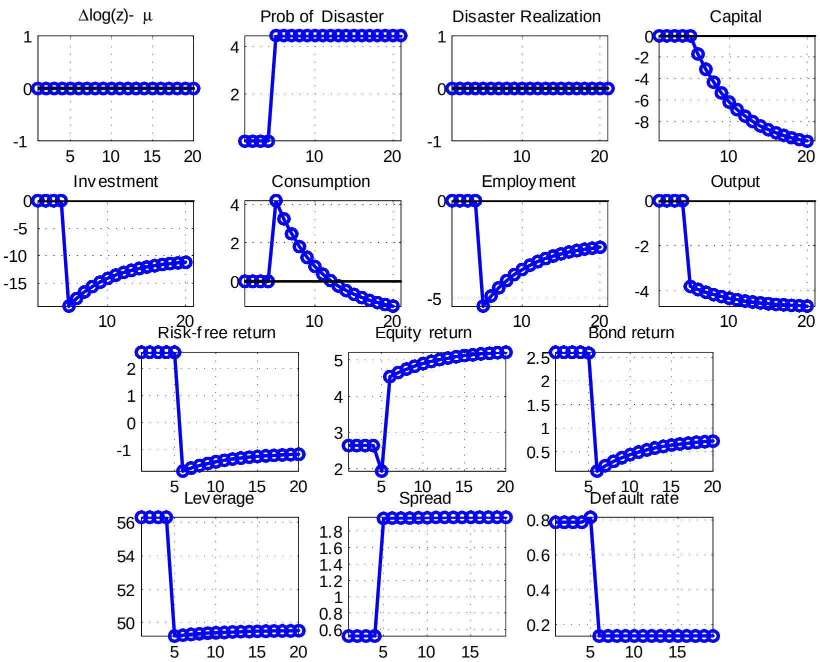

Figure 14: **Impulse response function of model quantities and returns to a shock to the probability of disaster**. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

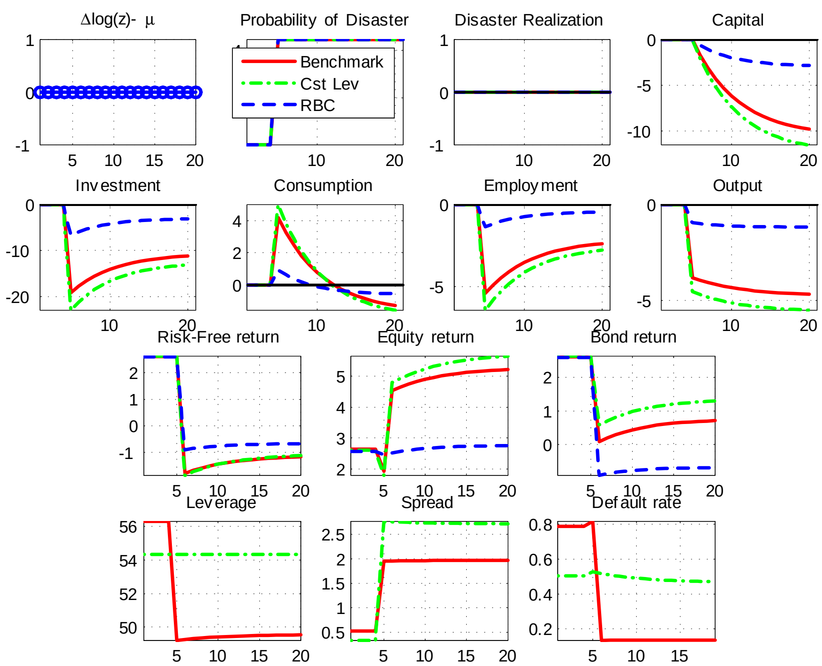

Figure 15: **Comparison of RBC model with and without financial friction.** This figure compares the impulse response of three models to a probability of disaster shock: the benchmark model (red full line), the model with constant leverage (green dot-dashed line), and the frictionless RBC model (blue dashed line). Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

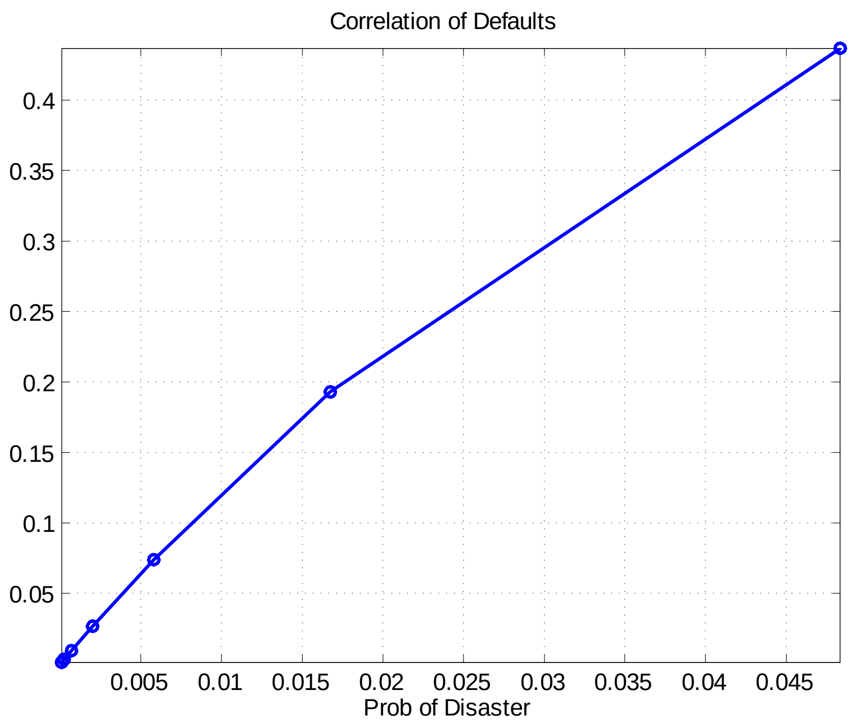

Figure 16: **Time-varying systematic risk: Correlation of defaults in the model.** This picture plots the correlation of default indicator between any two firms next period, i.e. $Corr_t(def_{it+1}, def_{jt+1})$, as a function of the disaster probability p.

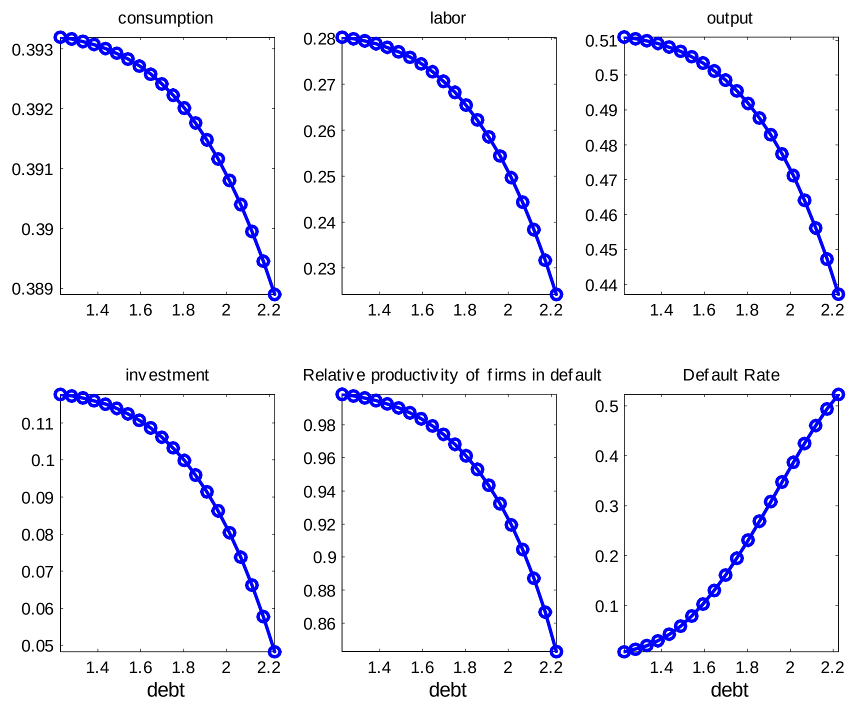

Figure 17: **Effect of outstanding debt on quantities, when firms in default are less productive.** The figure plots the policy functions for consumption, $c(k, b, p)$, employment $N(k, b, p)$, output $y(k, b, p)$, investment $i(k, b, p)$, the relative productivity of firms in default relative to firms not in default, and the share of firms in default, as a function of outstanding debt b (holding k and p fixed).

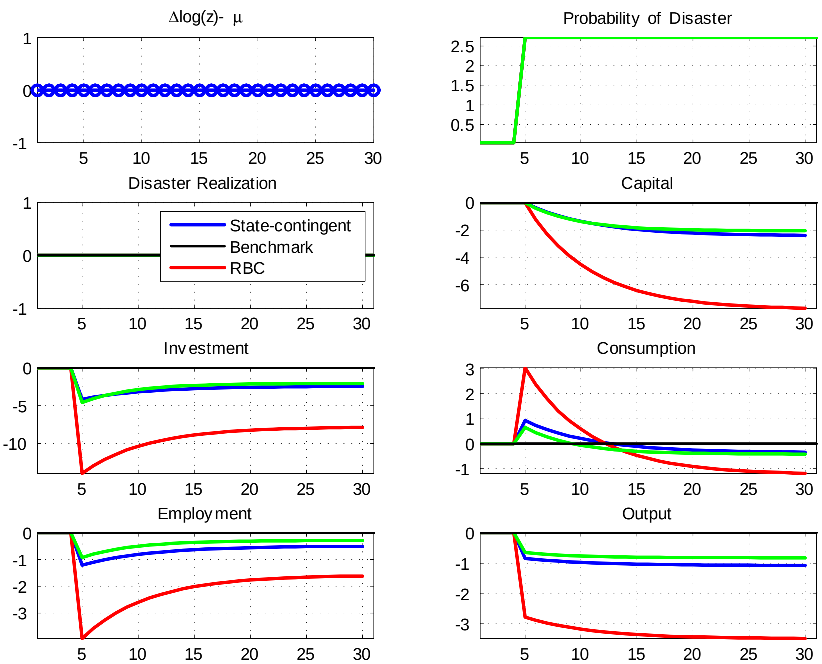

Figure 18: **Role of state-contingent debt.** The figure plots the impulse response function of model quantities to a shock to the probability of disaster. Blue full line = state-contingent debt, red line = benchmark model, green line = RBC frictionless model. Quantity responses are shown in % deviation from balanced growth path.