EUROPEAN CENTRAL BANK

WORKING PAPER SERIES
NO 621 / MAY 2006

ASSESSING PREDETERMINED EXPECTATIONS IN THE STANDARD STICKY-PRICE MODEL

A BAYESIAN APPROACH

by Peter Welz

EUROPEAN CENTRAL BANK

WORKING PAPER SERIES

NO 621 / MAY 2006

ASSESSING PREDETERMINED EXPECTATIONS IN THE STANDARD STICKY-PRICE MODEL

A BAYESIAN APPROACH [1]

by Peter Welz [2]

In 2006 all ECB publications will feature a motif taken from the €5 banknote.

1 I would like to thank the Department of Economics at Universitat Pompeu Fabra and the Research Department of the European Central Bank for their kind hospitality. In addition, I wish to thank Michal Brzoza-Brzezina, Fabio Canova, Nils Gottfries, Sune Karlsson, Jesper Lindé, Eric Mayer, Peter McAdam, Paul Söderlind, Mattias Villani, Ulrich Woitek and an anonymous referee. Thanks also go to seminar participants at the Ifo Institute, Uppsala University, De Nederlandsche Bank, Spring Meeting of Young Economists, Warsaw and the Annual Congress of the German Economic Association in Dresden for helpful discussions and comments. Meredith Beechey provided excellent editorial work and additional comments. Remaining errors are my own. Financial support from the Jan Wallander and Tom Hedelius Foundation, C. Borgström and C. Berch's Foundation and the Ifo Institute Munich is gratefully acknowledged. The opinions expressed are those of the author and do not necessarily reflect views of the European Central Bank. Earlier title: Bayesian Estimation of the Canonical New Keynesian Macromodel with Informational Delays.

2 European Central Bank, DG-Research, Kaiserstr. 29, 60311 Frankfurt am Main, Germany, Email: peter.welz@ecb.int; homepage: http://www.pwelz.net

Address
Kaiserstrasse 29
60311 Frankfurt am Main, Germany

Postal address
Postfach 16 03 19
60066 Frankfurt am Main, Germany

Telephone
+49 69 1344 0

Internet
http://www.ecb.int

Fax
+49 69 1344 6000

Telex
411 144 ecb d

ISSN 1561-0810 (print)
ISSN 1725-2806 (online)

CONTENTS

Abstract

This paper analyses the empirical performance of a New Keynesian sticky-price model with delayed effects of monetary impulses on inflation and output for the German pre-EMU economy. The model is augmented with rule-of-thumb behaviour in consumption and price setting. Using recently developed Bayesian estimation techniques, endogenous persistence is found to play a dominant role in consumption whereas forward-looking behaviour is greater for inflation. The model's dynamics following a monetary shock and a preference shock are comparable to those of an identified VAR model.

Key words: DSGE-Model, identified VAR, predetermined expectations, Bayesian estimation
JEL classification: E43, E52, C51

Non-technical summary

The purpose of the present study is to estimate a small-scale DSGE model with sticky prices for the German economy prior to European Monetary Union (EMU). A number of authors, have recently estimated medium- to large scale models and found that these models fit the data fairly well. This paper deviates from the existing literature in two respects. The model is kept much simpler and thus closer to the types of models commonly used for normative monetary policy analysis. In addition, it is assumed that consumption and price-setting decisions of optimising agents are determined one period in advance. In this way decisions are based on information up to and including the previous period which introduces a delayed effect of monetary impulses on output and inflation in the model. While not dictated by microfoundations, this assumption makes it possible to compare the impulse responses of the estimated DSGE model to those of a (recursively) identified VAR model, even when the two identification schemes differ otherwise. Many VAR studies of monetary policy have found that an identification scheme that leads to one-period delayed effects of monetary impulses on output and inflation fit the data quite well, at least in closed economies. It is therefore interesting to understand the effect of a similar identification strategy within a DSGE model.

The model in this paper also deviates from its simplest counterpart through the assumption of endogenous persistence on both the demand and supply side. This is introduced by assuming that the population can be divided into two types: one group solves an optimisation problem according to the rational expectations hypothesis whereas the other group deviates from fully rational behaviour and follows a rule-of-thumb. Specifically, rule-of-thumb individuals make decisions based on information from the previous period rather than optimising over an infinite horizon. This assumption may be justified because such forward-looking optimisation is complicated and costly and requires acquiring large amounts of information. On an empirical level these assumptions allow to account for the observed persistence in the data by assuming that the endogenous variables in the model are persistent rather than modelling much of the persistence through exogenous shock processes which would possibly be required in a purely forward-looking framework.

The model is estimated with the recently developed Bayesian estimation methodology that allows to formally incorporate prior information about the parameters of the model.

The estimated model features a high degree of persistence in consumption and output and sizeable backward-looking behaviour in inflation. Persistence of ex-

ogenous shock processes is only important for the technology process. Prices are estimated to be fixed for 6.5 quarters on average, quantitatively similar to finding based on Euro area data. Using a conventional output-gap measure, the model can account for the so-called acceleration phenomenon. In contrast, the output-gap measure suggested by the model, that is the deviation of output from its flexible price level, appears to be a poor estimate in this simple specification of the model.

The dynamics following monetary and preference shocks are qualitatively and quantitatively comparable between the DSGE model and the structural VAR model. The VAR, however, displays more persistence in inflation. Moreover, the data clearly favour a model with delayed effects on output and inflation when compared to a model that allows interest rate movements to have contemporaneous effects on these variables. This may justify the often used identification scheme in structural VAR models.

1 Introduction

In recent years a new paradigm has arisen in macroeconomics that combines elements of real business cycle theory (RBC) and New Keynesian Macroeconomics (NKM). The standard model involves a dynamic stochastic general equilibrium (DSGE) structure with intertemporally optimising agents who are assumed to make decisions based on rational expectations, an assumption that reflects the RBC origins of the paradigm. As a result, equilibrium conditions for aggregate variables can be computed from the optimal individual behaviour of consumers and firms. NKM features are introduced by explicitly allowing for monopolistic competition as well as costly - and therefore gradual - price and/or wage adjustment. In this environment, monetary policy takes on a stabilisation role because actions taken by the monetary authority have significant effects on real economic activity in the short- to medium run. Furthermore, due to the rigorous microfoundations on which such models are based, it is possible to evaluate the welfare implications of alternative policy regimes. Ideally, such evaluations should serve as the basis for economic policy advice.

The purpose of the present study is to estimate a small-scale DSGE model with sticky prices for the German economy prior to European Monetary Union (EMU). A number of authors, including Smets and Wouters (2003, 2004), Adolfson et al. (2005), Levin et al. (2005), have recently estimated medium- to large scale models and found that these models fit the data fairly well. This paper deviates from the existing literature in two respects. The model is kept much simpler and thus closer to the types of models commonly used for normative monetary policy analysis.[1] In addition, I introduce the assumption that the consumption and price-setting decisions of optimising agents are determined one period in advance. In this way decisions are based on information up to and including the previous period which introduces a delayed effect of monetary impulses on output and inflation in the model.[2] While not dictated by microfoundations, this assumption makes it possible to compare the impulse responses of the estimated DSGE model to those of a (recursively) identified VAR model, even when the two identification schemes differ. Many VAR studies of monetary policy have found that an identification scheme that leads to one-period delayed effects of monetary impulses on output and inflation fit the data quite well, at least in closed economies.[3] It is therefore interesting to understand the effect of a similar identification strategy within a DSGE model.

[1]But see Levin et al. (2005) for an exception.
[2]Woodford (2003, Chapter 4) also discusses delayed effects of monetary policy.
[3]Favero (2001) provides an overview of this literature.

The model in this paper also deviates from its simplest counterpart through the assumption of endogenous persistence on both the demand and supply side. I introduce endogenous persistence by assuming that the population can be divided into two types. One group solves an optimisation problem according to the rational expectations hypothesis whereas the other group deviates from fully rational behaviour and follows a rule-of-thumb. Specifically, rule-of-thumb individuals make decisions based on information from the previous period rather than optimising over an infinite horizon. This assumption may be justified because such forward-looking optimisation is complicated and costly and requires acquiring large amounts of information.

At a more general level, another motivation for introducing endogenous persistence is that the purely forward-looking sticky-price model cannot account for observed persistence in inflation and consumption (Fuhrer and Moore, 1995). Because the sticky price model is designed to analyse the short-run effects of monetary policy and to study optimal monetary policy, it is important that the model can account for the empirical regularities. Other studies have employed habit formation to introduce endogenous persistence on the demand side (McCallum and Nelson, 1999) or indexation of a fraction of prices to past inflation to generate persistence on the supply side (Christiano et al., 2005). In contrast, this paper provides a consistent modelling perspective by assuming rule-of-thumb behaviour in both consumption and price-setting, thus treating both sides of the economy symmetrically.

Several of the above-mentioned studies use a synthetic Euro area data set rather than data for individual countries within the EMU. However, analysis at the country level is important and the focus of this paper will be on the German economy. Germany deserves special attention not only because of its relative importance in the aggregate EMU economy but because of its unique monetary regime for the two decades prior to EMU.

The model is estimated with techniques developed by DeJong et al. (2000a,b) and Otrok (2001). The approach takes a Bayesian view that formally incorporates prior information about the parameters of the model. Smets and Wouters (2003, 2004) apply this technique to estimate the 34 parameters of a New Keynesian model with capital investment, sticky prices and wages using Euro area data. Adolfson et al. (2005) apply the same method to estimate an open-economy version of the model with a larger number of parameters. These larger models clearly have a greater chance of empirical success but deviate from the parsimonious sticky-price model commonly used for optimal monetary policy analysis. They also require a number of additional assumptions about the exact investment technology and the use of capital. The approach here is more modest in attempting to fit a small-scale New Keynesian model with 17 parameters. Thus, the model resembles more closely the standard class of models used in theory.

The remainder of the paper is structured as follows. Section 2 presents the theoretical model and discusses the solution method while Section 3 covers the estimation methodology and specification of priors. In Section 4, the results are presented and the DSGE impulse response functions are compared to those from an identified VAR. Section 5 summarises and draws some conclusions.

2 The Sticky Price Model

The theoretical model used for estimation purposes here is an extension of the standard sticky-price model with fixed capital commonly used for the analysis of optimal monetary policy (Galí, 2003; Woodford, 2003). Only a brief description is given in this section. No explicit reference is made to money balances because the central bank is assumed to follow an interest rate rule. Introducing money balances for instance into an additively separable utility function, would only add a money demand equation which endogenously determines the magnitude of money balances without affecting the general results.

2.1 Households

The economy consists of a continuum of infinitely-lived consumers of measure one where each individual is indexed by $j \in [0,1]$. It is assumed that expenditure decisions are made one period in advance and subsequently altered only due to disturbances to preferences. Following Amato and Laubach (2003) I assume that re-optimisation is costly due to information-gathering or information-processing constraints. Hence, every period a randomly chosen fraction of households $1-\alpha_y$ decides to base its consumption decision on optimising behaviour, whereas the remaining fraction α_y follows a rule-of-thumb that simply implies choosing the optimal consumption level from the previous period, i.e.

$$C_t^r = C_{t-1}. \tag{1}$$

Assuming that the individual household is too small to affect the level of consumption C_t^r, the re-optimisation problem is to find a sequence $\left\{C_{jt}^o\right\}_{t=1}^{\infty}$ that maximises the present discounted value of expected life-time utility

$$E_{t-1} \sum_{s=t}^{\infty} \beta^{s-t} e^{g_s} \left\{ \frac{C_{js}^{o\ 1-\sigma}}{1-\sigma} - e^{\varepsilon_s^{\pi}} \frac{N_{js}{}^{1+\varphi}}{1+\varphi} \right\}, \tag{2}$$

where β is a discount factor, g_t is a preference shock affecting the individual's time discount factor, and an individual's disutility derives from supplying work hours, N_{jt}, perturbed by ε_t^{π} (to be explained below). The intertemporal elasticity of substitution is defined by σ^{-1} and labour supply elasticity is denoted by φ^{-1}. Note that the expectation in (2) is conditional upon information up to and including time period $t-1$, reflecting the predetermined nature of the expenditure decision.

Aggregate consumption in the economy is given by the standard Dixit-Stiglitz aggregate

$$C_t = \left(\int_0^1 C_{it}{}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

where $\epsilon > 1$ denotes the elasticity of substitution among the varieties of goods. The associated aggregate price index that gives the minimum expenditure $P_{it}C_{it}$ for which the amount C_{it} of the composite consumption basket can be purchased is given by

$$P_t = \left(\int_0^1 P_{it}{}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{4}$$

This specification leads to the familiar isoelastic demand function for each variety of the consumption good

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} C_t. \tag{5}$$

Financial markets are assumed to be complete in this economy, that is, each household can insure against any type of idiosyncratic risk through purchase of the appropriate portfolio of securities. Since by assumption households are identical ex ante they are willing to enter such insurance contracts. The advantage of this assumption is that the representative agent framework can be preserved, avoiding the need to keep track of an additional state variable of households' wealth distribution. As a result of the homogeneity assumption, all optimising households choose the same level of consumption C_t^o, and per capita consumption in period t is given by $C_t \equiv \alpha_y C_t^o + (1-\alpha_y)C_t^r$. Each household then faces the same flow budget constraint

$$P_t C_t + (1+R_t)^{-1} B_t \leq B_{t-1} + W_t N_t + T_t, \tag{6}$$

i.e. households' income consists of security holdings from the previous period, B_{t-1}, labour income, W_t, and a transfer, T_t, that they receive in order to balance the wealth effects of choosing consumption according to the optimality condition instead of the rule-of-thumb (Amato and Laubach, 2003).

Since the model also abstracts from government expenditure, goods-market clearing requires that $C_t = Y_t$ in each time period. Thus the rule-of-thumb for consumption in (1) becomes $C_t^r = Y_{t-1}$ and output in period t is given by

$$Y_t = (1 - \alpha_y)C_t^o + \alpha_y Y_{t-1}. \tag{7}$$

Maximising (2) subject to the budget constraint (6) and substituting the market clearing condition and the output relation (7) yields an Euler equation whose log-linearised form leads to the following intertemporal IS equation:

$$y_t = \delta E_{t-1}\{y_{t+1}\} + (1-\delta)y_{t-1} - \frac{(1-\alpha_y)\delta}{\sigma}E_{t-1}\{i_t - \pi_{t+1}\} + \frac{(1-\alpha_y)\delta}{\sigma}(g_t - E_{t-1}\{g_{t+1}\}), \tag{8}$$

where $\delta \equiv (1+\alpha_y)^{-1}$. The equation is log-linearised around a zero inflation steady state, so $\pi_t \equiv \log(P_t/P_{t-1})$ is the inflation rate and i_t is the percent deviation from its steady-state level associated with zero inflation. Furthermore, y_t denotes the percent deviation of output from its steady state level. For the case in which all households base their consumption decisions on optimisation, i.e. $\alpha_y = 0$, and there are no implementation delays, the standard intertemporal IS equation is obtained:

$$y_t = E_t\{y_{t+1}\} - \sigma^{-1}(i_t - E_t\{\pi_{t+1}\}) - \sigma^{-1}E_t\{\Delta g_{t+1}\}. \tag{9}$$

Comparing (8) with (9) we notice that introducing rule-of-thumb behaviour in consumption generates a backward-looking term in the IS equation. This is appealing from an empirical point of view as will become clear below.

2.2 Firms

Firms indexed by $i \in [0,1]$ produce a continuum of goods in a monopolistically competitive market with a decreasing returns-to-scale technology perturbed by an exogenous labour productivity shock a_t that is common to all firms:

$$Y_{it} = (e^{a_t}N_{it})^{\alpha}, \tag{10}$$

Since $\alpha < 1$, firms with different production levels face different real marginal cost given by

$$MC_{it} = \frac{1}{\alpha(e^{a_t})^{\alpha}{N_{it}}^{\alpha-1}}\frac{W_t}{P_t} = \frac{N_{it}}{\alpha Y_{it}}\frac{W_t}{P_t},$$

which can be related to average marginal cost by

$$MC_t = \frac{N_t}{\alpha Y_t}\frac{W_t}{P_t}.$$

Using the production function, the demand function (5) and $Y_{it} = C_{it}$, the following relationship can be derived in log-linearised form

$$mc_{it} = mc_t - \left[\frac{\epsilon(1-\alpha)}{\alpha}\right](p_{it} - p_t).^4$$

Then, real marginal cost can be shown to be given by

$$\widetilde{mc}_t = \frac{1-\alpha+\alpha\sigma+\varphi}{\alpha} y_t - (1+\varphi)a_t + \varepsilon_t^\pi = mc_t + \varepsilon_t^\pi \tag{11}$$

where the first-order condition with respect to the labour decision has been substituted in.[5]

Turning to price setting, I make the same assumption as in Galí and Gertler (1999) and Amato and Laubach (2003) that a fraction of firms re-optimise their prices and another fraction sets prices following a rule-of-thumb. Those firms who are assumed to optimise follow the setup suggested by Calvo (1983); every period a random fraction $1-\theta$ of firms resets prices to the new optimal price whereas the remaining fraction of firms leaves prices unchanged from the period before. In addition, I assume that a fraction α_π does not act according to Calvo's price-setting mechanism but uses a backward-looking rule-of-thumb for setting their prices.[6] Analogous to the motivation for the rule-of-thumb behaviour on the demand side, this could be justified by the fact that it is time-consuming to gather information about the stance of the economy, costly to obtain this information and that firms possess limited information-processing capacity. In addition, in order to match the commonly-made assumption in identified VAR models that monetary disturbances do not have contemporaneous effects on inflation, I assume that newly chosen prices take effect one period later (Woodford, 2003, Chapter 4).

With these assumptions, the log-linearised aggregate price level evolves according to

$$p_t = \theta p_{t-1} + (1-\theta)\overline{p}_t^*, \tag{12}$$

where $\overline{p}_t^*$ is the (log-linearised) price index of prices set in period t,

$$\overline{p}_t^* = (1-\alpha_\pi)p_t^f + \alpha_\pi p_t^b. \tag{13}$$

[4]See for instance Sbordone (2002) or Walsh (2003), Chapter 5.

[5]This condition (in log-linearised form) is given by $w_t - p_t = \varepsilon_t^\pi + \varphi n_t + \sigma c_t$.

[6]This is the argument of Galí and Gertler (1999). Amato and Laubach (2003) use a slightly different motivation that leads to the same specification of the Phillips curve below.

The latter is a convex combination of the price p_t^f set by the forward-looking firms following the Calvo (1983) rule and the price p_t^b set by the remaining backward-looking firms that follow the rule-of-thumb. The forward-looking price can be derived from firms' profit maximisation and is given by[7]

$$p_t^f = (1-\beta\theta)E_{t-1}(\widetilde{mc}_t + p_t) + \beta\theta E_{t-1}p_{t+1}^f. \tag{14}$$

The backward-looking price setters are assumed to set their price equal to the average price in the previous period corrected for past inflation, i.e.

$$p_t^b = \overline{p}_{t-1}^* + \pi_{t-1}, \tag{15}$$

where, importantly, past inflation serves as the forecast for actual inflation. Equations (12)-(15) can be combined to yield the following 'hybrid'-Phillips curve

$$\pi_t = \gamma^b \pi_{t-1} + \gamma^f E_{t-1}\{\pi_{t+1}\} + \lambda(E_{t-1}\{mc_t\} + \varepsilon_t^\pi), \tag{16}$$

where the parameters are defined as follows

$$\begin{aligned}
\lambda &\equiv \Phi^{-1}(1-\alpha_\pi)(1-\theta)(1-\beta\theta)\mu \\
\gamma^f &\equiv \Phi^{-1}\beta\theta, \qquad \gamma^b \equiv \Phi^{-1}\alpha_\pi \\
\mu &\equiv \frac{\alpha}{1+(1-\alpha)(\epsilon-1)} \\
\Phi &\equiv \theta + \alpha_\pi\left[1-\theta(1-\beta)\right].
\end{aligned}$$

Thus, as first suggested by Fuhrer and Moore (1995), inflation is both forward- and backward-looking and depends on the forecastable component of real marginal cost. As in Clarida et al. (2001), the 'cost-push' shock ε_t^π derives from the random disturbance perturbing the labour supply decision in the utility function in (2). In effect, it introduces a wedge between the marginal rate of substitution between leisure and consumption and the real wage and can be interpreted as a stochastic wage markup.

Analogous, to the discussion of the IS equation, the purely forward-looking New Keynesian Phillips curve results when all firms follow the Calvo pricing rule, i.e. $\alpha_\pi = 0$, and prices are not preset one period in advance,

$$\pi_t = \beta E_t \pi_{t+1} + \widetilde{\lambda} mc_t + \widetilde{\lambda} u_t, \tag{17}$$

where $\widetilde{\lambda} = \frac{(1-\theta)(1-\beta\theta)}{\theta}\mu$. The lagged inflation term in (16) is again important to account for the empirically observed inflation persistence.

[7] The assumption is as in Galí and Gertler (1999) that all consumers choose consumption optimally so that the marginal utility of consumption is identical across consumers.

In the purely forward-looking specification, inflation would become a jump variable and the price level a state variable. Estrella and Fuhrer (2002) have shown that purely forward-looking specifications like (17) and the IS relation (9) imply counterfactual relationships. The former implies that inflation and the output gap[8] are positively correlated while the correlation between the *change* in inflation and the output gap is negative. This is at odds with the '*acceleration phenomenon*' according to which high economic activity should move hand-in-hand with positive movements in inflation. The argument is similar for the IS equation, that is equation (9) stipulates a negative correlation between the consumption level and the expected real interest rate and a positive correlation between consumption *growth* and the expected real interest rate. Hence, when the expected real interest rate rises above its steady state value, the level of consumption must decline but its growth rate remain positive. This is only possible when consumption 'jumps' down initially and approaches its lower level from below. To assess the predictions of the augmented model with rule-of-thumb behaviour, in section 4 I compare the characteristics of the actual data with those of simulated data from the estimated model.

2.3 Central Bank

The model is closed by assuming that the central bank follows a Taylor-type interest-rate rule. That is, it adjusts its instrument in response to deviations of inflation and output from their respective target levels of price stability and potential output. In addition, I include a lagged interest rate term to account for the fact that central banks generally do not move their instrument in large steps (Goodhart, 1997),

$$i_t = f_i i_{t-1} + (1 - f_i) \left[f_y (y_t - \overline{y}_t) + f_\pi \pi_t \right] + \varepsilon_t^i. \tag{18}$$

Here ε_t^i is a white-noise, exogenous shock to the interest rate that can be interpreted as the unsystematic component of monetary policy. All coefficients are assumed to be positive and the smoothing or partial-adjustment coefficient is assumed to obey the restriction $f_i \in [0, 1)$. Existence of a stable solution of the model requires certain restrictions on the policy coefficients (Clarida et al., 1999). Namely, in response to an increase in expected inflation, the central bank must increase the nominal interest rate sufficiently to achieve a rise in the real interest rate that dampens economic activity. I confine the analysis to stable unique solutions of the model in the estimation procedure. Specifically, stability and uniqueness of the model solution will be checked by the numerical solution algorithm.

[8]Under the assumptions made in this model, there is a proportional relationship between marginal cost and the output gap $y_t - \overline{y}_t$.

Against the background of the Bundesbank's official money growth-targeting strategy, it may be surprising that in this model central bank behaviour is modelled in terms of the interest rate. However, the instrument of the Bundesbank when conducting monetary policy has always been a short term interest rate. Clarida and Gertler (1996) argue that the behaviour of the Bundesbank in the post Bretton-Woods era can be described well by a Taylor-type rule that also incorporates the output gap. Furthermore, between 1975 and 1985 the Bundesbank announced a rate of 'unavoidable inflation' that ranged between 4.5% and 3%. From 1986 onwards the Bundesbank went a step further, announcing that an inflation rate of 2% was consistent with price stability (Deutsche Bundesbank, 1995). Also supporting the interest-rate rule formulation, it has been observed that the Bundesbank allowed deviations of money growth from target more often than deviations of inflation from its prescribed values; by analysing the effects of changes in forecasted money growth and forecasted inflation on the interest rate instrument, Bernanke and Mihov (1997) find that money growth plays a quantitatively unimportant role in explaining variations in the interest rate. This leads them to conclude that implementation of Bundesbank's monetary policy is described well with an interest-rate rule.

However a recent study by Gerberding et al. (2004) using real time data shows that a broad monetary aggregate enters significantly into a Taylor-type rule.

2.4 Solution of the Model

The three endogenous variables, y_t, π_t, i_t, are determined by three equations: the IS-equation (8), the Phillips curve (16) and the monetary-policy rule (18). The stochastics of this system of rational-expectations equations are assumed to be driven by four independent exogenous shocks: the preference shock g_t, the productivity shock a_t, the cost-push shock ε_t^π, and the monetary policy shock ε^i. The first two are assumed to follow stationary AR(1)-processes, while the monetary policy shock is assumed to be white noise. Because data for three series is employed, at least three shocks need to be specified in order to avoid a singular covariance matrix in the likelihood computation. However, Smets and Wouters (2003) note that allowing for richer stochastic specifications than dictated by the number of time series may be helpful in the estimation procedure.

The system has the following matrix representation[9]

$$\Gamma_0(\xi)s_t = \Gamma_1(\xi)s_{t-1} + \Psi z_t + \Pi\vartheta_t, \tag{19}$$

[9]See Appendix A for full details.

and is solved using the method developed by Sims (2002). ξ is a (17×1)-vector containing the parameters of the model including the autoregressive coefficients, γ_g, γ_a, and the standard deviations of the shock processes σ_d, $d \in \{g, \pi, a, i\}$

$$\xi = \left(\beta, \alpha, \epsilon, \sigma, \varphi, \theta, f_i, f_\pi, f_y, \alpha_\pi, \alpha_y, \gamma_g, \gamma_a, \sigma_g, \sigma_\pi, \sigma_a, \sigma_i\right)'.$$

The matrices $\Gamma_0(\xi)$, $\Gamma_1(\xi)$, Ψ and Π are the (12×12), (12×12), (12×4) and (12×6) coefficient matrices respectively, z_t is the (4×1)-vector of exogenous disturbances, $\vartheta_t = X_t - E_{t-1}X_t$ is a (6×1)-vector of expectational errors, i.e. $E_t(\vartheta_{t+1}) = 0_{(6\times1)}$ and

$$s_t = \left(y_t, \pi_t, i_t, mc_t, g_t, a_t, \widetilde{y}_t^1, \widetilde{y}_t^0, \widetilde{\pi}_t^1, \widetilde{\pi}_t^0, \widetilde{i}_t^0, \widetilde{mc}_t^0\right)',$$

where I have defined $\widetilde{x}_t \equiv E_t x_{t+1}$ for $x_t \in \{y_t, y_t^1, \pi_t, \pi_t^1, i_t, mc_t\}$ and added the six equations $x_t = \widetilde{x}_{t-1} + \vartheta_t^x$ to the system.[10]

The general solution to (19) has a VAR(1)-representation

$$s_t = T(\xi)s_{t-1} + R(\xi)\eta_t. \tag{20}$$

Note that the system is stochastically singular since s_t has dimension 12 but there are only four stochastic shocks, rendering the covariance matrix of the disturbances singular. Hence the series for output, inflation and the interest rate are selected via the measurement equation

$$Y_t = Zs_t, \tag{21}$$

where Y_t is a (3×1)-vector and Z a (3×12)-matrix. In the model the natural level of output - the level of output obtained when prices are flexible and no cost shocks are present - is driven by the unobservable stochastic technology process. Hence, it is treated as unobservable in the estimation procedure as well.

3 Estimation

3.1 Data

The data ranges from the first quarter of 1975 to the fourth quarter of 1998, covering the post Bretton-Woods era up until the launch of European Monetary Union. Real Gross Domestic Product (GDP) and the Consumer Price Index (CPI) are taken from the OECD Main Economic Indicators Database, and the interest-rate series

[10] See Sims (2002) for a thorough discussion of this method and again Appendix A for a brief description.

is constructed as the quarterly average of the monthly average of the bank call rate published in Deutsche Bundesbank's time-series database[11]. The raw data is transformed so that it is conformable with the theoretical model. GDP data for Western Germany is employed until 1991Q3, after which the GDP series is for unified Germany. I account for the level shift and the possible trend break by regressing each series on an individual constant and individual linear trend. An alternative method to treat the statistical effect of reunification on the output series would be to link the series for Western Germany, for which observations are available until 1994Q4, with the series for unified Germany for which data are available from 1991Q1 onwards. This strategy may understate the initial economic boom related to reunification that began shortly after the inner border was opened in the end of 1989 and would also assume that the Eastern and Western German economies had equal growth rates prior to reunification which seems implausible.

The inflation series is calculated as the difference between CPI-inflation and a quasi inflation-target series. This series, published in Gerberding et al. (2004), is comprised of announcements made by the Bundesbank about what they first called 'unavoidable inflation' and later termed inflation consistent with price-stability. A series for the nominal interest rate is obtained by regressing the interest rate on this inflation-target series and removing the mean of the resulting series.

3.2 Estimation Methodology

Traditionally, DSGE models are calibrated such that certain theoretical moments given by the model match as closely as possible their empirical counterparts.[12] However, this method lacks formal statistical foundations (Kim and Pagan, 1994) and makes testing the results difficult.[13] One approach used recently in the monetary-economics literature that has improved on this shortcoming is to minimise the distance between the theoretical impulse response functions of the model and the empirical impulse responses estimated from a structural VAR (Christiano et al., 2005; Rotemberg and Woodford, 1997, for example). Since DSGE models provide by construction only an abstraction of reality, one advantage of this method is that it allows the researcher to focus on that dimension of the model for which it was designed, for example, the effects of a monetary policy shock.

[11]http://www.bundesbank.de/stat/zeitreihen/index.htm, series code SU0101.

[12]For an overview see Favero (2001).

[13]See, however, Canova and Ortega (2000) for a discussion on how testing in calibrated DSGE models could be conducted.

Following Sargent (1989), it has become more common to estimate monetary DSGE models with maximum likelihood (ML) (Bergin, 2003; Kim, 2000). Well known problems that arise with this method are that parameters take on corner solutions or implausible values, and that the likelihood function may be flat in some dimensions. GMM estimation is a popular alternative for estimating intertemporal models (Galí and Gertler, 1999, and others). However, Christiano and Haan (1996) show by estimating a business cycle model on U.S. data that GMM estimators often do not have the distributions implied by asymptotic theory. In addition, Lindé (2005) finds that parameters in a simple New Keynesian model are likely to be estimated imprecisely and with bias. Parameters sometimes need to be fixed beforehand, implying that results are only valid conditional on these a priori 'calibrated' parameters. This aspect often remains undiscussed in the final assessment of the model, despite the fact that calibration calls for a careful sensitivity analysis.

The Bayesian approach taken in this paper follows work by DeJong et al. (2000a,b), Otrok (2001), Smets and Wouters (2003, 2004) [14] and can be seen as a combination of likelihood methods and the calibration methodology. Bayesian analysis allows uncertainty and prior information regarding the parametrisation of the model to be formally incorporated by combining the likelihood with prior information on the parameters of interest from earlier microeconometric or macroeconometric studies. In the Bayesian approach such values could be employed as the means or modes of the prior densities to be specified, while *a priori* uncertainty can be expressed by choosing the appropriate prior variance. For example, the restriction that AR(1)-coefficients lie within the unit interval can be implemented by choosing a prior density that covers only that interval, such as a truncated normal or a beta density. This strategy may help to mitigate such problems as a potentially flat likelihood as estimates of the maximum likelihood are pulled towards values that the researcher would consider sensible a priori. This effect will be stronger when the data carry little information about a certain parameter, that is the likelihood is relatively flat whereas the effect will only be moderate when the likelihood is very peaked. Uncertainty about the specification of the structural model can also be accommodated by the Bayesian approach. I do so in the robustness analysis in Section 4 when the model is compared to a model without delayed effects.

By Bayes' theorem, the posterior density $\varphi(\xi \mid Y)$ is related to prior and likeli-

[14] There are by now numerous applications of the approach, for example Adolfson et al. (2005), Justiniano and Preston (2004), Lubik and Schorfheide (2006), Rabanal and Rubio-Ramírez (2005).

hood as follows

$$\varphi(\xi \mid Y) = \frac{f(Y \mid \xi)\pi(\xi)}{f(Y)} \propto f(Y \mid \xi)\pi(\xi) = L(\xi \mid Y)\pi(\xi), \tag{22}$$

where $\pi(\xi)$ denotes the prior density of the parameter vector ξ, $L(\xi \mid Y) \equiv f(Y \mid \xi)$ is the likelihood of the sample Y and $f(Y) = \int f(Y \mid \xi)\pi(\xi)d\xi$ is the unconditional sample density. The unconditional sample density does not depend on the unknown parameters and consequently serves only as a proportionality factor that can be neglected for estimation purposes. In this context it becomes clear that the main difference between 'classical' and Bayesian statistics is a matter of conditioning. Likelihood-based non-Bayesian methods condition on the unknown parameters ξ and compare $f(Y \mid \xi)$ with the observed data. Bayesian methods condition on the observed data and use the full distribution $f(\xi, Y) = f(Y \mid \xi)\pi(\xi)$ and require specification of a prior density $\pi(\xi)$.

The likelihood function can be computed with the Kalman filter using the state-space representation of the above model, where (20) is the transition equation and (21) is the measurement equation. Denoting $\widehat{s}_t$ as the optimal estimator of s_t based on observations up to Y_{t-1} and $P_t = E\left[(s_t - \widehat{s}_t)(s_t - \widehat{s}_t)'\right]$ as the covariance matrix of the estimation error, the prediction equations are given by

$$\begin{aligned} \widehat{s}_{t|t-1} &= T\widehat{s}_{t-1} && (23) \\ P_{t|t-1} &= TP_{t-1}T' + RQR' && (24) \end{aligned}$$

and the updating equations are

$$\begin{aligned} \widehat{s}_t &= \widehat{s}_{t|t-1} + P_{t|t-1}Z'F_t^{-1}(Y_t - Z\widehat{s}_{t|t-1}) && (25) \\ P_t &= P_{t|t-1} - P_{t|t-1}Z'F_t^{-1}ZP_{t|t-1}, && (26) \end{aligned}$$

where $F_t = ZP_{t|t-1}Z'$ (Harvey, 1989, p. 106).

The updating equations describe the solution to the signal extraction problem based on information up to and including time $t-1$, the prediction equations are one-step ahead predictions and $Q = E\left(\eta_t\eta_t'\right)$. The recursions are then initialised with the values of the unconditional distribution $s_{1|0} = 0$ and $vec(P_{1|0}) = (I - T \otimes T)^{-1}vec(RQR')$[15] (Harvey, 1989, p. 121). Finally, the likelihood can be computed conditional upon the initial observation Y_0 using a prediction-error decomposition (Harvey, 1989, p. 125). The prediction error is defined as $\nu_t = Y_t - Z\widehat{s}_{t|t-1}$, and

[15] This is possible because the transition equation is stationary.

assuming that s_t is Gaussian, $\widehat{s}_{t|t-1}$ is also Gaussian with covariance matrix $P_{t|t-1}$. It follows that the log-likelihood can be written as

$$\log L(Y \mid \xi) = -\frac{NT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2}\sum_{t=1}^{T}\nu_t' F_t^{-1}\nu_t. \tag{27}$$

Computation of the posterior distribution $\varphi(\xi \mid Y)$ requires calculating the likelihood and then multiplying by the prior density. The likelihood itself is computed by applying the Kalman filter to the state space system in (20) and (21), after solving the model given values of the elements in the parameter vector ξ.

3.3 Specification of Priors

In specifying the prior density for the parameter vector I assume that all parameters are independently distributed of each other, i.e.

$$\pi(\xi) = \prod_{i=1}^{17}\pi_i(\xi_i), \tag{28}$$

where ξ_i, $i = 1, .., 17$ denotes elements in ξ. However, the solution set of the DSGE model is restricted to unique and stable solutions which may imply prior dependence.[16]

Table 1 provides an overview of the priors used in the estimation. However, a number of parameters are difficult to estimate given the available data and are fixed a priori. Because the discount factor in the model, β, is related to the steady state interest rate by $-log\beta = \bar{i}$ and the estimations are performed with demeaned data, an estimate for β cannot be pinned down. Hence I fix the discount factor to 0.99, implying an annual steady state interest rate of about 4 percent. From a Bayesian perspective this is equivalent to imposing a strict prior on β with zero variance. The intertemporal elasticity of substitution is assumed to equal one, guaranteeing a balanced growth path, as is the elasticity of labour supply. Labour's share in production is set to 0.67 and the markup is assumed to be 10 percent which implies $\epsilon = 11$. A sensitivity analysis with respect to some of these choices is provided in Section 4.7.

The price stickiness parameter is assumed to be characterised by a beta distribution with a mean that implies an average duration of fixed prices of about half a year. The interest-rate smoothing parameter should lie between zero and one and,

[16]Thanks to Sune Karlsson for pointing this out.

Table 1: Prior Specification and Posterior Estimates

Parameter		Prior Density	Mean	Std Dev	Posterior Estimates Mode	5%	Mean	95%
rule of thumb cons	α_y	Beta	0.50	0.25	0.97	0.91	0.96	0.99
rule of thumb infl	α_π	Beta	0.50	0.25	0.40	0.30	0.44	0.58
price stickiness	θ	Beta	0.50	0.20	0.83	0.76	0.84	0.91
Monetary policy rule								
interest rate	f_i	Beta	0.80	0.15	0.87	0.84	0.89	0.94
inflation	f_π	Normal	1.50	0.25	1.14	0.93	1.25	1.64
output gap	f_y	Normal	0.50	0.25	0.25	0.19	0.33	0.55
Shock persistence								
preference	γ_g	Beta	0.50	0.25	0.03	0.01	0.05	0.11
productivity	γ_a	Beta	0.50	0.25	0.93	0.72	0.87	0.96
Shock variances			Mode	Dof*				
preference	σ_g	Inv Gamma	0.80	2.00	1.11	0.99	1.12	1.26
cost push	σ_π	Inv Gamma	1.60	2.00	0.38	0.35	0.40	0.45
productivity	σ_a	Inv Gamma	0.80	2.00	0.78	0.42	1.21	2.39
monetary policy	σ_i	Inv Gamma	0.50	2.00	0.12	0.11	0.12	0.14

*Note: Dof = degrees of freedom

like all other autoregressive parameters in the model, is also assumed to follow a beta distribution. Its mean is chosen to be 0.8, whereas the prior densities of the shock processes are specified with a mean of 0.5 and fairly wide variance to account for the uncertainty about their persistence. Concerning the degree of rule-of-thumb behaviour in consumption, I take account of the findings in Campbell and Mankiw (1989) that the population can be divided into roughly equal shares of forward- and backward-looking agents. Thus a beta prior with mean of 0.5 and a relatively large standard deviation of 0.25 is specified to account for the a priori uncertainty of this value. The same prior is chosen for the fraction of rule-of-thumb price-setters. Finally, little is known about the standard deviations of the shock processes. I specify inverted gamma densities with infinite standard deviations to account for the lack of knowledge. The modes are based on simple AR(1)-regressions with data prior to the sample period.

4 Results

In this section the estimation results from the DSGE model are discussed and its empirical performance evaluated. Impulse response functions are compared to those of a VAR estimation. The results are also compared to a model without the one-period delay imposed on optimising consumers and price setters. Finally, one means of estimation diagnostic is discussed.

4.1 Parameter Estimates

Combining the joint prior with the likelihood leads to an analytically-intractable posterior density. In order to sample from the posterior, I employ a random-walk chain Metropolis-Hastings algorithm with a multivariate normal proposal density and generate 150 000 draws from the posterior.[17] A complete set of estimation results is reported in Table 1 while Figure 1 displays kernel estimates of the priors and the posteriors of each parameter. As the marginal posterior densities are reasonably symmetric I refer in the following discussion of the results to the means of the marginal posteriors.

Turning first to the consumption decision, on average nearly all agents employ the backward-looking rule-of-thumb. Forward-looking optimisation plays only a minor role. The preference shock is found to approximate white noise, with a persistence coefficient insignificantly different from zero. In their larger model containing nominal and real rigidities, Smets and Wouters (2003) also report a large fraction of backward-looking individuals in output[18] although not as high as is found here. In contrast, they find that preference shocks are highly persistent. Estimation of the posterior mode turned out to be sensitive to the starting values; sometimes the mode was estimated with a low rule-of-thumb fraction and a high persistence coefficient of the preference shock process. However, the marginal likelihood in these cases was lower than for those cases where rule-of-thumb behaviour is important and the preference shock process is close to white noise. This is, however, one example of potential identification problems in DSGE models that have recently been noted by Beyer and Farmer (2004), Canova and Sala (2006) and Lubik and Schorfheide (2005).

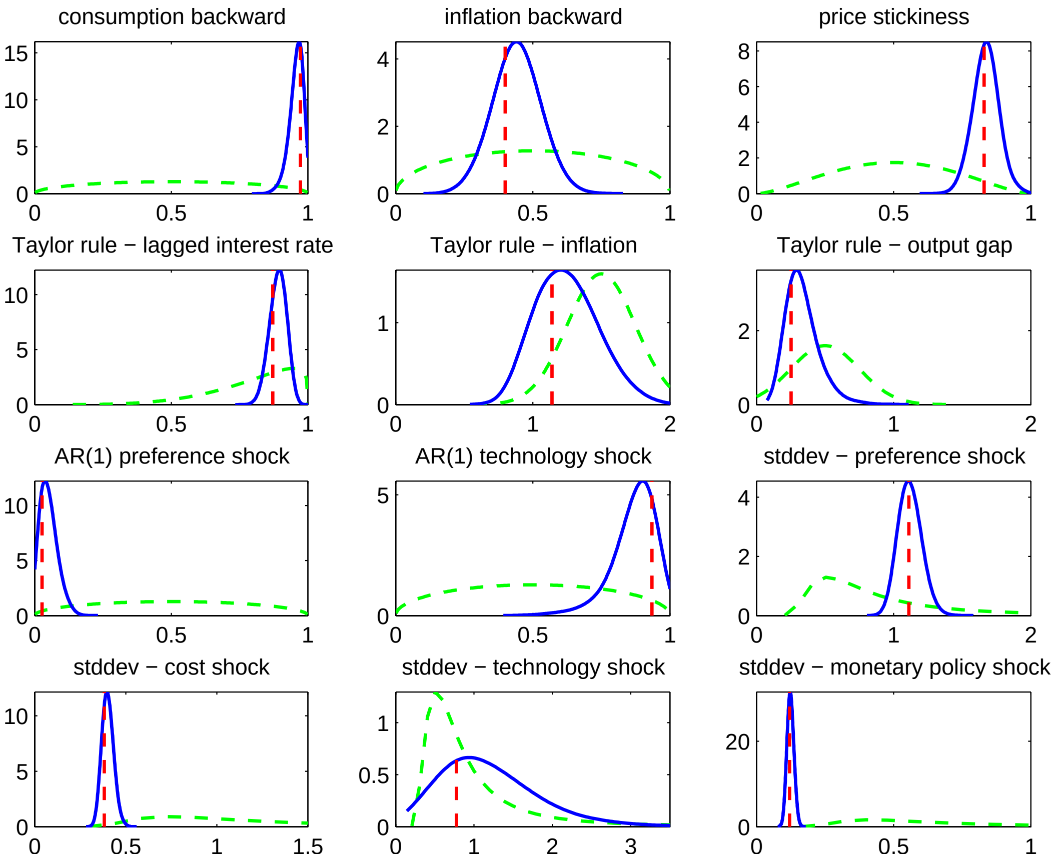
The supply side exhibits a considerable degree of forward-looking behaviour in inflation and stickier prices than a priori assumed. The estimated value of 0.84 implies that prices are fixed for 6.5 quarters on average.[19] This is consistent with Galí et al. (2001, 2003), who find a relatively low fraction of backward-looking price-setters using Euro area data. These studies estimate θ in the interval 0.77 to 0.87, varying with to the instruments used in their GMM approach. Similarly, Smets and Wouters (2003) estimate the Calvo price-stickiness parameter at 0.91 using Euro Area data. In order to clarify how backward-looking behaviour and

[17] Appendix B.1 reports details about this algorithm.

[18] They rationalise backward-looking behaviour with habit persistence in consumption which leads to a mathematical identical equation.

[19] The Calvo specification implies that the average duration of fixed prices is calculated as $\frac{1}{1-\theta} = \frac{1}{1-0.84} \approx 6.5$.

Figure 1: Delayed Effects Model: Prior- and Posterior Density



Notes: Prior (dashed lines) and posterior densities (solid lines) for the DSGE model with delayed effects.

price stickiness influence inflation dynamics, assume for a moment a purely forward-looking specification and consider a positive shock to marginal cost (or equivalently the output gap). The inflation rate jumps up instantaneously[20] to the maximum response and then reverts back to equilibrium. The degree of price stickiness governs the maximum response of inflation to cost shocks and the speed of convergence as it returns to equilibrium. The stickier prices are, that is, the fewer price setters who change price in a given period, the smaller is the inflation response and the more prolonged its convergence back to equilibrium. Allowing for a lagged inflation term heightens inflation persistence and produces a 'hump-shaped' inflation response so that the maximum impact on inflation is delayed somewhat. However, the reduced-form coefficient on marginal cost is very small (0.0018), indicating weak transmission from marginal-cost changes onto prices with respect to other shocks in the model.

The parameters of the Taylor rule display familiar values. The mean for the

[20]With one-period delayed effects this happens after one period, the first period in which inflation is allowed to move.

inflation coefficient is 1.25 and for the output gap coefficient 0.33 close to the values suggested by Taylor (1993) of 1.5 and 0.5 on inflation and the output gap, respectively. The partial-adjustment coefficient in the Taylor rule (mean of 0.89) is in line with results commonly found in most empirical studies irrespective of the method used. For example, Smets and Wouters (2003) estimate the mean of the lagged interest-rate term to be 0.93 using Euro area data. Finally, the technology shock is highly persistent, as has been found in other empirical work and commonly assumed in calibration studies. However, its standard deviation is not well identified.

4.2 Empirical Performance of the Model

4.2.1 Data Moments and Autocorrelation Functions

In this section I compare stylised facts from the actual data to those of simulated data from the model. Altogether, 10 000 sets of parameter values are drawn from the posterior distribution and used to simulate 96 observations for each of the three variables, equivalent to the number of observations of the actual data. The mean of the distribution of standard deviations and their 10- and 90-percentile values are calculated for each set of time series and compared to the standard deviations of the actual data. The results reported in Table 2 indicate that the simulated data series are a good match to the actual data, with inflation and the interest rate slightly more and output slightly less volatile than the actual data.

Table 2: Standard Deviations of Simulated and Actual Data

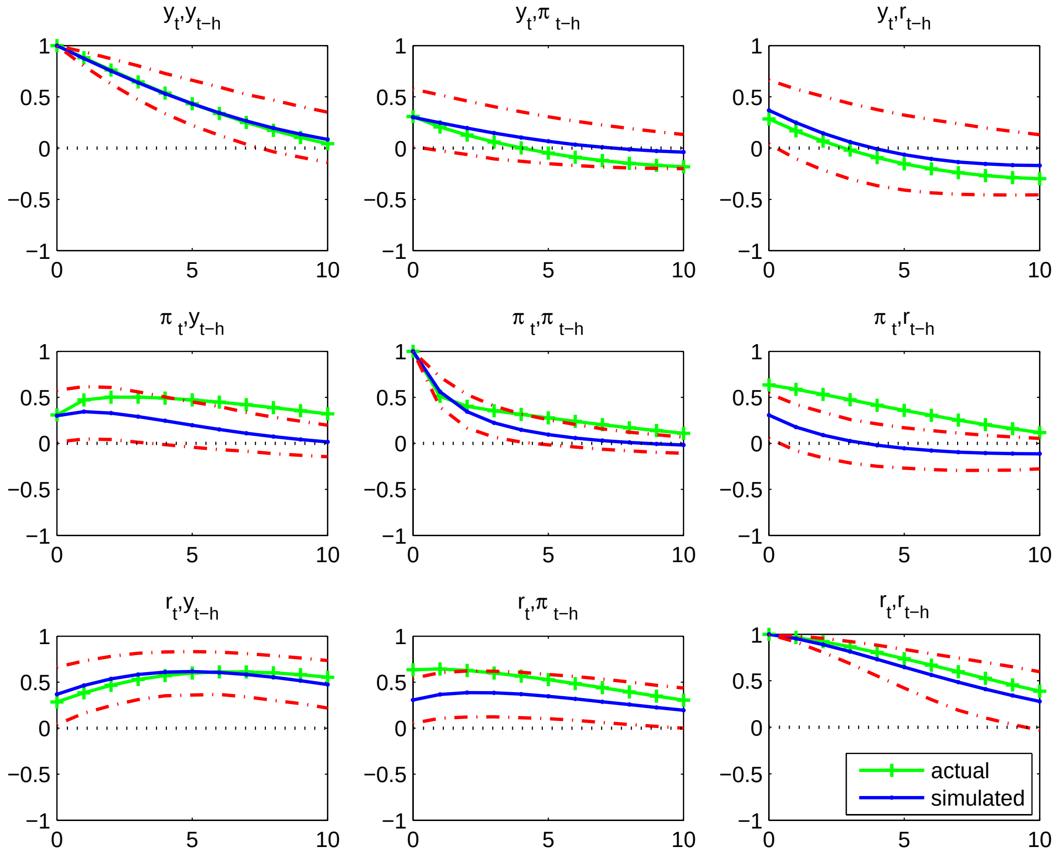
	Simulated Data			Actual Data
	10%	mean	90%	
GDP	1.81	2.51	3.34	2.38
Inflation	1.69	2.07	2.50	1.83
Interest rate	1.61	2.74	4.25	2.35

Autocorrelation functions for both the actual and simulated data are then estimated from a VAR(1).[21] Figure 2 summarises the results; the dashed lines indicate the 10- and 90-percentiles from the simulated data. The autocorrelations of the simulated data are typically in the vicinity of those of the actual data, but the DSGE model produces lower autocorrelations for inflation relative to the actual data. However, the wide error bands indicate that the autocorrelations from the DSGE model

[21]The Schwarz criterion selects one lag while the Akaike criterion selects a lag length of two for the VAR estimated on the actual data set.

are estimated with greater uncertainty. This exercise demonstrates the advantage of the Bayesian approach since the full (small-sample) distributions are available for all statistics.

Figure 2: Autocorrelation Functions



Notes: Autocorrelations for simulated data from the DSGE model with delays (solid lines) and the actual data (crossed lines) with 10- and 90%-tiles from DSGE-model (dash-dotted lines).

4.2.2 Acceleration Phenomenon

An interesting question is whether the estimated DSGE model can account for the acceleration phenomenon discussed in Section 2. The dynamic relationship between output and the expected real interest rate is also an open issue. In traditional empirical and theoretical analyses, the natural level of output is calculated as a deterministic trend, whereas New Keynesian models of the business cycle define natural output as the level of output obtained when all prices are flexible. In the simple model studied here, flexible-price output is simply proportional to the technology process, which may lead to a poor estimate of natural output. For this reason I use

Table 3: Acceleration Phenomenon and Output-Interest-Rate Dynamics

	Simulated data		Actual data
Output Measure	$\widetilde{y}_t = y_t - \overline{y}_t$	$\widetilde{y}_t = y_t$	
Panel A: Phillips Curve			
$\text{corr}(\Delta\pi_t, \widetilde{y}_t)$	-0.001	0.10	0.32
	[-0.15, 0.16]	[-0.06, 0.25]	
$\text{corr}(\pi_t, \widetilde{y}_t)$	-0.05	0.30	0.35
	[-0.28, 0.18]	[0.01, 0.58]	
Panel B: IS Equation			
$\text{corr}(\Delta\widetilde{y}_t, r_t)$	0.09	-0.26	-0.40
	[-0.12, 0.29]	[-0.44, -0.07]	
$\text{corr}(\widetilde{y}_t, r_t)$	-0.20	0.13	-0.59
	[-0.52, 0.14]	[-0.18, 0.45]	

Note: Mean of the model consistent output gap measure $y_t - \overline{y}_t$ and the classical (= linearly detrended output) output gap measure y_t. $\overline{y}_t$ is the natural level of output (under flexible prices) implied by the DSGE model. 10 and 90 percentiles in brackets below. The real interest rate is calculated as $r_t = i_t - \pi_t$.

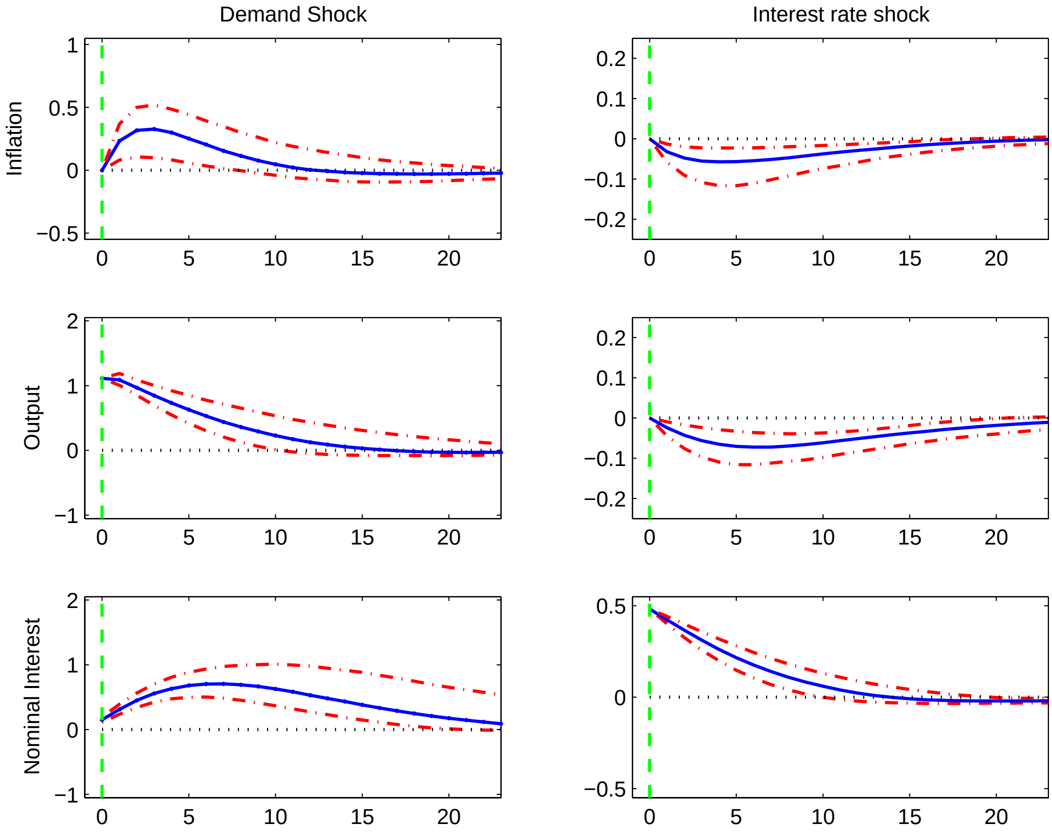
two measures of economic activity: (i) the theoretical output gap calculated from the DSGE model as the difference between output and output under flexible prices and (ii) the output variable measured as the deviation from steady-state output in the log-linearised model. These two measures are compared to those calculated from the data. Since mean and trend have already been removed from all variables, the empirical output measure can be interpreted as an output-gap measure in the classical sense. The same simulation procedure described above is used to generate data from the DSGE model. The real expected interest rate is approximated as the difference between the current nominal interest rate and current inflation and output growth and inflation growth are calculated as one year changes, that is $\Delta y_t = y_{t+2} - y_{t-2}$ and $\Delta\pi_t = \pi_{t+2} - \pi_{t-2}$. Panel A of Table 3 reports the results for the acceleration phenomenon and Panel B the correlation between output and the real expected interest rate.

As can be seen in the first data column, correlations based on the first measure of the output gap are not significantly different from zero. The results for the second output gap measure (2nd data column) are closer to the actual data (the final column of the table). As Panel A demonstrates, the model can generate positive correlations between inflation growth and output as well as between the level of inflation and output, namely the acceleration phenomenon observed in the actual data. However, there is considerable uncertainty around these correlations and the correlation between inflation growth and the output gap is significantly smaller in the generated data than in the actual data. Looking at Panel B, the DSGE model is able to generate a plausible negative correlation between output growth and the real expected interest rate but fails to do so with the level of output. In this respect the model is lacking.

4.3 Impulse-Response Analysis

Impulse response functions to each of the four shocks, together with 10 and 90 percentile error bands, are calculated from 10 000 draws of the posterior distribution and shown in Figure 3 and 4. All shocks are one standard deviation shocks.

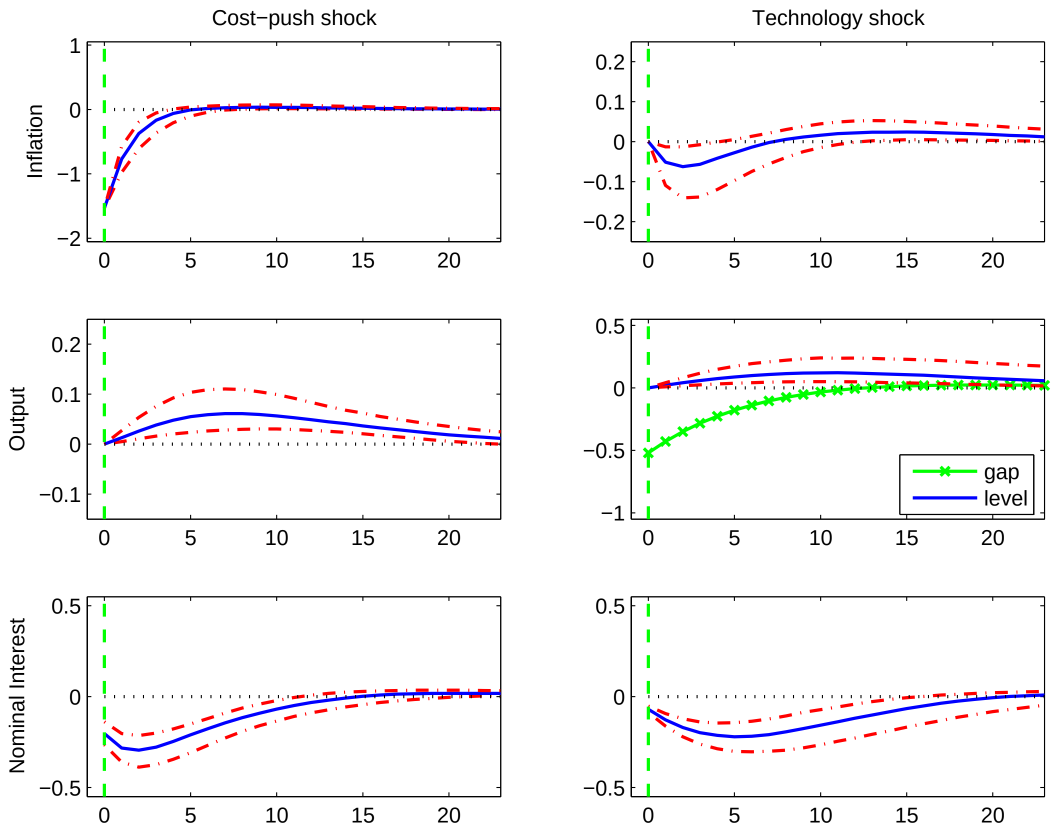
Figure 3: Demand and Monetary Policy Shock



Notes: Impulse responses (solid lines) from the DSGE model with delays with 10- and 90%-tiles (dash-dotted lines).

In response to a contractionary *monetary policy shock* (Figure 3, right column), the interest rate increases and output and inflation fall, consistent with the specification of the theoretical model. Both output and inflation show a hump-shaped and gradual reversion over time. The hump-shaped form of the impulse responses is due to significant backward-looking terms in both the Phillips-curve relation and the IS relation and is in line with stylised facts from VAR studies (Christiano et al., 2005). Note that in accordance with the specification of the theoretical model, output and inflation do not react in the period of the shock as is the case in recursively identified VAR models. A positive *preference shock* (Figure 3, left column) increases the discount factor in the intertemporal optimisation problem so that agents are willing to consume more, inducing a rise in output. In turn, excess demand triggers inflationary pressures due to increasing marginal cost. A positive output gap and inflation deviating from target consequently lead to an increase in the interest rate. Again, because expectations about marginal cost are predetermined, the rise in inflation begins with a one period delay.

Figure 4: Cost- and Technology Shock



Notes: Impulse responses (solid lines) from the DSGE model with delays with 10- and 90%-tiles (dash-dotted lines).

Following a positive *technology shock* (Figure 4, right column), output increases while inflation and the interest rate fall. Upon impact, marginal cost falls and natural output increases by more than the level of actual output, opening up a negative output gap. Since the monetary authority does not respond strongly enough to offset the shock, inflation falls. This result is in line with the New Keynesian literature on technology shocks (Galí, 1999).

Finally, a negative *cost-push shock* (Figure 4, left column) produces a qualitatively similar response to the technology shock. The fall in inflation causes the central bank to cut the interest rate which leads to a rise in output. However, the effect is quantitatively smaller.

4.4 Comparison to VAR

In order to gain insight into the quality of the results from the Bayesian estimation, I compare the monetary and preference shock impulse response functions of the DSGE model to those of an identified first order VAR estimated on the same data set. The VAR is ordered as inflation, output and nominal interest rate and takes the following form:

$$A_0 y_t = A_1 y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Omega). \tag{29}$$

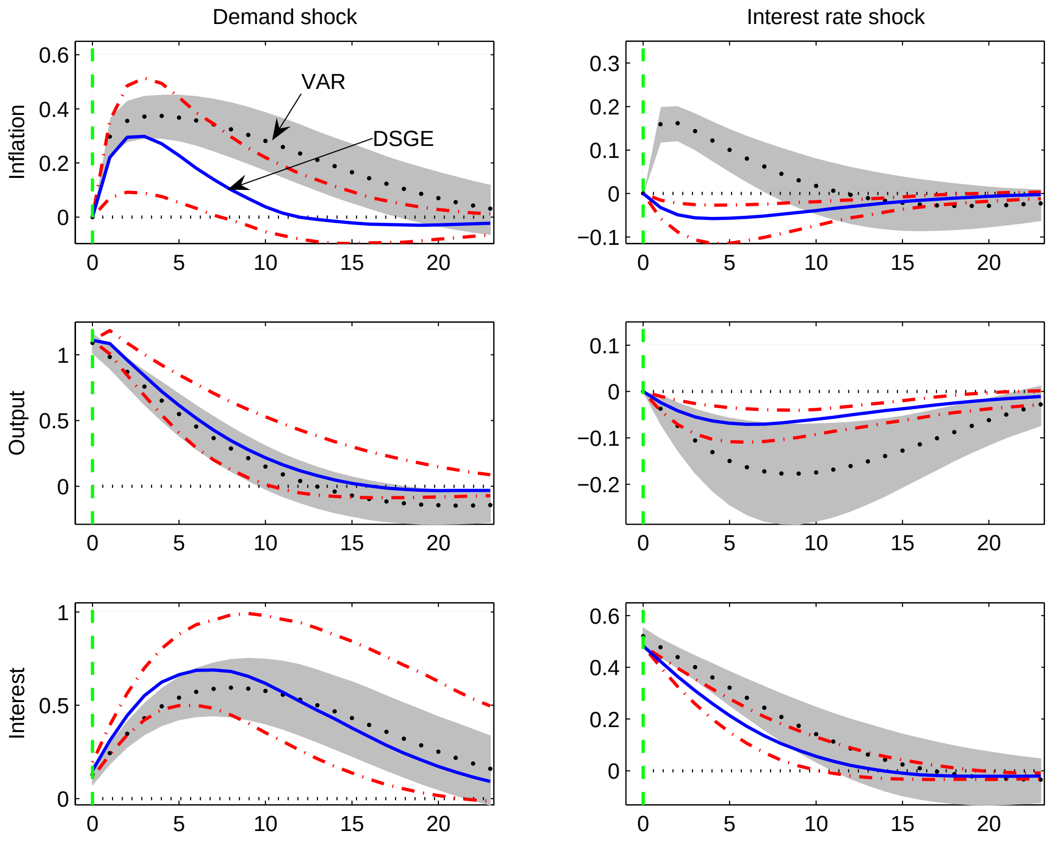
The identification is recursive with ones on the main diagonal and an additional zero entry at the (2,1) position,

$$A_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a_{31} & -a_{32} & 1 \end{bmatrix}. \tag{30}$$

That is, an inflationary shock does not have an immediate impact on output which allows identification of an aggregate demand shock in addition to the monetary policy shock. Given the assumptions about technology and cost shocks, these cannot be separately identified in this framework. In Figure 5, the impulse response functions for a contractionary one-standard-deviation monetary policy shock are shown (right column). Impulse responses from both the VAR and the DSGE model are shown, along with 10 and 90 percentile bands. For the VAR, the percentile bands are estimated with methods suggested by Sims and Zha (1999). Apart from the so called 'price puzzle', namely that inflation rises after a contractionary monetary policy shock, the estimated impulse response functions from the VAR are similar to those from the DSGE model. However, the high coefficient on the backward-looking term in the aggregate demand equation (8) implies weak transmission from real interest rate changes to consumption. This effect is mirrored in the weaker response of output in the DSGE model than in the VAR. The interest-rate dynamics match well between the DSGE model and the VAR. In both models the interest rate returns to zero after about twelve quarters.

Similar dynamics are also observed in both models in response to a preference shock (left column in figure 5). The response of inflation in the DSGE model is slightly less persistent than in the VAR model, which is in line with the evidence obtained from the autocorrelation functions, whereas there is almost a perfect fit with respect to the output response.

Figure 5: Dynamics of DSGE- and VAR Model



Notes: Impulse responses: DSGE model with delayed effects (thick solid line) with VAR (thin solid line); 10- and 90%-tiles of DSGE-impulses (dash-dotted lines) and error bands of VAR (shaded area).

In conclusion, despite its simple and stylised structure does the estimated DSGE model qualitatively resemble the identified VAR. The models are also quantitatively similar with respect to the monetary and preference shocks.

4.5 Comparison to a Model with Contemporaneous Effects

I have introduced delayed effects of monetary policy onto inflation and output in order to account for the assumptions often made in identified VAR studies. However, despite the fact that this recursive scheme has become the standard identification in the monetary-policy VAR literature, most DSGE models do not allow for such effects.[22] Rather, DSGE models postulate that monetary policy shocks have a contemporaneous impact on all variables. In this section I compare the results of the model in this paper to those of a baseline model in which all expectations are conditional on information up to and including period t. That is, optimising consumers' and price setters' decisions have immediate effects. Appendix C.1 presents the estimation results for such a model using the same prior specification as before; the estimation outcome is quite similar to the model with delayed effects (see Table 5). The prior and posterior density kernels are shown in Figure 7 in Appendix C.2 and the corresponding impulse responses are shown in Figures 9 and 8 in Appendix C.3. The coefficient estimates are similar to those from the model with delayed effects but in contrast, the contemporaneous responses of output and inflation to a monetary impulse are significantly different from zero.

As discussed in Geweke (1999), for example, the Bayesian approach to estimation allows a formal comparison of different models based on the marginal likelihood of the model. The marginal likelihood of a model M_i is defined by

$$f(Y_T|M_i) = \int_{\Xi} \varphi(\xi|M_i) f(Y_T|\xi, M_i) d\xi, \tag{31}$$

where $\varphi(\xi|M_i)$ is the prior density for model M_i and $f(Y_T|\xi, M_i)$ is the data density of model M_i given the parameter vector ξ. Integrating out the parameter vector, the marginal likelihood gives information about the overall likelihood of the model given the data.

Further, the posterior-odds ratio in favour of model M_i versus M_j is defined by

$$PO_i = \frac{p_i f(Y_T|M_i)}{p_j f(Y_T|M_j)}, \tag{32}$$

where p_i and p_j are the prior model probabilities for model i and j, respectively. Assuming that both models complete the model space and assigning equal prior probabilities of 1/2, the Bayes factor in favour of model i versus j can be calculated as

$$B_{ij} = \frac{f(Y_T|M_i)}{f(Y_T|Mj)}.$$

[22] Exceptions include Christiano et al. (2005) and Rotemberg and Woodford (1997).

Table 4: Model Comparison by Bayes Factors

	Model with Delayed Effects	Model without Delayed Effects
$log(f(Y_T \mid M_i))$	-421.8402	-427.2308
log(Bayes factor)	5.3906	

Note: Modified harmonic mean estimation with $p = 0.05$ (Gelfand and Dey, 1994)
See Appendix B.2 for more details.

Assuming that falsely choosing a model incurs equal losses for both models, a Bayes factor greater than 1 indicates that model i is more likely than model j after having observed the data.

The results in Table 4 show that the data favour the model with one-period delays over the model without delayed effects, as can be seen by the higher (log) marginal likelihood for the former as well as the magnitude of the Bayes factor. A value of the log-Bayes factor greater than 2 is decisive evidence against the alternative model (Kass and Raftery, 1995).

4.6 Estimation Diagnostics

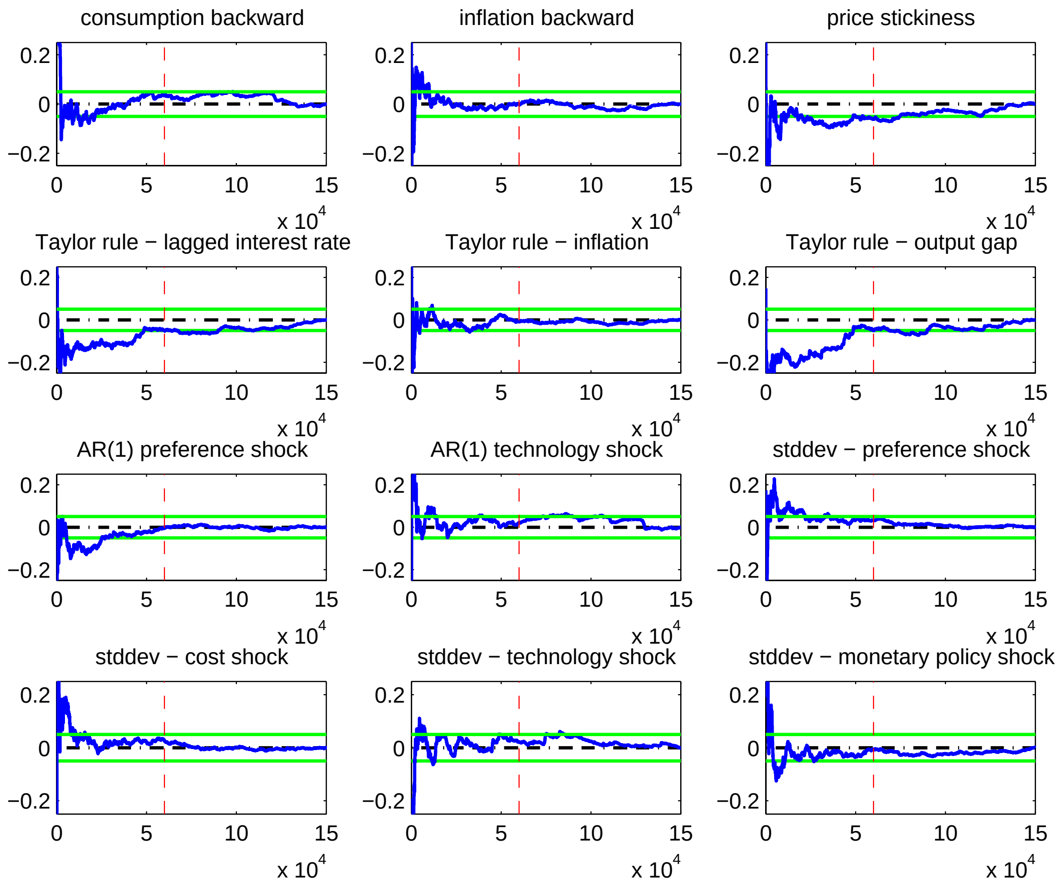
The Metropolis-Hastings sampler that is employed in order to generate random draws from the unknown posterior distribution falls in the class of Markov Chain Monte Carlo (MCMC) methods.[23] Essentially, the sampler generates draws from a *candidate generating density*[24] (a Markov chain) that is not identical to the posterior but one that 'wanders' over the posterior. The candidate draws are then accepted with a certain probability that is highest (lowest) in areas where the posterior probability is highest (lowest). The Markov chain is serially dependent but it can be shown that under some regularity conditions it converges asymptotically to the true posterior. Hence, convergence of the Markov chain becomes an important issue for validity of the results. One suggested diagnostic tool to analyse if the chain has converged is to look at the running means (CUSUM test) of the marginal posteriors.[25] The standardised statistic used to calculate these means given N draws of the Markov chain is (Bauwens et al., 1999)

$$CS_s = \left(\frac{1}{s} \sum_{i=1}^{s} \gamma_i - \mu_\gamma \right) / \sigma_\gamma \qquad s = 1, \ldots N, \tag{33}$$

[23]See Chib and Greenberg (1995) for an introduction to Metropolis-Hastings sampling.
[24]A random walk is often taken as the candidate generating density.
[25]See Koop (2003) for an overview of diagnostic methods for MCMC samplers.

where μ_γ and σ_γ are the mean and standard deviation of the N draws respectively, and $\frac{1}{s}\sum_{i=1}^{s}\gamma_i$ is the running mean for a subset of s draws of the chain. If the chain converges, the graph of CS_s should converge smoothly to zero. On the contrary, long and regular movements away from the zero line indicate that the chain has not converged. According to Bauwens et al. (1999), a CUSUM value of 0.05 after s draws means that the estimate of the posterior expectation deviates from the final estimate after N draws by 5 percent in units of the final estimate of the posterior standard deviation. The authors consider a value of 25 percent to be a good result. Figure 6 shows the CUSUM-paths along with 5 percent bands for each parameter for 150 000 draws (note that the overall interval captured by the figure corresponds to 25 percent bands). Overall, the figure points to a satisfactory degree of convergence.

Figure 6: CUSUM-Test



Notes: The horizontal grey lines indicate 5% bands, the vertical line indicates the 40%-burn-in of the Markov chain with 150 000 simulations.

4.7 Sensitivity Analysis

In this section I report briefly how sensitive the results are to the choice of a subset of a priori fixed parameters.[26] The data do not appear to be informative about the inverse of the intertemporal elasticity of substitution σ and the inverse labour supply elasticity φ. I estimated the benchmark model including these two parameters, where a normal prior density with mean one and standard deviation 0.25 was chosen. The marginal posteriors, however, are almost congruent to the priors indicating that nothing has been learned from the data. Also including these two parameters into the estimation process left the other parameters estimates nearly unchanged.

In addition, one version of the model was estimated under the assumption that the cost shock follows a stationary AR(1)-process. The estimated mean of this persistence parameter turned out to be close to zero again leaving other parameter estimates nearly unaltered. Therefore, in order to be able to better distinguish between the persistent technology shock and the cost shock, the latter was assumed to be a white noise process in the final specification of the model.

5 Summary and Conclusions

This paper has augmented a New Keynesian sticky-price model to include endogenous persistence in consumption and inflation as well as informational delays and then estimated using Bayesian methods for Germany pre-EMU. The estimated model features a high degree of persistence in consumption and output and sizeable backward-looking behaviour in inflation. Persistence of exogenous shock processes is only important for the technology process. Prices are estimated to be fixed for 6.5 quarters on average, quantitatively similar to those of Smets and Wouters (2003) based on Euro area data. Using a conventional output-gap measure, the model can account for the acceleration phenomenon. In contrast, the output-gap measure suggested by the model, that is the deviation of output from its flexible price level, appears to be a poor estimate in this simple specification of the model.

The data clearly favour the model with delayed effects on output and inflation when compared to a model that allows interest rate movements to have contemporaneous effects on these variables. This may justify the often used identification scheme in structural VAR models, even though both models could of course be

[26] Detailed results are available from the author upon request.

wrong. Moreover, the dynamics following monetary and preference shocks are comparable between the DSGE model and an identified VAR model. The VAR, however, displays more persistence in inflation.

The estimated Taylor rule in the model confirms earlier studies by Clarida and Gertler (1996) and Bernanke and Mihov (1997) that in practice the Bundesbank's behaviour can be described well as an inflation targeting strategy. On the other hand, this paper has not directly compared the Taylor rule to a monetary-policy strategy focussed exclusively on money balances as they do not play a meaningful role in the model. A potential way to introduce money balances into the model would be to specify a utility function that is non-additively separable in consumption and money balances as in Kim (2000). Kremer et al. (2003) find that estimating such a model with purely forward-looking agents leads to the conclusion that real money balances do play an important role for inflation and output dynamics.

Finally, unlike the VAR model does the DSGE model impose a restriction that produces the correct response of inflation to a contractionary monetary impulse, namely that inflation falls. The common empirical fix is to add a commodity prices index to the VAR as this may capture inflation expectations. However, it may be interesting to further investigate on theoretical grounds[27] why the VAR generates a positive response.

[27] Giordani (2004) is one contribution in this area.

References

Adolfson, M., Lasseén, S., Lindé, J., and Villani, M. (2005). Bayesian estimation of an open economy DSGE model with incomplete pass-through. Sveriges Riksbank Working Paper No 179.

Amato, J. D. and Laubach, T. (2003). Rule-of-thumb behaviour and monetary policy. *European Economic Review*, 47(5):791–831.

Bauwens, L., Lubrano, M., and Richard, J.-F. (1999). *Bayesian Inference in Dynamic Econometric Models*. Oxford University Press.

Bergin, P. (2003). Putting the New Open Economy Macroeconomics to a test. *Journal of International Economics*, 60:3–34.

Bernanke, B. S. and Mihov, I. (1997). What does the Bundesbank target? *European Economic Review*, 41(6):1025–1053.

Beyer, A. and Farmer, R. (2004). On the indeterminacy of New Keynesian economics. European Central Bank Working Paper No 323.

Calvo, G. (1983). Staggered prices in a utility maximising framework. *Journal of Monetary Economics*, 12(3):383–398.

Campbell, J. Y. and Mankiw, N. G. (1989). Consumption, income, and interest rates: Reinterpreting the time series evidence. *NBER Macroeconomics Annual*, pages 185–216.

Canova, F. and Ortega, E. (2000). Testing calibrated general equilibrium models. In Mariano, R., Schuerman, T., and Weeks, M., editors, *Simulation-based Inference in Econometrics: Methods and Applications*. Cambridge University Press.

Canova, F. and Sala, L. (2006). Back to square one: Indentification issues in DSGE models. European Central Bank Working Paper No. 583.

Chib, S. and Greenberg, E. (1995). Understanding the Metropolis-Hastings algorithm. *The American Statistician*, 49(4):327–335.

Christiano, L., Eichenbaum, M., and Evans, C. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 118(1):1–45.

Christiano, L. and Haan, W. D. (1996). Small sample properties of GMM for business cycle analysis. *Journal of Business and Economic Statistics*, 14(3):309–327.

Clarida, R., Galí, J., and Gertler, M. (1999). The science of monetary policy: A New Keynesian perspective. *Journal of Economic Literature*, 37(4):1661–1707.

Clarida, R., Galí, J., and Gertler, M. (2001). Optimal monetary policy in open versus closed economies: An integrated approach. *American Economic Review*, 91(2):248–252.

Clarida, R. and Gertler, M. (1996). How the Bundesbank conducts monetary policy. NBER Working Paper No. 5581.

DeJong, D., Ingram, B., and Whiteman, C. (2000a). A Bayesian approach to dynamic macroeconomics. *Journal of Econometrics*, 98(2):203–223.

DeJong, D., Ingram, B., and Whiteman, C. (2000b). Keynesian impulses versus Solow residuals: Identifying sources of business cycle fluctuations. *Journal of Applied Econometrics*, 15(3):311–329.

Deutsche Bundesbank (1995). *Die Geldpolitik der Bundesbank*. Deutsche Bundesbank, Frankfurt am Main.

Estrella, A. and Fuhrer, J. C. (2002). Dynamic inconsistencies: Counterfactual implications of a class of rational expectations models. *American Economic Review*, 92(4):1013–1028.

Favero, C. (2001). *Applied Macroeconometrics*. Oxford University Press.

Fuhrer, J. C. and Moore, G. R. (1995). Inflation persistence. *Quarterly Journal of Economics*, 110(1):127–159.

Galí, J. (1999). Technology, employment, and the business-cycle: Do technology shocks explain aggregate fluctuations? *American Economic Review*, 89(1):249–271.

Galí, J. (2003). New perspectives on monetary policy, inflation and the business cycle. In Dewatripont, M., Hansen, L., and Turnovsky, S., editors, *Advances in Economics and Econometrics: Theory and Applications. Eighth World Congress*, volume III, pages 151–197. Cambridge University Press.

Galí, J. and Gertler, M. (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics*, 44(2):195–222.

Galí, J., Gertler, M., and López-Salido, J. D. (2001). European inflation dynamics. *European Economic Review*, 45(7):1237–1270.

Galí, J., Gertler, M., and López-Salido, J. D. (2003). Erratum: European inflation dynamics. *European Economic Review*, 47(4):759–760.

Gamerman, D. (1997). *Markov Chain Monte Carlo. Stochastic Simulation for Bayesian Inference*. Chapman & Hall.

Gelfand, A. E. and Dey, D. K. (1994). Bayesian model choice: Asymptotics and exact calculations. *Journal of the Royal Statistical Society Series B*, 56(3):501–514.

Gerberding, C., Worms, A., and Seitz, F. (2004). How the Bundesbank really conducted monetary policy: An analysis based on real-time data. Discussion Paper Series 1: Studies of the Economic Research Center No 25/2004, Deutsche Bundesbank.

Geweke, J. (1999). Using simulation methods for Bayesian econometric models: Inference, development and communication. *Econometric Reviews*, 18(1):1–126.

Giordani, P. (2004). An alternative explanation of the price puzzle. *Journal of Monetary Economics*, 51(6):1271–1296.

Goodhart, C. A. E. (1997). Why do monetary authorities smooth interest rates? In Collignon, S., editor, *European Monetary Policy*. Pinter, London and Washington.

Harvey, A. C. (1989). *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge University Press.

Justiniano, A. and Preston, B. (2004). Small open economy DSGE models: Specification, estimation and model fit. Manuscript IMF.

Kass, R. E. and Raftery, A. E. (1995). Bayes factors. *Journal of the American Statistical Association*, 90(430):773–795.

Kim, J. (2000). Constructing and estimating a realistic optimising model of monetary policy. *Journal of Monetary Economics*, 45(2):329–360.

Kim, K. and Pagan, A. (1994). The econometric analysis of calibrated macroeconomic models. In Pesaran, H. and Wickens, M., editors, *Handbook of Applied Econometrics*, volume 1. Blackwell Press, London.

Koop, G. (2003). *Bayesian Econometrics*. John Wiley.

Kremer, J., Lombardo, G., and Werner, T. (2003). Money in a new Keynesian model estimated with german data. Discussion Paper Series 1, Studies of the Economic Research Center No 15/2003, Deutsche Bundesbank.

Levin, A. T., Onatski, A., Williams, J. C., and Williams, N. (2005). Monetary policy under uncertainty in micro-founded macroeconometric models. mimeo.

Lindé, J. (2005). Estimating new Keynesian Phillips curves: A full information maximum likelihood approach. *Journal of Monetary Economics*, 52(6):1135–1149.

Lubik, T. and Schorfheide, F. (2005). A Bayesian look at new open economy macroeconomics. *NBER Macroeconomics Annual (forthcoming)*.

Lubik, T. and Schorfheide, F. (2006). Do central banks respond to exchange rate movements? A structural investigation. *Journal of Monetary Economics (forthcoming)*.

McCallum, B. T. and Nelson, E. (1999). Nominal income targeting in an open-economy optimizing model. *Journal of Monetary Economics*, 43(3):553–578.

Otrok, C. (2001). On measuring the welfare cost of business cycles. *Journal of Monetary Economics*, 47(1):61–92.

Rabanal, P. and Rubio-Ramírez, J. F. (2005). Comparing new Keynesian models of the business cycle: A Bayesian approach. *Journal of Monetary Economics*, 52(6):1151–1166.

Rotemberg, J. J. and Woodford, M. (1997). An optimization-based econometric framework for the evaluation of monetary policy. *NBER Macroeconomics Annual*, 12:297–346.

Sargent, T. (1989). Two models of measurements and the investment accelerator. *Journal of Political Economy*, 97(2):251–287.

Sbordone, A. M. (2002). Prices and unit labor costs: A new test of price stickiness. *Journal of Monetary Economics*, 49(2):265–292.

Sims, C. A. (2002). Solving linear rational expectations models. *Computational Economics*, 20(1-2):1–20.

Sims, C. A. and Zha, T. (1999). Error bands for impulse responses. *Econometrica*, 67(5):1113–1155.

Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1(5):1123–1175.

Smets, F. and Wouters, R. (2004). Forecasting with a Bayesian DSGE model: An application to the euro area. *Journal of Common Market Studies*, 42(4):841–867.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214.

Walsh, C. E. (2003). *Monetary Theory and Policy*. The MIT Press, Cambridge, Massachusetts; London, England, 2 edition.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, New Jersey.

Appendices

A Model with Delays

A.1 Matrix Representation

Recall equation (19) from section 2 of the main text.

$$\Gamma_0(\xi)s_t = \Gamma_1(\xi)s_{t-1} + \Psi z_t + \Pi\vartheta_t$$

state vector $s_t = (y_t, \pi_t, i_t, a_t, g_t, E_t y_{t+1}, E_t y_{t+2}, E_t\pi_{t+1}, E_t\pi_{t+2}, E_t i_{t+1}, E_t mc_{t+1})'$

$$\Gamma_0 = \begin{array}{c}
\begin{array}{cccccccccccc} y_t & \pi_t & i_t & mc_t & a_t & g_t & y_t^1 & y_t^0 & \pi_t^1 & \pi_t^0 & i_t^0 & mc_t^0 \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & \frac{-\alpha_y\delta}{\sigma} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-(1-f_i)f_y & -(1-f_i)f_\pi & 1 & 0 & (1-f_i)f_y\psi_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1-\alpha+\alpha\sigma+\varphi}{\alpha} & 0 & 0 & 1 & 1+\varphi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}$$

Endogenous Errors

$y_t = y_{t-1}^1 + \eta_t^{y1}$, where $y_t^1 = E_t y_{t+1}$, $y_t^1 = y_{t-1}^0 + \eta_t^{y0}$, where $y_t^0 = E_t y_{t+1}^1$ $(= E_t y_{t+2})$

$\pi_t = \pi_{t-1}^1 + \eta_t^{\pi 1}$, where $\pi_t^1 = E_t\pi_{t+1}$, $\pi_t^1 = \pi_{t-1}^0 + \eta_t^{\pi}$, where $\pi_t^0 = E_t\pi_{t+1}^1$ $(= E_t\pi_{t+2})$

$mc_t = mc_{t-1}^0 + \eta_t^{mc}$, where $mc_t^0 = E_t mc_{t+1}$

$i_t = i_{t-1}^0 + \eta_t^i$, where $i_t^0 = E_t i_{t+1}$

$$
\Gamma_1 = \begin{array}{c}
\begin{array}{cccccccccccc}
y_{t-1} & \pi_{t-1} & i_{t-1} & mc_{t-1} & a_{t-1} & g_{t-1} & y^1_{t-1} & y^0_{t-1} & \pi^1_{t-1} & \pi^0_{t-1} & i^0_{t-1} & mc^0_{t-1}
\end{array} \\
\left[\begin{array}{cccccccccccc}
1-\delta & 0 & 0 & 0 & 0 & \frac{-\rho_g^2(1-\alpha_y)\delta}{\sigma} & 0 & \delta & 0 & \frac{(1-\alpha_y)\delta}{\sigma} & -\frac{(1-\alpha_y)\delta}{\sigma} & 0 \\
0 & \gamma^b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma^f & 0 & \lambda \\
0 & 0 & f_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \gamma_g & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\end{array}
$$

$$
\Psi = \begin{array}{c}
\begin{matrix} \varepsilon_t^i & \varepsilon_t^g & \varepsilon_t^u & \varepsilon_t^a \end{matrix} \\
\begin{bmatrix}
 & & & \\
1 & & & \\
 & & & \\
 & 1 & & \\
 & & 1 & \\
 & & & 1 \\
 & & & \\
 & & & \\
 & & & \\
 & & & \\
 & & &
\end{bmatrix}
\end{array}
\qquad
\Pi = \begin{array}{c}
\begin{matrix} \eta^{y1} & \eta^{y0} & \eta^{\pi 1} & \eta^{\pi 0} & \eta^{i0} & \eta^{mc0} \end{matrix} \\
\begin{bmatrix}
 & & & & & \\
 & & & & & \\
 & & & & & \\
 & & & & & \\
 & & & & & \\
1 & & & & & \\
 & 1 & & & & \\
 & & 1 & & & \\
 & & & 1 & & \\
 & & & & 1 & \\
 & & & & & 1
\end{bmatrix}
\end{array}
$$

state vector $s_t = (y_t, \pi_t, i_t, a_t, g_t, E_t y_{t+1}, E_t y_{t+2}, E_t \pi_{t+1}, E_t \pi_{t+2}, E_t i_{t+1}, E_t mc_{t+1})$

Aggregate Demand

$$
\begin{aligned}
y_t &= \delta E_{t-1} y_{t+1} + (1-\delta) y_{t-1} - \frac{(1-\alpha_y)\delta}{\sigma} (E_{t-1} i_t - E_{t-1} \pi_{t+1} - g_t + E_{t-1} g_{t+1}) \\
\delta &= \frac{1}{1+\alpha_y}
\end{aligned}
$$

Phillips Curve

$$
\begin{aligned}
\pi_t &= \gamma^b \pi_{t-1} + \gamma^f E_{t-1}(\pi_{t+1}) + \lambda E_{t-1} mc_t + \lambda \varepsilon_t^\pi \\
\gamma^b &= \Phi^{-1} \alpha_\pi, \qquad \gamma^f = \Phi^{-1} \beta \theta, \\
\lambda &= \Phi^{-1} (1-\beta\theta)(1-\theta)(1-\alpha_\pi)\mu, \\
\Phi &= \theta + \alpha_\pi \left[1 - \theta(1-\beta)\right] \\
\mu &= \frac{\alpha}{1 + (1-\alpha)(\epsilon - 1)}
\end{aligned}
$$

In the estimation procedure the coefficients on the shocks in the aggregate demand and Phillips equation are normalised to one.

Monetary Policy

$$
i_t = f_i i_{t-1} + (1-f_i) f_\pi \pi_t + (1-f_i) f_y (y_t - \overline{y}_t) + \varepsilon_t^i
$$

Marginal Cost (without cost shock)

$$
mc_t = \frac{1 - \alpha + \alpha\sigma + \varphi}{\alpha} y_t - (1+\varphi) a_t
$$

Potential Output

$$\overline{y}_t = \psi_a a_t \qquad \psi_a = \frac{\alpha}{1 - \alpha + \alpha\sigma + \varphi}$$

Demand Shock

$$g_t = \gamma_g g_{t-1} + \varepsilon_t^g$$

Productivity Shock

$$a_t = \gamma_a a_{t-1} + \varepsilon_t^a$$

Sims' method turns out to be convenient. The solution algorithm is very fast, which is convenient here because the model needs to be solved many times, and it can handle singular Γ_0 matrices. The algorithm uses the Schur decomposition to solve the generalised eigenvalue problem $\Gamma_0 s = \lambda \Gamma_1 s$, i.e. matrices Q and Z can be found such that $Q'\Lambda Z' = \Gamma_0$, $Q'\Omega Z' = \Gamma_1$ and $Q'Q = Z'Z = I$, where Q, Z, Λ and Ω are possibly conjugate complex and Λ and Ω are upper triangular.

To demonstrate this, consider the case where Γ_0 has full rank. The dynamics of the system are governed by the eigenvalues of the $\Gamma_0^{-1}\Gamma_1$-matrix. An eigenvalue-eigenvector decomposition $\Gamma_0^{-1}\Gamma_1 = C\Lambda C^{-1}$ is calculated in order to find the stable subspace of the system. The matrix C contains the eigenvectors that are associated with the eigenvalues of the system that are collected on the diagonal of the matrix Λ. By imposing the restriction $c_i s_t = 0$ for each eigenvector that is associated with an explosive eigenvalue (i.e. $\lambda > 1$), a stationary solution can be found. The information that the algorithm reports about existence and uniqueness of the solution is then used in the estimation procedure to restrict the admissible parameter space to unique and stable solutions.

B Bayesian Concepts

B.1 Metropolis-Hastings Algorithm

As in the main text denote the data set as Y, the prior density as $\pi(\xi)$ and the likelihood as $L(Y|\xi)$. In order to obtain N random draws from the posterior density, the following algorithm is implemented:

1. Start with an initial value ξ_0 and evaluate $\pi(\xi_0)L(Y|\xi_0)$
2. For each draw s,

$$\widehat{\xi}_s = \left\{ \begin{array}{ll} \xi_{s-1} & \text{with probability } 1-\alpha(\xi_{s-1}, \xi_s^*) \\ \xi_s^* & \text{with probability } \alpha(\xi_{s-1}, \xi_s^*) \end{array} \right\},$$

   where $\xi_s^* = \widehat{\xi}_{s-1} + \nu_s$, and ν_s is called the increment random variable which is multivariate normally distributed as $\nu \sim N(0, \widehat{\Omega}_M)$. The acceptance probability is calculated as (Gamerman, 1997, Chapter 6)

$$\alpha(\xi_{s-1}, \xi_s^*) = \min\left\{ \frac{\pi(\xi_s^*)L(Y|\xi_s^*)}{\pi(\xi_{s-1})L(Y|\xi_{s-1})}, 1 \right\}.$$

   This definition ensures that the chain moves in the appropriate direction, that is it is more likely that a draw in an area of high probability is accepted. Prior to running the Markov chain the posterior mode is estimated. A possible starting vector is the mode $\widehat{\xi}_M$ and $\widehat{\Omega}_M$ is taken to be the posterior covariance matrix.

B.2 Marginal Likelihood Computation

The presentation follows Koop (2003, Chapter 5). Given the posterior simulation output $\{\xi_s\}_{s=1}^N$ for model M_j defined on the region Θ, computation of the marginal likelihood makes use of the following relationship: for any p.d.f. $f(\xi)$ with support in Θ

$$E\left\{ \frac{f(\xi)}{\pi(\xi|M_j)L(Y|\xi, M_j)} \mid Y, M_j \right\} = \frac{1}{L(Y|M_j)}$$

Hence, using the posterior simulation output the empirical counterpart is

$$\frac{1}{L(Y|M_j)} = \frac{1}{N} \sum_{s=1}^{N} \frac{f(\xi_s)}{\pi(\xi_s|M_j)L(Y|\xi_s, M_j)}.$$

Following Geweke (1999), $f(\xi)$ is taken to be a truncated normal density in order to ensure that $\frac{f(\xi)}{\pi(\xi|M_j)L(Y|\xi,M_j)}$ is finite. Next, the support of $f(\xi)$ is defined as follows:

let $\widehat{\xi}_N$ and $\widehat{\Sigma}_N$ be estimates of $E\{\xi|Y,M_j\}$ and $Var(\xi|Y,M_j)$ from the posterior estimator. Then for some probability, $p \in (0,1)$, define the support, $\widehat{\Theta}$, of $f(\xi)$ as

$$\widehat{\Theta} = \left\{ \xi : (\widehat{\xi_N} - \xi)' \widehat{\Sigma}_N^{-1} (\widehat{\xi_N} - \xi) \leq \chi^2_{1-p}(k) \right\},$$

where $\chi^2_{1-p}(k)$ is the $(1-p)$th percentile of the Chi-squared distribution with k degrees of freedom and k is the dimension of ξ. Then $f(\xi)$ is given as

$$f(\xi) = p^{-1}(2\pi)^{-k/2} \left| \widehat{\Sigma}_N^{-1} \right| \exp \left\{ -\frac{1}{2} (\widehat{\xi_N} - \xi)' \widehat{\Sigma}_N^{-1} (\widehat{\xi_N} - \xi) \right\} \mathbf{1}(\xi \in \widehat{\Theta}),$$

where $\mathbf{1}(.)$ is the indicator function.

The algorithm is as follows

1. Calculate

$$\begin{aligned} \widehat{\xi}_N &= \frac{1}{N} \textstyle\sum_{s=1}^N \xi_s \\ \widehat{\Sigma}_N &= \frac{1}{N} \textstyle\sum_{s=1}^N (\widehat{\xi_N} - \xi_s)(\widehat{\xi_N} - \xi_s)' \end{aligned}$$

2. Choose p

3. Calculate $\frac{1}{L(Y|M_j)}$ using all $\xi_s \in \widehat{\Theta}$.

C Model with Contemporaneous Effects

C.1 Estimation Results

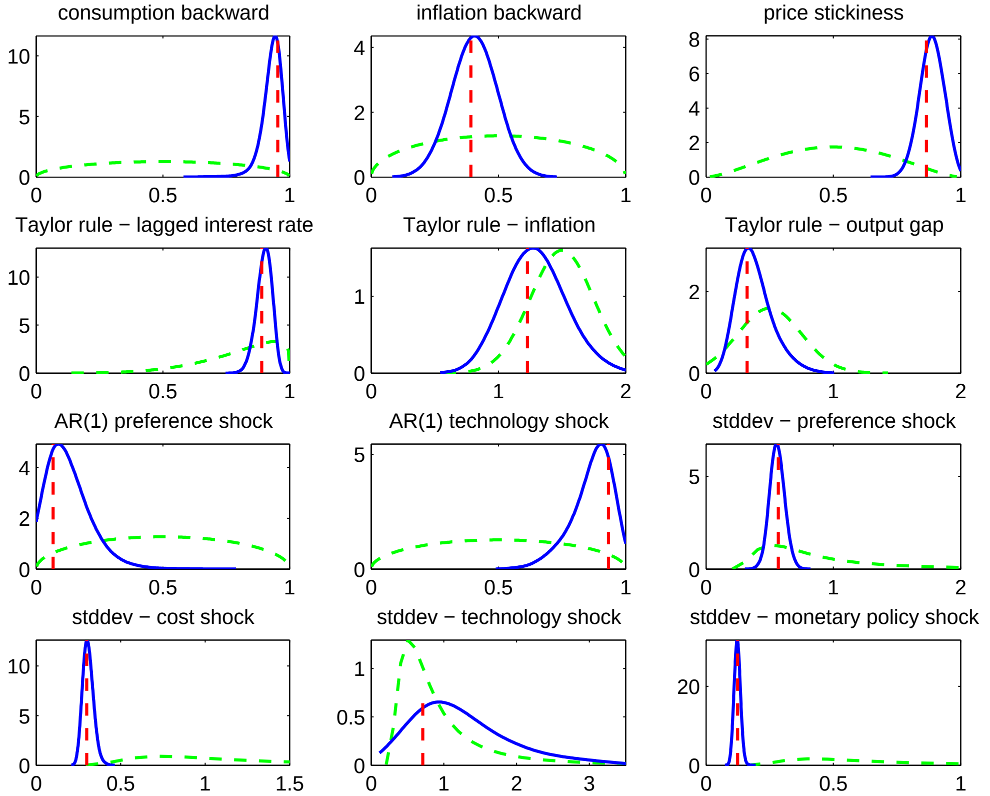
Table 5: Prior Specification and Posterior Estimates

Parameter		Prior Density	Mean	Std Dev	Posterior Estimates Mode	5%	Mean	95%
rule of thumb cons	α_y	Beta	0.50	0.25	0.95	0.87	0.93	0.98
rule of thumb infl	α_π	Beta	0.50	0.25	0.39	0.26	0.41	0.54
price stickiness	θ	Beta	0.50	0.20	0.87	0.81	0.89	0.96
Monetary Policy rule								
interest rate	f_i	Beta	0.80	0.15	0.89	0.85	0.90	0.94
inflation	f_π	Normal	1.50	0.25	1.23	0.93	1.29	1.69
output gap	f_y	Normal	0.50	0.25	0.32	0.21	0.38	0.64
shock persistence								
preference	γ_g	Beta	0.50	0.25	0.07	0.02	0.13	0.28
productivity	γ_a	Beta	0.50	0.25	0.93	0.71	0.87	0.97
shock variances			Mode	Dof*				
preference	σ_g	Inv Gamma	0.80	2.00	0.57	0.47	0.56	0.65
cost push	σ_π	Inv Gamma	1.60	2.00	0.30	0.26	0.31	0.36
productivity	σ_a	Inv Gamma	0.80	2.00	0.71	0.43	1.25	2.60
monetary policy	σ_i	Inv Gamma	0.50	2.00	0.12	0.10	0.12	0.14

*Note: Dof = degrees of freedom

C.2 Prior and Posterior Kernels

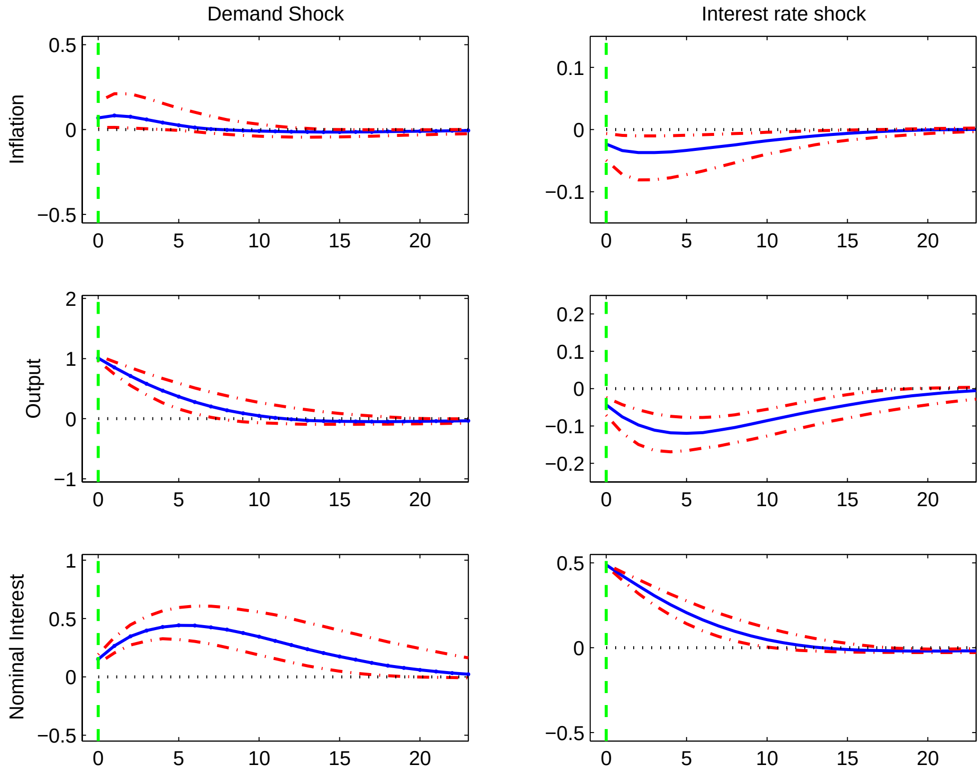
Figure 7: Contemporaneous Effects Model: Prior- and Posterior Density



Notes: Prior (dashed lines) and posterior densities (solid lines) for the DSGE model without delayed effects.

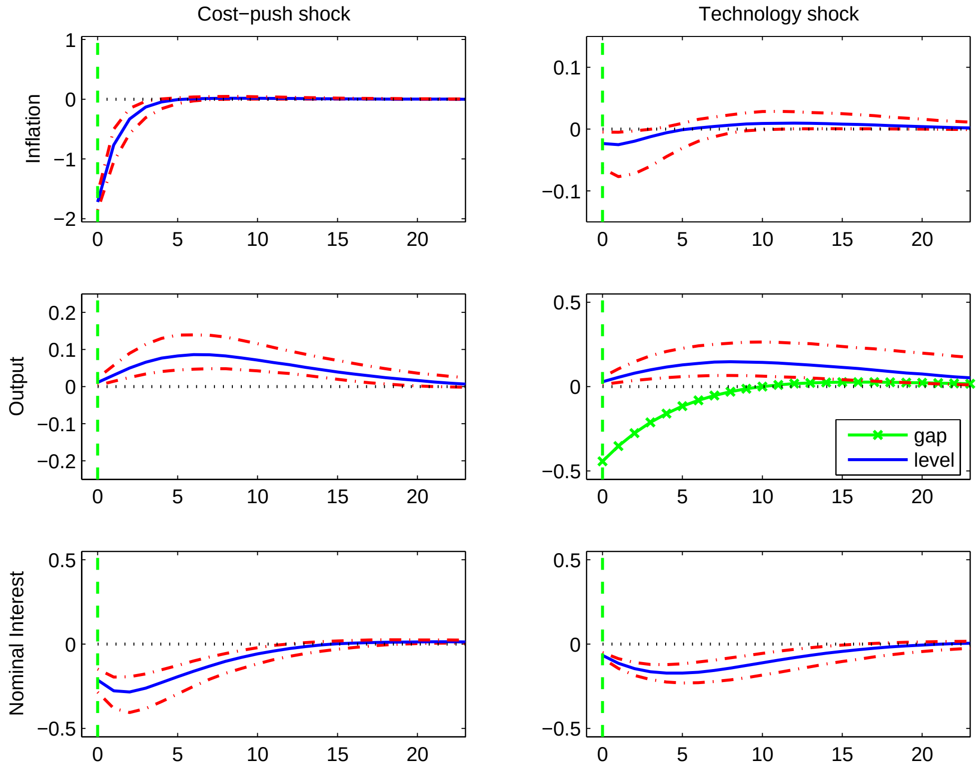
C.3 Impulse Responses

Figure 8: Demand- and Monetary Policy Shock



Notes: Impulse responses (solid lines) from the DSGE model without delays with 10- and 90%-tiles (dash-dotted lines).

Figure 9: Cost- and Technology Shock



Notes: Impulse responses (solid lines) from the DSGE model without delays with 10- and 90%-tiles (dash-dotted lines).

European Central Bank Working Paper Series

For a complete list of Working Papers published by the ECB, please visit the ECB's website (http://www.ecb.int)

585 "Are specific skills an obstacle to labor market adjustment? Theory and an application to the EU enlargement" by A. Lamo, J. Messina and E. Wasmer, February 2006.

586 "A method to generate structural impulse-responses for measuring the effects of shocks in structural macro models" by A. Beyer and R. E. A. Farmer, February 2006.

587 "Determinants of business cycle synchronisation across euro area countries" by U. Böwer and C. Guillemineau, February 2006.

588 "Rational inattention, inflation developments and perceptions after the euro cash changeover" by M. Ehrmann, February 2006.

589 "Forecasting economic aggregates by disaggregates" by D. F. Hendry and K. Hubrich, February 2006.

590 "The pecking order of cross-border investment" by C. Daude and M. Fratzscher, February 2006.

591 "Cointegration in panel data with breaks and cross-section dependence" by A. Banerjee and J. L. Carrion-i-Silvestre, February 2006.

592 "Non-linear dynamics in the euro area demand for M1" by A. Calza and A. Zaghini, February 2006.

593 "Robustifying learnability" by R. J. Tetlow and P. von zur Muehlen, February 2006.

594 "The euro's trade effects" by R. Baldwin, comments by J. A. Frankel and J. Melitz, March 2006

595 "Trends and cycles in the euro area: how much heterogeneity and should we worry about it?" by D. Giannone and L. Reichlin, comments by B. E. Sørensen and M. McCarthy, March 2006.

596 "The effects of EMU on structural reforms in labour and product markets" by R. Duval and J. Elmeskov, comments by S. Nickell and J. F. Jimeno, March 2006.

597 "Price setting and inflation persistence: did EMU matter?" by I. Angeloni, L. Aucremanne, M. Ciccarelli, comments by W. T. Dickens and T. Yates, March 2006.

598 "The impact of the euro on financial markets" by L. Cappiello, P. Hördahl, A. Kadareja and S. Manganelli, comments by X. Vives and B. Gerard, March 2006.

599 "What effects is EMU having on the euro area and its Member Countries? An overview" by F. P. Mongelli and J. L. Vega, March 2006.

600 "A speed limit monetary policy rule for the euro area" by L. Stracca, April 2006.

601 "Excess burden and the cost of inefficiency in public services provision" by A. Afonso and V. Gaspar, April 2006.

602 "Job flow dynamics and firing restrictions: evidence from Europe" by J. Messina and G. Vallanti, April 2006.

603 "Estimating multi-country VAR models" by F. Canova and M. Ciccarelli, April 2006.

604 "A dynamic model of settlement" by T. Koeppl, C. Monnet and T. Temzelides, April 2006.

605 "(Un)Predictability and macroeconomic stability" by A. D'Agostino, D. Giannone and P. Surico, April 2006.

606 "Measuring the importance of the uniform nonsynchronization hypothesis" by D. A. Dias, C. Robalo Marques and J. M. C. Santos Silva, April 2006.

607 "Price setting behaviour in the Netherlands: results of a survey" by M. Hoeberichts and A. Stokman, April 2006.

608 "How does information affect the comovement between interest rates and exchange rates?" by M. Sánchez, April 2006.

609 "The elusive welfare economics of price stability as a monetary policy objective: why New Keynesian central bankers should validate core inflation" by W. H. Buiter, April 2006.

610 "Real-time model uncertainty in the United States: the Fed from 1996-2003" by R. J. Tetlow and B. Ironside, April 2006.

611 "Monetary policy, determinacy, and learnability in the open economy" by J. Bullard and E. Schaling, April 2006.

612 "Optimal fiscal and monetary policy in a medium-scale macroeconomic model" by S. Schmitt-Grohé and M. Uribe, April 2006.

613 "Welfare-based monetary policy rules in an estimated DSGE model of the US economy" by M. Juillard, P. Karam, D. Laxton and P. Pesenti, April 2006.

614 "Expenditure switching vs. real exchange rate stabilization: competing objectives for exchange rate policy" by M. B. Devereux and C. Engel, April 2006.

615 "Quantitative goals for monetary policy" by A. Fatás, I. Mihov and A. K. Rose, April 2006.

616 "Global financial transmission of monetary policy shocks" by M. Ehrmann and M. Fratzscher, April 2006.

617 "New survey evidence on the pricing behaviour of Luxembourg firms" by P. Lünnemann and T. Y. Mathä, May 2006.

618 "The patterns and determinants of price setting in the Belgian industry" by D. Cornille and M. Dossche, May 2006.

619 "Cyclical inflation divergence and different labor market institutions in the EMU" by A. Campolmi and E. Faia, May 2006.

620 "Does fiscal policy matter for the trade account? A panel cointegration study" by K. Funke and C. Nickel, May 2006.

621 "Assessing predetermined expectations in the standard sticky-price model: a Bayesian approach" by P. Welz, May 2006.

ISSN 1561081-0