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When shadows grow longer: shadow banking with endogenous entry

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Abstract

Why did the shadow banking sectors in the US and the euro area expand in the decade before the financial crisis and what are the implications for systemic risk and macro-prudential policy? This paper examines these issues with a model of the financial sector where the size of the shadow banking sector is endogenous. In the model, shadow banking is an alternative banking strategy which involves greater risk-taking at the expense of being exposed to "fundamental runs" on the funding side. When such runs occur, shadow banks liquidate their assets in a secondary market. Entry into shadow banking is profitable when traditional banks provide sufficient secondary market demand to prevent these liquidations from causing a fire-sale. During periods of stability, the shadow banking sector expands to an excessively large size that ferments systemic risk. Its collapse then triggers a fire-sale that renders traditional banks vulnerable to "liquidity runs". The prospect of liquidity runs undermines market discipline and increases the risk-taking incentives of traditional banks. Policy interventions aimed at alleviating the fire-sale fuel further expansion of the shadow banking sector. Financial stability is achieved with a Pigouvian tax on shadow bank profits.

Keywords: Shadow Banking, Fire-Sales, Financial Crises, Macro-Prudential Regulation
JEL Codes: E44, G01, G11, G21, G28

Non-Technical Summary

In the decade before the financial crisis, the shadow banking sectors in the United States and the euro area grew rapidly based on a business model that combined highly-leveraged, short-term funding through repos and asset-backed commercial papers with risky long-term investments such as mortgage lending. In the wake of the rise in sub-prime mortgage delinquency rates in the US, shadow banks experienced a sharp contraction in their funding which bore a strong resemblance to a bank-run. The ensuing turmoil in the shadow banking sector quickly spread to the traditional banking sector as well, leading to widespread panic and resulting in the deepest recession experienced by advanced economies since the Great Depression. After a brief decline in 2007-8, the shadow banking sectors in the US and the euro area have continued to grow in recent years, both in absolute terms and relative to the traditional banking sectors in these economies. This has led to concerns about financial stability, as well as policy design due to the still largely unregulated nature of shadow banking.

A theoretical framework which can account for the existence and expansion of the shadow banking sector alongside the traditional banking sector is crucial for our understanding of these recent events and for our ability to forestall future threats to financial stability. This paper constitutes a first step towards such a framework by presenting a model of the financial sector where shadow banking emerges as an alternative business strategy that entails higher leverage and risk-taking than traditional banking. The size of the shadow banking sector relative to the traditional banking sector is determined endogenously in equilibrium.

Our model is capable of explaining the growth of the shadow banking sector and provides a novel perspective on its potential destabilizing impact on the financial sector as a whole. We find that during periods of stability such as the Great Moderation, the shadow banking sector grows to a size that makes it systemically important. A collapse of the shadow banking sector then triggers a sharp decline in asset prices that leaves traditional banks vulnerable to self-fulfilling bank runs.

In the model, shadow banks are vulnerable to negative signals about future asset returns. In such cases, they face a steep decline in their solvency prospects and are thus forced into early liquidation by their creditors. We refer to this as a “fundamental run” since it is triggered by a revision of expectations about banks’ financial health. The traditional banking strategy consists of forming a portfolio of assets that is safe enough to avert fundamental runs. Market discipline on traditional banks thus arises endogenously in this framework through the threat of early liquidation.

The relative size of the two sectors has systemic consequences due to the interaction of banks in a secondary market for assets. When the shadow banking sector is relatively small, secondary market purchases by traditional banks prevent fundamental runs from causing a downward spiral in asset prices. Shadow bank portfolios thus remain endogenously liquid and creditors reclaim most of their funds during a fundamental run. As a result, shadow banks retain the ability to borrow at low cost despite their high risk-taking and their profits surpass those of traditional banks, leading to further growth of the shadow banking sector.

When there is an extended period of stability without any fundamental runs, the shadow banking sector grows to a size where purchases by traditional banks are insufficient to prevent a collapse in asset prices in the case of its liquidation. When a fundamental run eventually takes place, the ensuing drop in asset prices leaves traditional banks illiquid and vulnerable to self-fulfilling bank-

runs which we refer to as "liquidity runs". The prospect of liquidity runs increases the borrowing costs of traditional banks and undermines market discipline as the promise of high interest rates raises creditors' tolerance to insolvency risk. This leads to greater risk-taking by traditional banks and increases the insolvency risk associated with them.

From a normative perspective, our paper offers two novel insights for policy design. First, we find that policy interventions may have significantly different implications when their impact on the size of the shadow banking sector is taken into account. We demonstrate this by considering an asset purchase scheme whereby the government leans against the collapse in asset prices by purchasing assets in the secondary market. We find that such an intervention is indeed effective when the size of the shadow banking sector is taken as given. However, the expectation of such asset purchases fuels further growth of the shadow banking sector in a manner that offsets the positive effects of the policy.

Second, we show that financial stability can be achieved through the taxation of shadow bank profits. We find that such a tax policy deters entry into the shadow banking sector and can be used to prevent it from reaching a size that is detrimental to financial stability.

Our findings are relevant for both the US and the euro area. The size of the shadow banking sector in the US is roughly equivalent to the traditional banking sector in terms of liabilities and assets. In the euro area, the shadow banking sector is smaller relative to the traditional banking sector but still significant, accounting for nearly half of the assets and a third of the lending of the banking system.

1 Introduction

In the decade before the financial crisis, the shadow banking sectors in the United States and the euro area grew rapidly based on a business model that combined highly-leveraged, short-term funding through repos and asset-backed commercial papers with risky long-term investments such as mortgage lending (see Figure 1). In the wake of the rise in sub-prime mortgage delinquency rates in the US, shadow banks experienced a sharp contraction in their funding which bore a strong resemblance to a bank-run. The ensuing turmoil in the shadow banking sector quickly spread to the traditional banking sector as well, leading to widespread panic and resulting in the deepest recession experienced by advanced economies since the Great Depression. After a brief decline in 2007-8, the shadow banking sectors in the US and the euro area have continued to grow in recent years, both in absolute terms and relative to the traditional banking sectors in these economies. This has led to concerns about financial stability, as well as policy design due to the still largely unregulated nature of shadow banking (see e.g. Constâncio, 2016).

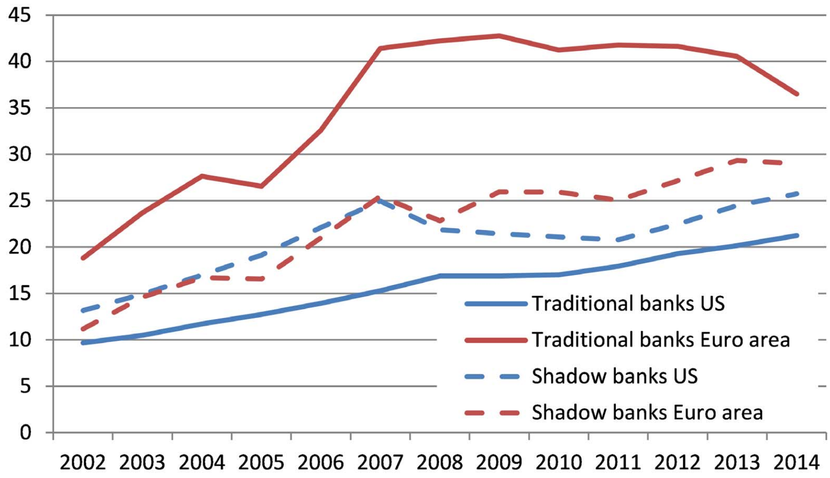
A theoretical framework which can account for the existence and expansion of the shadow banking sector alongside the traditional banking sector is crucial for our understanding of these recent events and for our ability to forestall future threats to financial stability. This paper constitutes a first step towards such a framework by presenting a model of the financial sector where shadow banking emerges as an alternative business strategy that entails higher leverage and risk-taking than traditional banking. The size of the shadow banking sector relative to the traditional banking sector is determined endogenously in equilibrium.

Our model is capable of explaining the growth of the shadow banking sector and provides a novel perspective on its potential destabilizing impact on the financial sector as a whole. We find that during periods of stability such as the Great Moderation, the shadow banking sector grows to a size that makes it systemically important. A collapse of the shadow banking sector then triggers a fire-sale that leaves traditional banks vulnerable to self-fulfilling bank runs.

In the model, banks collect deposits from households and invest in a portfolio of assets with aggregate risk. When forming their portfolios, banks may adopt either a shadow banking, or a traditional banking strategy. Since shadow banking involves high leverage and risk-taking, negative signals about future asset returns lead to a steep decline in shadow banks' solvency prospects which precipitates an early withdrawal by their depositors. We refer to this as a "fundamental run" since it is triggered by a revision of expectations about banks' financial health. The traditional banking strategy, on the other hand, consists of forming a portfolio of assets that is safe enough to avert fundamental runs. Market discipline on traditional banks thus arises endogenously in this framework through the threat of early liquidation.

In equilibrium, the relative size of the two sectors is determined by a free entry condition and has systemic consequences due to the interaction of banks in a secondary market for assets. When the shadow banking sector is relatively small, secondary market purchases by traditional banks prevent fundamental runs from causing a fire-sale. Shadow bank portfolios thus remain endogenously liquid and households reclaim most of their deposits during a fundamental run. As a result, shadow banks retain the ability to borrow at low cost despite their high risk-taking and their profits surpass those of traditional banks, leading to further entry into the shadow banking sector.

Figure 1: **Assets of traditional and shadow banks** (USD Trillions)



Source: Financial Stability Board. Global Shadow Banking Monitoring Report 2015.
Notes: Banks refer to the broader category of Deposit-taking Institutions. US banks include US Holding Companies. Shadow bank assets include the following sub-sectors under the European System of Accounts (ESA 2010): Money market funds, Non-MMF investment funds, Other financial intermediaries (except insurance corporations and pension funds), Financial auxiliaries, and Captive financial institutions and money lenders.

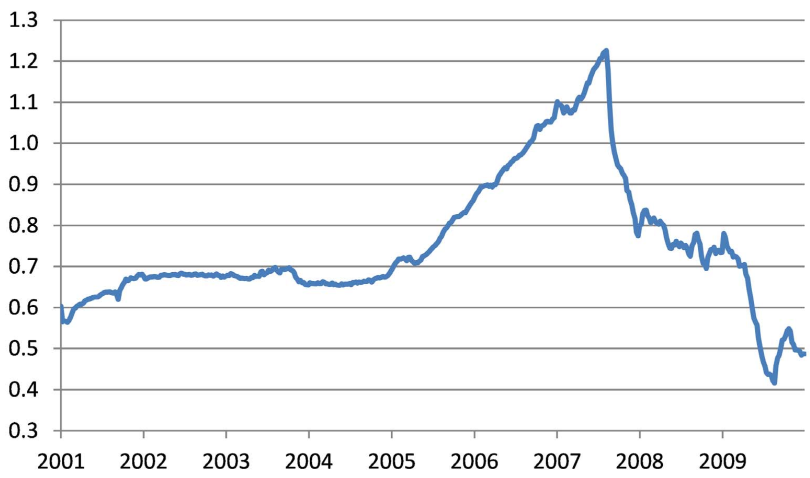
When there is an extended period of stability without any fundamental runs, the shadow banking sector grows to a size where purchases by traditional banks are insufficient to prevent a fire-sale in the case of its liquidation. When a fundamental run eventually takes place, the ensuing fire-sale leaves traditional banks illiquid and vulnerable to self-fulfilling bank-runs in the spirit of Diamond and Dybvig (1983), which we refer to as "liquidity runs". The prospect of liquidity runs increases the borrowing costs of traditional banks and undermines market discipline as the promise of high interest rates raises depositors' tolerance to insolvency risk. This leads to greater risk-taking by traditional banks and increases the insolvency risk associated with them.

From a normative perspective, our paper offers two novel insights for policy design. First, we find that policy interventions may have significantly different implications when their impact on the size of the shadow banking sector is taken into account. We demonstrate this by considering an asset purchase scheme whereby the government leans against fire-sales by purchasing assets in the secondary market. We find that such an intervention is indeed effective in alleviating fire-sales when the size of the shadow banking sector is taken as given. However, its ex-ante expectation fuels further growth of the shadow banking sector in a manner that exactly offsets the positive effects during its implementation. This creates the potential for time inconsistency as policymakers find it desirable to intervene once the fire-sale is underway.

Second, we show that financial stability can be achieved through the taxation of shadow bank profits. This can be considered as a Pigouvian tax since the adoption of a shadow banking strategy imposes a negative externality on the remainder of the financial sector through its contribution to fire-sales. We find that such a tax policy deters entry into the shadow banking sector and can be used to prevent it from reaching a size that is detrimental to financial stability.

Our paper focuses on interactions which played an important role in the financial crisis of 2008. Acharya et al. (2013) document that the market for asset backed commercial papers (ABCP), which constitute the main source of funding for the shadow banking sector, grew rapidly in the decade prior to the financial crisis. At their peak in July 2007, ABCP had become the largest money market instrument in the United States, accounting for approximately $1.2 trillion (see Figure 2). Following rising mortgage default rates and the suspension of withdrawals by a number of funds, the market for ABCP contracted by about $350 billion in the second half of 2007. Covitz et al. (2013) show that this sharp contraction of funding resembled a bank run on the shadow banking sector. Gorton and Metrick (2012) document a similar run on repo markets. In our model, the definitive characteristic of the shadow banking strategy is its vulnerability to fundamental runs which closely resemble these events.

Figure 2: **Asset-backed Commercial Paper Outstanding** (USD Trillions)



Source: FRED, Federal Reserve Economic Data, Federal Reserve Bank of St. Louis.

Our findings remain relevant for both the US and the euro area in present times. The size of the shadow banking sector in the US is roughly equivalent to the traditional banking sector in terms of liabilities and lending (see Figure 1). In the euro area, the shadow banking sector is smaller relative to the traditional banking sector but still significant, accounting for nearly half of the assets and a third of the lending of the banking system (Bakk-Simon et al., 2013).

To our knowledge, our study is among the first to incorporate shadow banking with an endogenous sector size into a general equilibrium framework suitable for policy analysis. Shadow banking is completely absent from the general equilibrium analysis of Kashyap et al. (2014). A number of recent studies including Goodhart et al. (2012, 2013) and Moreira and Savov (2014) evaluate macroprudential policies using general equilibrium models which feature shadow banking. Ordoñez and Piguillem (2015) use an overlapping generations model to quantify the role of shadow banking in increasing the credit supply in the United States in recent decades. In these studies, the scope for shadow banking is created either by the assumption of a difference in the intermediation technology or an opportunity for regulatory arbitrage.[1] As a result, the size of the shadow banking sector remains exogenous.

Another strand of the literature consists of studies which use stylized models to analyze the microfoundations of shadow banking. Gennaioli et al. (2013) develop a model which emphasizes the ability of shadow banks to generate safe assets through securitization. They find that shadow banks become excessively exposed to systemic risk when low probability tail events are neglected by investors. Ordoñez (2013), Harris et al. (2014) and Plantin (2015) focus on the role of regulatory arbitrage as a driving reason for shadow banking. Plantin (2015) shows that tighter capital requirements may drive banks to engage in a greater amount of shadow banking activities when regulators are unable to monitor this. Harris et al. (2014) find that competition by shadow banks creates an incentive for traditional banks to undertake investments that are conducive to risk-shifting.

Ordoñez (2013) analyses the interplay between regulatory arbitrage and reputation. He finds that regulatory arbitrage is welfare-enhancing when reputational concerns give shadow banks sufficient incentives to self-regulate. The value given to reputation, however, depends on expected future economic conditions. Thus, an adverse signal about future economic prospects leads to a loss of confidence in shadow banks and a run to the regulated traditional banking sector. Similar to Ordoñez (2013), our study entails a run on shadow banks following a negative signal about future fundamentals, but this is driven by the ex-ante risky nature of the optimal shadow bank portfolio rather than changes in the extent of self-regulation.

Our portrayal of shadow banking as an alternative banking strategy borrows from Hanson et al. (2015). They develop a partial equilibrium model based on the premise that traditional banks, which are secure from liquidity risk, have a comparative advantage in holding illiquid assets with low fundamental risk whereas shadow banks, which may be subject to early withdrawals, focus on liquid assets with greater fundamental risk. We adopt the same approach to incorporate a shadow banking sector into a general equilibrium model where endogenously determined sector sizes interact with fire-sales and bank-runs at an aggregate level.

The remainder of the paper is structured as follows: Section 2 provides a description of the model economy. Section 3 describes the calibration of key parameters and provides a numerical solution. Section 4 is dedicated to policy analysis. Section 5 extends the model to a dynamic set up. Section 6 concludes.

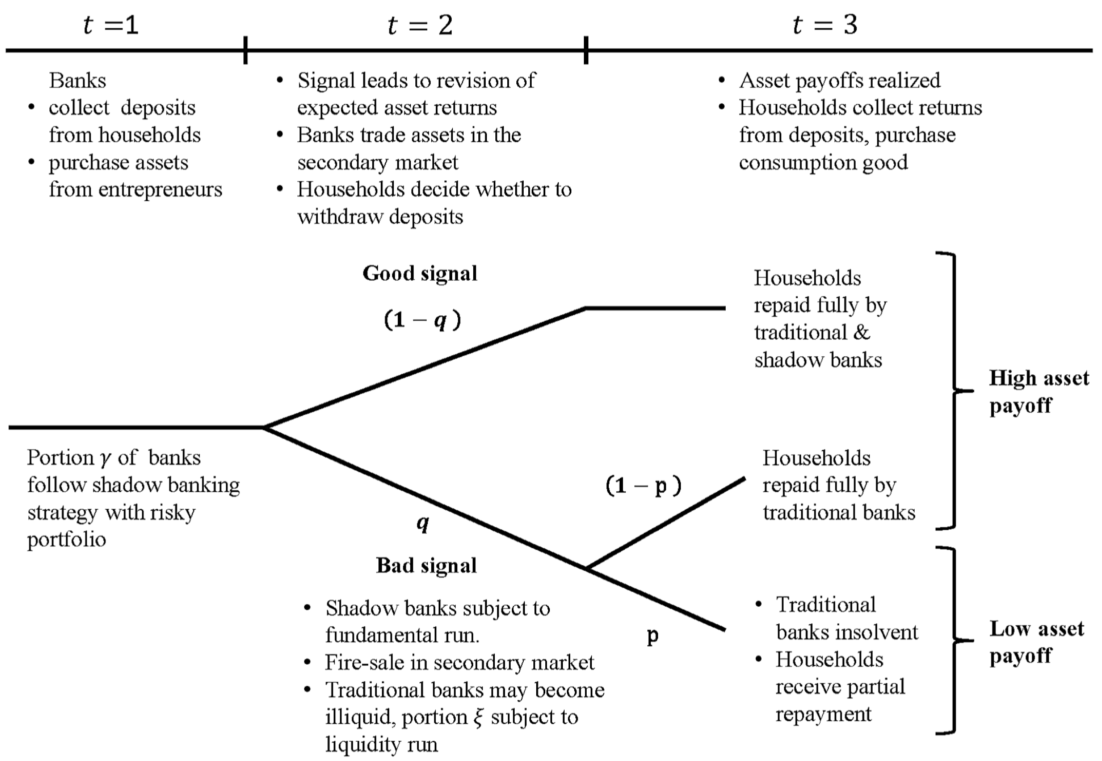
[1] See Adrian and Ashcraft (2012) and Claessens et al. (2012) for a comprehensive literature review on shadow banking and other proposed reasons for its existence.

2 Model Economy

The model economy is a stylized financial economy populated by three distinct types of agents – households, banks and entrepreneurs. Although we provide an in-depth discussion of each agent's behaviour in dedicated sections below, it is convenient to start with a brief summary of the progression of events.

Events in the model economy unfold over three stages, denoted as periods $t = \{1, 2, 3\}$ (see Figure 3 for a graphical representation). In the first period, banks collect deposits from households and purchase a portfolio of (aggregate) risky assets from entrepreneurs. Banks are ex-ante identical, but as we show below, optimally cluster into two distinct groups according to their portfolio strategies. An endogenously determined share $\gamma \in [0, 1]$ of banks follow a "shadow banking strategy" which entails a portfolio with high risk, while the remainder follow a "traditional banking strategy" and invest in a relatively safe portfolio of assets.

Figure 3: **Timeline**



The second period begins with a public signal which leads to a revision of expected asset returns. After observing the signal, banks have the opportunity to trade assets in a secondary market, while households decide whether to withdraw their deposits early. With probability q, the signal harbors "bad news", leading to a decline in expected asset returns. This propels households to optimally withdraw their deposits from shadow banks due to concerns about their future solvency prospects.

Shadow bank assets are then liquidated in the secondary market at an endogenous fire-sale discount, and there is a haircut on deposits accordingly. We refer to this process as a "fundamental run" since it is driven by a revision of expectations about asset fundamentals.

By construct, the portfolio of assets purchased under a traditional banking strategy is sufficiently safe to re-assure households about solvency prospects following a bad signal. Traditional banks are thus never subject to a fundamental run. When the share of shadow banks is above a threshold $\bar{\gamma}$, however, the liquidation of shadow bank assets leads to a steep fire-sale discount in the secondary market. This leaves traditional banks illiquid and susceptible to a "liquidity run" à la Diamond and Dybvig (1983). The liquidity run occurs with an idiosyncratic probability ξ which is increasing in the extent of the fire-sale discount, culminating in early withdrawals from a portion ξ of traditional banks.

In the third period, asset portfolios yield a high or low payoff contingent on economic fundamentals. Following a bad signal in the second period, there is (conditional) probability p of fundamentals turning out to be weak, leading to a low payoff from assets. Although this may leave the remaining traditional banks insolvent, their portfolios are constructed so as to ensure that the haircut imposed on their depositors remains below the level that would have made an early withdrawal optimal. This is precisely what distinguishes traditional banks from shadow banks.

When fundamentals turn out to be strong with (conditional) probability $(1-p)$, on the other hand, traditional banks remain solvent and households are repaid fully. Similarly, following a good signal in the second period, payoffs are high with certainty and households are repaid fully by both traditional and shadow banks. Finally, households spend their funds on a consumption good.

2.1 State and Asset Structure

We formalize the uncertainty about fundamentals and the signal in period 2 in a state structure. Economic fundamentals are contingent on the third period state realization $s_3 \in \{h, d\}$ where h and d respectively refer to strong and weak fundamentals. The probability density of s_3 is revealed with the signal in period 2. We formalize this signal as a second period state realization $s_2 \in \{g, b\}$ where g and b respectively stand for a good and bad signal. With probability q, there is a bad signal $(s_2 = b)$ and the (conditional) probability density for s_3 is given by

$$\Pr[s_3 = h | s_2 = b] = 1 - p$$
$$\Pr[s_3 = d | s_2 = b] = p$$

With probability $(1-q)$, on the other hand, there is a good signal $(s_2 = g)$ which eliminates the possibility of weak fundamentals, such that $s_3 = h$ with certainty. There are thus 3 possible combinations of states $s_2 s_3 \in \{gh, bh, bd\}$ which are respectively realized with probabilities $(1-q)$, $q(1-p)$ and qp.[2]

Without loss of generality, we limit the asset space to three different asset types; liquid, safe and risky, and normalize the unconditional (first period) expectation of asset payoffs to unity. The liquid asset l yields a certain payoff of 1 in period 2 while the safe and risky assets reach maturity in

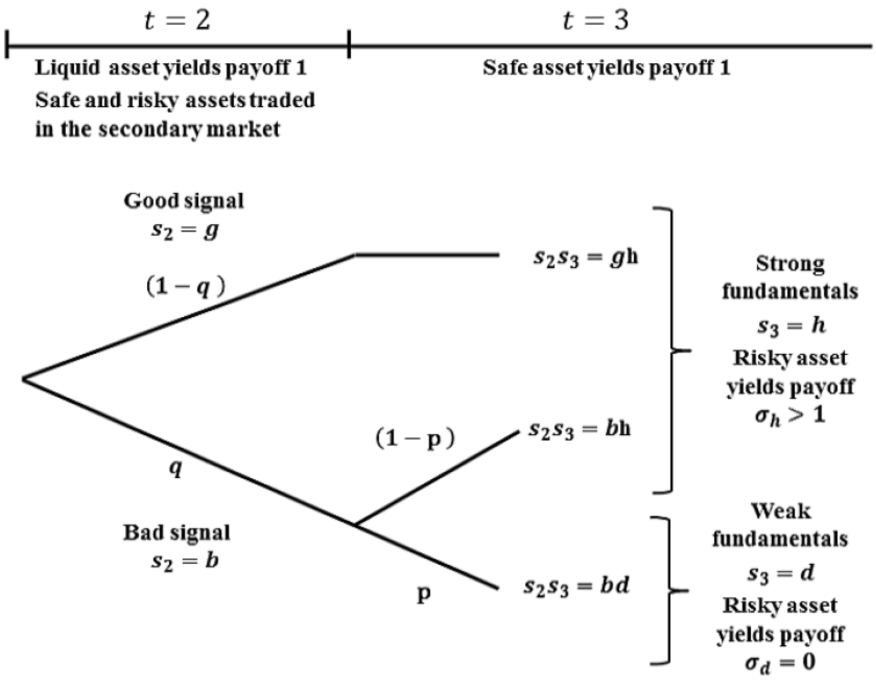
[2] The fundamental run on shadow banks creates history dependence in the model such that the allocations in gh and bh differ from each other.

the third period.[3] The safe asset s also yields a certain payoff of 1 whereas the risky asset r yields a payoff σ_{s_3} which is contingent on economic fundamentals. In the interest of a clear exposition, we assume that r yields zero payoff under weak fundamentals ($\sigma_d = 0$). Its payoff under strong fundamentals is then pinned down at a level consistent with an expected payoff of 1 as

$$\sigma_h = \frac{1}{1-qp}$$

Figure 4 provides a graphical depiction of the state structure and asset payoffs. Under the normalization, expected returns from assets are the reciprocal of their prices, which are determined endogenously as described in Section 2.6.

Figure 4: **State Structure and Asset Payoffs**



2.2 Banks

There is a unit continuum $j \in [0,1]$ of ex-ante identical, monopolistically competitive banks. In the first period, banks collect deposits $D(j)$ from households and purchase a portfolio of assets $\{I_1(i,j), i = l,s,r\}$ from entrepreneurs. Their first period budget constraint can then be written as

$$\sum_{i\in\{l,s,r\}} P_1(i,j)\, I_1(i,j) = D(j) \tag{1}$$

[3] Although it is possible to liquidate these assets early at a secondary market, cash-in-market pricing applies such that excessive liquidations may culminate in a fire-sale. The fire-sale discount is determined endogenously as described in Section 2.7.

where $I_1(i,j)$ is the amount purchased of an asset i and $P_1(i,j)$ is the corresponding asset price.[4] In the second period, banks collect the payoff from their holdings $I_1(l,j)$ of the liquid asset and trade their safe and risky asset holdings $\{I_1(s,j), I_1(r,j)\}$ in the secondary market to form a new portfolio allocation $\left\{I_{2|s_2}(s,j), I_{2|s_2}(r,j)\right\}$. This yields the set of second period budget constraints

$$\sum_{i\in\{s,r\}} P_{2|s_2}(i)\left[I_{2|s_2}(i,j) - I_1(i,j)\right] = I_1(L,j) \quad \forall\ s_2 \in \{g,b\} \tag{2}$$

where the secondary market prices $P_{2|s_2}(i)$ and the new portfolio allocations are contingent on the signal s_2. Profits can then be defined as the difference between the payoff from the bank's asset holdings at the end of period 2 and payments made to depositors

$$\Pi_{s_2 s_3}(j) = I_{2|s_2}(s,j) + \sigma_{s_3} I_{2|s_2}(r,j) - V_{s_2 s_3}(j)\, D(j)\, R(j) \geq 0 \tag{3}$$

where $R(j)$ is the interest paid to depositors in the final period. Although $R(j)$ is not state-contingent, due to limited liability the recovery rate $V_{s_2 s_3}(j) \in [0,1]$ adjusts to ensure that $\Pi_{s_2 s_3}(j)$ is weakly positive. Under the structure of asset payoffs described in Section 2.1, partial default may only occur under weak fundamentals ($s_3 = d$) such that

$$\begin{aligned} V_{s_2 s_3}(j) &= 1\ \forall\ s_2 s_3 \in \{gh, bh\} \\ V_{bd}(j) &= \min\left[1, \frac{I_{2|s_2}(s,j)}{D(j)\, R(j)}\right] \end{aligned} \tag{4}$$

We also define $\theta_{s_2}(j) \in [0,1]$ as the bank's liquidation value which reflects the ratio of funds it can raise through an early liquidation of its assets to its deposit repayment obligations.

$$\theta_{s_2}(j) = \min\left[1, \frac{I_1(l,j) + P_{2|s_2}(s)\, I_1(s,j) + P_{2|s_2}(r)\, I_1(r,j)}{D(j)}\right] \tag{5}$$

Following a good signal, all uncertainty is resolved and there are no bank-runs. As such, it follows directly from $V_{gh}(j) = 1$ that $\theta_g(j) = 1$ as well. To streamline the notation, we drop the state subscripts from $\{\theta_b(j), V_{bl}(j)\}$ in the remainder of the text. Before we can describe the optimal strategy of banks in, it is necessary to elaborate further on the environment in which they operate. Thus, we provide a description of bank-runs, entrepreneurs, households and secondary markets and then return to bank strategies in Section 2.8.

2.3 Fundamental Runs and Strategy Selection

A fundamental run occurs when a bad signal makes it optimal for households to withdraw their deposits early rather than facing the prospect of a partial repayment $V(j) < 1$ if fundamentals come out to be weak. We refer to this as a fundamental run since the withdrawals are driven by the increased probability of weak fundamentals rather than a coordination failure among households. In other words, households find it optimal to withdraw their deposits even in the absence of other

[4] $P(i,j)$ is specific to each bank j due to specific financial frictions described in Section 2.6. Since shadow and traditional banks value assets differently, we require this feature in order to preclude a corner solution in first period asset holdings.

withdrawals.

In Section 2.5, we show that banks may avert a fundamental run by investing in a portfolio of assets that ensures that their recovery rate $V(j)$ is above an incentive compatible level $\bar{V}$. In choosing their portfolio allocations, banks may then follow two alternative strategies.

Under a **shadow banking** strategy, they invest in a risky portfolio and accept the prospect of a fundamental run following a bad signal. Under a **traditional banking** strategy, on the other hand, they preclude the possibility of fundamental runs by investing in a relatively safe portfolio which satisfies the incentive compatibility condition

$$V(j) \geq \bar{V} \tag{6}$$

This allows them to remain in business after a bad signal and make a positive profit if fundamentals turn out to be strong (in the state realization $s_2 s_3 = bh$), but reduces their profits following a good signal (in $s_2 s_3 = gh$).

As banks are completely homogenous in all aspects other than the strategy they follow, we replace the indicator j with the superscript "SB" for shadow banks and "TB" for traditional banks. Banks determine their strategy before collecting deposits and purchasing assets in the first period. The strategy selection process takes the form of a sequential game where each bank adopts the strategy that yields the highest expected profit and has the opportunity to alter its choice after observing the strategies of other banks. A pure strategy Nash equilibrium may then occur only when the expected profits under the two banking strategies are equivalent. This leads to the free entry condition

$$E\left[\Pi^{SB}\right] = E\left[\Pi^{TB}\right] \tag{7}$$

which determines the share $\gamma \in [0,1]$ of shadow banks within the financial sector.

2.4 Liquidity Runs

Banks may also be subject to a liquidity run in the spirit of Diamond and Dybvig (1983). When a bank has a liquidity shortfall $\theta(j) < 1$, the sequential-service constraint leads to the emergence of a bank run equilibrium where households find it optimal to withdraw their deposits given that everyone else is withdrawing.

The liquidation value $\theta(j)$ is defined by (5) and depends on the banks' portfolio as well as the secondary market prices determined in Section 2.7. In general, liquidity shortfalls occur only when there is a fire-sale in the secondary market which prevents banks from liquidating their assets at their expected payoff. We show in Section 2.7 that there are no fire-sales after a good signal, since expected payoffs from risky assets increase and there is no fundamental run on shadow banks. Following a bad signal, on the other hand, the fundamental run on shadow banks causes a fire-sale on safe assets when the size of the shadow banking sector is above the threshold $\gamma \geq \bar{\gamma}$. This leads to a liquidity shortfall $\theta^{TB} < 1$ for traditional banks, leaving them susceptible to a liquidity run which occurs with probability ξ. We follow Kashyap et al. (2014) in adopting the functional form

$$\xi = \left(1 - \theta^{TB}\right)^2 \tag{8}$$

as an approximation to the global games solution of Goldstein and Pauzner (2005). The functional form captures two desirable properties; it ensures that ξ is decreasing in the liquidation value θ^{TB} and only positive when there is a liquidity shortfall $\theta^{TB} < 1$. It is also important to note that liquidity runs are idiosyncratic. In other words, each traditional bank perceives a probability ξ of experiencing a liquidity run but only a share ξ actually suffers from this.

2.5 Households

Households are risk neutral and derive utility only from final period consumption, making their utility maximization problem equivalent to maximizing expected consumption[5]

$$E\left[c_{s_2 s_3}\right] = (1-q)\, c_{gh} + q\,(1-p)\, c_{bh} + qpc_{bd} \tag{9}$$

In the first period, households receive an endowment E and allocate it between deposits in traditional and shadow banks $\left\{D^{TB}, D^{SB}\right\}$ and a safe storage technology M which transfers funds to the next period at a zero net return.[6] The first period budget constraint can thus be written as

$$D^{SB} + D^{TB} + M_1 = E$$

The second period budget constraint is contingent on the signal in period 2. Following a good signal $(s_2 = g)$, there are no withdrawals and thus households simply set $M_{2|g} = M_1$. For the case after a bad signal $(s_2 = b)$, we first solve the optimization problem taking it as a given that there will be a fundamental run on shadow banks but not traditional banks, and then derive the incentive compatible repayment rate $\bar{V}$ for traditional banks.[7] The second period budget constraint under a bad signal is then given by

$$M_{2|b} = M_1 + \theta^{SB} D^{SB} + \xi\theta^{TB} D^{TB}$$

where the second term accounts for the fundamental run on shadow banks and the third term represents deposits withdrawn from traditional banks in a liquidity run.[8] Finally, the third period budget constraints give us an expression for consumption for each state realization $s_2 s_3 \in \{gh, bh, bd\}$

$$\begin{aligned} c_{gh} &= M_{2|g} + D^{TB} R^{TB} + D^{SB} R^{SB} \\ c_{bh} &= M_{2|b} + (1-\xi)\, D^{TB} R^{TB} \\ c_{bd} &= M_{2|b} + \bar{V}\,(1-\xi)\, D^{TB} R^{TB} \end{aligned}$$

[5] These assumptions are solely in the interest of tractability. The mechanisms we describe retain their validity under risk aversion and period-by-period discounting of consumption

[6] Banks also have access to M but never find it optimal to allocate a positive amount of funds to it. This is not the case for households which are barred from investing directly in the assets $\{l, s, r\}$ by financial frictions described in Section 2.6.

[7] A consistency check for the fundamental run on shadow banks is also provided in the appendix.

[8] Under the sequential service constraint, a portion θ^{TB} of depositors are able to withdraw all of their funds during a liquidity run while the remainder receive no payment. We assume that the outcome of an attempted withdrawal is idiosyncratic to the household-bank pairing. Households then find it optimal to secure the successful withdrawal of a portion θ^{TB} of their total deposits by diversifying their holdings across a large number of banks. This does not affect our results but serves to streamline the model by preventing households from becoming heterogenous in whether they have been repaid.

The representative household chooses $\left\{D^{SB}, D^{TB}, M_1, M_{2|g}, M_{2|b}\right\}$ to maximize its expected consumption given by (9) subject to the budget constraints listed above. The first order conditions for $\left\{D^{TB}, D^{SB}\right\}$ yield the following expressions for the interest rates on traditional and shadow bank deposits.

$$R^{TB} = 1 + q\frac{\xi\left(1-\theta^{TB}\right) + p\left(1-\xi\right)\left(1-\bar{V}\right)}{1-q\left(\xi + p\left(1-\xi\right)\left(1-\bar{V}\right)\right)} \tag{10}$$

$$R^{SB} = 1 + \frac{q}{1-q}\left(1-\theta^{SB}\right) \tag{11}$$

Notably, R^{TB} is increasing in the liquidity run probability ξ and decreasing in the incentive compatible repayment rate $\bar{V}$ as well as the liquidation value θ^{TB}, while R^{SB} is decreasing in θ^{SB}. We can also observe that traditional banks borrow at a risk-free rate $R^{TB} = 1$ when $\theta^{TB} = \bar{V} = 1$ and similarly $R^{SB} = 1$ when $\theta^{SB} = 1$.

Incentive Compatibility The incentive compatible repayment rate $\bar{V}$ required to avert a fundamental run can be determined by comparing expected consumption after a bad signal across two different scenarios. On the one hand, when the household does not withdraw its funds as assumed above, expected consumption is given by

$$\begin{aligned} E\left[U\left(c_{s_2 s_3}\right)|s_2 = b\right] &= (1-p)\,c_{bh} + pc_{bd} \\ &= M_1 + \theta^{SB}D^{SB} + \left[\left(1-p\left(1-\bar{V}\right)\right)(1-\xi)\,R^{TB} + \xi\theta^{TB}\right]D^{TB} \end{aligned}$$

On the other hand, when the representative household withdraws its deposits from both shadow and traditional banks, its consumption is

$$c_b^w = M_1 + \theta^{SB}D^{SB} + \left(1-\left(1-\theta^{TB}\right)\xi\right)D^{TB}$$

with certainty. $\bar{V}$ is defined as the repayment rate at which the household is indifferent between the two scenarios

$$\begin{aligned} (1-p)\,c_{3|bh} + pc_{3|bd} &= c_b^W \\ \therefore \bar{V} &= \frac{1}{p}\left[\frac{1}{R^{TB}} - (1-p)\right] \end{aligned}$$

Observe that when $R^{TB} = 1$, households require a complete repayment $\bar{V} = 1$ from traditional banks in order not to withdraw their deposits early. This is because, when $R^{TB} = 1$ deposits yield no return in excess of the return from safe storage $M_{2|b}$. As such, households are not compensated for the prospect of an incomplete repayment $V < 1$ under weak fundamentals, and thus react to such a possibility by causing a fundamental run. This imposes market discipline on traditional banks, driving them to form a portfolio of assets consistent with remaining solvent at all times. Since we have $R^{TB} = 1$ only when traditional banks are fully liquid with $\theta^{TB} = 1$, a functioning secondary market for assets is a prerequisite for effective market discipline. We can see this more clearly by combining the expression for $\bar{V}$ with (10) which yields

$$\begin{aligned}\bar{V} &= 1 - \frac{q}{p}\frac{\xi\left(1-\theta^{TB}\right)}{1-q\left(1-\xi\left(1-\theta^{TB}\right)\right)} \\ R^{TB} &= 1 + \frac{q}{1-q}\xi\left(1-\theta^{TB}\right)\end{aligned}$$

such that $\theta^{TB} = 1$ leads to $R^{TB} = \bar{V} = 1$. When traditional banks have a liquidity shortfall $\theta^{TB} < 1$, on the other hand, liquidity run risk (ξ) leads to a rise in promised interest rates $\left(R^{TB}\right)$. This in turn increases the household's tolerance for insolvency risk as well, undermining market discipline and reducing the incentive compatible repayment rate to $\bar{V} < 1$.

2.6 Entrepreneurs

In the first period, each bank has access to a separate but ex-ante identical island j of entrepreneurs which can exert costly effort $h(i,j)$ to produce assets $I_1(i,j)$. Each entrepreneur can produce all three asset types $i \in \{l,s,r\}$ with a Cobb-Douglas production function that is additively separable in the asset type

$$I_1(i,j) = Ah(i,j)^{\alpha} \quad \forall\, i \in \{l,s,r\} \tag{12}$$

where $A > 0$ is a productivity parameter and $\alpha \in (0,1)$ is the standard Cobb-Douglas elasticity. We assume that specific financial frictions make it prohibitively costly for banks to purchase assets from islands other than their own.[9] The representative entrepreneur's problem can then be written as

$$\max_{I_1^{i,j}} \sum_{i\in\{l,s,r\}} P_1(i,j)\, I_1(i,j) - h(i,j)$$

subject to the production technology (12), where $P_1(i,j)$ is the asset price. This yields the set of first order conditions

$$P_1(i,j) = \frac{1}{\alpha A^{\alpha}} I_1(i,j)^{\frac{1-\alpha}{\alpha}} \quad \forall\, i \in \{l,s,r\}$$

which can be interpreted as an upward-sloping asset supply schedule. As such, assets which are demanded in larger quantities by banks are sold at a premium and risk and liquidity premia emerge endogenously. The information frictions associated with the production of assets constitute barriers to entry and it is natural for banks to wield market power over entrepreneurs in this environment. In the interest of tractability, we do not explicitly model the outside options of entrepreneurs but rather reflect the outcome of this process in a markup $\mu \in \left(0, \frac{1-\alpha}{\alpha}\right)$ which is positively related to banks' market power. The extent of market power can then be defined in a simple relationship

$$\frac{\partial P_1(i,j)}{\partial I_1(i,j)} = \mu \frac{P_1(i,j)}{I_1(i,j)} \quad \forall\, i \in \{l,s,r\} \tag{13}$$

[9] This is a relationship lending assumption which can easily be microfounded by allowing entrepreneurs to effortlessly produce a pseudo-asset that pays zero return. If banks may costlessly monitor the quality of assets produced in their own island but find it prohibitively costly to do so in other islands, this leads to complete specialization across islands between financial institutions. The same friction also bars households from investing directly in these projects. As such, banks act as intermediaries that channel funds from households to productive investment opportunities.

where each bank behaves like a monopoly under $\mu = \frac{1-\alpha}{\alpha}$ and there is perfectly competitive behaviour in the limiting case $\mu \to 0$.

2.7 Secondary Market

In the second period, the long assets $\{s, r\}$ are traded in a secondary market at prices $\{P_{2|s_2}(s),$ $P_{2|s_2}(r)\}$ which adjust to ensure that the market clears. First, we consider the case following a good signal $(s_2 = g)$ which eliminates the possibility of weak fundamentals. This dispels all uncertainty and increases the expected payoff from portfolios such that there is no prospect of bankruptcy. Banks then price assets efficiently, finding it optimal to purchase assets priced below their expected payoff and liquidate those priced above. The market clearing prices then reflect the corresponding asset's expected payoff as follows

$$P_{2|g}(s) = 1 \ , \ P_{2|g}(r) = \sigma_h \tag{14}$$

Although trade between banks may take place, the pricing of assets at their expected payoff ensures that it is inconsequential for the equilibrium allocation. Moreover, the rise in the expected payoff of the risky asset ensures that there is no liquidity shortfall and hence no risk of a liquidity run after a good signal. Following a bad signal $(s_2 = b)$, on the other hand, the liquidation of shadow bank assets may lead to an excess supply of assets from the banking sector, defined as

$$\tilde{I}(i) \equiv \gamma I_1^{SB}(i) + (1-\gamma)\left[I_1^{TB}(i) - (1-\xi)\, I_{2|s_2}^{TB}(i)\right] \geq 0 \quad \forall i \in \{s, r\} \tag{15}$$

where the second part of the expression reflects the liquidity run on a portion ξ of traditional banks, and the (optimal) portfolio reallocation of the remainder. When traditional banks absorb all liquidated assets of type i such that $\tilde{I}(i) = 0$, the asset is priced at its expected payoff as in the above case. When there is excess supply $\tilde{I}(i) > 0$, on the other hand, the asset is subject to a fire-sale. We find it convenient to define the fire-sale discount relative to the assets' expected payoffs, yielding the price schedules

$$\begin{aligned} P_{2|b}(s) &= \left\{ \begin{array}{c} 1 \text{ if } \tilde{I}(s) = 0 \\ \phi \text{ if } \tilde{I}(s) > 0 \end{array} \right\} \\ P_{2|b}(r) &= \left\{ \begin{array}{c} (1-p)\,\sigma_h \text{ if } \tilde{I}(r) = 0 \\ \phi\,(1-p)\,\sigma_h \text{ if } \tilde{I}(r) > 0 \end{array} \right\} \end{aligned} \tag{16}$$

The fire-sale discount $\phi \in (0, 1]$ is determined endogenously in a framework similar to that of Stein (2012). Specifically, we assume that the excess supply of assets is absorbed by outside investors which have limited resources and optimally allocate their funds between secondary market purchases and an outside project with decreasing returns to scale. As outside investors place a lower value on assets than banks, ϕ decreases until they find it optimal to purchase an amount $\tilde{I}(i)$ consistent with secondary market clearing.

We relegate the outside investor's optimization problem to the appendix but provide the resulting functional form for ϕ in Proposition 1.

Proposition 1 *When $\tilde{I}(i) > 0$, the fire-sale discount consistent with market clearing is given by the expression*

$$\phi = 1 - \tilde{I}(s) - (1-p)\,\sigma_h \tilde{I}(r) \ \ , \ \phi \in [0,1)$$

which is common across both assets and decreasing in both $\tilde{I}(s)$ and $\tilde{I}(r)$

Proof. Provided in Appendix Section B ■

For our findings to retain their validity, it is sufficient that the secondary market price of an asset decreases in its own excess supply. The commonality characteristics and the specific functional form only serve to simplify the exposition and can be relaxed without any impact on the mechanisms we focus on. In the next section, we show that traditional banks need to purchase safe assets in the secondary market in order to avert a fundamental run following a bad signal. The numerical results in Section 3 indicate that this prevents a fire-sale on safe assets when the share of shadow banks is below a threshold $\gamma < \bar{\gamma}$ while there is always a fire-sale on risky assets.

2.8 Bank Strategies

In this section, we evaluate the optimal behaviour of the banks described in Section 2.2 under the two alternative strategies of shadow and traditional banking. The only common thread running between the two strategies is through the secondary market prices $\left\{P_{2|b}(s), P_{2|b}(r)\right\}$ which suffer from a fire-sale when there is an excess supply of liquidated assets as described in Section 2.7. We show below that anticipated fire-sales affect the optimal strategy and expected payoff of banks under both strategies. We describe each strategy under a separate heading and simplify our notation by dropping the superscripts "SB" and "TB" for the remainder of this section. The variables $\{D, \theta, \Pi, I_1(i), P_1(i), i \in \{l,s,r\}\}$ should thus be taken to refer to the strategy being evaluated under each heading.

Shadow Banking Banks that follow the shadow banking strategy are not constrained by the incentive compatibility condition (6) while forming their portfolio. They are thus subject to a fundamental run following a bad signal $s_2 = b$ and disregard the states of nature $s_2 s_3 = \{bh, bd\}$ under limited liability.[10] The representative shadow bank chooses its deposits D and first period portfolio allocation $\{I_1(i), i \in \{l,s,r\}\}$ in order to maximize its expected payoff[11]

$$E[\Pi] = (1-q)\left[I_1(l) + I_1(s) + \sigma_h I_1(r) - DR\right]$$

subject to the budget constraint

$$\sum_{i \in \{l,s,r\}} P_1(i)\, I_1(i) = D$$

[10] While solving the model, we treat limited liability as an occasionally binding constraint. First, we solve for the case without a liquidity shortfall and carry out a consistency check by using (5) to confirm that $\theta = 1$. If this check fails, we solve for the case with a liquidity shortfall $\theta < 1$. The numerical results in Section 3 indicate that, across a range of calibrated parameters and policy interventions, there is always a liquidity shortfall in equilibrium.

[11] We attain this expression by using the budget constraint (2) and secondary market prices (14) under a good signal ($s_2 = g$) to substitute for $\left\{I_{2|gh}(s), I_{2|gh}(r)\right\}$ in (3). We are able to do this without explicitly optimizing the portfolio allocation following a good signal due to the indifference result in Section 2.7.

Under the financial frictions described in Section 2.6, shadow banks wield market power over entrepreneurs and thus internalize the impact of their first period asset purchases on the corresponding asset prices through the relationship given by (13). The representative shadow bank is also monopolistically competitive in the deposit market due to the dependence of its borrowing costs R^{SB} on its liquidation value

$$\theta = \frac{I_1(l) + P_{2|b}(s) I_1(s) + P_{2|b}(r) I_1(r)}{D} \tag{17}$$

and internalizes the effects of its choices on its borrowing cost R through (11). The optimal portfolio allocation is then determined by the set of first order conditions

$$(1+\mu) P_1(i) - 1 = (1-q)\left(P_{2|g}(i) - 1\right) - q\left(1 - P_{2|b}(i)\right)$$

where $\left\{P_{2|b}(i), P_{2|g}(i)\right\}$ are the asset's value on the secondary market respectively after good and bad signals.[12] The first order condition can be interpreted as asset pricing equations as $P_1(i)$ adjusts to ensure that the condition holds with equality. The LHS represents the wedge between the asset's expected payoff (which is equal to 1 for all three asset types) and the price at which the bank is willing to purchase a positive amount of assets, adjusted by a mark-up μ that arises from the bank's market power over entrepreneurs. A relevant benchmark is

$$\tilde{P}_1 = \frac{1}{1+\mu}$$

which is the price that would be set by a risk-neutral agent that also has market power over entrepreneurs, but is not protected by limited liability or prone to an early liquidation. The terms on the RHS represent distortions in shadow banks' incentives. The first term is a risk-shifting motive which indicates that shadow banks favour assets that appreciate in value after a good signal ($s_2 = g$), since this is the state where they remain solvent. The second term reflects a liquidity motive. Shadow banks realize that a low liquidation value during a fundamental run increases their borrowing costs ex-ante. As such, they favour assets which have a higher secondary market price after a bad signal ($s_2 = b$).

For the liquid asset l, which yields a certain payoff in the second period, we can set $P_{2|b}(l) = P_{2|g}(l) = 1$ such that the two motives exactly offset each other, and terms in the RHS are eliminated. Its price $P_1(l)$ is then equivalent to the efficient benchmark. This is also the case for the safe and risky assets $\{s, r\}$ when they are not subject to a fire-sale. When there is a fire-sale discount ϕ, on the other hand, the demand for $\{s, r\}$ decreases such that

$$\begin{aligned} P_1(s) &= \frac{1 - q(1-\phi)}{1+\mu} < \tilde{P}_1 \\ P_1(r) &= \frac{1 - q(1-p)(1-\phi)\sigma_h}{1+\mu} < \tilde{P}_1 \end{aligned}$$

It is notable, however, that the above expressions indicate that $P_1(r) > P_1(s)$. This yields the following pricing order: When there is no fire sale, all assets are priced at the benchmark such that $P_1(r) = P_1(s) = P_1(l)$. When only the risky asset is subject to a fire-sale, we have $P_1(r) <$

[12] $\left\{P_{2|b}(i), P_{2|g}(i)\right\}$ are determined as in Section 2.7. The first order conditions pin down $\{I_1(i), i \in \{l, s, r\}\}$ and D can then be backed out from the budget constraint.

$P_1(s) = P(l)$. When both safe and risky assets are subject to a fire-sale, on the other hand, the order becomes $P_1(s) < P_1(r) < P_1(l)$ such that risky assets are preferred over safe ones. In other words, the prospect of illiquidity leads to a rise in risk-taking among shadow banks. The explanation for this lies in the skew of the risky asset payoff towards the state with the good signal ($s_2 = g$) which is not affected by a potential fire-sale. The expected payoff from the risky asset thus decreases less than that of the safe asset when they are both subject to a fire-sale.

Finally, although we are not able to obtain a tractable expression for $E[\Pi]$ in terms of the fire-sale discount ϕ, the intuition from above indicates that a fire-sale leads to a fall in the expected payoff from a shadow banking strategy as asset purchases (from which a mark-up is collected) decrease and borrowing costs increase. The numerical results in Section 3 confirm this intuition.

Traditional Banking Banks that follow the traditional banking strategy avert fundamental runs by satisfying the incentive compatibility condition (6). Using (4), we can write this condition as an occasionally binding constraint in terms of the traditional bank's safe asset holdings after a bad signal ($s_2 = b$)

$$I_{2|b}(s) \geq \bar{V}DR$$

Proposition 2 shows that this constraint holds with equality at all times.

Proposition 2 *Following a bad signal, the incentive compatibility condition is a binding constraint on traditional banks' safe asset holdings such that*

$$I_{2|b}(s) = \bar{V}DR \tag{18}$$

$$I_{2|b}(r) = \frac{I_1(l) + P_{2|b}(s)\left(I_1(s) - \bar{V}DR\right)}{P_{2|b}(r)} + I_1(r) \tag{19}$$

Proof. Provided in Appendix Section C. ■

Traditional banks are reliant on limited liability under two particular cases. First, they become insolvent in the state with weak fundamentals ($s_2 s_3 = bd$).[13] Second, they are subject to a liquidity run with probability ξ as described in Section 2.4.[14] Traditional banks disregard the outcomes of both of these cases such that their expected payoff can be written as

$$\begin{aligned} E[\Pi] = {} & (1-q)\left[I_1(l) + I_1(s) + \sigma_h I_1(r)\right] \\ & + q(1-p)(1-\xi)\left(\frac{I_1(l) + P_{2|b}(s)I_1(s) - \left(P_{2|b}(s) - P_{2|b}(r)\right)\bar{V}DR}{P_{2|b}(r)} + I_1(r)\right)\sigma_h \\ & - \left[(1-q) + q(1-p)(1-\xi)\right]DR - \tau \end{aligned} \tag{20}$$

where we have used (18) and (19) to substitute for $\{I_{2|b}(s), I_{2|b}(r)\}$. The first two lines represent the payoff from assets in the states where the bank remains solvent while the first term in the third line reflects payments to depositors. We introduce the lump-sum cost $\tau > 0$ in order to capture

[13] The incentive compatibility condition requires a minimum repayment rate $\bar{V} \leq 1$ after insolvency. Although the bank is technically solvent when $\bar{V} = 1$, it is left without any additional profits which is equivalent to coming under the protection of limited liability.

[14] We follow Kashyap et al. (2014) in assuming that ξ is an aggregate variable such that banks take it as given.

the additional overhead costs associated with traditional banking, which tend to operate a greater number of branches and come under greater regulatory burden than shadow banks.

It is notable that the payoff in the second line is *increasing* in the fire-sale on the risky asset. This follows directly from (19) as a decline in $P_{2|b}(r)$ increases the return from risky asset purchases in the secondary market following a bad signal ($s_2 = b$). In other words, traditional banks profit from purchasing risky assets at a fire-sale discount. The extent to which they can profit from this is limited by the incentive compatibility condition which requires them to hold a minimum amount $\bar{V}DR$ of safe assets, however.

The representative traditional bank chooses its deposits D and first period portfolio allocation $\{I_1(i), i \in \{l, s, r\}\}$ in order to maximize (20) subject to the budget constraint

$$\sum_{i \in \{l,s,r\}} P_1(i) I_1(i) = D$$

As with shadow banks, traditional banks internalize the effects of their market power over entrepreneurs given by (13). When there is a positive probability of a liquidity run ($\xi > 0$), traditional banks' borrowing costs R also depend on their liquidation value θ, and this is internalized through (10). The optimal portfolio allocation is then determined by the set of first order conditions

$$P_1(i) = \frac{1}{(1+\mu)\psi}\left[(1-q) P_{2|g}(i) + \left(q(1-\xi)\frac{(1-p)\sigma_h}{P_{2|b}(r)} + q\xi\psi\right) P_{2|b}(i)\right] \; \forall i \in \{l, s, r\}$$

where ψ is defined as the marginal cost of raising funds through deposits

$$\psi \equiv \frac{(1-q) + q(1-p)(1-\xi)\left(1 + \bar{V}\left(\frac{P_{2|b}(s)}{P_{2|b}(r)}\sigma_h - 1\right)\right)}{1 - q\left(\xi + p(1-\xi)\left(1-\bar{V}\right)\right)}$$

which accounts for repayments to depositors, the rise in borrowing costs due to the associated reduction in liquidation value θ, and the tightening of the incentive compatibility condition as per (18). Specifically, the last term in the numerator reflects the fact that additional safe asset purchases are necessary after a bad signal ($s_2 = b$) in order to satisfy (18). The costs associated with this are increasing in $\frac{P_{2|b}(s)}{P_{2|b}(r)}$, the price of safe assets relative to risky ones in the secondary market.

The first order condition itself can be interpreted as reflecting two different considerations by traditional banks. The first term on the RHS shows that traditional banks prefer assets which become more valuable after a good signal ($s_2 = g$) as they are solvent with certainty after a good signal. The second term, on the other hand, shows that traditional banks have a preference for assets with a high secondary market price after a bad signal ($s_2 = b$). This is due to two reasons. First, as we explained above, traditional banks profit from risky asset purchases $I_{2|b}(r)$ in the secondary market, and assets with a higher value in this state can be converted to a larger amount of risky assets. We can see that this channel is strengthened as the fire-sale on risky assets deepens and reduces $P_{2|b}(r)$. Secondly, a higher secondary market value helps increase the bank's liquidation value and lower its borrowing costs. This channel becomes stronger as ψ increases.[15]

[15] The relative importance of these two channels depends on the liquidity run probability ξ. When $\xi = 0$, liquidation value concerns are eliminated. As ξ increases however, they become increasingly important relative to the first channel, which yields a payoff only in the absence of a liquidity run.

Finally, although $E[\Pi]$ is not tractable, we briefly discuss the effects of a fire-sale on traditional banks' expected payoffs. There are two conflicting effects in this case. On the one hand, traditional banks profit from a fire-sale as it provides them with an opportunity to purchase assets at a price below their expected payoff. Note that this applies to both safe and risky assets, as a fire-sale on safe assets relaxes the incentive compatibility condition while a fire-sale on risky assets increases risk-shifting opportunities as described above. On the other hand, fire-sales reduce traditional banks' liquidation value and leave them susceptible liquidity runs. An increase in ξ leads to a decline in $E[\Pi]$ since the probability of solvency decreases and borrowing costs increase. In the numerical analysis in Section 3, we find that the first effect dominates such that traditional banking becomes more attractive under a fire-sale.

2.9 Equilibrium

The equilibrium solution is given by a collection of endogenous variables

$$\left\{ \begin{array}{c} P_1^j(i), P_{2|s_2}(i), \phi, I_1^j(i), I_{2|s_2}^j(i), \tilde{I}(i), \\ \theta^j, \bar{V}, \xi, \Pi_{s_2 s_3}^j, \gamma, R^j, D^j, i \in \{l, s, r\}, \\ j \in \{SB, TB\}, s_2 \in \{g, b\}, s_3 \in \{h, d\} \end{array} \right\}$$

which are consistent with the equilibrium conditions given by

1. The definitions and budget constraints for banks given in Section 2.2.
2. The determination of liquidity runs through the mechanism described in Section 2.4.
3. Households optimally determining their deposits and early withdrawals as in Section 2.5.
4. Entrepreneurs optimally determining the supply of assets as described in Section 2.6.
5. Secondary market clearing as described in Section 2.7.
6. The optimal strategies of shadow and traditional banks as described in Section 2.8.
7. The share of shadow banks γ being determined through the free entry condition (7).

3 Numerical Solution

In this section, we provide a calibrated numerical solution to the model as an illustrative example. Table 1 reports the calibrated parameters. We calibrate the probabilities q and p according to the frequency of recessions and severe recessions in OECD countries and match the markup μ to data on bank concentration in high income countries (HICs). Regarding the entrepreneurs, we calibrate α to 0.3 in line with the convention for Cobb-Douglas production functions and set the productivity parameter A to normalize total bank assets to 1. Operating costs τ can then be calibrated in accordance with the ratio of overhead costs to total bank assets in HICs.

Table 1: **Calibration**

Parameter	Calibrated Value	Source
q	0.50	Claessens et al. (2009)
p	0.50	Claessens et al. (2009)
μ	0.15	World Bank (2015)
τ	0.025	World Bank (2015)
α	0.30	-
A	0.02	-

Figure 5 plots the solution across a range $\gamma \in [0,1]$ of shadow banking sector sizes. The equilibrium sector size is denoted by the vertical bar labelled γ^* where traditional and shadow banking strategies yield the same expected payoff as shown in the centre panel of Figure 5. This indicates that there is a unique mixed equilibrium at an interior value $\gamma^* \in (0,1)$ where shadow and traditional banks co-exist. The equilibrium is globally stable. When $\gamma < \gamma^*$, the shadow banking strategy yields higher expected profits $E\left[\Pi^{SB}\right] > E\left[\Pi^{TB}\right]$ leading to the expansion of the shadow banking sector while the opposite is true for $\gamma > \gamma^*$.

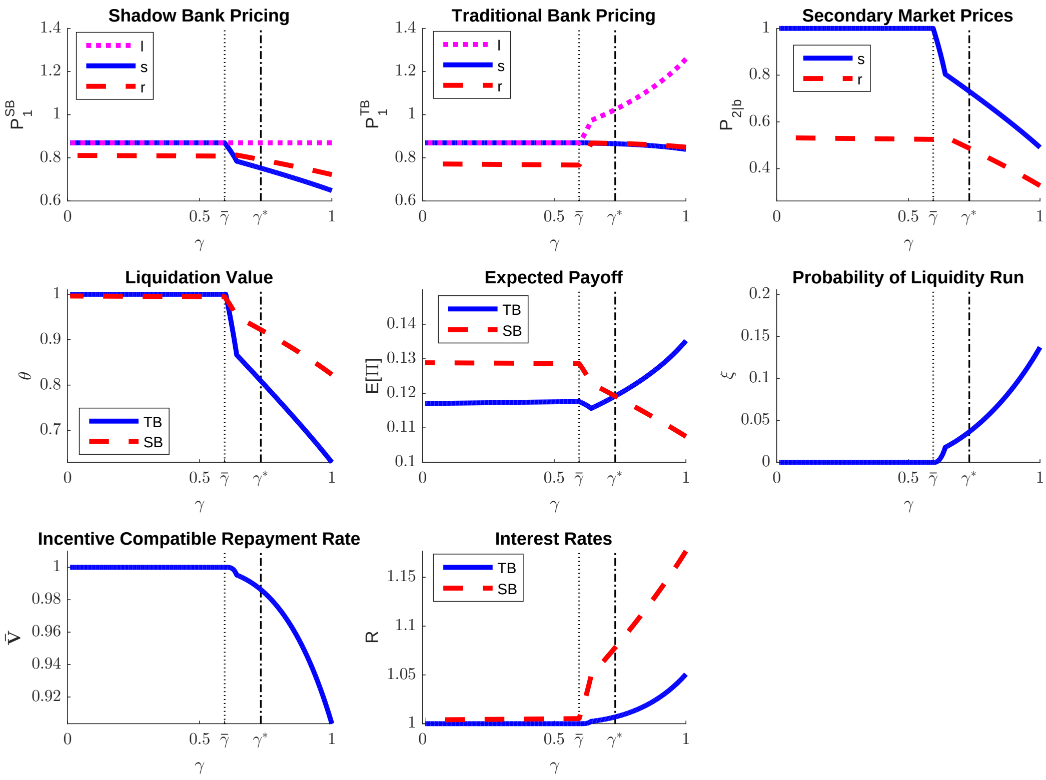
The bar labelled $\bar{\gamma}$ denotes the threshold above which the safe asset suffers from a fire-sale following a bad signal (as shown in the upper right panel). It is notable that $\gamma^* > \bar{\gamma}$ such that there is a fire-sale on safe assets in equilibrium. This reflects the key mechanism of the model. When the shadow banking sector is relatively small, shadow bank portfolios are endogenously liquid due to the ability to sell their assets to traditional banks during a fundamental run. This increases the profitability of shadow banking, and leads to the growth of the sector until secondary market purchases by traditional banks are insufficient to prevent a fire-sale, and shadow bank portfolios become endogenously illiquid.

We can observe this mechanism in Figure 5 as well. When the size of the shadow banking sector is below the threshold $\gamma < \bar{\gamma}$, there is no fire-sale on safe assets as shown in the top right panel. This keeps the liquidation value θ^{SB} of shadow banks close to unity (as indicated by the centre left panel). As such, households receive almost complete repayment from their deposits D^{SB} in shadow banks during a fundamental run, and the borrowing costs R^{SB} of shadow banks remain close to the risk-free rate (as shown in the lower centre panel). This makes the shadow banking strategy relatively profitable and fuels the expansion of the shadow banking sector until $\gamma > \bar{\gamma}$ such that there is a fire-sale on safe assets. The consequent fire-sale increases the borrowing costs of shadow banks and reduces the expected payoff associated with the shadow banking strategy.

The fire-sale also affects traditional bank profits through two conflicting channels. On the one hand, traditional banks profit from the fire-sale by purchasing assets at a discount after a bad signal ($s_2 = b$), and then making a positive return in case fundamentals come out to be strong ($s_2 s_3 = bh$). As shown in the upper centre panel of Figure 5, liquid assets become particularly attractive for traditional banks as the fire-sale deepens, since their payoff can be used to purchase assets in the secondary market at an increasingly favourable rate.

On the other hand, the fire-sale reduces traditional banks' liquidation value θ^{TB} and leaves them susceptible to liquidity runs. Indeed, the centre right panel shows that the liquidity run probability ξ becomes positive after γ exceeds the threshold $\bar{\gamma}$, and thereafter increases in γ. This has a negative effect on traditional banks' profits as it reduces the probability that they make a positive profit in the state $s_2 s_3 = bh$, and increases their borrowing costs R^{TB} as shown in the lower centre panel.

Figure 5: **Numerical Solution**



The centre panel indicates that, with the exception of a small region immediately to the right of the threshold $\bar{\gamma}$, the former effect dominates such that traditional bank profits increase as the fire-sale deepens. The equilibrium is thus reached at $\gamma^* > \bar{\gamma}$ when the expected payoffs from traditional and shadow banking strategies are equivalent. The key point here is that a degree of illiquidity (achieved through the fire-sale on safe assets) is necessary to equilibrate the model, since shadow bank profits are decreasing in the fire-sale discount on safe assets while the opposite is true for traditional banks.

Although traditional banks profit from the fire-sale, the prospect of a liquidity run on traditional banks has profound consequences for financial stability. In the absence of liquidity run risk ($\xi = 0$) when $\gamma < \bar{\gamma}$, traditional banks borrow at the risk-free rate $\left(R^{TB} = 1\right)$ as shown in lower centre panel. Since deposits in traditional banks yield no excess return to households above that of safe storage, households respond to any prospect of incomplete repayment with an early withdrawal such that the incentive compatible repayment rate is $\bar{V} = 1$ (as shown in the lower left panel). This imposes market discipline on traditional banks, driving them to form a portfolio of assets consistent with solvency even under weak fundamentals.

When there is liquidity run risk $\xi > 0$, on the other hand, the interest rate R^{TB} on traditional bank deposits increases, leading to a rise in households' tolerance for insolvency risk which undermines market discipline by reducing the incentive compatible repayment rate $\left(\bar{V} < 1\right)$. The upper centre panel shows that this leads to an increase in traditional banks' valuation of the risky asset. The expansion of the shadow banking sector thus engenders greater risk-taking by traditional banks and creates the prospect of their insolvency in the state with weak fundamentals.

In contrast, the upper left panel shows that shadow banks reduce their demand for both safe and risky assets as the fire-sale deepens due to concerns about their liquidation value θ^{SB}. They place a higher valuation on risky assets relative to safe assets in equilibrium, however, since the illiquidity of risky assets is counterbalanced by their high payoff following a good signal.

4 Policy analysis

In this section, we extend to model to evaluate two specific policy interventions. First, we consider the implications of asset purchases during a fire-sale. Second, we evaluate a Pigouvian tax on shadow bank profits.

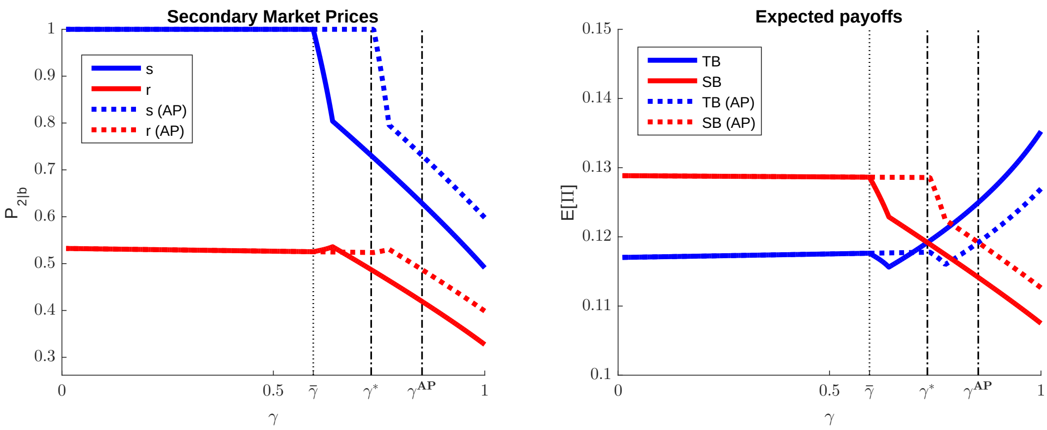
4.1 Asset Purchases

The government can lean against the fire-sale by purchasing safe assets in the secondary market. In this case, the expression for the excess supply of safe assets becomes

$$\tilde{I}\left(s\right) \equiv \gamma I_1^{SB}\left(s\right) + \left(1-\gamma\right)\left[I_1^{TB}\left(s\right) - \left(1-\xi\right) I_{2|s_2}^{TB}\left(s\right)\right] - I^{AP}$$

where $I^{AP} > 0$ refers to asset purchases by the government. Figure 6 shows the outcome of an asset purchase scheme where I^{AP} is set to absorb the excess supply of safe assets in the secondary market when the size of the shadow banking sector is at the initial equilibrium γ^*. The dashed lines show the effects of the asset purchases.

Figure 6: **Asset Purchases**



From an ex-post perspective where the size of the shadow banking sector is taken as given, the intervention succeeds in preventing the fire-sale on safe assets. This is achieved by shifting out the fire-sale threshold from $\bar{\gamma}$ to γ^* as shown on the left panel of Figure 6. Consequently, all of the negative effects on financial stability described in the previous section are offset, including the susceptibility of traditional banks to liquidity runs.

From an ex-ante perspective, however, the anticipation of asset purchases serves to fuel further growth of the shadow banking sector. As shadow banks retain their high liquidation value and low borrowing costs in the absence of a fire-sale, the expected payoffs associated with the shadow banking strategy remain high beyond γ^* as shown on the right panel. The shadow banking sector thus continues to grow until new equilibrium sector size $\gamma^{AP} > \gamma^*$, where the asset purchases are insufficient to offset the fire-sale.

The left panel of Figure 6 indicates that the extent of the fire-sale at this new equilibrium is identical to the fire-sale at the initial equilibrium without an intervention. The remaining features of the equilibrium are thus also identical to the initial case, such that the asset purchase scheme is completely ineffective in anything but increasing the size of the shadow banking sector.

It is important to stress that the growth of the shadow banking sector stems from the ex-ante anticipation of asset purchases. From an ex-post perspective where the size of the shadow banking sector is fixed, the asset purchase scheme is successful in offsetting the fire-sale and the vulnerability of traditional banks to liquidity runs. This creates the potential for time inconsistency issues as policymakers would naturally find it desirable to intervene once the fire-sale is underway.

4.2 Tax on Shadow Bank Profits

The second policy intervention we consider is the taxation of shadow bank profits with the purpose of deterring entry into the shadow banking sector. This can be considered as a Pigouvian tax since the adoption of a shadow banking strategy imposes a negative externality on the remainder of the financial sector through its contribution to fire-sales in the secondary market. Accordingly, we extend the model to allow for the imposition of a percentage tax T on shadow bank profits which reduces their expected payoff to

$$E\left[\Pi^{SB,T}\right] = (1-T)\, E\left[\Pi^{SB}\right]$$

From the numerical solution in Section 3, we know that the expected payoff schedules of shadow and traditional banks are nearly horizontal in a large region of lower γ values below the fire-sale threshold $\bar{\gamma}$. A constant tax rate T is thus either largely ineffective, or leads to the complete elimination of the shadow banking sector which is undesirable as shadow banks provide additional financial intermediation between households and entrepreneurs that the traditional banks cannot provide due to the incentive compatibility constraint. Instead, we envision the optimal tax on shadow bank profits as one that reduces the equilibrium size of the shadow banking sector to the highest level that is compatible with financial stability. Given the findings from Section 3, this corresponds to the fire-sale threshold $\bar{\gamma} < \gamma^*$.[16] It is possible to achieve this with a tax schedule

[16] Our findings from Section 3 indicate that, absent a fire-sale on safe assets, the liquidation values of both shadow and traditional banks are maximized, leaving the latter invulnerable to liquidity runs. Moreover, in this case the incentive compatibility constraint ensures that traditional banks remain solvent even when fundamentals turn out to be weak.

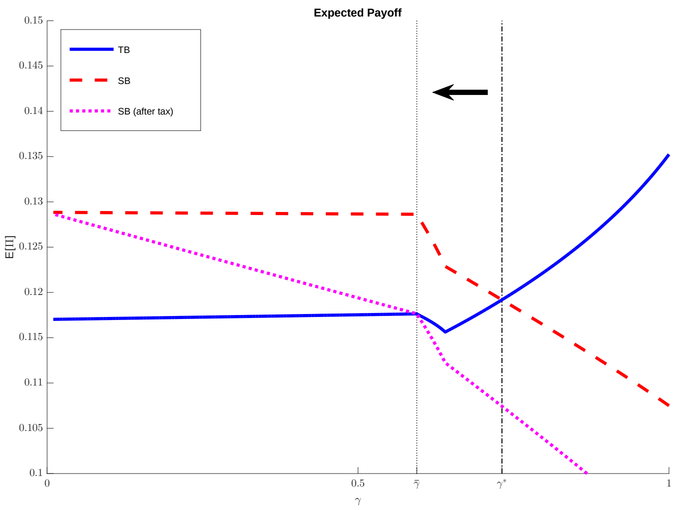
that is increasing in the share of shadow bank liabilities among total financial liabilities[17]

$$T = \chi \left(\frac{\gamma D^{SB}}{\gamma D^{SB} + (1-\gamma) D^{TB}} \right)$$

where $\chi > 0$ is a constant and $\left(D^{SB}, D^{TB}\right)$ are aggregate variables. Figure 7 demonstrates the expected payoffs under a tax schedule where χ is set to reduce the equilibrium shadow banking sector size γ^* to the fire-sale threshold $\bar{\gamma}$. Since the tax is conditional on aggregate variables, it does not distort the behaviour of existing banks and outcomes across γ values are identical to those shown in Figure 5. The sole effect of the tax is to deter entry into the shadow banking sector by altering the free entry condition which can now be written as

$$(1-T) E\left[\Pi^{SB}\right] = E\left[\Pi^{TB}\right]$$

Figure 7: **Tax on Shadow Bank Profits**



5 Dynamics

We extend the model to a dynamic setting in the form of a repeated game where the three stages described in Section 2 unfold as sub-periods within a single time period. In order to do this, we abstain from the net worth accumulation process by assuming that banks consume their expected payoff at the end of each period.[18] This leaves γ, the size of the shadow banking sector, as the only endogenous state variable. To bring about sluggish adjustment of γ, we also introduce an adjustment cost into the entry process such that the free entry condition becomes

[17] The important feature here is that the tax schedule is conditional on a quantity that is increasing in γ, which we do not directly target due to concerns with observability. The effectiveness of the policy is robust to changing the specific measure that is targeted.

[18] While we do not dispute that net worth accumulation is an important process in financial dynamics, its inclusion would significantly complicate the extension of the model to a dynamic setting without playing an integral role in the mechanism.

$$E\left[\Pi^{SB}\left(\gamma'\right)\right] = E\left[\Pi^{TB}\left(\gamma'\right)\right] + \mathcal{C}\left(\gamma' - \gamma\right) \tag{21}$$

where the expected payoffs are now written as functions of γ', which represents the new sector size and $\mathcal{C}(.)$ is the adjustment cost function. We motivate the adjustment cost through financial frictions which constitute a barrier to entry. Specifically, we posit that when a bank changes its strategy, it needs to form new lending relationships with the entrepreneurs described in Section 2.6. The costs associated with this are positively related to the number of banks that are switching to the same strategy due to competition in forming these relationships. This leads to the following restrictions on $\mathcal{C}(.)$

$$\mathcal{C}(0) = 0, \mathcal{C}'\left(\gamma' - \gamma\right) > 0 \; \forall \gamma \in [0,1] \tag{22}$$

which imply that $\mathcal{C}(\gamma' - \gamma) \gtreqless 0$ for $\gamma' - \gamma \gtreqless 0$ and are satisfied by a purely linear function as well as any odd degree polynomials. We can attain an expression for the evolution of γ' by re-arranging (21) such that

$$\gamma' = \gamma + \mathcal{C}^{-1}\left(E\left[\Pi^{SB}\left(\gamma'\right)\right] - E\left[\Pi^{TB}\left(\gamma'\right)\right]\right) \tag{23}$$

The restrictions given by (22) then ensure convergence to a quasi-steady state at $\gamma = \gamma^*$ characterised by $E\left[\Pi^{TB}\left(\gamma^*\right)\right] = E\left[\Pi^{SB}\left(\gamma^*\right)\right]$.[19] We refer to this as a quasi-steady state as γ evolves according to (23) only when there is a good signal, which occurs with probability $(1-q)$ as in the static model. With probability q, on the other hand, a bad signal leads to the liquidation of the shadow banking sector in a fundamental run and we have $\gamma' = 0$. We formalize the signal realization as an exogenous state variable ϕ which takes the value 1 under a good signal and 0 under a bad signal. The law of motion for γ can then be written as

$$\gamma'(\gamma, \phi) = \left\{ \begin{array}{ll} \gamma + \mathcal{C}^{-1}\left(E\left[\Pi^{SB}\left(\gamma'\right)\right] - E\left[\Pi^{TB}\left(\gamma'\right)\right]\right) & \text{if } \phi = 1 \\ 0 & \text{if } \phi = 0 \end{array} \right\}$$

Under this set up, the numerical solution shown in Figure 5 remains the same, but can now be interpreted as a global solution across the state space $\gamma \in [0,1]$ with convergence dynamics as described above.

Figure 8 demonstrates the dynamic properties of the model under a simple linear adjustment cost function. The left panel displays the expected payoffs associated with traditional and shadow banking strategies across $\gamma \in [0,1]$, which remain identical to the static case. As before, the vertical lines respectively denote $\bar{\gamma}$, the fire-sale threshold above which the liquidation of shadow banks leads to financial instability, and γ^*, which is now interpreted as a quasi-steady state as explained above.

The centre panel shows the law of motion for γ under a good signal $\phi = 1$. This shows that, as in the static case, the shadow banking sector has a tendency to expand to a size $\gamma^* > \bar{\gamma}$ where it poses a threat to financial stability. An important insight from the dynamic extension, however, is that this expansion is interrupted by a bad signal which leads to the liquidation of the shadow banking sector. A series of good signal realizations is thus required for the size of the shadow banking sector to exceed the fire-sale threshold $\bar{\gamma}$.

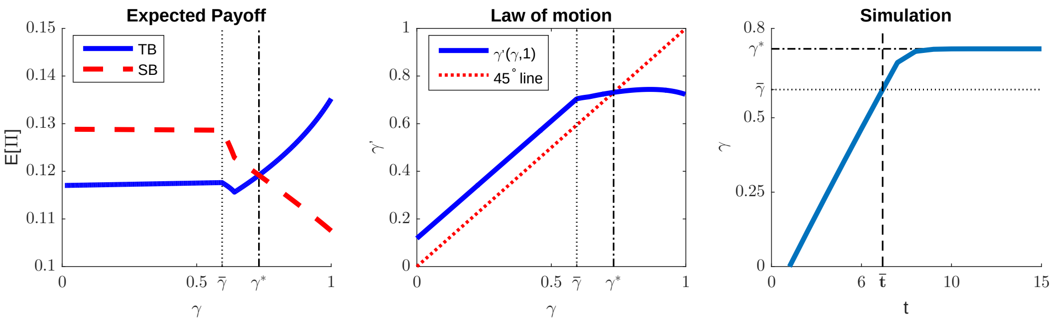
[19] The speed of convergence is positively related to the curvature of $\mathcal{C}(\gamma' - \gamma)$. To rule out fluctuations and ensure a smooth convergence to the quasi-steady state, we also need to make the speed of adjustment sufficiently low so that $\gamma' \in (\gamma, \gamma^*) \; \forall \; \gamma \in [0,1]$

The right panel shows the evolution of γ in a dynamic simulation where an initial value $\gamma = 0$ is subjected to a series of good signal realizations $\phi = 1$. The vertical line denoted as $\bar{t}$ indicates the number of periods it takes for the size of the shadow banking sector to exceed $\bar{\gamma}$. When a bad signal realization occurs at time $t \leq \bar{t}$, the shadow banking sector is small enough for traditional banks to absorb all of the liquidated assets. The fundamental run on shadow banks thus does not lead to a fire-sale, and there are no negative implications on financial stability.

However, when there is a succession of good signals which lasts for $t > \bar{t}$ periods, the shadow banking sector grows too large for traditional banks to absorb the assets liquidated during a fundamental run. A bad signal realization that occurs after $t = \bar{t}$ thus leads to a fire-sale which leaves traditional banks illiquid and vulnerable to liquidity runs. Moreover, as explained in Section 3, the anticipation of liquidity run risk undermines market discipline on traditional banks. This leads to an increase in risk-taking by traditional banks which leaves them insolvent when fundamentals come out to be weak with probability p after the bad signal.

In short, the dynamic model indicates that long periods of stability lead to the growth of the shadow banking sector and a consequent increase in financial fragility. When a negative shock eventually occurs, its effects on asset prices are endogenously amplified. The ensuing financial turmoil then engulfs the traditional banking sector as well as the shadow banking sector, culminating in a particularly destructive financial crisis. In view of the experience of the United States in the last two decades, this suggests that the period of stability associated with the Great Moderation was one of the drivers of the growth of the US shadow banking sector, which in turn increased the severity of the 2008 financial crisis.

Figure 8: **Dynamic Model**



6 Conclusion

We have presented a model of the financial sector where the size of the shadow banking sector is endogenous. In the model, shadow banking constitutes an alternative banking strategy which involves greater leverage and risk-taking at the expense of an unstable funding structure. Following a bad signal about prospective asset returns, shadow banks are liquidated early by their depositors in a fundamental run, while traditional banks avoid this fate by forming a more conservative portfolio. Traditional banks then act as secondary buyers for the assets liquidated by shadow banks.

Our main finding is that the shadow banking sector has a tendency to expand to a size where it ferments systemic risk. When the shadow banking sector is relatively small, secondary market purchases by traditional banks cushion the liquidation value of shadow banks and help keep their borrowing costs low. In periods of stability, this culminates in the expansion of the shadow banking sector until it reaches a size where purchases by traditional banks cannot prevent a fire-sale in the event of a fundamental run. The liquidation of shadow banks then leaves traditional banks susceptible to liquidity runs. This increases the borrowing costs of traditional banks and weakens market discipline on them, engendering greater risk-taking and a rise in insolvency risk.

An important contribution of our paper to the literature is the incorporation of shadow banking with an endogenously determined sector size into a general equilibrium framework. This yields novel insights regarding the interaction between policy interventions and the size of the shadow banking sector. We find that, while asset purchases by the government in the secondary market are effective in alleviating the fire-sale ex-post, their ex-ante expectation fuels further growth of the shadow banking sector in a manner that exactly offsets these gains.

Our findings regarding the destabilizing consequences of a large shadow banking sector also lend support to the imposition of a tax on shadow bank profits with the purpose of reducing the size of the shadow banking sector to a level compatible with financial stability. This can be considered as a Pigouvian tax since the adoption of a shadow banking strategy imposes a negative externality on the remainder of the financial sector through its contribution to fire-sales.

There are two potential extensions to this paper. First, it can be extended to a business cycle model with the introduction of a more sophisticated real sector. This would permit a quantitative analysis of the effects of shadow banking on the real economy. Second, the inclusion of a wider range of funding and investment instruments can be used to evaluate the effects of a richer set of regulatory policies. While these extensions would certainly yield many interesting findings, the core insights from the model would be robust to these changes.

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A Consistency check for fundamental runs

We can confirm that shadow banks are subject to a fundamental run following a bad signal by comparing expected consumption when households withdraw their deposits early to the a counterfactual case where they do not. Expected consumption under withdrawals is given by

$$(1-p)\, c_{3|bh} + p c_{3|bd} = M_1 + \theta^{SB} D^{SB} + \left[\left(1 - p\left(1 - \bar{V}^{TB}\right)\right)\left(1 - \xi^{TB}\right) R^{TB} + \xi \theta^{TB}\right] D^{TB}$$

in line with the expressions for consumption in Section 2.5. When households do not withdraw their funds, on the other hand, we have

$$\begin{aligned} c_{bh}^{nw} &= \left(1 - \xi^{TB}\right) D^{TB} R^{TB} + \xi^{TB} \theta^{TB} D^{TB} + \left(1 - \xi^{SB}\right) D^{SB} R^{SB} + \xi^{SB} \theta^{SB} D^{SB} + M_1 \\ c_{bd}^{nw} &= \left(1 - \xi^{TB}\right) \bar{V}^{TB} D^{TB} R^{TB} + \xi^{TB} \theta^{TB} D^{TB} + \left(1 - \xi^{SB}\right) \bar{V}^{SB} D^{SB} R^{SB} + \xi^{SB} \theta^{SB} D^{SB} + M_1 \end{aligned}$$

such that expected consumption is

$$\begin{aligned} (1-p)\, c_{bh}^{nw} + p c_{bd}^{nw} = &\ \xi^{SB} \theta^{SB} D^{SB} + \xi^{TB} \theta^{TB} D^{TB} + M_1 + \left(1 - p\left(1 - \bar{V}^{SB}\right)\right)\left(1 - \xi^{SB}\right) D^{SB} R^{SB} \\ &+ \left(1 - p\left(1 - \bar{V}^{TB}\right)\right)\left(1 - \xi^{TB}\right) D^{TB} R^{TB} \end{aligned}$$

where we now place the superscripts $\{TB, SB\}$ on $\{\xi, \bar{V}\}$ in order to distinguish between shadow and traditional banks. The incentive compatible repayment rate for shadow banks is then given at the point of indifference

$$\begin{gathered} (1-p)\, c_{3|bh} + p c_{3|bd} = (1-p)\, c_{bh}^{nw} + p c_{bd}^{nw} \\ \therefore \bar{V}^{SB} = \frac{\theta^{SB}}{p R^{SB}} - \frac{1-p}{p} \end{gathered}$$

and using (11) to substitute for R^{SB} yields

$$\bar{V}^{SB} = 1 - \frac{1 - \theta^{SB}}{p\left(1 - q \theta^{SB}\right)}$$

The fundamental run on shadow banks will be confirmed when we have a lower repayment rate $V^{SB} < \bar{V}^{SB}$.

B Outside Investors

The representative outside investor is risk-neutral with the same preferences as households. During a fire-sale, she optimally allocates her endowment $\tilde{E}$ between secondary market purchases $\left\{\tilde{I}(s), \tilde{I}(r)\right\}$ and an outside project $\tilde{K}$ which yields a certain payoff $f\left(\tilde{K}\right)$ with diminishing returns to scale $f'(.) > 0$, $f''(.) < 0$. The representative outside investor's optimization problem can then be written as

$$\max_{\left\{\tilde{I}(s), \tilde{I}(r), \tilde{K}\right\}} (1-p)\, \tilde{c}_{bh} + p \tilde{c}_{bd}$$

subject to the budget constraint

$$\tilde{K} + \phi(s)\tilde{I}(s) + \phi(r)(1-p)\sigma_h\tilde{I}(r) = \tilde{E}$$

where $\{\phi(s), \phi(r)\} \in [0,1]^2$ are the fire-sale discounts on assets and consumption $\tilde{c}_{s_3}$ is defined in a state-contingent manner due to the uncertainty of the payoff from her risky asset holdings $\tilde{I}(r)$.

$$\begin{aligned}\tilde{c}_{bh} &= f\left(\tilde{K}\right) + \tilde{I}(s) + \sigma_h\tilde{I}(r) \\ \tilde{c}_{bd} &= f\left(\tilde{K}\right) + \tilde{I}(s)\end{aligned}$$

The first order conditions indicate that the fire-sale discounts $\{\phi(s), \phi(r)\}$ are equal such that $\phi \equiv \phi(s) = \phi(r)$ and implicitly defined by the expression

$$\phi = \frac{1}{f'\left(\tilde{E} - \phi\left(\tilde{I}(s) + (1-p)\sigma_h\tilde{I}(r)\right)\right)}$$

Since $f(.)$ has diminishing returns to scale, the fire-sale discount ϕ will be a decreasing function of $\left(\tilde{I}(s) + (1-p)\sigma_h\tilde{I}(r)\right)$. In the interest of a clear exposition, we let $f\left(\tilde{K}\right) = \ln(K)$ and $\tilde{E} = 1$ which yields

$$\phi = 1 - \tilde{I}(s) + (1-p)\sigma_h\tilde{I}(r)$$

C Proof for Proposition 2

Since banks are protected by limited liability in the state with weak fundamentals ($s_2s_3 = bd$), they find it optimal to maximize their holdings of the asset with the highest return in the state with strong fundamentals ($s_2s_3 = bh$). This comes out to be the risky asset when secondary market prices satisfy the condition

$$\frac{P_{2|b}(s)}{P_{2|b}(r)} > \frac{1}{\sigma_h}$$

It follows from (16) that the condition is satisfied when there is a positive probability $p > 0$ of weak fundamentals. Using the second period budget constraint (14), we can then also attain an expression for $I_{2|b}(r)$.

Note that this result is not driven by the commonality of the fire-sale discount ϕ. When we allow the fire-sale discount to differ between $\{s, r\}$, the relevant condition becomes $\phi(s) > (1-p)\phi(r)$ such that the result is strengthened as long as the discount on the risky asset is greater than that of the safe asset. This is indeed the case in a variant of the model with risk averse outside investors.

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