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OPTIMAL LEVEL OF GOVERNMENT DEBT

MATCHING WEALTH INEQUALITY AND THE FISCAL SECTOR

Edgar Vogel

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Edgar Vogel
European Central Bank; e-mail: edgar.vogel@ecb.europa.eu

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Address	Kaiserstrasse 29, 60311 Frankfurt am Main, Germany
Postal address	Postfach 16 03 19, 60066 Frankfurt am Main, Germany
Telephone	+49 69 1344 0
Internet	http://www.ecb.europa.eu
Fax	+49 69 1344 6000

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Abstract

We calibrate an incomplete markets large scale OLG model to the US income and wealth distribution and examine the effects of alternative government debt levels and adjustment policies on macroeconomic aggregates and welfare. We find that the government should hold negative debt. Due to the high degree of wealth and income dispersion ex ante lifetime utility increases with increasing wages (falling interest rates) by around 6% of lifetime consumption at optimal debt levels. The optimal level depends on the adjustment policy can vary by up to 70% of GDP (between -180% and -110%).

With lower government debt, high income/wealth agents are always worse off. Adjusting transfers benefits the lowest income/wealth group. The largest gains are, however, experienced by agents in the middle of the income/wealth distribution: they benefit from higher wages and transfers but do not lose too much capital income.

JEL classification: C54, D52, D6, E2, H2, H6
Keywords: Government Debt, Redistribution, Incomplete Markets, Ricardian Equivalence

Non-Technical Summary

What is the effect of government debt on macroeconomic aggregates and individual welfare? To answer this question, the paper uses a calibrated general equilibrium Overlapping Generations Model with uninsurable labor market risk, detailed demographics and a carefully modelled public sector. The model is calibrated to the US and specifically targets the income and wealth distribution which plays a key role in the analysis. The main finding is that instead of holding debt, the government should accumulate assets. The intuition behind this result is as follows. As the government decreases debt (or equivalently, increases assets) it also increases the total capital stock. The higher capital stock increases wages and decreases returns to capital. However, the observed wealth distribution is such that a large part of assets (capital) is held by a few. Hence, the majority of the population would benefit from higher wages but only a minority would loose from falling returns on capital. Consequently, a high aggregate capital stock (increasing wages) driven by low government debt is welfare improving. It is important to note that the analysis focuses on the general equilibrium effect in steady state. The paper does not model the transition to that new (hypothetical) value of debt. The main reason is that there are a variety of options how to get to that new level which are largely of political (economy) nature and beyond the scope of this paper.

Theoretical literature has shown that under certain assumptions, government debt does not have an effect on the macroeconomy. This is the case if an increase in government debt today, say to increase transfers, must be reversed at some point in the future and hence financed by higher taxes. If agents perfectly predict this future increase, they will save the additional transfers today in order to pay back debt in the future. However, this result was proven by empirical research to be largely of theoretical nature. In practice government debt matters at least for three reasons. First, the burden and benefits of a change in government debt often do not accrue to the same generation. Second, taxes are distortionary and hence non-neutral. Third, the presence of frictions (e.g. borrowing constraints) inhibits agents to adjust their behavior even if they perfectly foresee future policy.

While the literature agrees that debt matters, there is no consensus as regards the optimal level. On the one hand, higher taxes to finance more debt are clearly lowering welfare. Also, higher government debt crowds out capital and decreases production and income. However, in models with uninsurable income risk, like the present one, agents want to put money aside for the case of unemployment or low wages. Then, a benevolent social planner might want to remunerate those precautionary savings at a high rate in order to increase welfare. This can be done by increasing debt. However, as in general equilibrium high interest rates are the mirror image of low wages, this hurts wage earners. The total effect is unknown and is ultimately a quantitative question.

In line with other papers accounting for the skewed income distribution, we find that governments should accumulate assets. The welfare maximising policy is one where labor taxes are adjusted. We find that the lowest income group benefits mostly from higher transfers and less from lower taxes on capital (as they have little savings) and lower taxes on labor (as they pay low taxes initially). The largest winner is the group of agents with a mix of not to low labor income and some asset holdings. They are able to reap the benefits of higher wages, benefit also from insurance by higher transfers but do not have sufficiently high asset holdings to lose a sizable portion of their income due to falling interest rates. Agents in the high income/wealth group are always losing as a higher capital stock depresses their returns from assets. While they benefit from higher wages, these gains are eroded by the high income tax bracket.

1 Introduction

The question whether government debt has an effect on allocations and prices (and who benefits or loses) is as old as the economics profession. Earlier literature (Ricardo (1888) or Barro (1974) for classical references) showed that under specific conditions the level of government debt is neutral and any change in government borrowing will be completely offset by agents' saving decisions. Since then, the profession converged to the conclusion that in reality debt and hence the timing of taxes and spending matters (Bernheim (1987) or Seater (1993) for an overview). While earlier results were based on frictionless representative agent frameworks, recently the research focus has shifted to models explicitly modeling wealth (income) inequality, preference heterogeneity and frictions.

This paper follows the latter approach and builds a large scale general equilibrium OLG model with uninsurable income risk, endogenous labor supply and a detailed public sector. We explicitly target the observed US wealth and income distribution and match the usual macroeconomic targets.

We contribute to the literature in several ways. Firstly, the OLG structure captures the life-cycle behavior and the associated incentives and constraints in a realistic way. For instance, agents are likely to own little assets (or being borrowing constraint) early in life but will have a sizable amount of assets when the retire. Hence, policies without affecting the net present value of agents' lifetime income (e.g. lower transfers to the young and higher pensions) will matter. Secondly, we carefully model a detailed public sector with government debt, taxes on capital, consumption, labor (non-linear), transfers, and a pension system. While some of these channels can also be modeled with infinitely lived agents, an OLG model uses the "correct" population weights and provides a more realistic picture of the underlying population. Thirdly, we are the first paper to bring detailed life-cycle and fiscal modeling together with the observed income and wealth distribution. We use these two elements to make statements about who benefits or loses (young or old, asset poor or rich) from which policy (changing transfers or capital taxes).

Reasons why Ricardian equivalence fails to hold are threefold: finite planning horizons, distortionary taxation, and incomplete markets hindering agents to borrow against future income. Firstly, if government policies are such that the burden and benefits do not accrue to the same generation, any change in government spending or taxation will be non-neutral. Secondly, changing the time profile of distortionary taxation and spending will have an effect on intertemporal allocations (Auerbach and Kotlikoff (1987), McGrattan (1994), Trostel (1993a)), even if both policies affect the same generation. Effects of taxation are magnified if it also affects reproducible factors benefiting future generations; like human capital in Trostel (1993b). Thirdly, if agents cannot borrow against future income, government policies will have non-trivial effects. Modeling these channels, Heathcote (2005) finds that the combination of uninsurable risk and distortionary taxation have quantitatively the largest role in explaining the breakdown of Ricardian

equivalence.

While the literature agrees that debt matters, there is no consensus as regards the optimal level. On the one hand, in dynamically inefficient economies higher debt can bring the economy to the optimal capital level and increase welfare.[1] On the other hand, distortions caused by financing debt are clearly lowering welfare.[2] Further, in models with heterogeneous agents and incomplete markets government bonds (debt) can be an instrument to self-insure against unfavorable labor market outcomes to smooth consumption over time. Hence, in this class of models, increasing debt (to crowd out capital) and higher interest rates can be welfare improving as savers are remunerated with higher returns. However, by the inverse relationship of wages and interest rates in general equilibrium, welfare gains from higher capital returns might be outweighed by losses from lower wages. The total effect is unknown and is ultimately a quantitative question. As Dávila, Hong, Krusell, and Ríos-Rull (2011) show, a constrained planner's choice depends on the model calibration and can give rise to quantitatively *and* qualitatively very different answer to the question what the optimal capital level is. The crucial condition is whether the consumption-poor ("unlucky") consumers have lower labor earnings or asset holdings relative to the average. In a nutshell, it is the income composition of the majority which determines the optimal capital intensity. Depending on the main source of income in the economy, high (low) wages (interest rates) might be more or less desirable from an ex ante view of a benevolent government.

Echoing the ambiguity from theory, the quantitative literature provides a set of mixed results. Aiyagari and McGrattan (1998) find that the welfare maximizing level of debt is positive and around 60% of GDP with a very flat welfare function. Flodén (2001) calibrates a model to the US and argues that debt increases welfare only if transfers are sup-optimal. If transfers are at the optimal level (23% of output) welfare maximizing government debt is -100% of GDP. He argues that it is more efficient to provide insurance by transfers rather than crowding out capital to push up the interest rate. However, both models have to little inequality compared to the data. Calibrating the income process to match earnings and wealth distribution, Röhrs and Winter (2012) show that the optimal level of debt is negative (more than -100%) and welfare gains from moving to the optimum are non-negligible. However, an implementation might be not feasible as the welfare change including the transition is negative. Combining idiosyncratic and aggregate risk and also matching the wealth distribution, Desbonnet and Kankanamge (2011) estimate the optimal level of debt to be around 5%. Also, Dávila, Hong, Krusell, and Ríos-Rull (2011) find that with a high degree of inequality (as in the data) government debt should be negative to increase wages. The practical importance of accounting for the

[1] The basic ideas dates back to Diamond (1965) which was extended to endogenous labor supply by Lopez-Garcia (2008). For quantitative applications see Aiyagari (1995) who uses capital taxes or İmrohoroğlu, İmrohoroğlu, and Joines (1995) who use social security to crowd out capital.

[2] A non-linear tax code can provide an insurance against fluctuating pre-tax wage income. However, Conesa and Krüger (2006) find that the insurance effect is not large enough to justify the increasing distortions and hence argue that a linear tax rate increases welfare.

skewed income and asset distribution is also demonstrated by Gomes, Michaelides, and Polkovnichenko (2010). They show that the fiscal "bailout package" in the US had a very different effect on consumers' welfare depending on their position in the wealth and income distribution and their age with old (wealthy) agents bearing the majority of the costs.[3]

In line with other papers accounting for the skewed income distribution, we find that the optimal level of government debt is in fact negative: governments should accumulate assets. The steady state welfare maximizing level of government debt (assets) depends on the adjustment instrument and the optimal debt level can vary by as much as 70% of GDP. Among the considered instruments, adjusting labor taxes such that the tax code becomes more linear is maximizing welfare. Our results also show that providing insurance by increasing transfers provides relatively large welfare gains of around 6% of lifetime consumption around the calibrated value of debt/GDP-ratio (50%). However, this policy is not the global maximizer in the (restricted) policy space as it does not create incentives to increase savings or labor supply. More labor supply by highly productive agents due to lower labor taxes spills also over to lower income agents.

A higher steady state capital stock is *ex ante* welfare increasing as young and low asset agents are more likely to be borrowing constraint and at the same time have a higher remaining lifetime labor income risk. More specifically, we find that the lowest income group benefits mostly from higher transfers and less from lower taxes on capital (as they have little savings) and lowering labor taxes (as they pay low taxes initially). The largest winner is the group of agents with a mix of not to low labor income and some asset holdings. They are able to reap the benefits of higher wages, benefit also from insurance by higher transfers but do not have sufficiently high asset holdings to lose a sizable portion of their income due to falling interest rates. Agents in the high income/wealth group are always losing as a higher capital stock depresses their returns from assets. While they benefit from higher wages, these gains are eroded by the high income tax bracket. In general, we find that the role of changing wages and interest rates (due to general equilibrium effects) is more important than changing taxes.

The plan for the paper is as follows. Section 2 presents the formal model, section 3 the calibration strategy, and section 4 presents the results. Concluding remarks are in section 5. Technical details are provided in appendix A and additional results in appendix B.

[3]There is also a large literature focusing mostly on representative agents but giving a more active role to government debt and fiscal policy. In these models debt is often used to smooth distortions over time (Lucas and Stokey (1983), Aiyagari, Marcet, Sargent, and Seppälä (2002)), to stabilize aggregate consumption over the cycle (Hiebert, Pérez, and Rostagno (2009)), to loosen private borrowing constraints to crowd in investment (Woodford 1990) or to crowd out capital in dynamically inefficient economies (Aiyagari (1995), İmrohoroğlu, İmrohoroğlu, and Joines (1995)).
For heterogenous agent models past research (e.g. (Krusell and Smith 1998)) has shown that including aggregate risk in a standard setup without adding additional elements interacting with it (e.g. pro-cyclicallity of idiosyncratic shocks) does not help much to explain the wealth dispersion in the data. Nevertheless, aggregate risk would help to generate a more dispersed wealth distribution and would enable a calibration of the idiosyncratic income process with less risk.

2 The Model

We use a large scale OLG model following Auerbach and Kotlikoff (1987) with endogenous labor supply, a standard consumption-saving decision, and uninsurable idiosyncratic shocks to labor endowment in the spirit of Aiyagari (1994). There is no aggregate uncertainty. The population structure is exogenously given.

Firms produce with a standard constant returns to scale production function using capital and efficient labor as inputs. Factor markets are perfectly competitive.

The government collects labor taxes, social security contributions, capital taxes, and consumption taxes. It spends revenues on government consumption, lump-sum transfers, debt servicing and social security liabilities (pensions) and uses lump-sum transfers to make the budget constraint hold. Following the related literature, we abstain from modeling a financial sector.

2.1 Households

Time is discrete and there are J generations at each point in time. In each period t new agents are born. Households have one unit of time and choose consumption, saving, and labor supply until the age of jr and retire afterwards. Agents are heterogenous along three dimensions: age, asset holdings and the realization of the labor endowment shock. At the beginning of economic life ($j = 0$), households maximize

$$V = \mathbb{E}\left[\sum_{j=0}^{J} \beta^j \pi_{t,j} u(c_{t+j,j}, 1 - \ell_{t+j,j})\right] \qquad \sigma > 0 \tag{1}$$

$$u(\cdot) = \frac{1}{1-\sigma}\{c_{t+j,j}^{\phi}(1-\ell_{t+j,j})^{1-\phi}\}^{1-\sigma},$$

where we use V to denote expected lifetime utility (before the first shock realization), $c_{t,j}$ and $\ell_{t,j}$ to denote consumption and labor supply. Expectations are taken with respect to the realizations of labor endowment shocks. β is the discount factor, ϕ is the weight of consumption and σ is the inverse of the intertemporal elasticity of substitution with respect to the consumption-leisure aggregate. We use $\pi_{t,j}$ to denote the unconditional probability to survive up to age j in year t with $\pi_{t,j} = \prod_{k=0}^{j-1} \varphi_{t+k,k}$, for $j > 0$ where $\varphi_{t+j,j}$ is the conditional survival probability. Agents die with certainty at $j = J + 1$.

Workers supply effective labor and receive a pension when retired, earn interest payments on their assets, and receive lump sum transfers. With missing annuity markets the government redistributes accidental bequests as lump-sum payments to all living households. Labor income is subject to a linear social security contribution rate τ_t and a non-linear labor income tax rate. We use $T_t(y_t)$ to denote the tax income code T_t applied to the tax base y_t of period t. The tax code T_t is indexed by a time index t as the parametrization of the tax code is allowed to vary. Net labor income of an agent in year t of age j with labor endowment shock η is given by

$$w_{t,j}^{n} \quad = \quad \ell_{t,j} w_t (1 - \tau_t) h_j \eta - T_t(y_{t,j}) \tag{2}$$

where $w^n_{t,j}$ is net labor income, w_t is gross wage per unit of effective labor supply, h_j is the deterministic age-dependent efficiency profile and η is the current realization of the labor endowment shock. The dynamic budget constraint of the household is given by

$$a_{t+1,j+1} = \begin{cases} (a_{t,j} + b_t)(1 + r_t(1 - \tau^k_t)) + w^n_{t,j} + tr_t - c_{t,j}(1 + \tau^c_t) & \text{if } j \leq jr \\ (a_{t,j} + b_t)(1 + r_t(1 - \tau^k_t)) + p_t + tr_t - c_{t,j}(1 + \tau^c_t) & \text{if } j > jr, \end{cases} \tag{3}$$

where $a_{t,j}$ denotes assets, b_t denotes accidental bequests, tr_t are lump sum transfers, p_t pension payments, r_t is the gross interest rate, τ^k_t is the capital tax rate, and τ^c_t is the tax rate on consumption. Agents start with zero assets ($a_{t,0} = 0$) and optimality requires $a_{t,J+1} = 0$. As common in the literature without annuity markets and survival risk we impose a strict borrowing constraint with $a_{t+1,j+1} \geq 0$.

We define $\mathcal{E}$ to be the set of all possible realizations of the labor endowment shock with $\eta \in \mathcal{E}$. The transition process between states is time-invariant and independent across agents and can be characterized by a finite-state Markov chain with a transition probability matrix

$$Q_t(\eta, \mathcal{E}) = Pr(\eta' \in \mathcal{E}|\eta) = Q(\eta, \mathcal{E}) \tag{4}$$

with only positive entries in Q to ensure a unique and time-invariant distribution $\bar{Q}$. For each newborn cohort, the initial distribution of labor productivity corresponds to the time-invariant distribution $\bar{Q}$ and the associated realizations η with $\eta \in \mathcal{E}$. Hence, agents enter economic life with identical conditions but they experience different sequences of labor market outcomes generating endogenously a distribution of income, assets, labor supply, and consumption.

2.2 Firms

Firms operate in a perfectly competitive environment and produce one homogenous good used for private consumption, government consumption, and investment according to

$$Y_t = K^\alpha_t (A_t H_t)^{1-\alpha}, \tag{5}$$

where α is the share of capital. Y_t, K_t, H_t, and A_t are output, physical capital, effective labor and the level of technology. We assume that labor input of agents of different cohorts are perfectly substitutable.[4] Competition ensures that factor prices are given by

$$w_t = (1 - \alpha) A_t k^\alpha_t \qquad r_t = \alpha k^{\alpha-1}_t - \delta \tag{6}$$

$$k_t = \frac{K_t}{A_t H_t}, \tag{7}$$

with w_t being the gross wage per unit of effective labor, r_t is the gross return on capital, and δ is the depreciation rate. Technology A_t is growing at the exogenous rate g_t with $A_{t+1} = A_t(1 + g_t)$.

[4] See Prskawetz and Fent (2007) for modeling with imperfect substitutability of age groups.

2.3 Government

The role of the government is to adjust either revenues or spending such that its consolidated intertemporal budget constraint is satisfied in all periods. To do this, it runs a social security system collecting contributions and paying pensions, and it levies taxes in order to finance government expenditure and service debt obligations. The consolidated budget constraint is

$$G_t + Tr_t + (1 + r_t)D_t = BPS_t + T_t^c + T_t^k + T_t^\ell + D_{t+1} \tag{8}$$

$$T_t^c = \tau_t^c \int c(a, \eta, j)\Phi(da \times d\eta \times dj) \tag{9}$$

$$T_t^k = \tau_t^k r_t \int a(a, \eta, j)\Phi(da \times d\eta \times dj) \tag{10}$$

$$T_t^\ell = \int T[w_t h_j \eta \ell(a, \eta, j)]\Phi(da \times d\eta \times d\{1 \dots jr\})$$

$$Tr_t = \sum_{j=0}^{J} N_{t,j} tr_t \tag{11}$$

where G_t is government consumption, Tr_t are aggregate lump-sum transfers to households, BPS_t is the balance of the pension fund (social security system), T_t^c are consumption taxes, T_t^a are capital income taxes, T_t^ℓ are labor income taxes, and D_t denotes the amount of government assets. If necessary, the government must adjust its taxation or spending policy such that it is able to honor its debt obligations. Hence, no default is possible and government policies are such that the debt to GDP ratio must converge to a finite value.

2.4 Social Security

We assume that pensions p_t are a share of current average net wages after the deduction of social security (but before taxes),

$$p_t = \rho_t(1 - \tau_t)w_t\bar{h}_t \tag{12}$$

where τ_t is the contribution rate, ρ_t is the replacement rate – an indicator for the generosity of the pension system – and $\bar{h}_t$ is average effective labor supply of a working age person. Then, for all t the budget of the social security system can be described by

$$w_t\tau_t H_t - \rho_t(1 - \tau_t)w_t\bar{h}_t R_t = BPS_t \tag{13}$$

where the LHS is income minus spending (pension payments, outflows) of the social security system and, BPS_t is the balance of the pension fund at time t where for the

sake of a more compact notation we used

$$H_t = \int N_t h_j \ell_t(a,\eta,j)\Phi_t(da \times d\eta \times d\{1\dots jr\}) \tag{14}$$

$$L_t = \sum_{j=0}^{jr} N_{t,j} \tag{15}$$

$$\bar{h}_t = \frac{H_t}{L_t} \tag{16}$$

$$R_t = \sum_{j=jr+1}^{J} N_{t,j} \tag{17}$$

Adjusting either ρ_t or τ_t for a given balance BPS_t leads to the following solutions

$$\rho_t = \frac{\bar{\tau}H_t - bps_t}{(1-\bar{\tau})R_t\bar{h}_t}, \tag{18}$$

$$\tau_t = \frac{bps_t + \bar{\rho}\bar{h}_t R_t}{H_t + \bar{\rho}\bar{h}_t R_t}. \tag{19}$$

where $bps_t = \frac{PBS_t}{w_t}$ is the budget balance scaled by wages. Holding ρ_t or τ_t fixed will affect the balance if either the number of effective contributors H_t or recipients R_t changes. Beyond adding realism to the model, pensions serve the purpose of correctly capturing saving incentives for old-age. While pensions provide disincentives for low income agents to save (hence helping us to generate enough agents with low savings), pension payments do not matter for agents at the top of the income distribution. However, by ignoring the link between pensions and lifetime earning we increase the tax distortions in the economy.

2.5 Labor Income Taxation

We use the widely used tax function proposed and estimated by Gouveia and Strauss (1994).[5] The taxable income mimics the US tax system where the tax base is defined as total labor income without the deduction of social security contributions.[6] Formally, the tax base $y_{t,j}$ is thus

$$y_{t,j} = \ell_{t,j} w_t h_j \eta \tag{20}$$

and the tax function (dropping the subscripts t, j) is

$$T(y) = a_0\left(y - (y^{-a_1} + a_2)^{-\frac{1}{a_1}}\right) \tag{21}$$

For $y \to \infty$ we have $\frac{T(y)}{y} = T'(y) = a_0$, for $a_1 = -1$ the system turns into a lump-sum tax system with $T(y) = -a_0 a_2$ and for $a_1 \to 0$ the system is a proportional with $T(y) = a_0 y$

[5] Recent examples include Conesa and Krüger (2006), Castaneda, Diaz-Giménez, and Ríos-Rull (2003), and Conesa, Kitao, and Krüger (2009).

[6] See Carey and Rabesona (2002) for a cross-county review of tax systems and the treatment of social security contributions.

and $T'(y) = a_0$. For the general case we obtain

$$t(y) = \frac{T(y)}{y} = a_0 \left(1 - (1 + a_2 y^{a_1})^{-\frac{1}{a_1}}\right) \tag{22}$$

$$T'(y) = a_0 \left(1 - (1 + a_2 y^{a_1})^{-1-\frac{1}{a_1}}\right) \tag{23}$$

Note that a_2 is not scale-invariant and has to be adjusted in case of trend growth.[7]

2.6 Market Structure

We assume that workers neither can buy insurance against the idiosyncratic labor market risk nor are annuity markets available to insure against longevity risk. Hence, agents can only trade one period risk-free bonds a in order to self-insure themselves against unfavorable labor market outcomes. We follow the literature and impose a tight borrowing constraint in order to prevent households (who would like to borrow) to die with negative assets.[8]

2.7 Welfare Criterion

To quantify the effects of different policies on welfare, we follow the literature (Conesa and Krüger (1999), Attanasio, Kitao, and Violante (2007)) and compute the consumption equivalent variation, that is, we ask how much an individual's consumption has to change (holding leisure constant) in all future periods under the alternative policy to make the individual indifferent between the two alternatives. When comparing welfare of households between two policies (i.e. living in two different steady states), we will use V to denote expected total lifetime utility *before* the first labor market shock is realized. Given the assumptions on the functional form of utility, this can be computed by

$$g = (V(\{c, 1-\ell\})_1 / V(\{c, 1-\ell\})_0)^{\frac{1}{\phi(1-\sigma)}} - 1 \tag{24}$$

where the subscript 0 indicates utility from the baseline policy and the subscript 1 is utility from the alternative policy. Positive numbers for g indicate welfare gains from moving to the alternative policy 1. In addition to the quantification of overall welfare changes we quantify the role of uncertainty and changes in life cycle profiles following the methodology of Pries (2007).[9]

[7] Define $y_t = y_0 A_t$ where A_t is growing at a constant gross rate $1+g$ with $A_t = A_0(1+g)^t$. Define $a_{2,t}$ as a_2 in the period t corresponding to the level of A_t. Then we have $a_{2,t} y_t^{a_1} = a_{2,0} y_0^{a_1}$ which can be rewritten to $a_{2,t} = a_{2,0} A_t^{-a_1}$ implying that marginal and average tax rates are independent of A_t.

[8] Alternatives would include a set of financial contracts with interest rates adjusted for the survival risk or introducing annuity markets in combination with some natural borrowing limit (Aiyagari 1994). See e.g. also Kehoe and Levine (2001) and Ábrahám and Carceles-Poveda (2010) for models with endogenous market participation constraints and Japelli and Pagano (1999) for welfare effects of borrowing constraints in OLG models.

[9] Formally, we compute

$$g(\eta, 0, 0) = \left(\frac{\sum_{\eta \in \mathcal{E}} \bar{Q}(\eta) V(\eta, 0, 0)_1}{\sum_{\eta \in \mathcal{E}} \bar{Q}(\eta) V(\eta, 0, 0)_0}\right)^{\frac{1}{\phi(1-\sigma)}} - 1 \tag{25}$$

$$\tag{26}$$

Let $V(\{\bar{c}, 1-\bar{\ell}\})_k$ denote total lifetime utility (for policy k) of an agent who is given the *average* consumption and leisure life cycle profiles. The associated welfare compensation (denoted $\bar{g}$) is computed as

$$\bar{g} = \left(V(\{\bar{c}, 1-\bar{\ell}\})_1 / V(\{\bar{c}, 1-\bar{\ell}\})_0\right)^{\frac{1}{\phi(1-\sigma)}} - 1 \tag{27}$$

and measures how well agents are insured against wage shocks. If welfare gains from moving away from the steady state policy are higher in the deterministic case relative to optimization under uncertainty (i.e. $\bar{g} > g$), it must hold that policy 0 offers a better protection against income shocks. Further, we assess the role of changes in levels (wages, transfers, pensions) and the role of a shift in life cycle profiles. First, we rescale total lifetime consumption and leisure such that they are identical to averages observed under the baseline policy. That is, we compute the scaling factors γ^c and γ^ℓ by solving $\gamma^c \mathbb{E}\left[\sum_{j=0}^{J} c_j^1\right] = \mathbb{E}\left[\sum_{j=0}^{J} c_j^0\right]$ and $\gamma^\ell \mathbb{E}\left[\sum_{j=0}^{J} 1-\ell_j^1\right] = \mathbb{E}\left[\sum_{j=0}^{J} 1-\ell_j^0\right]$. The consumption needed to make agents indifferent between two bundles with identical averages but different life cycle profiles is

$$\tilde{g} = \left(V(\{\gamma^c\bar{c}, \gamma^\ell(1-\bar{\ell})\})_1 / V(\{\gamma^c\bar{c}, \gamma^\ell(1-\bar{\ell})\})_0\right)^{\frac{1}{\phi(1-\sigma)}} - 1 \tag{28}$$

2.8 Thought Experiments

We use the targets reported in section 3 to calibrate the structural parameters of our model in the steady state. As the driving force of the model we take exogenous variation in the steady state level of government debt. In a general equilibrium setting, this induces changes in prices and taxation (or transfers). Both will affect household behavior which in turn changes welfare. We measure these changes. We then report also changes in factor prices, aggregates, inequality, and the required adjustments of government policies. We compare only steady states with each other and do not model the transition; the main reason being ambiguity about the adjustment path. Hence, we focus more on the conceptual issue of what the optimal level is and not so much how we get there. For instance, the optimal transitional policy of a benevolent government could even be a *higher* level of debt. Hence, the model would never reach the optimal lower level of debt.[10] We also ignore the issue of implicit debt; a major source of concern for many countries. The reason is that implicit debt is only of temporary nature until today's promises turn into actual pension claims. As this is not the focus of this paper, we delegate a detailed discussion and additional material to section B.3 in the appendix.

where we sum up over all possible realizations of η using the time-invariant distribution $\bar{Q}(\eta)$. Obviously, this can be done for any age, labor market status or asset holdings separately and thus computing $g(\eta, a, j)$. For alternative welfare decompositions see also Flodén (2001) or Bénabou (2002).

[10] Essentially all research using infinitely lived agent models shows that the costs of decreasing debt (forgone consumption) is higher than the expected gains. This result will be even stronger in an OLG model as the costs of the reform (debt reduction) will be borne by today's generations and the benefits will materialize only in the future. Given that OLG agents are "short lived", they will reap only a very small share of this benefits making the reform unfeasible. In fact, Desbonnet and Weitzenblum (2011) show that it is rather likely that if initial debt is not to high, going even deeper into debt may be optimal as higher consumption in the short run is likely to overcompensate long run losses.

While performing the experiments, we adjust only one policy parameter (tax rate, transfers, etc.) and keep all other instruments at their calibrated values. We also keep G/Y constant. We do this in order to keep the overall level of distortions that arise due to collection of taxes constant.[11] Further, when we adjust tax rates, we have to take a stand on whether per capita transfers are kept constant (in absolute numbers) or whether they vary with GDP (i.e. keep Tr/Y constant). Flodén (2001) shows that transfers can be an effective alternative to insure agents against bad shocks. The baseline is to keep Tr/Y constant and thereby increase transfers with GDP. As robustness check we run the experiments also solving for the equilibrium with a constant *level* of transfers (changing Tr/Y) and will discuss the differences between the two alternatives.

2.9 Market Clearing and Equilibrium

In this section we define the competitive equilibrium of the economy and the associated state variables. Firms and households are price takers and maximize profits and utility given expected future wages and interest rates. Individual households' state variables are asset holdings a, individual realization of the labor market endowment η, and age j. Given the individual state variables, the aggregate economy is then characterized by a joint measure Φ over asset holdings, labor endowment, and age.

Definition 1. *Given the exogenous population distribution and survival rates* $\{\{N_{t,j}, \pi_{t,j}\}_{j=0}^{J}\}_{t=0}^{T}$, *an initial physical capital stock* K_{-1}, *an initial level of government assets* D_{-1}, *and an initial measure* Φ_{-1}, *an approximate competitive equilibrium are sequences of individual decision variables* $\{\{c_{t,j}, \ell_{t,j}, a_{t+1,j+1}\}_{j=0}^{J}\}_{t=0}^{T}$, *government expenditures* $\{G_t\}_{t=0}^{T}$, *consumption tax rates* $\{\tau_t^c\}_{t=0}^{T}$, *capital tax rates* $\{\tau_t^k\}_{t=0}^{T}$, *labor income tax codes* $\{T_t\}_{t=0}^{T}$, *social security contribution and replacement rates* $\{\tau_t, \rho_t\}_{t=0}^{T}$ *with corresponding pension payments* $\{p_t\}_{t=0}^{T}$, *lump-sum transfers* $\{tr_t\}_{t=0}^{T}$, *government assets* $\{D_t\}_{t=0}^{T}$, *prices* $\{w_t, r_t\}_{t=0}^{T}$ *with corresponding firm plans* $\{H_t, K_t\}_{t=0}^{T}$, *and measures* $\{\Phi_t\}_{t=0}^{T}$ *such that*

1. *given prices, transfers, government tax policies, and initial individual conditions, households solve the maximization problem as described in equation (1) subject to the constraints in equation (3) and the strict borrowing constraint* $a_{t+1,j+1} \geq 0$,
2. *factors of production are paid marginal products according to equations (6),*
3. *per capita transfers from accidental bequests are determined by*

$$b_t = \frac{\int (1 - \varphi_{t,j}) a_{t+1,j+1}(a, \eta, j) \Phi_t(da \times d\eta \times dj)}{\Phi_{t+1}(da \times d\eta \times dj)}, \tag{29}$$

4. *the budget of the social security system as specified in (13) holds with equality in every period and pensions are given by (12),*

[11] Keeping the absolute level of government spending constant would imply a declining G/Y ratio over time which would be an implicit modeling of structural change. Further, the lower distortions would increase the optimal capital stock further and welfare gains would be higher for any debt/GDP ratio.

5. *government policies are such that the consolidated dynamic budget of the government as specified in (8) holds every period,*

6. *allocations are feasible for all periods with aggregates defined as*

$$K_{t+1} = \int a_{t+1,j+1}(a,\eta,j)\Phi_t(da \times d\eta \times dj) - D_{t+1} \tag{30}$$

$$H_t = \int N_t h_j \ell_t(a,\eta,j)\Phi_t(da \times d\eta \times d\{1 \dots jr\}) \tag{31}$$

$$Y_t = \int c_t(a,\eta,j)\Phi_t(da \times d\eta \times dj) + K_{t+1} + G_t \tag{32}$$

with Y_t as defined in (5),

7. *and the joint measure Φ over asset holdings, labor endowment shocks and age evolves according to*

$$\Phi_{t+1} = \int P_t((a,\eta,j),\mathcal{A},\mathcal{Q},\mathcal{J})\Phi_t(da \times d\eta \times dj) \tag{33}$$

for all sets $\mathcal{A},\mathcal{Q},\mathcal{J}$ where transition probabilities P_t for agents $j \geq 1$ are defined as

$$P_t((a,\eta,j),\mathcal{A},\mathcal{Q},\mathcal{J}) = \begin{cases} Q(\eta,\mathcal{E})\pi_{t,j} & \text{if } \ a_{t+1,j+1}(a,\eta,j) \in \mathcal{A}, \quad j+1 \in \mathcal{J} \\ 0 & \text{else} \end{cases} \tag{34}$$

and for newborns with $j = 0$

$$\Phi_{t+1}(\mathcal{A},\mathcal{Q},1) = N_{t+1,0} \times \begin{cases} \bar{Q}(\mathcal{E}) & \text{if } \ 0 \in \mathcal{A} \\ 0 & \text{else} \end{cases} \tag{35}$$

Definition 2. *A stationary equilibrium is a competitive equilibrium in which per capita variables grow at the gross rate $1 + g^A$, de-trended prices and polices are constant, and aggregate variables grow at the rate $(1 + g^A)(1 + n)$.*

3 Calibration

To calibrate the model we chose parameters such that we match selected moments in the data. Our calibration targets are the "great ratios" well established in the literature and the empirically observed income and wealth distribution. The time window for macroeconomic data used for calibration is 1975-2000. This choice is motivated by the availability of data: effective tax data provided by Carey and Rabesona (2002) and Gouveia and Strauss (1994) and the fact that this period is characterized by rather stable economic environment.

In the sequel we describe our procedure in more detail. Although we associate each parameter with the calibration target most closely linked to it, the calibration is done in one step with all coefficients having an effect on all targets. All parameters are summarized in table 1 and more details on the data construction is provided in the appendix.

Table 1: Model Parameters

		Value	Target
	Preferences		
σ	Inverse Intertemporal Elasticity of Substitution	2.00	IES = 0.5
β	Pure Time Discount Factor	0.992	$K/Y = 2.7$
ϕ	Weight of Consumption	0.45	Avg. Hours = 1/3
	Production		
α	Share of Physical Capital in Production	0.33	Data
δ	Depreciation Rate of Physical Capital	5.9%	$I/Y = 19.9\%$
g^A	Exogenous Growth Rate	1.5%	Data
	Demographics		
jr	Retirement Age	65	assumption
J	Maximum Age	90	assumption
π	Survival Probabilities	Data	Bell and Miller (2005)
	Working Age Population Ratio	83.2%	Data
	Old Age Dependency Ratio	20.2%	Data
	Government		
D/Y	Debt to GDP Ratio	50%	Data
G/Y	Government Spending to GDP Ratio	16.3%	Data
Tr/Y	Transfers to GDP Ratio	7.6%	Implicit
τ	Social Security Contribution	11.4%	Data
τ^c	Consumption Tax	6.5%	Carey and Rabesona (2002)
τ^k	Capital Tax	39.4%	Carey and Rabesona (2002)
a_0	"Marginal" Labor Tax	0.258	Gouveia and Strauss (1994)
a_1	Shape Parameter	0.768	Gouveia and Strauss (1994)
a_2	Scaling Parameter	4.35	$T^\ell/GDP = 14.3\%$
	Endowment Process		
η	Level of Labor Endowment	Table 2	Calibrated
$\bar{Q}$	Invariant Distribution	Table 2	Calibrated

Notes: Population data describes the demographic situation used for calibration.

Table 2: Endowment Process

	Transition Matrix				$\bar{Q}$	η
	$\eta' = 1$	$\eta' = 2$	$\eta' = 3$	$\eta' = 4$		
$\eta = 1$	96.3%	1.6%	2.0%	0.0%	43.2%	0.12
$\eta = 2$	2.4%	96.7%	0.9%	0.0%	27.7%	0.45
$\eta = 3$	3.2%	0.2%	95.6%	0.9%	27.5%	0.67
$\eta = 4$	2.8%	9.6%	5.5%	82.1%	1.6%	5.38

3.1 Demographics and Households

Cohort survival probabilities are taken from Bell and Miller (2005). The calibrated cohort (born in 1950) has a life expectancy of 75.4 years at birth and a remaining life expectancy at the age of 20 of 68.7 years. We report key demographic statistics in table 1.[12]

On the household level we choose the discount factor $\beta = 0.992$ in order to match the capital output ratio of 2.7 from the NIPA tables. Our calibration target for hours worked is 1/3 of the available time which pins down $\phi = 0.45$.[13] The coefficient of relative risk aversion σ is set equal to 2; a choice within the range of estimates reported in Browning, Hansen, and Heckman (1999). We show calibrated life cycle profiles for household decisions in figure 17 in the appendix.

3.2 Production

We chose α to match the labor share in the data which requires $\alpha = 0.33$. We calibrate the depreciation rate δ such that we match the observed share of investment in the national accounts (20%) which delivers $\delta = 0.059$. The average growth rate of technological progress is set to $g_t = 0.015$.

[12] Population data is an average over males and females. For the cross sectional aggregation during calibration we take the data from United Nations (2007). Note that as the cross sectional population is not in a steady state during the 20th century, cohort survival rates computed are not compatible with the ones computed from cross sectional data. Computing survival rates from the cross section would underestimate the life expectancy of any cohort and thus bias cycle profiles. Using cohort survival rates to construct the cross sectional population distribution would overstate the OADR (i.e. make the population older) and thus introduce a bias in the calibration of the social security system. Hence, we use cohort survival rates to solve the household problem but use the actual cross sectional distribution for aggregation.

[13] The Frisch labor supply elasticity in a model without borrowing constraints is given by $\frac{1-\phi(1-\sigma)}{\sigma}\frac{1-\ell}{\ell}$. However, as Contreras and Sinclair (2011) show, this is a misleading measure for models with incomplete markets and overestimates the true elasticity substantially. Domeij and Flodén (2006) show that neglecting liquidity constraints biases results downwards whereas Wallenius (2011) shows that the direction of the bias goes into the same direction when neglecting endogenous human capital accumulation. Both papers show that these biases can be quantitatively large.

3.3 Labor Endowment Process and Life Cycle Labor Efficiency

The deterministic life cycle efficiency profile is taken from Huggett, Ventura, and Yaron (2012) and displayed in figure 1. It peaks around the age of 50 and decreases thereafter.[14] To calibrate the stochastic part of the endowment process we follow Castaneda, Diaz-Giménez, and Ríos-Rull (2003). Instead of following the traditional approach of taking a microeconometric estimate for the endowment process and discretize it according to some technique (e.g. Tauchen (1986) or Kopecky and Suen (2010)), we directly calibrate jointly the Markov transition matrix and the realizations of the shocks (i.e. the levels).[15] In order to be able to come close to the empirically observed income and wealth distribution we choose a transition matrix with 4 states. We take the data on the Gini coefficients and the quintile of the income and wealth distribution from Diaz-Giminez, Quadrini, and Rios-Rull (1997) (based on data from the SCF). We report these numbers (together with the calibrated distributions) in table 3.

The calibrated transition matrix, the time-invariant distribution (which we use to initialize the newborns' entry into the labor market) and the associated realizations of the endowments can be found in table 2. The structure of the matrix is similar to Castaneda, Diaz-Giménez, and Ríos-Rull (2003). Relative to a matrix generated by more standard procedures, this matrix is not symmetric around the diagonal. The intuition behind this structure – and why it helps to match the skewed distributions – is that it is for all states more likely to move to a lower income realization than to receive a better realization of the income shock. Hence, compared to a symmetric matrix, this income process entails more downward risk and agents have a stronger incentive to insure themselves against bad shocks. This is reinforced by the fact that the differences between the income levels are relatively large. We are aware that the choice of this calibration strategy may not be innocuous. However, as Dávila, Hong, Krusell, and Ríos-Rull (2011) show, the result of low optimal debt depends primarily on the highly unequal income and wealth distribution and not how this distribution is generated. All models rely on some incentive to accumulate assets because of significant uncertainty (e.g. about income or health outcomes) or because of the initial conditions and preferences (e.g. for bequests) lead to a divergence of wealth paths but it is not possible to judge how the results would change without a proper model comparison.

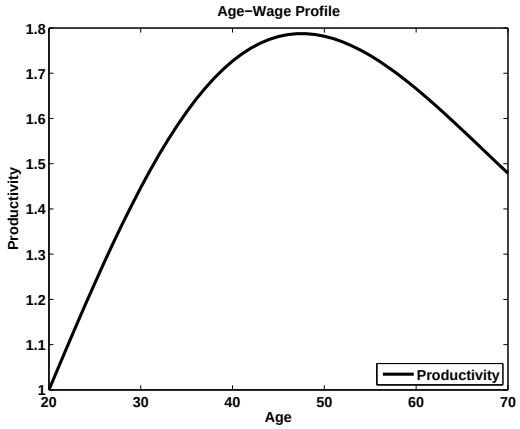
3.4 Social Security System

We use data on contributions and expenditures from the NIPA tables (Government Social Insurance Funds Current Receipts and Expenditures) and divide them by total compensation per employee to compute contribution rates and benefits. Figure 2(a) shows the

[14] The original estimates are available only from age 23 to 60. In order to get also data for younger and older agents, we estimate a 3rd degree polynomial and use this to generate the smooth profile (instead of using original data where it is available).

[15] See Cagetti and De Nardi (2008) for a review of the literature focusing on the replication of the observed income and wealth distribution.

Figure 1: Efficiency Profile



Notes: Data smoothed by a 3rd order polynomial and standardized by the wage at the age 20.
Source: Huggett, Ventura, and Yaron (2012), own calculations.

Table 3: Distribution of Wealth and Income

			Quintile				
		Gini	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
Earnings	Data	0.63	0.00	0.03	0.12	0.23	0.61
	Calibration	0.62	0.01	0.03	0.12	0.23	0.61
Wealth	Data	0.78	0.00	0.02	0.06	0.14	0.80
	Calibration	0.78	0.00	0.00	0.05	0.17	0.79

Notes: Source: Diaz-Giminez, Quadrini, and Rios-Rull (1997), own calculations.

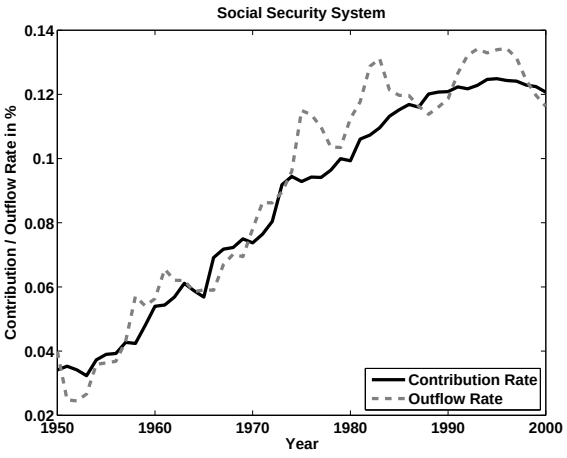
evolution of the contribution rate τ_t with the average over the calibration period being $\bar{\tau}_t = 11.4\%$. As can also seen in figure 2(a), the ratio of outflows (pension payments, "outflow rate") to total contributions ("contributions rate") closely follows contributions and averages 12.1%. As over the calibration period the system does not show a strong tendency for large surpluses or deficits we assume for calibration a balanced budget ($BPS_t = 0$), keep τ_t fixed, and back out the replacement rate ρ_t. The model is calibrated such that the economy is dynamically efficient (as shown by Abel, Mankiw, Summers, and Zeckhauser (1989) for the U.S.). Hence, the role of social security is not to decrease inefficiently high savings.

3.5 Labor Income Taxation

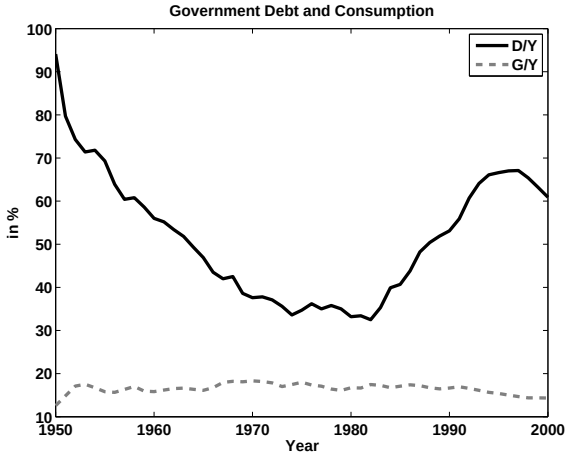
We calibrate the tax function in two steps. Firstly, we use the baseline estimates for for a_0 and a_1 as computed by Gouveia and Strauss (1994). Secondly, we adjust a_2 such that the share of labor income taxes in aggregate output corresponds to the ratio computed from the data setting $T^{\ell}/Y = 14.3\%$. As the data used for estimation of a_0 and a_1 is from 1989 we have a coefficient estimate generated by the middle of our calibration period.

Figure 2: Calibration Public Sector

(a) Social Security System



(b) Government Debt and Spending



Notes: Source: NIPA-tables, own calculations.

Further, note that this procedure captures the curvature of the tax function but also matches the average tax by the choice of a_2. To give the reader a quick overview about the calibration, we plot marginal and average tax rates for a hypothetical 20 year old agent (h_j=1) with $\ell = 1/3$ for the four different realizations of η in figure 3. Adjusting a_0 has a linear effect on marginal and average tax rates for all types.[16] However, due to the fact that tax rates are a function of the *level* of income, changing a_0 has a relatively larger effect on high income earners, relative to low wage agents (the tax functions are steeper). A change in a_2 has a stronger effect on low income earners but leaves tax rates unchanged for a broad range of a_2 for high-income agents.[17]

3.6 Government Sector

Given the calibration of labor income taxes and social security, we have to choose the remaining free parameters: capital taxes, consumption taxes, and variables related to government spending, transfers, and debt.

For the consumption and capital taxes we use the numbers provided by Carey and Rabesona (2002) for the period 1975-2000.[18] This gives $\tau^c = 6.5\%$ and $\tau^k = 39.4\%$. Together with the calibration of the capital-output ratio and the depreciation rate, this gives a net interest rate of 3.9%. For the government debt to GDP ratio we use the numbers on gross federal debt provided by the US Office of Management and Budget. This gives a calibration target of $D/Y = 50\%$. Further, we divide gross government

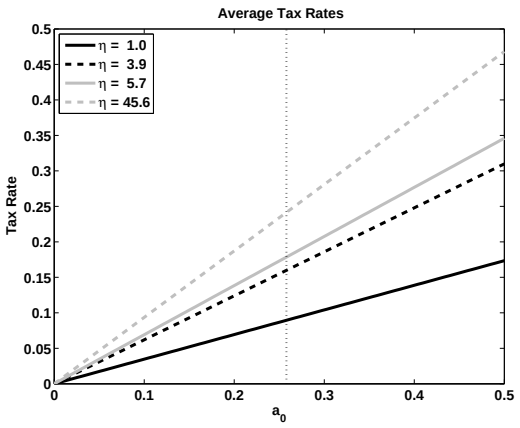
[16] We would obtain a qualitatively similar behavior of the tax system by changing a_1. Decreasing a_1 makes the tax system more linear with a linear tax system for $a_1 \to 0$.

[17] To be precise, this effect depends on the calibration of the tax system. In our case, decreasing a_2 has a larger effect on lower incomes.

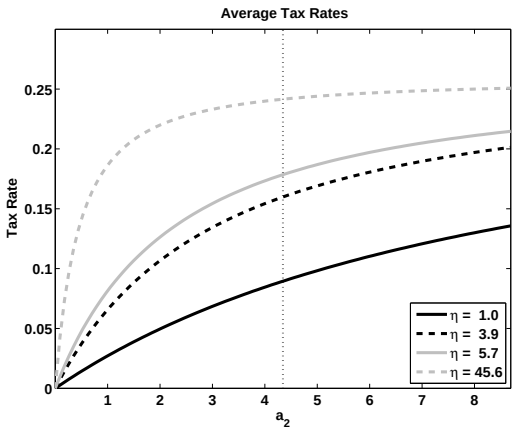
[18] Carey and Rabesona (2002) follow closely the approach of Mendoza, Razin, and Tesar (1994) but refine the calculations by taking country specific regulations and a longer data set into account.

Figure 3: Calibration of Tax System

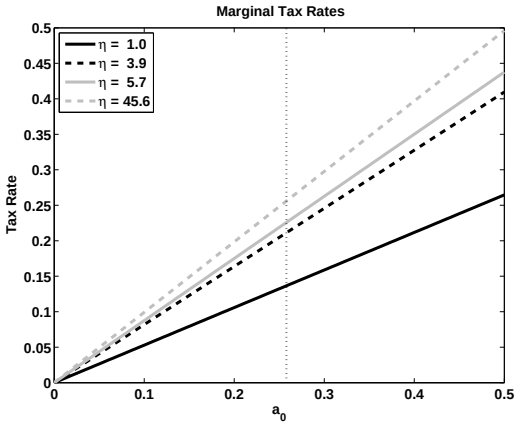
(a) Adjust a_0 (Average Tax Rate)



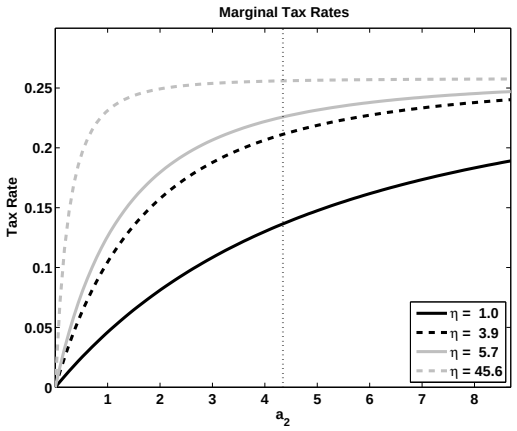
(b) Adjust a_2 (Average Tax Rate)



(c) Adjust a_0 (Marginal Tax Rate)



(d) Adjust a_2 (Marginal Tax Rate)



Notes: The vertical dotted line indicates the calibrated value of the respective parameter.

consumption spending from the NIPA by GDP to calibrate $G/Y = 16.3\%$. We plot the evolution of both series over time in figure 2(b). Given these numbers, we adjust lump-sum government transfers Tr such that the consolidated government budget constraint is satisfied with equality. This gives implicitly $Tr/Y = 7.6\%$. These choices are broadly in line with the numbers used in other studies, e.g. Uhlig and Trabandt (2012), Conesa, Kitao, and Krüger (2009) or Heathcote (2005).

3.7 Computational Strategy

For the solution of the individual household we adapt the procedure developed by Carroll (2006). We use 75 gridpoints and a non-linear grid for cash-on-hand with a higher density in the regions where marginal utility has more curvature. On the aggregate level we make a guess for the relevant state variables (depending on the model variant) and use the procedure developed by Ludwig (2007) to update our guess until convergence. For the calibration procedure, we make an initial guess for the vector of structural parameters,

compute the equilibrium as described above, and update the initial vector until we have matched our specified calibration targets. The computation of the "outer-outer-loop" is the most expensive part of the procedure as it requires to solve the model for equilibrium many times for each guess of the candidate vector of structural parameters.

4 Results

In section 4.1 we examine the effects of government debt on factor prices, welfare, and the implicitly required policy changes to support an exogenously given government asset level in general equilibrium. We start with a global overview and examine then each policy in a separate sub-section. When looking at welfare, we will show the changes *ex ante* and *after* the realization of the first labor market shock. Later in the section we will also report changes in aggregates.

Section 4.2 sheds light on the role of risk, income levels, and reallocation over the life cycle for the changes in welfare. We show that the welfare gains come from higher income levels (higher capital intensity) and a reallocation over the life cycle due to lower interest rates and the partial loosening of liquidity constraints. Steady states with less government debt carry a higher cost in terms of income risk (decreasing welfare).

By focusing on the general equilibrium, we mix effects stemming from changes in prices (equilibrium capital intensity) and changes in taxation or transfers. To account for this we conduct two counterfactual experiments to isolate price effects and effects from budgetary adjustments. In section 4.3.1 we keep wages and interest rates at calibrated levels and use tax rates and transfers obtained from the general equilibrium analysis to recompute welfare. This tells us something about the quantitative relevance originating from the budget constraint. Then, in section 4.3.2 we reverse the exercise and keep tax rates and transfers at calibrated levels and use the equilibrium capital intensity to compute welfare changes caused by adjusting prices.[19] Lastly, a more saddle effect stems from keeping the ratio of transfers over GDP constant and hence varying the level of transfers. However, the insurance provided by transfers has been shown (e.g. Flodén (2001)) to be an important element, especially for low wealth agents. To control for this effect we re-compute the general equilibrium solution fixing the level of transfers at calibrated values and report the main differences to the baseline (section 4.4).

In our policy experiments we consider four different adjustment mechanisms as a reaction to changing government debt: adjustment of transfers, capital taxes and two scenarios for labor taxation. In our first scenario, we keep all tax rates constant and adjust lump-sum transfers such that the government's intertemporal budget constraint holds. In our second simulation we trace out the effects of changing debt but adjust the capital tax rate to balance the budget. Finally, when we change labor taxes we change the parameters

[19] Yet another possibility is to compte the equilibrium of a small open economy with fixed capital intensity. Then, welfare gains from decreasing government debt are higher as interest rates are not falling (so wealthier agent do not lose) but taxes are declining or transfers are rising. However, this exercise is not interesting as there is no trade-off and welfare increases monotonically with government assets.

a_0 and a_2 (one at a time) but keep a_1 unchanged. We use these two adjustment options as this allows us to change marginal and average taxation specifically targeting different parts of the income distribution. In equation 36 we visualize our strategy by spelling out the steady state budget constraint of the government. When changing the steady state debt/GDP-ratio ($\bar{d}$) we adjust government spending such that G/Y and Tr/Y stay at their calibrated values (16.3% and 7.6% respectively) but keep all other tax *rates* constant except the policy variable. Keeping the tax rate constant implies that the ratios can change as the structure of the economy endogenously adjusts to the varying capital intensity.

$$\frac{G}{Y} + \frac{Tr}{Y} - \left(\frac{BPS}{Y} + \frac{T^c}{Y} + \frac{T^k}{Y} + \frac{T^\ell}{Y} \right) = \bar{d}(g - r) \tag{36}$$

When interpreting the results, one should keep in mind that changing policy affects agents' welfare in three ways (Flodén 2001). A change in government debt, by changing capital intensity, will affect the level of income. Higher debt will decrease capital, wages and welfare. On the other hand, lower wages decrease the share of risky income increasing welfare. Thirdly, for any given income distribution, increasing (decreasing) the wage/interest rate ratio will make wage (capital) earners better (worse) off. Hence, the distribution of the income source matters. Taking all effects together, it is a priori not clear which effect dominates and whether the government should increase or decrease capital intensity by changing its net asset position.

To keep the presentation format constant, we present for each adjustment scenario two graphs: the left figure presents CEV, the net interest rate, and the share of that instrument in GDP (e.g. capital taxes over GDP when changing capital taxation). Panel b) in the figure then reports the welfare change according to the initial labor market shock. All welfare numbers are measured in percent. A dotted vertical line indicates the calibrated debt to GDP ratio of 50%.

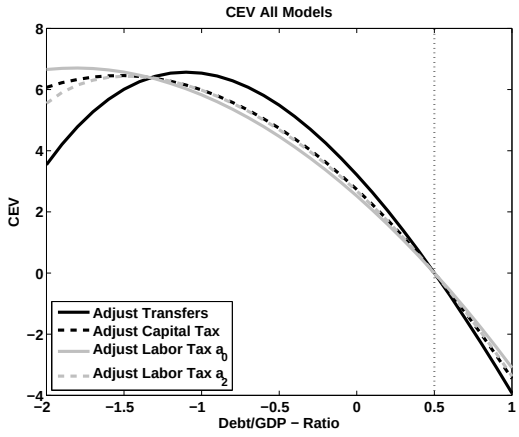
4.1 A Global Comparison Across Policies

We find that the global welfare maximum is achieved by adjusting labor income taxes in such a way that the marginal tax rate is decreased linearly, i.e. by lowering a_0 (figure 4). Pushing capital intensity beyond welfare maximizing debt decreases welfare via the government's budget channel. As a higher capital intensity pushes the rate of return down, income from capital taxes decreases. The government is then forced to make up for that shortfall in revenues by increasing some other tax or decreasing transfers. Despite relatively small differences in welfare levels, we observe relatively large differences in the optimal debt/GDP ratio ranging from -180% to -110%.

Furthermore, the slope of the welfare function when adjusting transfers is much steeper compared to the other options. The reason for this is that by increasing transfers, young and low income agents reap the largest benefits: they are less likely to hit the borrowing constraint, must hence work less and can also consume more. At very high aggregate

capital levels, transfers fall below calibrated levels. Low income and poor agents will work more and accumulate more assets in order to self-insure against shocks. For measures also increasing pure economic efficiency (i.e. adjusting taxes), the welfare function is less steep and flatter after the peak.

Figure 4: Comparing Welfare for All Scenarios



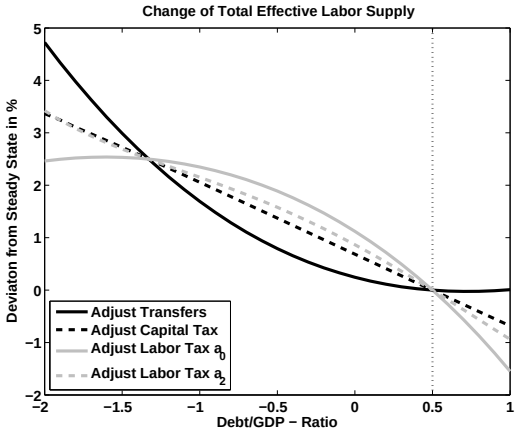
We summarize the effects of debt on aggregate labor supply (hours worked) and total effective labor supply in figure 5 and offer three main findings. Firstly, increasing transfers has initially a relatively small effect on effective labor supply. This is due to low income agents now receiving higher transfers. More insurance reduces their labor supply as the incentive to work hard to build up a buffer stock of savings (in order to stay away from the borrowing constraint) decreases. However, as transfers must decrease at higher levels of capital intensity, the effect is reversed and low income agents work longer hours when government assets increase further. This squares with the observation that the peak of the welfare curve for adjusting transfers is at much lower levels of government assets and drops faster thereafter. Secondly, decreasing a_0 has a stronger effect on effective labor supply than on hours worked. This is due to the fact that lowering a_0 comes with relatively larger benefits for agents with high productivity. This increases the incentive for that group to work harder. The ranking is reversed for adjusting a_2: here incentives for low income earners (which constitute a large part of the economy) are stronger and hence total hours worked increase. However, as firms employ effective labor (and not just raw labor), the effect on the capital/labor ratio is small. Thirdly, the effect of the adjustment of capital taxes on effective labor supply is similar to an adjustment of a_0. This should not come as a total surprise as these two adjustment mechanisms have a similar target group: wealthy and high-income agents. In line with the argument from above, decreasing tax rates have initially the strongest effect on capital and output as the incentive component is relatively strong. Higher transfers provide less incentives to accumulate capital and work more. However, this changes at higher levels of aggregate as assets and transfers decrease and the necessity for more self-insurance via saving (and more labor supply) increases. Hence, output is highest when transfers are adjusted. We

show these results in figure 6.

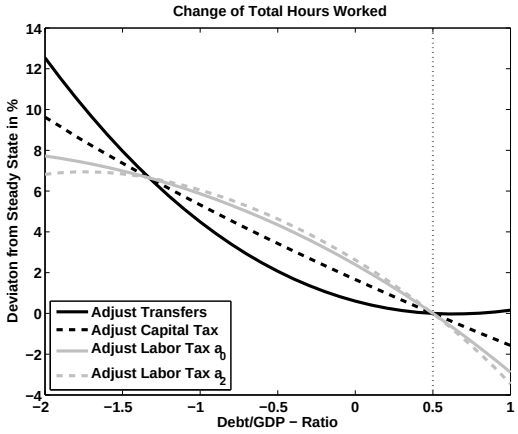
In tables 4 and 5 we provide an overview of changes in selected aggregates, prices, and welfare at the optimal debt ratio for each adjustment scenario individually. Echoing the arguments from above, we observe that similar welfare results are achieved for rather different aggregate outcomes. This underlines the need to examine and properly model heterogeneity instead of focusing exclusively on aggregates.

Figure 5: Change in Effective Labor and Hours

(a) Change of Effective Labor Supply



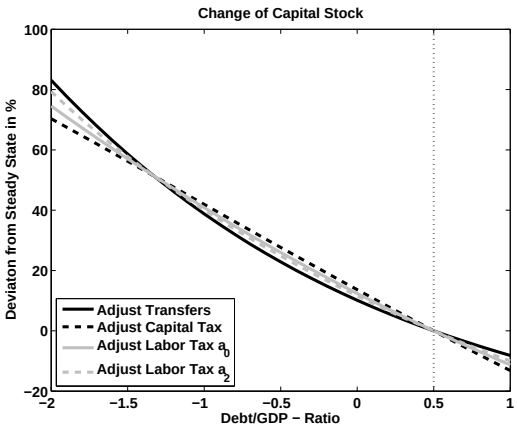
(b) Change of Aggregate Hours



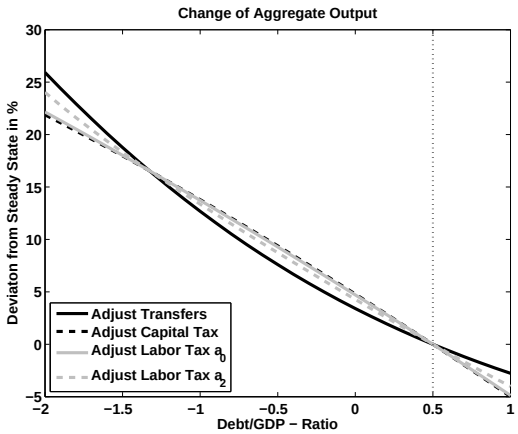
Notes: Numbers are deviations from calibrated value.

Figure 6: Change in Capital Stock and Output

(a) Change in Aggregate Capital



(b) Change in Output



Notes: Numbers are deviations from calibrated value.

Table 4: Equilibrium values for welfare maximizing debt ratios

	r^n	K/Y	Tr/Y	T^ℓ/Y	T^a/Y	τ^k	CEV	D/Y
Calibration	3.9%	2.7	7.7%	14.2%	8.0%	39.4%		0.5
Transfers	2.4%	3.4	8.1%	14.4%	3.5%	39.4%	6.6%	-1.1
Capital Tax	1.9%	3.6	7.7%	14.5%	3.0%	42.8%	6.5%	-1.5
Labor Tax a_0	1.8%	3.7	7.7%	15.5%	2.3%	39.4%	6.7%	-1.8
Labor Tax a_2	2.0%	3.6	7.7%	14.8%	2.7%	39.4%	6.4%	-1.5

Table 5: Equilibrium values for welfare maximizing debt ratios

	ΔY	ΔK	ΔH	PBB/Y	$\sum T^{\ell/c/a}/Y$	D/Y
Calibration	0.0%	0.0%	0.0%	2.4%	26.4%	0.5
Transfers	13.8%	42.4%	1.9%	-2.7%	21.7%	-1.1
Capital Tax	18.0%	56.2%	2.7%	-2.8%	21.2%	-1.5
Labor Tax a_0	20.5%	67.4%	2.5%	-2.6%	21.4%	-1.8
Labor Tax a_2	18.3%	57.8%	2.7%	-2.7%	21.3%	-1.5

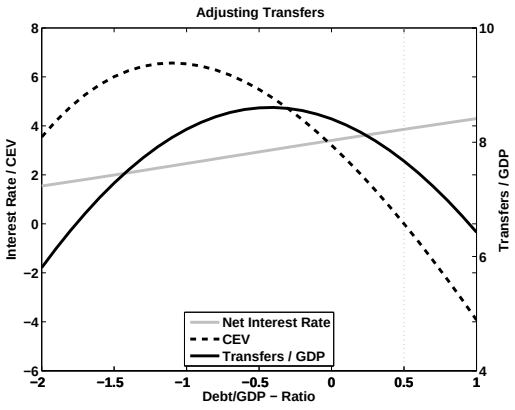
Notes: Δ H, Δ K, and Δ Y refer to the percentage difference of total effective labor supply, capital stock, and aggregate output relative to the calibration. PBB denotes primary budget balance, and $\sum \ldots$ denotes total taxes as a share of GDP.

4.1.1 Adjusting Transfers

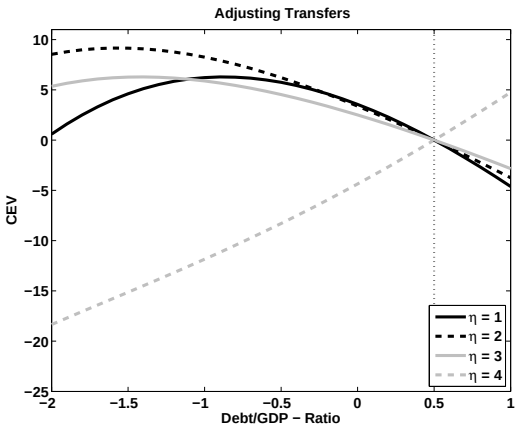
We find that the welfare maximizing level of debt adjusting *transfers* is around -110% of GDP with considerable gains in welfare (figure 7). Starting from the calibrated value of a debt to GDP ratio of 50%, decreasing debt increases the capital stock, hence depresses the rate of return. Additionally, welfare gains come from increasing transfers (providing more insurance) and rising wages. However, as the government's income from capital taxation decreases due to lower interest rates and all other expenditures and tax rates are fixed, transfers must eventually decrease (starting at about 40% of net government assets) to balance the budget. Panel b) of the same figure decomposes the aggregate welfare change for the different initial labor market shock realizations. What stands out is that agents with the best shock ($\eta = 4$), experience large welfare losses. The reason is that they start their lives as wealthy agents and suffer from the drop in interest rates. On the other hand, low types gain from decreasing debt. The group with the lowest starting income level has initially the largest gains. This is because initially transfers increase, providing these agents with more insurance and income. However, when transfers start to decrease at around 40% (panel a), they fall behind groups 2 and 3 (in relative terms). The latter groups still gain as they do not rely so much on the insurance effect from transfers but rather benefit from increasing wages.

Figure 7: Adjust Transfers

(a) Aggregates



(b) Welfare Decomposition



Notes: Left axis is the interest rate and CEV. Right axis is in %.

4.1.2 Adjusting Capital Taxes

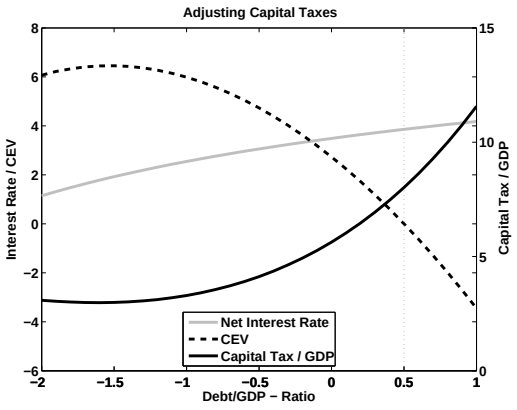
If we adjust *capital taxation* the welfare maximizing level of debt is around -150% of GDP (figure 8). As a consequence of lower interest payments on the stock of debt, the government can initially decrease capital taxes to about 33% when the public sector holds approximately 60% of GDP as net assets. However, as the tax base becomes smaller (due to falling interest rates), capital taxes must be increased again to pay for exogenous government expenditures and transfers. Decomposing aggregate welfare effects delivers a similar message as above. The main losers are asset-rich agents ($\eta = 4$). However, the largest gains are reaped by agents with $\eta = 2$. The reason is that these agents rely to a large extent on labor income (which increases) but also have some assets (benefitting from initially lower capital taxes). Further, as the share of transfers in GDP is constant but total GDP increases, they also benefit to some extent from higher absolute transfer payments. Welfare effects for groups 1 and 3 are relatively similar as gains from higher wages and losses from falling interest rates roughly compensate each other.

4.1.3 Adjusting Labor Taxes

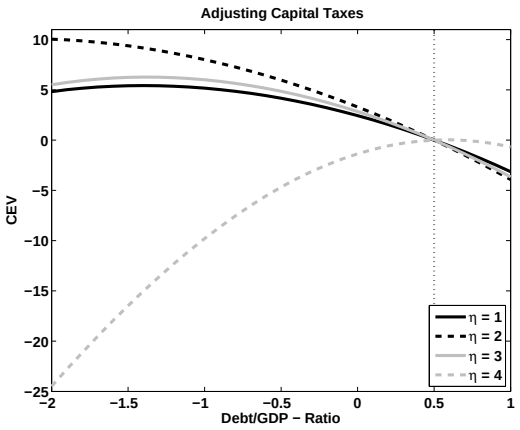
Turning to an adjustment of *labor taxation* by changing a_0, the welfare maximizing level of debt amounts to around -180% of GDP (figure 9). Starting again from the calibrated steady state, labor taxes can be decreased, increasing labor supply and production. However, for the same reasons as outlined above, labor taxes must be eventually increased again, leading to higher distortions partly undoing the positive effect of higher wages on labor supply. Therefore, welfare is not monotonically increasing. Looking at the decomposition (panel b), the results are similar to the scenario with adjusting capital taxes: agents receiving a good initial shock are the losers are they suffer from falling interest

Figure 8: Adjust Capital Taxes

(a) Aggregates



(b) Welfare Decomposition

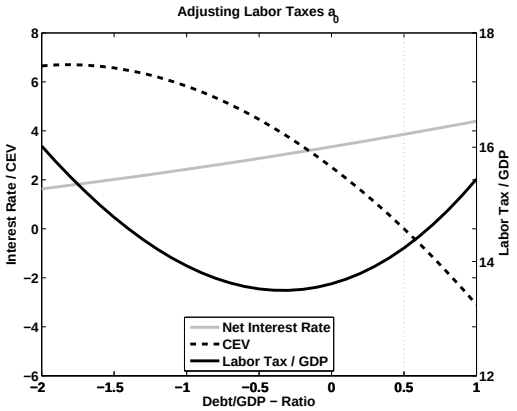


Notes: Left axis is the interest rate and CEV. Right axis is in %.

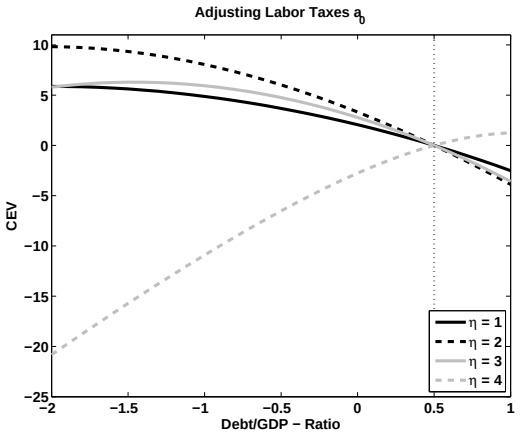
rates. The "unlucky" agents ($\eta = 1$) fare worst among the other types as the labor tax cut is rather limited for them (section 2.5). Conversely, agents with better shock realizations can benefit from higher wages and lower taxation as their marginal and average taxes decrease by an economically significant amount.

Figure 9: Adjust Labor Taxes (a_0)

(a) Aggregates



(b) Welfare Decomposition



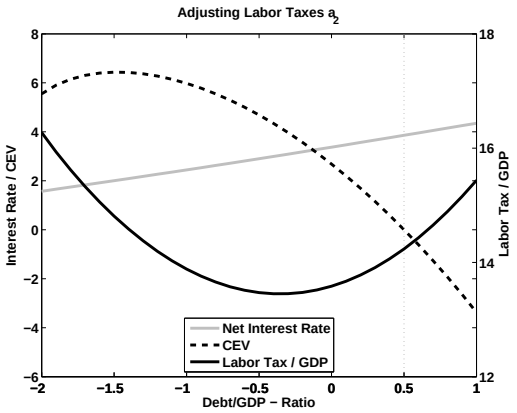
Notes: Left axis is the interest rate and CEV. Right axis is in %.

The second adjustment option for *labor taxation* by changing a_2 delivers remarkably different results (figure 10). First, the optimal level of government assets is much lower (150% of GDP) compared to the alternative labor tax adjustment option. Second, adjusting a_2 leads to decreasing welfare beyond the optimal debt/GDP ratio for *all* groups while the adjustment of a_0 increases welfare for low income groups (1 and 2) over the

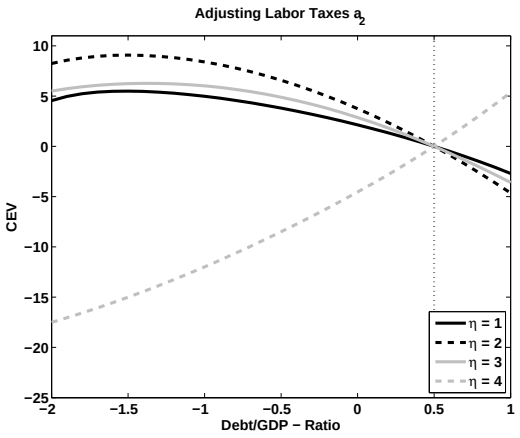
entire range of debt levels. The difference can be explained by a complex interaction of labor taxes, insurance due to transfers and labor supply. As lowering a_2 benefits rather low income agents, incentives to increase labor supply for high productivity agents are low. Hence, a given change in government asses generates a smaller increase in effective labor supply and output. The difference in incentives depending on the labor market productivity can be best seen by looking at aggregates. Although changing a_2 increases total hours worked more than when adjusting a_0, the aggregate response of effective hours is much weaker: the order is even reversed (figure 5). Hence, the same change in government assets generated lower aggregate income due to the relatively low labor supply of high income agents. This feeds then back to low income agents who have, as a consequence of lower transfers, less insurance from transfers. This is strong evidence for spill-over effects from highly productive agents to the rest: decreasing a_0 benefits more productive agents as a first order effect but makes everybody better off as these agents increase their labor supply relatively more. Changing a_2 increases the welfare of poor agents initially much more but the global maximum is nevertheless lower.

Figure 10: Adjust Labor Taxes (a_2)

(a) Aggregates



(b) Welfare Decomposition



Notes: Left axis is the interest rate and CEV. Right axis is in %.

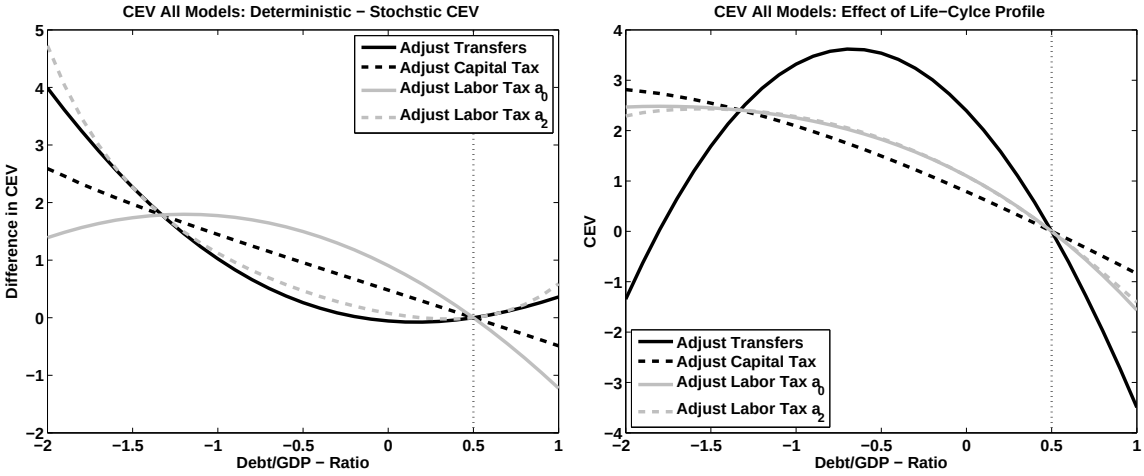
4.2 The Role of Levels, Life Cycle Decisions, and Risk

As argued earlier, a change in the capital intensity raises wages and depresses interest rates. While *average* income will be higher, agents will earn a larger portion of their income from wages and the risky component of their income will also increase. This is detrimental for welfare. In the remainder of this section use the notation established in section 2.7.

We find that for the case of adjusting transfers and labor taxes by changing a_2, lower debt levels offer a better protection against income risk. This is shown in figure 11 as $\bar{g} - g < 0$ for debt between -10% and 50% of GDP. For adjusting capital taxes and

Figure 11: Welfare Evaluation: Risk Protection and Life Cycle Effects

(a) Risk Protection Effect: $\bar{g}$-g (b) Life Cycle Profile Effect: $\tilde{g}$



Notes: definitions as in section 2.7.

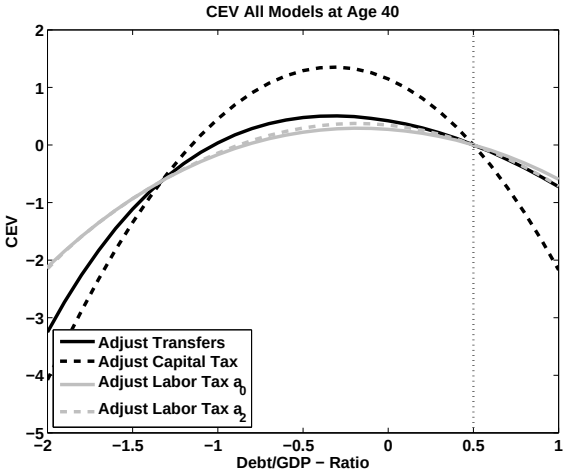
making the labor tax system more linear (change in a_0) we find that lower debt levels provide less protection from income risk ($\bar{g} - g > 0$). This is because higher capital levels are associated, everything else equal, with a higher income risk due to the shift in the share of income sources. However, this effect is counterbalanced in the case of higher transfers or lower taxes for poor agents. For the two other cases economic efficiency is enhanced (so is income) but this is not sufficient to overcompensate the risk via the "income composition channel".

On the other hand, even after controlling for the level effects on consumption and leisure, agents prefer the life cycle profiles chosen in the steady states with lower levels of debt. This is shown in panel b) of figure 11 which corresponds to $\tilde{g}$. The reason is twofold. First, higher income levels and also higher transfers loosen borrowing constraints earlier in life. Second, a higher capital stock decreases the interest rate and brings consumption forward. Hence, for the same *average* level of consumption and leisure agents obviously prefer the flatter profile.

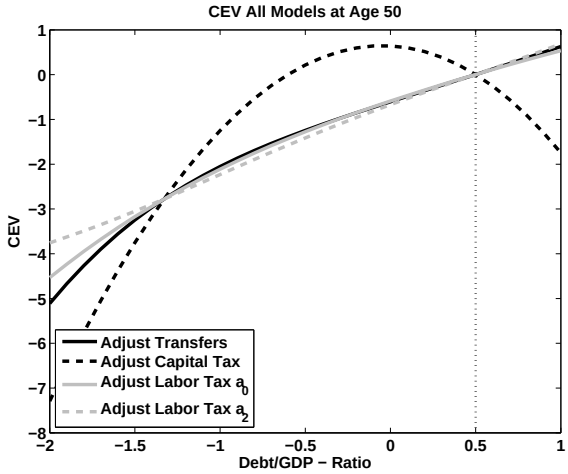
The role of labor income risk and asset position can also be assessed by computing the CEV at different ages. In contrast to newcomers to the labor market, agents aged 40 prefer initially cuts in capital tax rates. However, as at that age, agents are in the middle of their working live and/or might have not accumulated a lot of assets, they also derive gains from the other three policy adjustments (figure 12a). At age 50, agents unanimously prefer cuts in capital tax rates and are worse off under all other alternative (figure 12b). This is because they have built up a considerable amount of assets, need less insurance provided by transfers and the share or (remaining) risky labor income has decreased considerably.

Figure 12: Welfare Evaluation at age 40 & 50

(a) CEV at age 40



(b) CEV at age 50



Notes: CEV computed unconditional (i.e. averaging over asset position and age) at that specific age.

4.3 Welfare Results for Partial Equilibrium

This section contains a partial equilibrium analysis with two scenarios: keeping either prices (section 4.3.1) or transfers/taxes (section 4.3.2) at their calibrated values and take the value of the other variables from the general equilibrium solution. Finally, we quantify the contribution of the simultaneous adjustment in section 4.3.3.

4.3.1 Welfare Results keeping Prices at Calibrated Levels

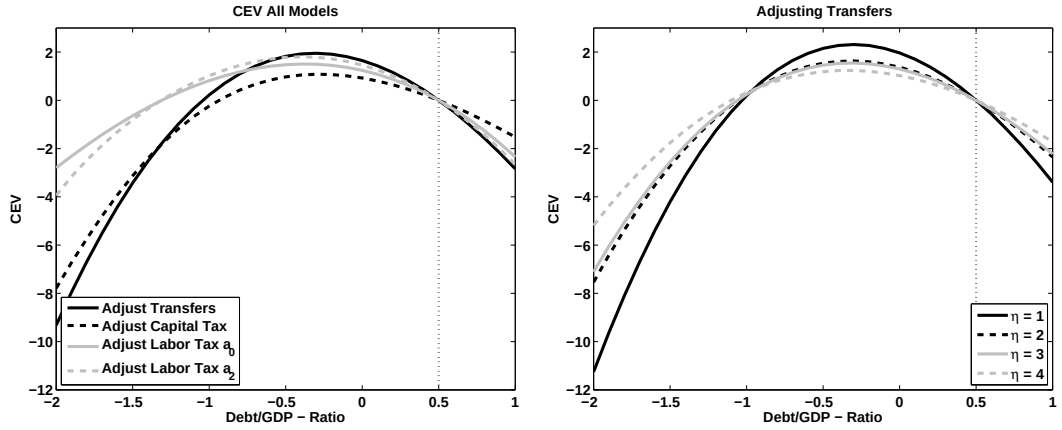
In this section we recompute household welfare by holding the capital intensity at the calibrated level. All other variables correspond to the respective values from the general equilibrium simulation. Hence, changes in government debt have no effect on the marginal product of labor (gross wages) or capital (gross interest rate) but affect household decisions only via changing taxes and transfers. Figure 13a) shows expected welfare as a function of government debt. The most important observation is that welfare gains are much smaller than in the general equilibrium scenario and reached at much lower capital levels. This can be explained by two counteracting factors. Firstly, wages are not increasing and agents relying on wage income are worse off compared to the general equilibrium situation. Secondly, as interest rates are not falling, welfare losses off asset-rich agents are much lower. This is shown in figure 13b) where we perform the familiar welfare assessment after the first shock. For the sake of brevity, we show only the decomposition from the scenario where transfers are adjusted. The other three scenarios are qualitatively similar and only shown in the appendix. For instance, remaining lifetime utility of agent $\eta = 4$ at a debt/GDP ratio of -100% almost unchanged compared to the calibrated value when holding prices fixed. With falling general equilibrium interest rates, the same agent's loss is around 15% of total lifetime consumption as shown in fig-

ure 7b). However, as the majority of the economy consists of low type agents, the losses due to the constant wage level outweigh the relative gains by the asset-rich households.

Figure 13: Welfare with Prices from Calibration

(a) Comparing Welfare for All Scenarios

(b) Welfare Decomposition for Adjusting Transfers



Notes: CEV computed with taxes and transfers from the general equilibrium and gross wages and interest rates (prices) from the calibration.

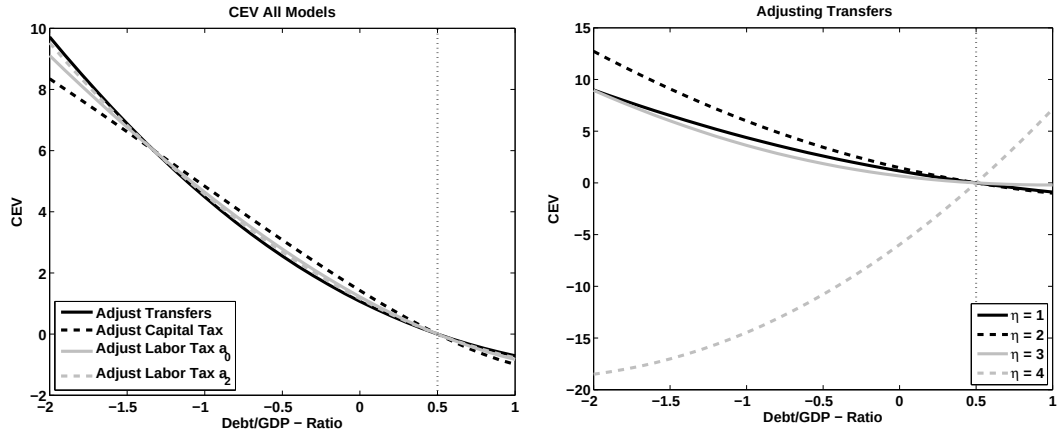
4.3.2 Welfare Results keeping Taxes and Transfers at Calibrated Levels

To cancel income effects from higher transfers, we evaluate welfare holding taxes and transfers at their calibrated levels. Hence, the only source of variation is the change in (gross) wages and (gross) interest rates. The intention of this exercise is to isolate welfare effects originating purely from general equilibrium feedback of prices. We employ the strategy from the previous section. Figure 14a) shows welfare changes for all four policy adjustment variants. Panel b) decomposes welfare after the first shock for the case when prices are taken from the simulation adjusting transfers. The other three cases are qualitatively similar and hence only shown in the appendix. The results are – not surprisingly – in stark contrast to the results from the previous section. Keeping taxes and transfers at calibrated levels and letting the capital intensity go up, increases welfare monotonically. This is due to rising wages with large gains for wage earners. Wealthy agents' ($\eta = 4$) losses, however, increase as the interest rates drops monotonically. Nevertheless, even this group's loss is smaller (compared to the general equilibrium simulation) as for high levels of assets the government does not have to increase capital taxes. For instance, welfare losses for the agent with $\eta = 4$ are about 15% in this counterfactual scenario but roughly 20% in general equilibrium at a debt/GDP ratio of -200%. Smaller losses for capital owners and higher gains for wage earners raise total welfare gains.

Figure 14: Welfare with Taxes and Transfers from Calibration

(a) Comparing Welfare for All Scenarios

(b) Welfare Decomposition for Adjusting Transfers



Notes: CEV computed with gross wages and interest rates (prices) from the general equilibrium and taxes and transfers from the calibration.

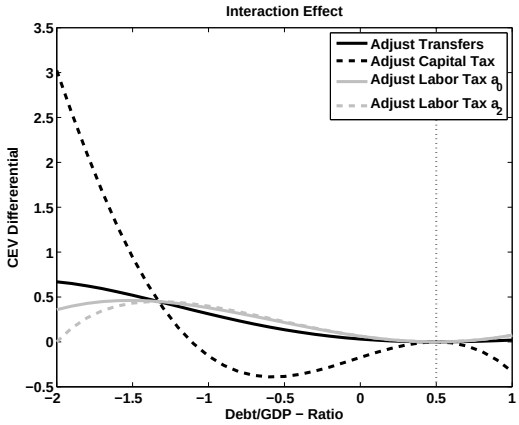
4.3.3 Interaction Effects

Results from the previous sections show that welfare gains come to a large extent from variations in prices. To quantify the interaction between changing prices, taxes and transfers we compute the difference between the CEV obtained in the general equilibrium and the sum of the two "partial" results obtained from the previous sections. Positive numbers in figure 15 indicate that the CEV from the general equilibrium is larger than the sum from the two counterfactual experiments. The interaction effect is economically significant with the interpretation being that effects due to changes in household income are not symmetric. Higher wages make especially poor agents much better off but decreasing their income by a similar amount pushes lots of agents closer to the borrowing constraint. This is reflected in the fact that gains from wages are relatively evenly distributed (holding taxes and transfers constant) but the variance of losses when e.g. transfers are cut are much larger (and concentrated) for low types (see figures 18 and 19 in the appendix).

4.4 Welfare Maximizing Debt in General Equilibrium: Fixed Level of Transfers

For this experiment we compute the general equilibrium solution keeping the per capita transfers constant and adjusting only labor or capital taxes. We summarize welfare effects in figure 16 and macroeconomic aggregates in figure 20 in the appendix. The main message can be summarized as follows. Firstly, welfare gains are about 1.5 percentage points lower compared to the case with adjusting transfers. Secondly, as transfers are not growing with GDP, lowering labor taxes for low income agents (adjusting a_2) is the optimal policy. This serves as a substitute to increasing transfers and provides insurance.

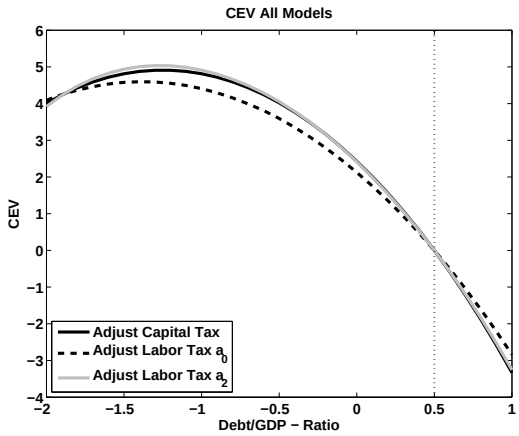
Figure 15: Interaction Effect



Notes: CEV differential computed as: CEV from GE less sum of the two counterfactual experiments.

One can conclude that while higher transfers make up a non-negligible part of the welfare gains from lower debt, they are not the most important drivers of our earlier results. On the aggregate level, the main difference to the baseline now is that differences in output across scenarios are lower.

Figure 16: Comparing Welfare for All Scenarios (Constant Transfers)



5 Conclusion

How does government debt affect prices, macroeconomic aggregates and the distribution of wealth and income? To answer this question, we calibrated a large scale overlapping generations model to the US economy with a focus on the skewed income and wealth distribution and a detailed fiscal sector.

We find that the optimal level of government debt to maximize *ex ante* welfare is negative, i.e. the government should accumulate assets. This is because young and low

asset agents are more likely to be borrowing constraint with a higher remaining lifetime labor income risk. The welfare maximising policy is one where labor taxes are adjusted and the corresponding level of government debt is -180% of GDP. However, the range for the optimal debt level varies according to the adjustment policy between -180% and -110% of GDP. Conditioning on the realization of the labor market status upon entry we find that wealthy agents incur losses while poorer agents are in general winners. On the aggregate level, output, consumption and effective labor supply will increase.

The results give rise to several lessons for the effect of debt on the *distribution* of welfare. First, as agents live in a risky environment, providing unlucky or low income agents with a thicker insurance pillow via increasing *transfers* improves expected welfare.[20] However, as later transfers have to be decreased, the lowest income group suffers most with almost no welfare gains at high aggregate capital levels. In contrast, agents with intermediate labor market outcomes benefit essentially over the whole range. This is because they still benefit from increasing wages at constant tax rates and are not affected so much by falling transfers. Second, for the case of adjusting *capital or labor taxes*, the lowest and the third income group experience very similar welfare gains. For the low income group, transfers and wages are increasing with GDP making them better off while changes in taxes and lower capital income does not affect them much. On the other hand, the third income group is already wealthy enough to lose due to falling interest rates and does not derive a significant benefit from higher transfers as the insurance channel does not play such a prominent role. Also, due to their relatively high initial wages and further wage increases, lowering labor taxes does not have a large effect. This all adds up to a small positive welfare gain. Third, the second income group incurs the largest welfare gains. It benefits from higher transfers, increasing wages, lower taxes (as it's absolute wage level is lower) but does not yet suffer much from falling returns on capital. These results show that the group with a balanced capital/labor income ratio will benefit most from lower government debt.

The complexity of the interaction can be best seen by looking the two partial equilibrium counterfactual simulations. There, we show that in absence of a simultaneous change in prices and taxes/transfers the effects are "monotonic" and more intuitive. Further, the same comparison shows that welfare gains come to a large extent from variations in prices. While higher wages obviously increase welfare, a lower interest rate is also welfare enhancing by re-allocating consumption over the life cycle. On the other hand, higher wages increase the share of the risky income component with negative welfare effects if not counterbalanced by increasing transfers.

This paper provides many essential ingredients needed for a quantification exercise but many are still missing. Hence, some caveats should be considered. For instance, we have neglected aggregate shocks and the potential of government debt to act as a shock absorber or tax smoother over the business cycle. Also, the paper takes one specific

[20]This finding is qualitatively consistent with Flodén (2001) and Flodén and Lindé (2001) who find that optimal transfers in the US are much higher than currently.

approach to replicate the income and wealth dispersion observed in the data. This may not be an unimportant choice but can be ultimately only verified with a proper model comparison. Further, the role of transfers for low income people and the incentives for their labor supply should be modeled in more detail. While not everything can be put into one model, some of these extensions are on the future research agenda.

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A Technical Appendix

A.1 Data Construction

In this section we describe the data used to calibrate the model in more detail

Government Debt

We use the data from the web-site of U.S. Office of Management and Budget (http://www.whitehouse.gov/omb/budget/Historicals, table 7.1) to compute the average gross government debt for the calibration period. We show the data from 1950-2000 in figure 2.

Social Security System

We use the statistics provided in the NIPA table 3.14 (Government Social Insurance Funds Current Receipts and Expenditures) and table 2.1 (Compensation of employees, received). When computing contributions and payments, we exclude the interest payments received on assets but include only items related to payroll contributions and administrative expenses (the latter being relatively small). To obtain the model-equivalent numbers we divide contributions by total compensation. The corresponding time series is show in figure 2.

Human Capital Efficiency Profile

As the raw data from Huggett, Ventura, and Yaron (2012) only covers the age range of 23-60 we extrapolate it to the relevant range of 20-23 and 60-65. To to this, we first estimate of polynomial of degree 3 and use the predicted values in the model. Other frequently used profiles (e.g. Hansen (1993) are very similar to our profile.

A.2 Technical Appendix

Here we describe the technical implementation of the model. We first present the solution to the individual problem. Then we briefly talk about the solution of the aggregate model.

A.2.1 Household Solution

To solve the household problem we use the technique of endogenous gridpoints developed by Carroll (2006). To economize on notation in this section, we will drop the age and time index and denote future variables with a prime. We also will assume that the age specific efficiency $h_j = 1$ for all ages (or equivalently, $w = wh_j$). Further, we use r to denote the *net* interest rate (or equivalently, there are no capital taxes) and assume that the efficiency wage w and the net interest rate r are constant over time have (hence they

carry no time index). In case we want to denote taxes or income in the case the agent is working full time, we use $|_{\ell=1}$. The rest of the notation follows the paper.

The dynamic programming problem the household has to solve in each period is then

$$V(a,\eta,j) = \max_{c,\ell,a'} \left\{ u(c,1-\ell) + \beta\pi\mathbb{E}\left[V(a',\eta',j+1)\right] \right\} \tag{37}$$

where the maximization is subject to the constraints specified in (3) and the borrowing constraint $a' \geq 0$. The household's individual state variables are current asset holdings a, labor market productivity η, and age j. Expectations are taken with respect to future realizations of the productivity shocks. Further, following Deaton (1991) we define cash on hand x as total resources available with maximum labor supply (i.e. $\ell = 1$) net of labor taxes and social security contributions. To shorten notation we summarize all other (lump-sum) income sources of the agent as net transfers and denote it with ntr. Then, total maximum available resources in a given period can be written as

$$\begin{aligned}
x &= a(1+r) + ntr + w\eta(1-\tau)|_{\ell=1} - T(w\eta)|_{\ell=1} && (38)\\
x' &= a'(1+r) + ntr' + w\eta'(1-\tau)|_{\ell'=1} - T(w\eta')|_{\ell'=1} && (39)\\
x' &= \left[x - c(1+\tau^c) - w\eta(1-\tau)\tilde{T}(w\eta)|_{\ell=1} \left(1 - \ell \frac{\tilde{T}(w\eta\ell)}{\tilde{T}(w\eta)|_{\ell=1}} \right) \right] (1+r) && (40)\\
&\quad + ntr' + w\eta'(1-\tau)|_{\ell'=1} - T(w\eta')|_{\ell'=1} && (41)
\end{aligned}$$

where we have replaced a' by the following expression

$$\begin{aligned}
a' &= a(1+r) + ntr + w\eta(1-\tau)\ell - T(w\eta\ell) - c(1+\tau^c) && (42)\\
a' &= x - c(1+\tau^c) + w\eta(1-\tau)\ell - T(w\eta\ell) - [w\eta(1-\tau)|_{\ell=1} - T(w\eta)|_{\ell=1}] && (43)\\
a' &= x - c(1+\tau^c) - w\eta(1-\tau)\tilde{T}(w\eta)|_{\ell=1} \left(1 - \ell \frac{\tilde{T}(w\eta\ell)}{\tilde{T}(w\eta)|_{\ell=1}} \right) && (44)
\end{aligned}$$

and used the following definition

$$\begin{aligned}
\tilde{T}(w\eta\ell) &= 1 - \frac{T(w\eta\ell)}{w\eta(1-\tau)\ell} && (45)\\
&= 1 - \bar{t}(w\eta\ell), && (46)
\end{aligned}$$

where $\bar{t}(w\eta\ell)$ can be interpreted as the average tax rate at the optimal solution. Note that with a linear tax system $\bar{t}$ is the tax rate and $a' = x - c(1+\tau^c) - w\eta(1-\tau)(1-\bar{t})(1-\ell)$. Hence, the last term is now leisure $1-\ell$ valued at the net wage (independent of the labor supply decision). Using the new notation re-write now the Bellman equation as

$$V(x,\eta,j) = u(c,1-\ell) + \beta\pi\mathbb{E}[V(x',\eta',j+1)] \tag{47}$$

where the new state variable is x. Optimality conditions with respect to consumption and labor supply are

$$\begin{aligned}
c: &\quad u_c = \beta\pi(1+\tau^c)(1+r)\mathbb{E}[V_{x'}(x',\eta',j+1)] && (48)\\
\ell: &\quad u_{1-\ell} = \beta\pi(1+r)w\eta\mathbb{E}[V_{x'}(x',\eta',j+1)]\left(1-\tau-T'(y)\right) && (49)\\
&\quad T'(y) = \frac{\partial T(w\eta\ell)}{\partial w\eta\ell}
\end{aligned}$$

The envelope condition is then

$$V_x = \beta\pi(1+r)\mathbb{E}[V_{x'}(x',\eta',j+1)] \tag{50}$$

which can be combined with (48) to get

$$u_c = (1+\tau^c)V_x(x,\eta,j) \quad \text{and} \quad u_{c'} = (1+\tau'^c)V_{x'}(x',\eta',j+1) \tag{51}$$

Combing this with optimality conditions delivers the standard Euler equation

$$\begin{aligned} u_c &= (1+\tau^c)\beta\pi(1+r)\mathbb{E}[V_{x'}(x',\eta',j+1)] &(52)\\ u_c &= \frac{1+\tau^c}{1+\tau'^c}\beta\pi(1+r)\mathbb{E}[u_{c'}]. &(53)\end{aligned}$$

To solve for optimal labor supply we combine the two first order optimality conditions (for interior solutions) and get the intratemporal optimality condition

$$u_{1-\ell} = u_c w\eta\frac{1-\tau-T'(y)}{1+\tau^c}. \tag{54}$$

Using the Cobb-Douglas utility function gives

$$c = \frac{\theta}{1-\theta}(1-\ell)w\eta\frac{1-\tau-T'(y)}{1+\tau^c}. \tag{55}$$

Given that we have a closed form expression for u_c we can write

$$u_c = \left(c^\theta(1-\ell)^{1-\theta}\right)^{-\sigma}\theta(1-\ell)^{1-\theta}c^{\theta-1} \tag{56}$$

where we can substitute c from (55) into the above equation. Given that we know u_c (which is a scalar), we can now solve for ℓ by solving one (nonlinear) equation in one variable. Alternatively, one can also solve the two by two system simultaneously.

Remark 1. *For the case of linear labor taxation we avoid using numerical solution techniques and solve for labor supply and consumption as follows. We compute the leisure-consumption ratio lcr from the intratemporal optimality conditions and from the Cobb-Douglas utility function which gives*

$$\frac{u_{1-\ell}}{u_c} = w\eta\frac{(1-\tau-T'(y))}{1+\tau^c} = \frac{\theta}{1-\theta}\frac{c}{1-\ell} \tag{57}$$

where $T'(y)$ is a known constant independent of labor supply. Then

$$lcr = \frac{1-\ell}{c} = \frac{1-\theta}{\theta}\frac{1+\tau^c}{w\eta(1-\tau-T'(y))}. \tag{58}$$

Further, using the utility function we can write

$$\begin{aligned} u_c &= \theta lcr^{(1-\sigma)(1-\theta)}c^{-\sigma} &(59)\\ c &= \left(\frac{u_c}{\theta lcr^{(1-\sigma)(1-\theta)}}\right)^{-\frac{1}{\sigma}} &(60)\\ \ell &= 1 - lcr\cdot c. &(61)\end{aligned}$$

the case that $\ell < 0$, we set $\ell = 0$ and compute consumption as below.

For the case that labor supply is binding, i.e. $\ell = 0$ ($\ell = 1$ is ruled out by the Inada-Condition) we operate only on the intertemporal Euler-equation for consumption. Hence, by taking u_c from (48) as given and by the choice of the utility function we can compute consumption as

$$c = \Big((1+\tau^c)\beta\pi(1+r)\mathbb{E}[V_{x'}(x',\eta',j+1)]\Big)^{\frac{1}{\theta(1-\sigma)-1}} \tag{62}$$

with $\ell = 0$.

For the case that the agent is borrowing constrained (i.e. we solve the first gridpoint), we proceed as follows. By assuming that the agent's savings are zero ($a' = 0$), we rewrite the cash-on-hand equation to

$$c = \frac{x - leis}{1+\tau^c} \tag{63}$$

where $leis$ is

$$leis = w\eta(1-\tau)\tilde{T}(w\eta)|_{\ell=1}\left(1 - \ell\frac{\tilde{T}(w\eta\ell)}{\tilde{T}(w\eta)|_{\ell=1}}\right) \tag{64}$$

substituting this into the utility function and taking derivatives with respect to ℓ gives the first order condition for labor supply of borrowing constrained agents. With ℓ at hand now we can compute consumption.

Using the solution procedure described above we proceed as follows

1. Define an exogenous grid of savings G^a.
2. Starting with the last generation $j = J$, define a grid for cash on hand G^x by computing cash on hand as $\mathrm{G}^x_J = \mathrm{G}^a + y_J$ where y stands for total income in that period.
3. We know that for $j = J$, it holds that $c = x$ and $a' = 0$ for all gridpoints. Using equation 50 we can update $V_x(x,\eta,j)$ and $V_{x'}(x',\eta,j)$ for all gridpoints.
4. Given that we know expected marginal utility for period J for all gridpoints x, we can now work backwards from $j = J-1$. However, as future cash on hand x' is generally not on the grid $\mathrm{G}^{x'}$, we linearly interpolate on a transformation of future marginal utility. Given that interpolated expected marginal utility, we can compute today's consumption c and labor supply ℓ. Taking these numbers we can update the grid of cash on hand G^x as

$$\mathrm{G}^x = \begin{cases} a' + c(1+\tau^c) + w\eta(1-\tau)\tilde{T}(w\eta)|_{\ell=1}\left(1 - \ell\frac{\tilde{T}(w\eta\ell)}{\tilde{T}(w\eta)|_{\ell=1}}\right) & \text{if } j \leq jr \\ a' + c(1+\tau^c) + p & \text{if } j > jr, \end{cases} \tag{65}$$

   for all gridpoints $i > 1 \in \mathrm{G}^x$. For $i = 1$ we set x to the lowest possible realization (if this is smaller than x_2) or with an arbitrary fraction of x_2.
5. After having updated policy functions and grids, we update the value functions again by interpolating.

A.3 Model Rescaling

For computational reasons we solve the household problem in its de-trended version where we have to transform c, a, tr, b, and w (i.e. all trending variables) by diving through A_t and denote the de-trended terms by a $\tilde{\cdot}$. Due to the functional form assumption we can write the utility function as

$$u(c_{t,j}, 1 - \ell_{t,j}) = A_t^{\phi(1-\sigma)} u(\tilde{c}, 1 - \ell), \tag{66}$$

the budget constraint must be rewritten to

$$\tilde{a}' = \frac{1}{1+g}\left((\tilde{a} + \tilde{b})(1+r) + \tilde{w}^n + \tilde{tr} - \tilde{c}(1+\tau^c)\right), \tag{67}$$

and the discount factor has to be adjusted to $\tilde{\beta} = \beta\pi(1+g)^{\phi(1-\sigma)}$. The de-trended version of the problem is then to maximize

$$V(\tilde{a}, \eta, j) = \max_{\tilde{c},\ell,\tilde{a}'} \left\{ u(\tilde{c}, 1-\ell) + \tilde{\beta}\mathbb{E}V(\tilde{a}', \eta', j+1) \right\} \tag{68}$$

subject to the budget constraint from equation (67) and the borrowing constraint $\tilde{a}' \geq 0$. The rest of the procedure is identical.

A.4 Solution of the Aggregate Model

To solve for the general equilibrium of the model we proceed as follows: for a given vector of structural parameters we iterate on a $m \times 1$ vector of endogenous state variables. Depending on the model variant, some variables (D/Y-ratio, tax rates, etc.) may be exogenous or endogenous (see more on this below). Our mechanical steps to find the equilibrium are as follows

1. Make an initial guess P_q for the endogenous aggregate state variables, derive the relevant state variables for the household and solve the household problem
2. Update the initial guess P_q then by
   (a) Aggregate household assets and household effective labor supply
   (b) Given the exogenous D/Y ratio, private assets, and labor supply compute an update for capital intensity
   (c) Using private assets, compute an update for the bequest ratio
   (d) Update social security contributions/replacement rates by using new effective labor supply
   (e) Update average human capital decision
   (f) Using the new guess on total output and ratios for government spending, transfers and exogenous taxes, compute aggregate government variables and from the budget constraint pins down the remaining free tax parameter.
3. Collect the updated state variables into the vector P^{q+1}.
4. Note that formally we have $P^{q+1} = H(P^q)$ where H is a vector valued function with m equations. Define the distance function $\Delta(P) = P^q - H(P^q)$ and iterate over P until $\Delta(P)$ is sufficiently small, i.e. solve for the root of the function $\Delta(P)$.

B Additional Material

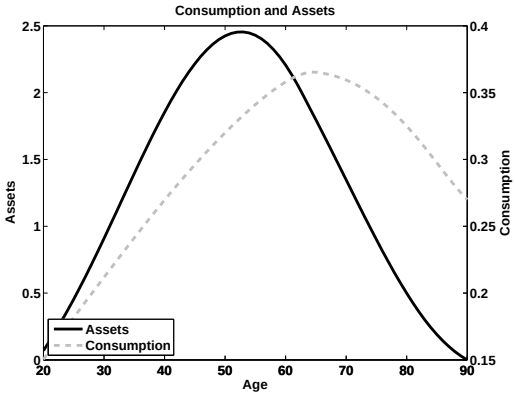
This section contains material which could be not included into the main text due to space limitations.

B.1 Life cycle Profiles for Household Decisions and Decomposition of Welfare Gains

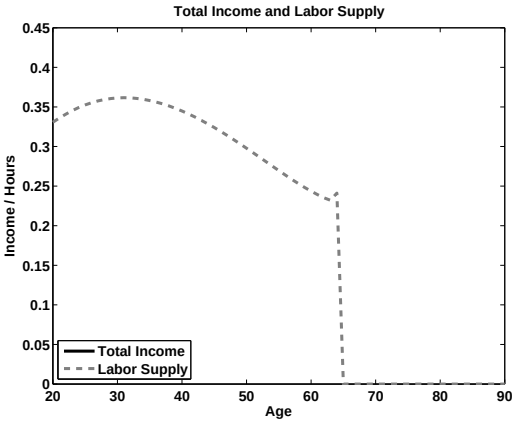
Figure 17 shows the life cycle profiles for labor supply (hours), total labor income (excluding transfers), consumption, and asset holdings. Hours worked is shaped by two considerations. Firstly, the deterministically increasing productivity profile links hours with wages. Secondly, as toward the beginning of the working life agents are subject to a considerable amount of risk, they work more. However, remaining lifetime risk vanishes as the number of uncertain periods decreases and agents decrease their "precautionary" labor supply.

Figure 17: Life Cycle Decisions

(a) Consumption and Assets



(b) Labor Supply and Income



Notes: Labor supply is in share of available time.

Welfare Decomposition: Keeping Prices at Calibrated Levels

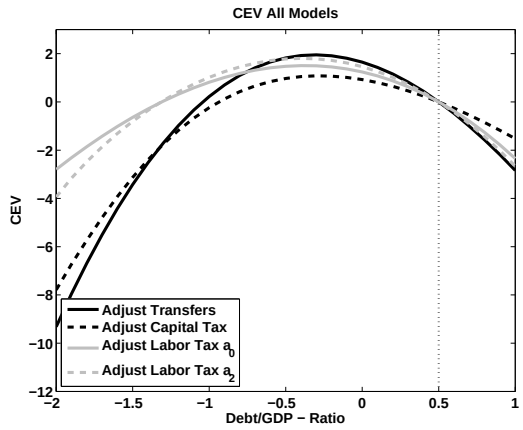
Here we show the welfare decomposition after the first labor market shock has hit and keep prices at calibrated levels. We show once again aggregate welfare changes for all for variants in panel a) of figure 18. The other panels show the welfare decomposition after all four policies.

Welfare Decomposition: Keeping Taxes and Transfers at Calibrated Levels

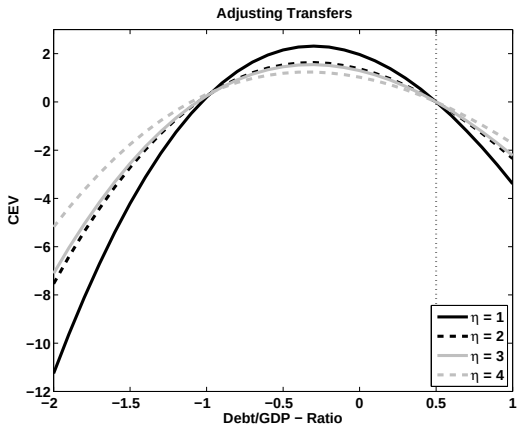
Here we show the welfare decomposition after the first labor market shock has hit and keep taxes and transfers at calibrated levels. We show once again aggregate welfare

Figure 18: Changes in Welfare

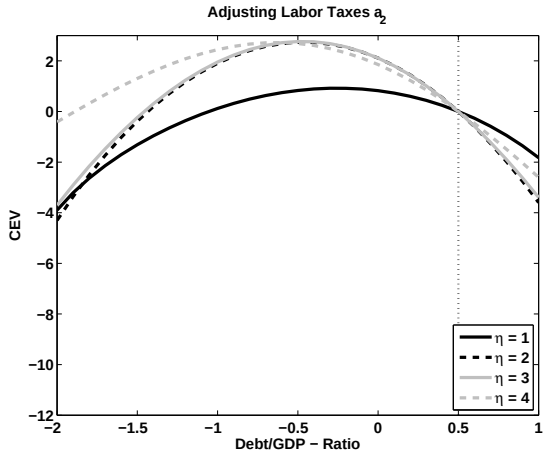
(a) Comparing Welfare for All Scenarios



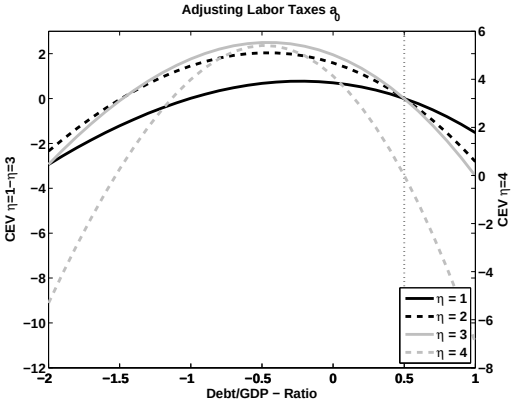
(b) Welfare Decomposition: Adjust Transfers



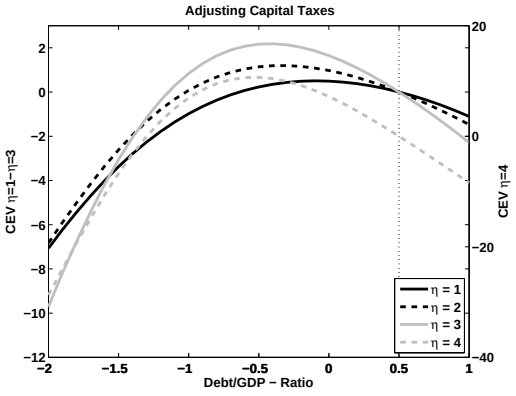
(c) Adjust Labor Taxes (a_2)



(d) Adjust Labor Taxes (a_0)



(e) Adjust Capital Taxes

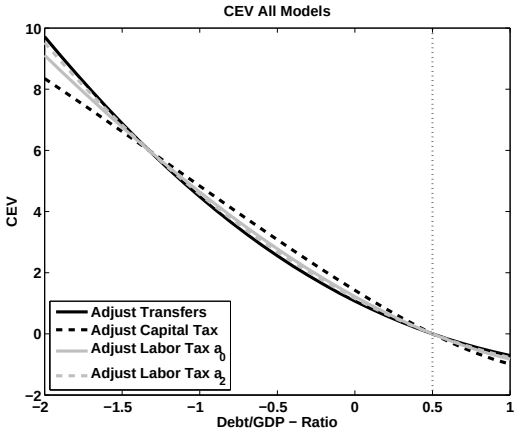


Notes: In panels d) and e) the results for $\eta = 4$ are shown on the right x-axis.

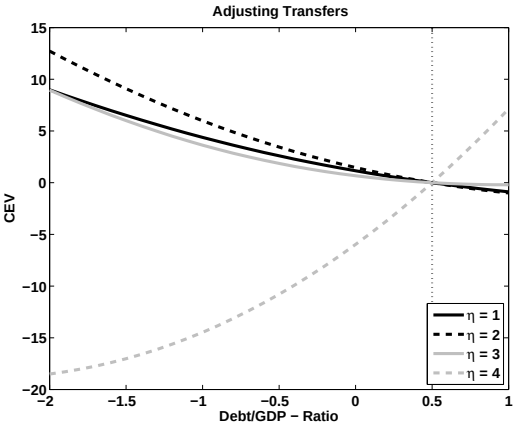
changes for all for variants in panel a) of figure 19. The other panels show the welfare decomposition after all four policies.

Figure 19: Changes in Welfare

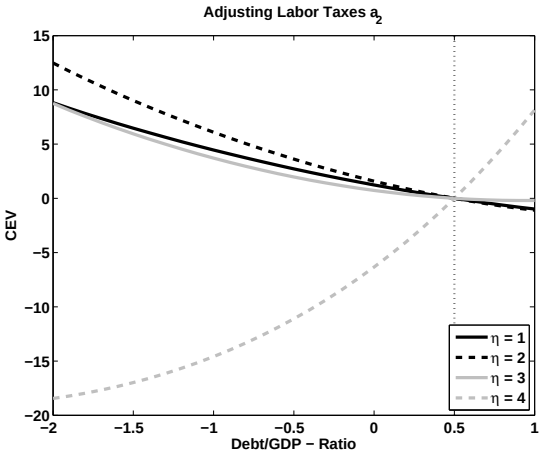
(a) Comparing Welfare for All Scenarios



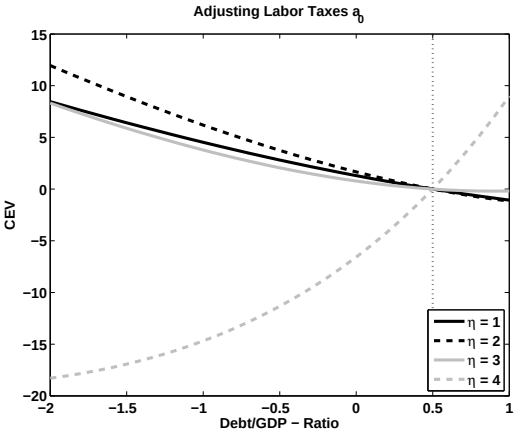
(b) Welfare Decomposition: Adjust Transfers



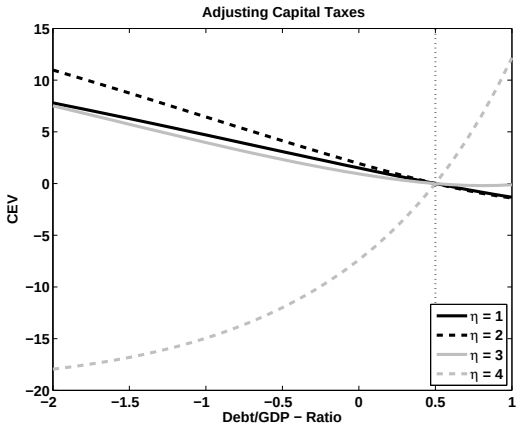
(c) Adjust Labor Taxes (a_2)



(d) Adjust Labor Taxes (a_0)



(e) Adjust Capital Taxes

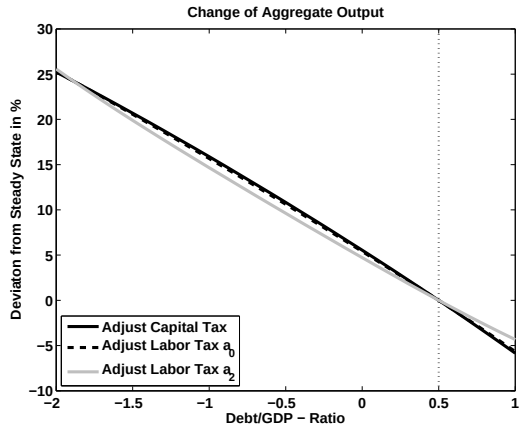


Notes: In panels d) the results for $\eta = 4$ are shown on the right x-axis.

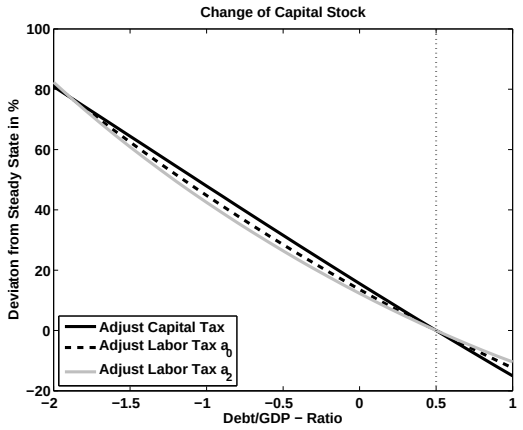
B.2 Changes in Aggregates with Constant Transfers

Figure 20: Change in Aggregates (Constant Transfers)

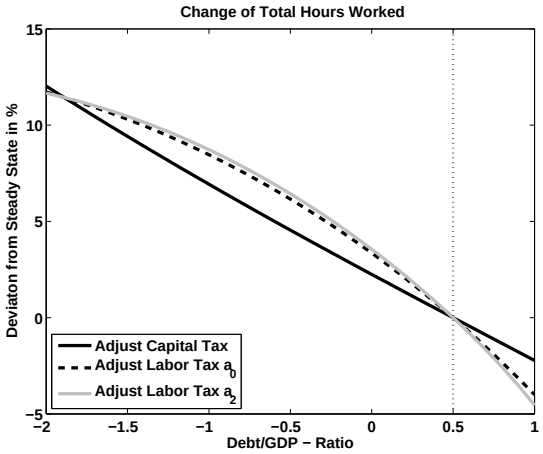
(a) Change of Output



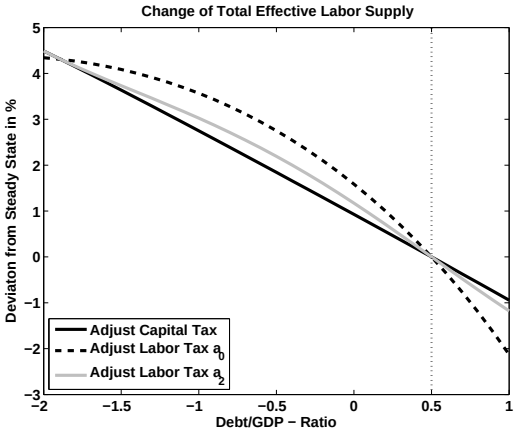
(b) Change of Capital



(c) Change of Aggregate Hours



(d) Change of Effective Labor Supply



Notes: Results from general equilibrium simulation with fixed level of transfers.

B.3 Accounting for Implicit Debt

While this paper deals only with explicit debt, many countries' quantitatively larger fiscal burden is looming in the implicit debt accumulated in the social security system. Implicit debt are liabilities due at some later point in time. The traditional interpretation is that as the population ages, currently prevailing contribution and replacement rates are not at levels such that revenues and expenses balance each other. However, this is conceptually difficult to model in a steady state environment as the meaning of future is not well defined. Implicit debt is "only" a transitional phenomenon until these liabilities are due in the form of payoffs (pensions) promised to future generations. Hence, the more pragmatic question is how – given the current state of the world (i.e. *explicit* debt, population structure, taxes, etc.) – the government would have to adjust policies to cover the expenses when the implicit debt is transformed into actual pension claims. It is important to note that after all transitional dynamics have played out (assuming a non-exploding system) there is no implicit debt in the new steady state. Three basic ways to cover the deficit during the transition are a) to reduce pensions (implicit default) and/or increase contributions, b) cross-subsidize the social security system or c) to increase explicit debt. A radically alternative view is that by abolishing the social security system, the government will mechanically eliminate implicit debt.

The two feasible ways to approximate the notion of implicit debt in the steady state is to follow b) or to shut down the pension system. The latter implies that the government promises not to pay any pensions and hence mechanically eliminates the possibility of future liabilities. For scenario b) we keep the contribution and replacement rates constant at calibrated levels but feed in an older population. This older population is generated by the survival rates of the cohort born in 2050 (life expectancy 85.7 years) and is taken from Bell and Miller (2005). The gap in the pension system is then financed by higher labor taxes (a_0). We re-scale the population size such that total population in the baseline case and the alternative case are identical. For the alternative without a pension system we set contribution and replacement rates both to zero and use the population structure from the baseline scenario. As in the previous case, we adjust a_0 to make the government's budget constraint hold. We chose to adjust a_0 as this policy option was the global maximizer in the baseline experiments.

Welfare results and optimal debt levels are markedly different from the baseline case. For the case without social security ("No Social Security") the optimal policy is to have no government debt (0%) with welfare gains at the optimum being relatively small (0.2% CEV). For the case where we finance the gap in the pension system with labor taxes ("Implicit Debt") there is no debt level maximizing ex ante welfare in the range considered. Welfare gains are above 15% of lifetime consumption if the government holds 200% of GDP as assets. This stark contrast can be explained by the different incentive and distortional effects of taxes and pensions. Without a social security system, agents have to rely on their own assets for old-age consumption. This will increase private savings and hence the capital stock (figure 22). On the other hand, as the capital stock increases, the

government's revenue from capital taxes decreases which is counterbalanced by higher labor taxes (figure 23). To sum up, while there is no government support when old, the decrease in net taxes (i.e. social security contributions and labor taxes) is smaller than the decrease in social security. Hence, overall savings motives (precautionary and old-age) are strengthened pushing private capital accumulation high enough such that the government does not have to accumulate a large amount of assets.

The case is different for the scenario with the older population. Here, agents receive still relatively generous pension benefits which then need to be financed by higher labor taxes (figure 23). Labor tax income relative to GDP jumps by about 9 percentage points, corresponding to the deficit of the pension system. As agents receive pensions but the tax distortion increases, incentives to supply labor and private capital accumulation decrease sharply (figure 22). Hence, capital intensity implied by only private savings is much lower and must be counterbalanced by public action. Note also that this scenario corresponds to increasing contribution rates; the only difference being the non-linear nature of the labor tax code.

Figure 24 provides the usual decomposition of welfare changes after the realization of the first labor market shock. Low income agents (η=1) do not gain much from moving to the optimal level of debt if we consider the "No Social Security" scenario. The reason is that while they have to save for retirement, available funds when young (i.e. net wages) are not increasing substantially. Agents with the highest productivity shock (η=4) incur large losses as they did not benefit from pension payments (loosing them does not hurt) but they now earn a much lower return on their assets. On the contrary, when looking at the subsidized pension system agents with a low income realization gain more than 15% of lifetime consumption at high government asset levels. The reason is that while they still receive pension payments, the majority of the increase in labor taxes is borne by middle and high income agents. In addition, they also benefit from higher wages.

Figure 21: CEV All Scenarios

(a) CEV All Scenarios

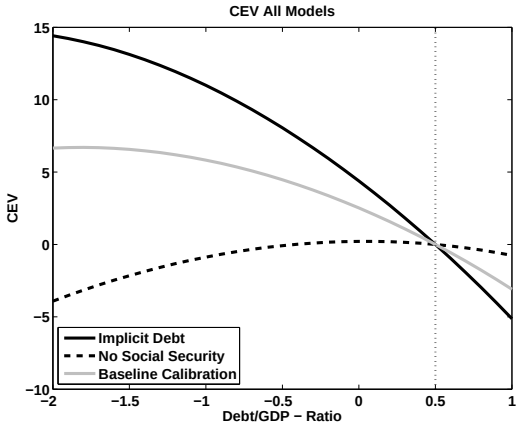
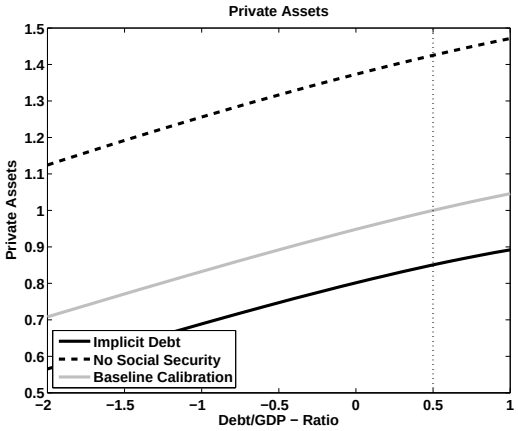
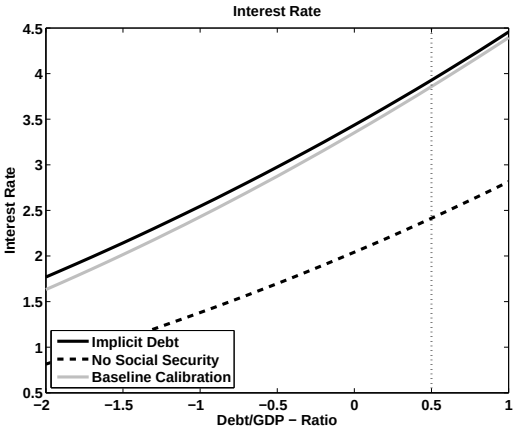


Figure 22: Private Assets and Net Interest Rate

(a) Change in Private Assets



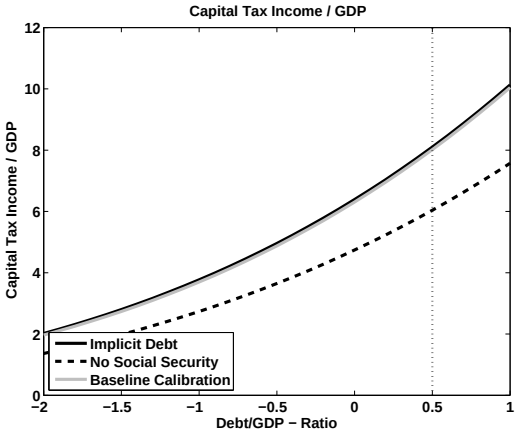
(b) Interest rate



Notes: When comparing private assets, population size is standardized to one for all scenarios. All asset levels are relative to the calibrated case (with 50% debt/GDP and baseline calibration).

Figure 23: Labor and Capital Tax Income as % of GDP

(a) Capital Tax Revenues / GDP



(b) Labor Tax Revenues / GDP

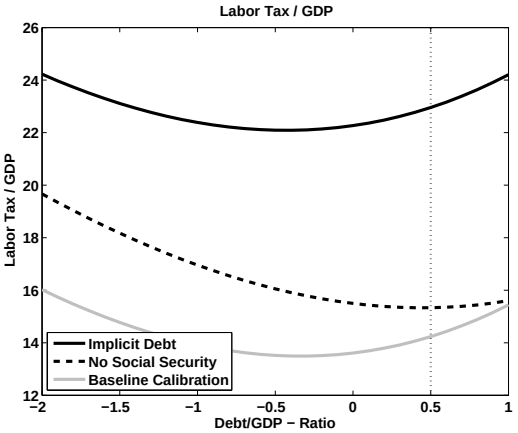
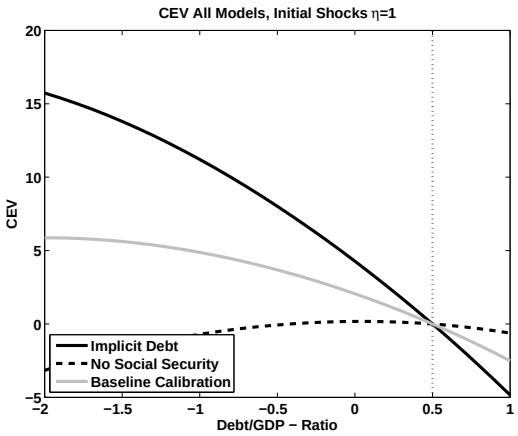
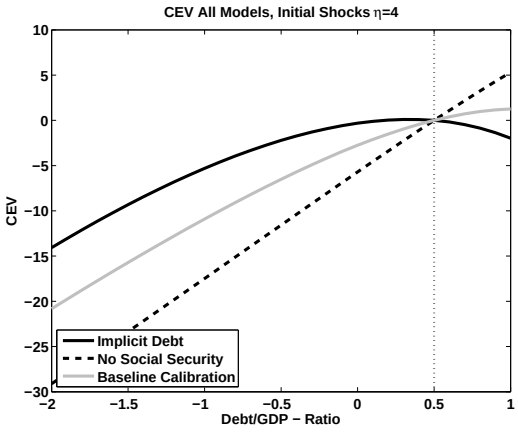


Figure 24: CEV All Scenarios

(a) CEV for Initial Shocks η=1



(b) CEV for Shock η=4



B.4 Changes in Inequality

With more capital, interest rates decline and agents have incentives to frontload their consumption decisions. Young agents rather prefer to consume now and defer building up a capital stock later in life. This is also due to higher wages effectively loosen borrowing constraints and hence carrying less capital into the next period is a less risky strategy. Hence, intuitively one should expect a more dispersed wealth distribution. We confirm this and find that as the capital stock increases, there is an even larger concentration of wealth in the highest quintile. On the other hand, as wages increase, the income distribution becomes more equal with more agents at the lower end of the distribution. This is of course also a consequence of the non-linear tax system which compresses the post-tax income distribution mechanically and the one before taxes by also changing incentives to work. In a nutshell, asset inequality rises while income inequality decreases.

We report the corresponding numbers in table 6 where the first line reports deviations of the Gini coefficient while the rest of the table shows changes along the Lorenz curves. All numbers are for the policy specific optimal debt/GDP-ratio. Figure 25 shows the deviation of the Gini coefficient from the calibrated values for all simulated debt ratios.

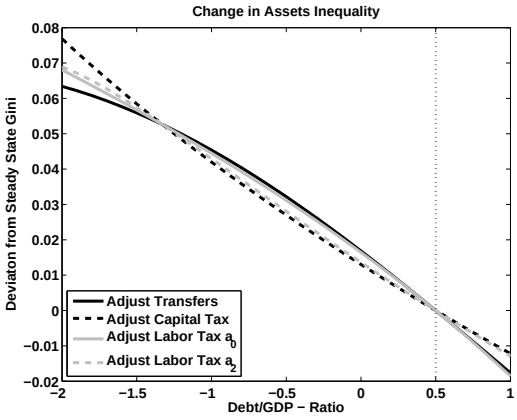
Table 6: Decomposition in Change of Inequality

	Assets				Earnings			
	Tr	τ^k	a_0	a_2	Tr	τ^k	a_0	a_2
Gini	0.05	0.06	0.06	0.06	-0.01	-0.02	-0.02	-0.02
Q1	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01
Q2	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00
Q3	-0.02	-0.03	-0.03	-0.03	0.00	0.00	0.00	0.00
Q4	-0.03	-0.04	-0.04	-0.04	0.00	0.00	0.00	0.00
Q5	0.06	0.07	0.07	0.07	-0.01	-0.01	-0.01	-0.01

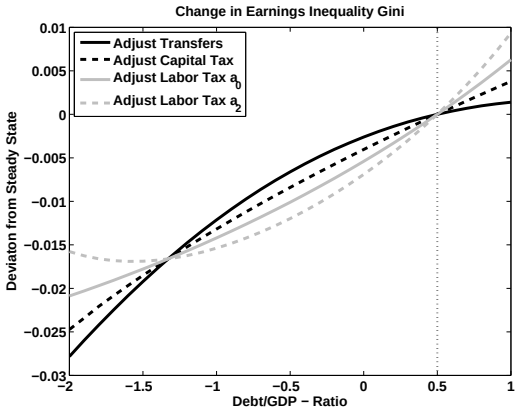
Notes: Numbers are deviations from the calibrated values at the policy specific optimal D/Y-ratio.

Figure 25: Changes in Inequality

(a) Change in Asset Gini



(b) Change in Earnings Gini



Notes: Numbers are deviations from the calibrated value.