



EUROPEAN CENTRAL BANK

EUROSYSTEM

**WORKING PAPER SERIES**

**NO 740 / MARCH 2007**

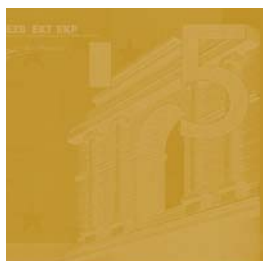
**TRANSITION ECONOMY  
CONVERGENCE IN A  
TWO-COUNTRY MODEL  
IMPLICATIONS FOR  
MONETARY INTEGRATION**

by Jan Brůha  
and Jiří Podpiera



EUROPEAN CENTRAL BANK

EUROSYSTEM



## WORKING PAPER SERIES

NO 740 / MARCH 2007

# TRANSITION ECONOMY CONVERGENCE IN A TWO-COUNTRY MODEL IMPLICATIONS FOR MONETARY INTEGRATION <sup>1</sup>

by Jan Brůha <sup>2</sup>  
and Jiří Podpiera <sup>2</sup>



In 2007 all ECB publications feature a motif taken from the €20 banknote.

This paper can be downloaded without charge from <http://www.ecb.int> or from the Social Science Research Network electronic library at [http://ssrn.com/abstract\\_id=971017](http://ssrn.com/abstract_id=971017).

<sup>1</sup> This research was partially conducted during the authors' fellowship at the ECB DG Research under the ESCB/IO programme. We thank to Michal Andrlé, Michal Kejak, Andrea Ricci, Frank Smets, and Philippe Weil for helpful discussions over earlier drafts presented on various occasions. The opinions expressed herein are those of the authors and do not necessarily reflect those of the European Central Bank or the Czech National Bank.

<sup>2</sup> External Economic Relations Division, Czech National Bank, Na Příkopě 28, I 15 03 Praha 1, Czech Republic; e-mail addresses: [jan.bruha@cnb.cz](mailto:jan.bruha@cnb.cz); [jiri.podpiera@cnb.cz](mailto:jiri.podpiera@cnb.cz)

© European Central Bank, 2007

**Address**

Kaiserstrasse 29  
60311 Frankfurt am Main, Germany

**Postal address**

Postfach 16 03 19  
60066 Frankfurt am Main, Germany

**Telephone**

+49 69 1344 0

**Internet**

<http://www.ecb.int>

**Fax**

+49 69 1344 6000

**Telex**

411 144 ecb d

*All rights reserved.*

*Any reproduction, publication and reprint in the form of a different publication, whether printed or produced electronically, in whole or in part, is permitted only with the explicit written authorisation of the ECB or the author(s).*

*The views expressed in this paper do not necessarily reflect those of the European Central Bank.*

*The statement of purpose for the ECB Working Paper Series is available from the ECB website, <http://www.ecb.int>.*

ISSN 1561-0810 (print)  
ISSN 1725-2806 (online)

# CONTENTS

Abstract	4
Non-technical summary	5
1 Introduction	6
2 Some stylized facts	9
3 The two-country model	12
3.1 Firms	12
3.1.1 Market structure	16
3.1.2 Optimal plans	17
3.2 Household behavior	20
3.3 General equilibrium	22
3.4 Note on real exchange rate	24
4 Calibration and projections	24
5 Policy implications	28
6 Conclusion	31
Annex: detailed derivation of the model	32
References	44
Figures	47
European Central Bank Working Paper Series	60

---

## Abstract

In this paper we present a two-country dynamic general equilibrium model of ex ante unequally developed countries. The model explains a key feature recently observed in transition economies – the long-run trend real exchange rate appreciation – through investments into quality. Our exchange-rate projections bear important policy implications, which we illustrate on the collision between the price and nominal exchange rate criterion for the European Monetary Union in a set of selected transition economies in Central and Eastern Europe.

*Key words:* Two-country modeling, Convergence, Monetary Policy, Currency area  
*J.E.L. Classification:* E58, F15, F43

---

## Non-technical summary

The recently observed symptom of economic convergence of the European transition economies – the real exchange rate appreciation – has triggered a new stream of literature that concentrates on the role of quality and innovations in the convergence process. Namely, many empirical studies have recently suggested that the quality improvements might be in majority responsible for the rapid and continuing real exchange rate appreciation of transition countries. This, however, contrasts with the predictions of standard macroeconomic models of international trade, which would predict the opposite effect due to decreasing scarcity of products.

We devise a model that is an extension of the two-country models of the New Open Economy Macroeconomics by introducing the investment into quality explicitly and non-trivial cross-border assets ownership. On a sample calibration for five Central and Eastern European transition economies vis à vis the Euro Area we demonstrate that the model with quality investment and product variety can coherently explain both, the pace of real convergence as well as the pace of real appreciation of local currency.

Conditionally on the parameterization of the model in the past, we present the long-run convergence transition dynamics of the key endogenous variables, i.e., the real exchange rate and output, in the future. In particular, our central question is: what will be the pace of real appreciation in the future, if the countries will continue in the type of convergence observed in the past.

This question is of particular interest from the point of view of the timing of monetary union integration of the transition economies. Namely, conditionally on our model, we evaluate the probabilities of jointly fulfilling the exchange rate and inflation stability criteria for the countries in our sample. We conclude that countries with intensive growth (quality improvements) will likely encounter a policy challenge if applying for monetary union prematurely. On the contrary, countries that exhibit extensive growth would have little policy challenges to accept single currency and sustain price stability under a single currency area.

## 1 Introduction

The purpose of this paper is to analyze the potential of two-country dynamic general equilibrium modeling initiated by the so-called New International Macroeconomic (*henceforth NIM*) literature for understanding the convergence processes of emerging market economies. The paper offers a promising extension to the canonical NIM framework, which can be useful for the assessment of convergence of emerging market economies. The structural story, behind the presented extension, is shown to have important policy implications.

The NIM models have become increasingly popular in recent past. The reason is that they are able to provide a rigorous microfoundation for a bulk of observations, which are puzzling from the perspective of the standard DSGE models (such as persistent deviations from the PPP or low volatility in the relative price of nontraded goods). Thus, this type of models may be a suitable tool not only for academic curiosity to explaining certain puzzling phenomena, but also for policy purposes. Typical features of the NIM framework include monopolistic competition, heterogeneity of production entities and tradeness self-selectiveness, as in Melitz (2003). The framework is used, for example, by Ghironi and Melitz (2005) to explain international business-cycle dynamics, by Naknoi (2006) to decompose real exchange rate movements, by Bergin and Glick (2005) to study the behavior of price dispersion during episodes of international economic integration, or by Bergin and Glick (2006) to explain low degree of volatility in the relative price of nontraded goods. Since the NIM framework seems to be better microfounded than standard open-economy dynamic general equilibrium models, it seems to be more promising as a tool for welfare evaluation of policy regimes. Naknoi et al. (2005) use the NIM framework to compare benefits and costs of fixed versus flexible exchange rate regimes and Baldwin and Okubo (2005) integrate the NIM approach to a New Economic Geography model and derive a set of useful normative assessments and positive political-economy predictions of economic integration.

Recently, Bayoumi et al. (2004) construct a DSGE model with the NIM features and calibrate it for a transition economy (the Czech Republic). This is an important step, since macroeconomic dynamics of transition economies is even more puzzling from the perspectives of standard DSGE models than in the case of advanced economies. Unfortunately, the model of Bayoumi et al. (2004) does not address any specific transition feature and thus its applicability for convergence projections or policy prescriptions may be limited<sup>1</sup>. Nevertheless,

---

<sup>1</sup> Thus, it is not surprising that the model is not able to replicate the significant observed pace of the real exchange rate appreciation in Central and Eastern European countries.

the NIM framework may still be a promising tool for explaining the pace of transition countries if the framework is married with structural issues relevant for transition economies. Structural stories are better suited for understanding important phenomena of external position of emerging market economies and can provide a more solid bases for understanding and explaining the real exchange rate development.

Explaining (and forecasting) the observed trend equilibrium development in real exchange rate in many transition economies is of crucial relevance for conduct of monetary policy. The monetary policy conditions in a small open economy are composed from exchange rate and interest rate conditions. Thus, the equilibrium exchange rate development is a crucial input into the policy debate. Similarly, having joined the EU, a transition economy should at some point in time pursue membership in the monetary union. However, a significant trend real appreciation of the local currency will be likely incompatible with joint fulfilment of the nominal exchange rate stability and inflation criterion, especially for those countries who will implement a peg regime in the ERM II (mandatory Exchange Rate Mechanism prior the Euro adoption).

Recently, many authors suggest that quality improvements might play a role among determinants of real exchange rate appreciation of transition economies and the symptoms of quality investments in transition economy are found in empirical studies. Studies appealing to quality driven real exchange rate for tradables, such as Broeck and Slok (2006) or Égert and Lommatzsch (2004), find that quality improvements of tradable goods in catching-up economies is a source of the real exchange rate appreciation. Also, on the example of the Czech Republic, Podpiera (2005) shows that the large gains in physical quantities exported were concurrently observed with an improving terms of trade, which mirrors the quality improvements. At the same time, quality improvements are not accounted for by the statistical offices in transition economies, such as the Czech Republic, Hungary, Poland, Slovakia, or Slovenia, see Ahnert and Kenny (2004) for a comprehensive survey. In addition, according to the assessment of the quality bias of consumer price index in the Czech Republic, see for instance Hanousek and Filer (2004), the inflation overstatement could have been as high as 5 percentage points a year in the first decade of economic transformation. Therefore, quality-unadjusted price indexes might be well responsible for a substantial part of the pace of the real exchange rate development in a transition economy.

In order to capture the key features of emerging market economy and simulate the transition dynamics in the key macroeconomic variables in the consistent framework of general equilibrium we use a deterministic model in aggregate variables. We build our model on postulates developed by Ghironi and Melitz (2005) and extend the framework. Canonical NIM models, such as by Ghironi and Melitz (2005), can give only a limited insight to the understanding



of external position of emerging market economies. The reason is that the production side operates with one production factor (labor) only. This feature does not address additionally important factors of production capacities. Mainly, in this paper we argue that for successful replication of the pace of relative prices of goods produced in the emerging, converging, economy, in terms of goods prices of the advanced economy, it is necessary to enrich the production structure by an additional factor, which we interpret as *investments to quality*. In addition, the model allows for non trivial cross-border assets ownership, i.e., modeling foreign direct and portfolio investment. Our model is solved for the transition dynamics of a transition country, which converges to its more advanced counterpart. Thus, it contrasts with the standard DSGE models, which aim at explaining deviations from exogenously given long-run trends.

We present a calibration of the model to a set of selected transition economies of the Central and Eastern European Transition (*henceforth CEET*) Countries, comprising the fully homogenous group of Visegrad-4 countries, i.e., the Czech Republic, Hungary, Poland, and Slovakia. In addition, in order to demonstrate the differences in extensive (quantity) vs. intensive (quality) growth and convergence for the development in real exchange rate, we also calibrate the model for Slovenia, which differed from the Visegrad-4 mainly by less significant inward direct investment and real exchange rate development. We calibrate the model on a decade 1995-2005 and simulate the projections of economic convergence until 2030. We start with the period after 1995, since by that time all considered countries had completed the initial phase of transition, i.e., full liberalization. Besides in the periods prior 1995, data quality was often questioned, see Bayoumi et al. (2004).

Based on the projections, we evaluate the probability of joint fulfillment of the inflation criterion under the stability of the nominal exchange rate. We find that countries converging through intensive margin (quality improvements), such as the group of Visegrad-4 countries are likely to encounter difficulty in joint fulfillment of the two criteria by 2020. Nevertheless, if a country will pursue single currency earlier, a nominal exchange rate appreciation throughout the period of ERM II evaluation would be necessary in order to alleviate the difficulty to fulfil the inflation criterion. After the single currency adoption, however, such a country is likely to experience a positive inflation differential within the Euro Area countries. On the contrary, a country that converges through extensive economic growth, such as Slovenia, has proved to have no significant collision in stability criteria even after adoption of Euro in 2007, i.e., no significant inflation differential is predicted.

The rest of the paper is organized as follows: Section 2 describes some relevant stylized facts and Section 3 presents the two-country model. Section 4 contains calibration and explains dynamics of some of the endogenous variables. Sec-

tion 5 features policy implications and Section 6 concludes. The Appendix A contains a detailed derivation of the model, its reformulation using a recursive form and discusses numerical techniques used to solve the model.

## 2 Some stylized facts

The crucial prerequisite for a successful economic convergence of a transition country is the implementation of reforms in political, legal, and institutional infrastructure. Among the most prominent reforms that facilitate the speed-up of economic convergence is the full liberalization, i.e., price, current account, and financial account liberalization.

The external liberalization permits for trade and financial integration of the emerging economy and thus affects selectivity to trade of goods, and creates, in some circumstances, pressures on real exchange rate appreciation.

The evidence on the positive effects of current account liberalization for economic convergence is largely documented in the literature. Fischer *et al.* (1996), who used De Melo *et al.* (1996) liberalization index, which comprises degree of liberalization of internal markets, of external markets, and of private sector entry, established a positive link between the cumulative liberalization and the output dynamics in a panel of twenty transition economies. Similarly, Sachs (1996) confirms the aforementioned relation by employing index of reform constructed by the EBRD. Kaminski *et al.* (1996) also report that among other factors, liberalization and openness to international trade were the key factors underpinning the export performance in a large sample of transition economies.

In relation to income differentials elimination among less and more developed countries, liberalization is often cited as a prominent factor. For instance, Ben-David (1993) studied the income differentials within the European Economic Community and concluded that the income disparities started to diminish only after removal of the trade barriers among member countries. Similar empirical support can be found in the literature in the case of the financial account. Henry (2003) provides a sample evidence on eighteen emerging markets and shows that following financial account liberalization, the cost of capital declines and both the capital stock growth as well as output growth per worker accelerate.

The economic transformation of the countries in Central and Eastern Europe started in early 1990. The economic but mainly institutional transformation took place in the initial phase. This phase was characterized by price liberalization, liberalization of trade and foreign exchange, and privatization. The key elements of the economic transition have been completed relatively quickly.

Taking the example of the Slovenia and Visegrad-4 countries, i.e., the Czech Republic, Hungary, Poland, and Slovakia, the EBRD index of price liberalization shows that all these countries have liberalized prices to the level 3 that is comparable to advanced industrialized countries by 1993. Similarly, all these countries liberalized trade and foreign exchange until 1995 to the extent (level 4, EBRD index) considered as standard for advanced industrialized countries. And finally, the small as well as large scale privatization has been completed (level 4, EBRD index) by 1995 and 1997, respectively. For more discussion on reforms implementation, see Roland (2004).

While the prerequisites for economic convergence have been implemented relatively quickly, the actual economic convergence has been relatively slow in the initial period. Nevertheless, starting 1995, i.e., after the initial period of institutional transformation, the economic convergence speeded up. As we can see from the plots in Figure 1, since 1995 the average output per capita in PPP comparatively to the EU15 average have been uniformly steadily increasing in all countries in our sample, which is in congruence with the observation of significant growth in the GDP per employee (a proxy for labor productivity), since following 1996 the productivity started to rise quite rapidly in all five countries.<sup>2</sup>

The trend real exchange rate appreciation (also in tradables) observed in the majority of CEET economies, see Cincibuch and Podpiera (2006) for recent empirical evidence, constitutes a puzzle and renders the standard models incomplete for explanation of the transition economy dynamics. Indeed, the observed inconstancy of the real exchange rate for tradables seems to be in contradiction with the view of the traditional models of Open Economy Macroeconomics, where the purchasing power parity condition in tradable goods is a standard assumption, see for instance Edison and Pauls (1993) or Obstfeld and Rogoff (1995).

The New Open Economy Macroeconomics of two-country models, such as by Ghironi and Melitz (2005), provides a solid base for tackling some of the issues, for instance that of inconstancy of the real exchange rate for tradables and endogenously determined foreign trade. It basically allows for an endogenous short-run, and possibly persistent, deviation from purchasing power parity, i.e., for an endogenously generated Harrod-Balassa-Samuelson (HBS) effect. However, the permanent, equilibrium, trend in real exchange rate remains unaddressed. Besides the empirical evidence of small HBS type of convergence in CEETC dominates the recent literature, see for instance Mihaljek and Klau (2006) or Flek et al. (2003). As already noted, the trend equilibrium in the

---

<sup>2</sup> Note on data: All data have been taken from the IMF financial statistics and AMECO, European Commission, Eurostat database.

real exchange rate is also a puzzle for the alternative stream of two-country modeling in recent literature. Even a model applied to CEETC with various real and nominal rigidities, see Bayoumi et al. (2004), does not predict a long-run appreciation of the real exchange rate, despite its relatively rich structure.

There are also other typical macroeconomic patterns of the investigated countries other than the real exchange rate development. As we can further see from the plots, all countries exhibited relatively constant consumption to absorption ratio (around 75 per cent) and negative trade balance to output ratio (about 5 per cent on average), with a tendency to diminish towards the end of our sample period. This observation speaks for consumption smoothing, when initial borrowing took place to finance consumption. This also suggests that the supply side of the economy was insufficient at the beginning of transition, however it slowly started to improve. This could be seen from the export to output ratio. All five economies experienced an increasing economic integration and openness of their economies in terms of foreign trade. Export to GDP ratio increased by 10-20 percent over the sample period in all considered countries.

At the same time, financial integration started to deepen and all Visegrad-4 countries experienced substantial net inflow of the foreign direct investment after 1995. An exception in timing was Hungary, where the inflow started few years earlier. The direct investment was to large extent directed to export oriented industries, which had an effect on improvements in quality of products (due to competition pressures in tradable goods markets worldwide) and at the same time on improvements in trade balance of the investment receiver countries. Consequently, we document a high correlation of the size of the direct investment inflow with the size of the real appreciation of the local currency, which can be seen from the plots in Figure 1.

Thus, the Visegrad-4 countries appears as a homogenous group of transition countries that converge to the EU15 in terms of GDP per capita in PPP, real exchange rate and its financial and foreign trade integration is deepening. Slovenia also exhibits similar pattern in majority of characteristics, however is distinct in two of them, the foreign direct investment inflow and real exchange rate development. Therefore, we design a model of a transition economy that would explain the paths of convergence in major variables and address the differing observed development in real exchange rate and financial flows, which proves to be of high policy relevance.



### 3 The two-country model

This section presents the core of the two-country model. A more detailed discussion is provided by Brůha and Podpiera (2007).

The two countries are modeled in a discrete time that runs from zero to infinity. The home country is populated by a representative competitive household who has recursive preferences over discounted streams of period utilities. The period utility is derived from consumption. A similar household inhabits the foreign country. Production takes place in heterogeneous production entities called firms.<sup>3</sup>

#### 3.1 Firms

There is a continuum of firms in the domestic country. In each period there is an unbounded mass of new, ex-ante identical, entrants. Firms ex-post differ by the total factor productivity: upon entry, it draws a shock  $z$  from a distribution  $G(z)$ , which has the support on  $[z_L, z_U)$  with  $0 \leq z_L < z_U < \infty$ . This shock determines the idiosyncratic part of the firm productivity. At the end of each period, there is an exogenous probability that a firm is hit by an exit shock  $\delta$ , which is assumed to be independent on aggregate as well as individual states. Hit firms shut down.

The production function maps two inputs into two outputs. The one of the input is fixed and we label it as ‘capital’, the second of the input is variable and is labeled as ‘labor’. The variable input – labor – is available in inelastic supply in each country and is immobile between countries.

One of the output is quality  $h$  and if the firm  $j$  uses  $k_j$  units of capital, then the quality of its product is given simply as  $h_j = k_j$ . Capital investment can be thus considered as an improvement in quality. The second output is the physical quantity of produced goods  $x$ . The production function is given as follows:  $x_{jt} = z_j A_t \ell(l_{jt}, k_j)$ . The production function  $\ell$  is strictly increasing in the first argument (labor), but strictly decreasing in the second argument (capital)<sup>4</sup>. This implies that investments into quality increase the needed labor inputs to

<sup>3</sup> The production entities are called firms, however, since we aim at understanding equilibrium convergence of a transition economy, which is likely to experience a significant change in production structure, it would be appropriate to associate production entities with *production projects*.

<sup>4</sup> We require that the function  $\ell$  is strictly decreasing in the capital. If the function  $\ell$  were not decreasing in capital, the linearity of  $h_j$  in  $k_j$  would imply endogenous growth, as in Young (1998) or Baldwin, Forslid (2000).

produce physical quantities. One may think that the production of a better good requires more labor or more skilled labor. Thus, quality investment is costly for two reasons: first, it requires fixed input  $k_j$ , second more labor is required to produce better goods.

The production of the physical quantities is increasing in the level of firm total factor productivity  $A_t z_j$ , which has two components: (a) idiosyncratic component  $z_j$ , which is i.i.d. across firms and which follows distribution  $G(z)$  introduced above, and (b) the common component  $A_t$ . The total factor productivity  $A_t$  pertains to the ownerships: firms owned by the domestic household enjoy at time  $t$  the productivity  $A_t^H$ , while firms owned by the foreign household enjoy the productivity  $A_t^F$ . The productivity does not depend on the location of production or on the time of entry (the time of entry is henceforth called *vintage*) of firms.

We assume that the final output of the firm is given by the product of quality and quantity:  $q_{jt} = h_j x_{jt}$  and that this final quality-quantity bundle is what is sold at the market. This assumption reflects the nowadays standard approach of growth theoreticians, for example Young (1998). Thus, the production of the final bundle can be described as  $q_{jt} = z_j A_t f(k_j, l_{jt})$ , where  $f$  is given as  $f(k_j, l_{jt}) \equiv k_j \ell(l_{jt}, k_j)$ . We assume that the final bundle production function is increasing in both arguments and is homogenous of degree one. This places some restrictions on the quantity production function  $\ell$ ; the most important restriction is that  $\ell$  should be homogenous of degree zero.

The quality investment is a fixed factor, set at the time of entry, while labor can be freely adjusted. Given a realization of the productivity shock  $z_j$ , the probability of the exit shock  $\delta$ , and a chosen production plan, the value of a firm is determined by the stream of discounted profits.

Since the presented model involves several kinds of goods and firms, we will use indexes to distinguish among them. To make reading of the paper easier, we introduce the following convention. Firms differ by location, ownerships, and vintage. Location of firms is distinguished by superscripts  $d$  and  $f$ , where the former stands for the *domestic* and the latter for the *foreign* country. Firms owned by household from the foreign country are denoted by the superscript  $*$ , while the ownership of domestic household is given no special superscript. The vintage is denoted by Greek letters  $\tau, \sigma$ , while the real time is denoted by the Latin character  $t, v$ .

Firms produce differentiated goods, which are labeled as follows: the good produced by the firm located in the country in which the good is also sold is denoted by the superscript  $d$ , while goods imported (produced in the non-resident country) are denoted by the superscript  $m$ . The sale market is denoted by the superscript  $*$ . Namely, goods consumed by the domestic household are

without superscript, while goods consumed by the foreign household do have it.

Similarly,  $p_{jt}^d$  will denote the price of a good produced by a firm  $j$  located in the domestic country at time  $t$  sold to the domestic market,  $p_{jt}^m$  is the price of a good  $j$  imported to the domestic market from the foreign country, while  $p_{jt}^{m*}$  would be the price of a good from the domestic country to the foreign household. We further assume that prices are denominated in the currency of the market of sale.

According to the introduced convention,  $\Pi_{j\tau t}^d$  denotes a  $t$ -period profit of the firm located in the domestic country of vintage  $\tau$  and owned by the domestic household. The nominal profit  $\Pi_{j\tau t}^d$  is given as follows:

$$\Pi_{j\tau t}^d = \left[ \kappa_{jt} p_{jt}^d + (1 - \kappa_{jt}) \frac{s_t}{1 + \mathbf{t}} p_{jt}^{m*} \right] A_t^H z_j f(k_j, l_{jt}) - w_t l_{jt},$$

where  $0 \leq \kappa_{jt} \leq 1$  is a share of product  $q_{jt}$  sold in the domestic market,  $s_t$  being a nominal *foreign exchange rate*, and  $\mathbf{t} \geq 0$  represents unit iceberg exporting costs. Firms of different vintages and different ownership have different levels of investment into quality, that is why  $\Pi_{j\tau t}^d$  will be naturally different along these dimensions. Similar definitions apply to the remaining types of firms as well.

Firms may export only if special fixed costs are sunk. If a firm at the time of entry decides to sunk the fixed export costs, then it becomes eligible to export in all subsequent periods, otherwise it is for all periods not eligible to export. The export decisions of the *eligible* firms are taken on a period-by-period basis. Thus an eligible firm may decide not to export in a given period.

Unit iceberg exporting costs  $\mathbf{t}$  represents transportation costs and policy barriers such as tariffs, while the fixed export eligibility costs may represent expenditures associated with acquiring necessary expertise such as legal, business, or accounting standards of the foreign market. It is worth to note that the unit iceberg costs  $\mathbf{t}$  is related to the degree of trade frictions, while the ratio  $c^e/c^n$  speaks for the trade openness. Obviously, non-eligible firms have  $\kappa_{jt} \equiv 1$  regardless of the state of the world.

We assume that nominal investment costs take the following form:  $P_t(k + c^\xi)$ ,  $\xi \in \{e, n\}$ , where  $P_t$  represents the ‘ideal’ price index, which is the price of both consumption and investment goods. We assume that:

$$c^e > c^n > 0,$$

where the superscript refers to eligibility, i.e.  $e$  – *eligible* or  $n$  – *noneligible*: eligible firms pay larger fixed costs. This implies – as in Melitz (2003) – that in equilibrium there is an endogenous cut-off productivity value  $\bar{z}$ , such that

firms with lower idiosyncratic productivity  $z_j < \bar{z}$  will not invest to become eligible, while firms with a sufficiently high productivity level  $z_j \geq \bar{z}$  will do. In the calibration exercise, we assume that eligibility costs differ by location and ownerships. Thus, when necessary we will distinguish eligibility costs by ownerships and location indexes.

We assume that firm's manager maximizes the expected stream of discounted profits. The discounting respects the ownerships. Thus the value of the profit stream of the firm of vintage  $\tau$ , enjoying the idiosyncratic productivity level  $z_j$  and owned by the domestic household in real terms is:

$$V_\tau^d(z_j) = \max_{\xi, k, \{l_\tau\}} \sum_{t=\tau}^{\infty} (1 - \delta)^{t-\tau} \mu_\tau^t \frac{\Pi_{j\tau t}^d}{P_t} - (c^\xi + k), \quad (1)$$

where  $\frac{\Pi_{j\tau t}^d}{P_t}$  is the  $t$ -time real profit of a firm of vintage  $\tau$ , enjoying the productivity level  $j$  under the optimal production plan (derived later in Subsubsection 3.1.2), and the effective discount factor is given as  $(1 - \delta)^{\tau-t} \mu_\tau^t$ , where  $\mu_\tau^t$  is the marginal rate of intertemporal substitution between dates  $\tau$  and  $t$ . The rate of the intertemporal substitution is defined in Subsection 3.2.

The value of the firm owned by the foreign household is defined analogously with the exception that the marginal rate of the intertemporal substitution is taken from the perspective of the foreign household.

To summarize the sequencing, the timing proceeds first with the domestic and foreign households' decision about a number of new entrants in both countries. Then, each new entrant draws a productivity level from the distribution  $G$  and the owner decides the amount of investment into quality and whether to invest for export eligibility. Then labor demand and production (of both entrants and incumbents) take place<sup>5</sup>. At the end of the period, some firms experience the exit shock and shut down.

Even firms located in the same country and owned by the same household differ along two dimensions: idiosyncratic productivity variance  $z_j$  and vintage  $\tau$ . The ownership within each country affects the amount of investment into quality, since both households have different rates of the intertemporal substitution along the transition path. Likewise the vintage affects incentives to invest. This implies that firms of different vintages and ownership will invest different amounts into quality, even if they experience the same idiosyncratic productivity level. Therefore we shall define the time-varying distribution measure over firms:  $\Gamma_t^d(j, \tau)$  for the firms in the home country owned by the domestic household and the star version  $\Gamma_t^{d*}(j, \tau)$  will denote the analogous measure for

<sup>5</sup> The capital is firm specific and the model lacks the usual one-lag time-to-build assumption. The time-to-build is not needed in our model since we aim at long-run dynamics, not at short-run fluctuations.



the firms owned by the foreign household. The counterparts of firms located in the foreign country are denoted by  $\Gamma_t^f(j, \tau)$ , and  $\Gamma_t^{f^*}(j, \tau)$ . The superscript convention applied to the distributions follows the one applied to firms.

### 3.1.1 Market structure

The final good  $Q$  in home country<sup>6</sup> is composed of a continuum of intermediate goods, some of them are produced in the home country and some are imported. There is an imperfect substitution among these goods. The parameter  $\theta > 1$  measures substitution among goods. The limit case  $\theta \rightarrow \infty$  implies perfect substitution and hence perfect competition. The aggregate good in the domestic country is defined as:

$$Q_t = \left( \sum_{\xi \in \{d, d^*\}} \int_{\Omega^\xi} (q_{jt}^d)^{\frac{\theta-1}{\theta}} d\Gamma_t^\xi(j, \tau) + \sum_{\xi \in \{f, f^*\}} \int_{\Omega_e^\xi} (q_{jt}^m)^{\frac{\theta-1}{\theta}} d\Gamma_t^\xi(j, \tau) \right)^{\frac{\theta}{\theta-1}}, \quad (2)$$

where,  $q_j$  is the output of the firm  $j$ ,  $\Omega^d$  denotes the set of products of firms located in the domestic country and owned by the domestic household and  $\Omega^{d^*}$  denotes the set of products of firms located in the domestic country and owned by the foreign household. Analogously, for sets of firms located in the foreign country we have:  $\Omega^f, \Omega^{f^*}$ . If a set is labeled by the subscript  $e$ , it reads as a subset of eligible firms: thus  $\Omega_e^{f^*} \subset \Omega^{f^*}$  is the subset of goods produced by *eligible* firms owned by the foreign household located in the foreign country.<sup>7</sup> The final good in the foreign country is defined similarly. The market structure implies the following definition of the aggregate price index:

$$P_t = \left( \sum_{\xi \in \{d, d^*\}} \int_{\Omega^\xi} (p_{jt}^d)^{1-\theta} d\Gamma_t^\xi(j, \tau) + \sum_{\xi \in \{f, f^*\}} \int_{\Omega_e^\xi} (p_{jt}^m)^{1-\theta} d\Gamma_t^\xi(j, \tau) \right)^{\frac{1}{1-\theta}},$$

where  $p_{jt}$  is the price of products of firm  $j$  at time  $t$ .

The CES market structure implies that the demand for individual producer's products in the domestic market satisfies:

$$\begin{aligned} q_{jt}^d &= \left( \frac{p_{jt}^d}{P_t} \right)^{-\theta} Q_t, \\ q_{jt}^m &= \left( \frac{p_{jt}^m}{P_t} \right)^{-\theta} Q_t. \end{aligned}$$

<sup>6</sup> The final good is consumption as well as investment good, so that  $Q$  can be interpreted as domestic absorption.

<sup>7</sup> It holds that  $q_j^d \in \Omega^d$  or  $p_j^d \in \Omega^{d^*}$  and  $q_j^{m^*} \in \Omega_e^d, q_j^{m^*} \in \Omega_e^{d^*}$ , but  $q_j^{m^*} \notin \Omega^d \setminus \Omega_e^d$  nor  $q_j^{m^*} \notin \Omega^{d^*} \setminus \Omega_e^{d^*}$ .

Analogous formulae apply to the demand for the products in the foreign market as well.

### 3.1.2 Optimal plans

The optimal production and investment plans are derived using backward induction. We present the derivation for a firm located in the domestic country and owned by the domestic household. The reader can then similarly derive optimal plans for other types of firms.

Thus, let us assume the problem of maximizing the value of a firm, under given location, ownership, and sunk investments. Since there are no labor adjustment costs, labor decisions are made on a period-by-period basis. Standard results of monopolistically competitive pricing under the CES market structure suggest that prices are set as a mark-up over marginal costs. Nevertheless, an important issue here is that the standard assumption of symmetric equilibrium is given up: firms enjoying identical productivity levels  $z_j$  and identical capital levels  $k_j$  are supposed to price identically, but firms with different characteristics charge different prices  $\{p_{jt}^d, p_{jt}^{m*}\}$ , and obviously produce different output  $q_{jt}$ .

Simultaneously with prices, firms also decide  $\kappa_j$ . Brůha and Podpiera (2007) show that – for a general neoclassical production function  $f$  – eligible firms would produce goods for both markets, i.e.,  $0 \leq \kappa_{jt} \leq 1$  for an eligible firm. This part of the paper derives the optimal production plan for such a general production function. See Appendix A.1 for the derivation of the model for the specific parametrization used in calibration and policy scenario. We denote *real* quantities by the *blackboard* scripts:  $\mathbb{P}_{j\tau t}^d \equiv \Pi_{j\tau t}^d / P_t$  is the real profit of a domestic firm and  $\mathbb{W}_t \equiv w_t / P_t$  is the real domestic wage.

Now, let us take the perspective of a non-eligible firm of vintage  $\tau$  and productivity level  $A_t^H$ . Its real profit  $\mathbb{P}_{j\tau t}^{dn}$  in a period  $t$  is given – conditional on non-eligibility status, aggregate productivity, idiosyncratic productivity  $z_j$ , – as a solution to the following program:

$$\begin{aligned} \mathbb{P}_{j\tau t}^{dn} &= \max_{l_{jt}} \left\{ \frac{p_{jt}}{P_t} A_t^H z_j f(k_j, l_{jt}) - \mathbb{W}_t l_{jt} \right\} = \\ &= \max_{l_{jt}} \left\{ \left[ A_t^H z_j f(k_j, l_{jt}) \right]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathbb{W}_t l_{jt} \right\}. \end{aligned} \quad (3)$$

The second row of Expression in (3) and in the subsequent expression follows from the CES market structure. Similarly, the real profit of an eligible firm

$\mathbb{P}_{j\tau t}^{de}$  of vintage  $\tau$  in a period  $t$  is given by:

$$\begin{aligned}\mathbb{P}_{j\tau t}^{de} &= \max_{l_{jt}} \left\{ \left( \kappa_{jt} \frac{p_{jt}}{P_t} + (1 - \kappa_{jt}) \frac{\eta_t}{1+t} \frac{p_{jt}^*}{P_t^*} \right) A_t^H z_j f(k_j, l_{jt}) - \mathbb{W}_t l_{jt} \right\} = \quad (4) \\ &= \max_{l_{jt}} \left\{ \left( \kappa_{jt} Q_t^{\frac{1}{\theta}} + (1 - \kappa_{jt}) \frac{\eta_t}{1+t} Q_t^{*\frac{1}{\theta}} \right) [A_t^H z_j f(k_j, l_{jt})]^{\frac{\theta-1}{\theta}} - \mathbb{W}_t l_{jt} \right\}.\end{aligned}$$

The expected present value of profit streams is as follows

$$\mathbb{P}_{j\tau}^{d\xi} = \sum_{t=\tau}^{\infty} \mu_{\tau}^t (1 - \delta)^{t-\tau} \mathbb{P}_{j\tau t}^{d\xi},$$

where  $\xi \in \{n, e\}$ . The expected present values depend on idiosyncratic productivity  $z_j$ , invested capital  $k_j$ , and the future path of productivities, real wages, and demands.

The optimal investment decision of an eligible firm located in the domestic country and owned by the domestic household, which enjoys a productivity level  $z_j$ , maximizes the value of the firm, which is given as

$$\mathbf{V}_{\tau}^{de}(k_j|z_j) = \mathbb{P}_{j\tau}^{de} \left( z_j, k_j, \left\{ \mathbb{W}_{t+\tau}, Q_{\tau+t}, Q_{\tau+t}^*, A_{\tau+t}^H, \eta_{\tau+t} \right\}_{t=0}^{\infty} \right) - (c^e + k_j) \quad (5)$$

and similarly for a non-eligible firm:

$$\mathbf{V}_{\tau}^{dn}(k_j|z_j) = \mathbb{P}_{j\tau}^{dn} \left( z_j, k_j, \left\{ \mathbb{W}_{t+\tau}, Q_{\tau+t}, A_{\tau+t}^H \right\}_{t=0}^{\infty} \right) - (c^n + k_j). \quad (6)$$

Maximization of  $\mathbf{V}_{\tau}^{de}(k_j|z_j)$  (resp.  $\mathbf{V}_{\tau}^{dn}(k_j|z_j)$ ) yields the optimal demand for quality investment (capital) for eligible (resp. non-eligible) firms, and the value of a firm is:

$$V_{\tau}^{d\xi}(z_j) = \max_{k_j \geq 0} \mathbf{V}_{\tau}^{d\xi}(k_j|z_j),$$

where  $\xi \in \{e, n\}$ . The value functions  $V_{\tau}^{dn}(z_j)$ ,  $V_{\tau}^{de}(z_j)$  implicitly define the cut-off value  $\bar{z}$ , which is the least idiosyncratic shock, which makes the export-eligibility investment profitable<sup>8</sup>. Thus it is defined as

$$\bar{z}_{\tau}^d = \min_{z_j} (V_{\tau}^{de}(z_j) \geq V_{\tau}^{dn}(z_j)).$$

<sup>8</sup> It is worth to mention that the cut-off value differs across locations and vintages (since firms located in different location or firms appeared in different times face different relative prices) and across ownership (because the marginal rate of substitution is – in general – different).

The value of a firm is given by

$$V_{\tau}^d(z_j) = \max_{\xi \in \{n,e\}} V_{\tau}^{d\xi}(z_j) = \begin{cases} V_{\tau}^{de}(z_j) & \text{if } z_j \geq \bar{z}_{\tau}^d \\ V_{\tau}^{dn}(z_j) & \text{if } z_j < \bar{z}_{\tau}^d \end{cases},$$

and the expected value of a new entrant, owned by the domestic household, of vintage  $\tau$ ,  $\mathcal{V}_{\tau}^d$  is:

$$\mathcal{V}_{\tau}^d = \int_{z_L}^{z_U} V_{\tau}^d(z) G(dz). \quad (7)$$

This completes the backward induction.

The, just derived, optimal production plan naturally induces a measure over firms. We denote  $\tilde{\mathbb{P}}_{\tau,t}^d$  as the  $t$ -time expected profit of a domestically-owned firm, which enters in time  $\tau$ , expectation being taken with respect to that measure  $\tilde{\mathbb{P}}_{\tau,t}^d = \int_{z_L}^{z_U} \mathbb{P}_{j\tau t}^d G(dz_j)$ , and  $\tilde{c}_{\tau}^d$  the expected investment costs under such measure. Then:

$$\mathcal{V}_{\tau}^d = \sum_{\sigma \geq 0} \mu_{\tau}^{\tau+\sigma} (1 - \delta)^{\sigma} \tilde{\mathbb{P}}_{\tau,\tau+\sigma}^d - \tilde{c}_{\tau}^d.$$

Similarly, one can express the expected real investment costs as

$$\tilde{c}_{\tau}^d = G(\bar{z}_{\tau}^d) c^n + (1 - G(\bar{z}_{\tau}^d)) c^e + \int_{z_L}^{\bar{z}_{\tau}^d} k_j^{opt,n} G(dz) + \int_{\bar{z}_{\tau}^d}^{z_U} k_j^{opt,e} G(dz).$$

The first two terms correspond to the expected fixed costs, while the last two terms correspond to the expected costs of capital investment. The expected investment costs differ across locations, vintages and ownerships and this is because (i) the cut-off values differ across these dimensions too (as was already described) and (ii) these dimensions also variate the optimal amount of invested capital  $k_j^{opt,e}$  and  $k_j^{opt,n}$ . When calibrating the model, we allow that  $c^e$  to be different for domestic and foreign firms. It is intuitive to expect that it may be comparatively less costly to sunk export costs for foreignly-owned firms operating in the domestic markets than for domestically-owned firms. This adds an additional reason, why the expected investment costs differ across locations and ownerships.

Therefore – in accordance to the convention introduced above – we will denote expected investment costs in the domestic country from the perspective of the domestic household  $\tilde{c}_t^d$  and from the perspective of the foreign household  $\tilde{c}_t^{d*}$ . The counterpart of these costs in the foreign country will be denoted as  $\tilde{c}_t^f$  (from the perspective of the domestic household) and as  $\tilde{c}_t^{f*}$  (when foreign household's perspective is taken).

### 3.2 Household behavior

The home country is populated by a representative competitive household who has recursive preferences over discounted streams of period utilities. The period utilities are derived from consumption of the aggregate good. Leisure does not enter the utility and so labor is supplied inelastically. The aggregate labor supply in the domestic country is  $\mathcal{L}$ , while  $\mathcal{L}^*$  is the aggregate labor supply in the foreign country. Households can trade bonds denominated in the foreign currency.

The domestic household maximizes

$$\max U = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

subject to

$$\begin{aligned} B_t = (1+r_{t-1}^*)B_{t-1} + \frac{1}{\eta_t} \left( -C_t + \frac{w_t}{P_t} \mathcal{L} \right) + \frac{1}{\eta_t} \left( \sum_{\sigma \leq t} (1-\delta)^{t-\sigma} n_{\sigma}^d \frac{\tilde{\Pi}_{\sigma,t}^d}{P_t} - \tilde{\chi}(n_t^d) \right) + \\ + \left( \sum_{\sigma \leq t} (1-\delta)^{t-\sigma} n_{\sigma}^f \frac{\tilde{\Pi}_{\sigma,t}^f}{P_t^*} - \tilde{\chi}(n_t^f) \right) - \frac{\Psi_B}{2} B_t^2 + \mathcal{T}_t, \end{aligned} \quad (8)$$

where  $B_t$  is the real bond holding of the domestic household. Bonds are denominated in the foreign currency by our convention, however, since the model is deterministic, this assumption is completely innocent.  $C_t$  denotes consumption and  $r_{t-1}^*$  is the real interest rate of the internationally traded bond.  $\Psi_B$  represents adjustment portfolio costs as in Schmitt-Grohe and Uribe (2003) to stabilize the model<sup>9</sup> and  $\mathcal{T}_t$  is the rebate of these costs in a lump-sum fashion to the household.

The momentary utility function  $u(C)$  is assumed to take the conventional constant-relative-risk-aversion form:  $u(C) = \frac{C^{1-\varepsilon}}{1-\varepsilon}$ , with the parameter of intertemporal substitution  $\varepsilon$ . As usually, the case of  $\varepsilon = 1$  is interpreted as  $\log(C)$ .

The number of new domestically located entrants owned by the domestic household in time  $t$  is  $n_t^d$ , while  $\tilde{\chi}(n_t^d)$  presents the investment cost associated

<sup>9</sup> In a strict sense, the model is stable even without portfolio adjustment costs (i.e. under  $\Psi_B = 0$ ). The model is deterministic and therefore it would not exhibit the unit-root behavior even under  $\Psi_B = 0$ . On the other hand, if  $\Psi_B = 0$ , then the model would exhibit the steady state dependence on the initial asset holding. Therefore we use the nontrivial adjustment costs  $\Psi_B > 0$  to give up the dependence of the steady state on the initial asset holding.

with entry of  $n_t^d$  entrants. These costs are given as follows:

$$\tilde{\chi}(n_t^d) = \tilde{c}_t^d n_t^d + \frac{\Psi_d}{2} (n_t^d)^2.$$

The first term gives the expected<sup>10</sup> investment cost (where the expectation is taken with respect to the measure induced by the optimal production plan). The second term may be interpreted as adjustment costs (e.g. due to limited supply of skills needed to run firms, such as legal experts), and its purpose is to mitigate knife-edge conditions on household investments. These adjustment costs are assumed to be rebated by the lump-sum fashion to households (e.g. they are included in  $\mathcal{T}_t$ ).

Similarly,  $n_t^f$  denotes number of new entrants in the foreign country owned by the domestic household. The associated costs are given as

$$\hat{\chi}(n_t^f) = \tilde{c}_t^f n_t^f + \frac{\Psi_f}{2} (n_t^f)^2.$$

The two functions  $\tilde{\chi}$ ,  $\hat{\chi}$  differ by the terms  $\Psi_d$  and  $\Psi_f$  only. The parameter  $\Psi_d$  is the adjustment cost of investing in the resident country (i.e., in the domestic country for the domestic household and in the foreign country for the foreign household), while the parameter  $\Psi_f$  is the adjustment cost of investing in the non-resident country. While the parameter  $\mathbf{t}$  and the ratio  $c^e/c^n$  (conditionally on the values of the remaining parameters) model trade friction and the degree of trade openness, respectively, the ratio  $\Psi_f/\Psi_d$  and the parameter  $\Psi_B$  are used to model the degree of financial openness.

The first order conditions for the domestic household are as follows:

$$u'(C_t) (1 + \Psi_B B_t) = \frac{\eta_{t+1}}{\eta_t} (1 + r_t^*) \beta u'(C_{t+1}), \quad (9)$$

$$\lim_{t \rightarrow \infty} B_{t+1} = 0, \quad (10)$$

$$\tilde{\chi}'(n_t^d) u'(C_t) = \sum_{v \geq 0} (1 - \delta)^v \beta^v u'(C_{t+v}) \frac{\tilde{\Pi}_{t,t+v}^d}{P_{t+v}},$$

$$\eta_t \hat{\chi}'(n_t^f) u'(C_t) = \sum_{v \geq 0} (1 - \delta)^v \beta^v u'(C_{t+v}) \eta_{t+v} \frac{\tilde{\Pi}_{t,t+v}^f}{P_{t+v}^*}.$$

In a strict sense, the equation (10) should read as  $\lim_{t \rightarrow \infty} B_t u'(C_t) \beta^t = 0$ , as a combination of the transversality condition and the non-Ponzi game conditions. However, because of nontrivial bond adjustment costs  $\Psi_B > 0$ , such a

<sup>10</sup> Because of the law of large numbers and of perfect foresight, the *ex-ante* expected values of the key variables for household decisions (such as investment costs or profit flows) coincide with *ex-post* realizations.

condition reduces to a simpler form of (10). The last two optimality conditions read as:

$$\tilde{c}_t^d + \Psi_d n_t^d = \sum_{v \geq 0} (1 - \delta)^v \mu_t^{t+v} \frac{\tilde{\Pi}_{t,t+v}^d}{P_{t+v}}, \quad (11)$$

$$\eta_t(\tilde{c}_t^f + \Psi_f n_t^f) = \sum_{v \geq 0} (1 - \delta)^v \eta_{t+v} \mu_t^{t+v} \frac{\tilde{\Pi}_{t,t+v}^f}{P_{t+v}^*}. \quad (12)$$

The marginal rate of substitution between times  $t_1$  and  $t_2$  is defined as:

$$\mu_{t_1}^{t_2} \equiv \beta^{t_2-t_1} \frac{u'(C_{t_2})}{u'(C_{t_1})}.$$

Although there is an idiosyncratic variance at the firm level, the model is deterministic at the aggregate level, thus the dynasty problem is deterministic too. Therefore the marginal rate of substitution does not involve the expectation operator.

The part of the model related to the foreign household is defined analogously and details of the derivations are given in Brůha and Podpiera (2007).

### 3.3 General equilibrium

The general equilibrium is defined as a time profile of prices and quantities such that all households optimize and all markets clear. Since there are no price stickiness, nominal prices are indeterminate. Therefore, only the relative prices matter. The general equilibrium requires that the market-clearing conditions hold.

The aggregate resources constraint is given as follows:

$$C_t + n_t^d \tilde{c}_t^d + n_t^{d*} \tilde{c}_t^{d*} = Q_t, \quad (13)$$

$$C_t^* + n_t^f \tilde{c}_t^f + n_t^{f*} \tilde{c}_t^{f*} = Q_t^*, \quad (14)$$

Similarly, the labor market equilibrium requires:

$$\int l_{jt} d\Gamma_t^d(j, k) + \int l_{jt} d\Gamma_t^{d*}(j, k) = \mathcal{L}, \quad (15)$$

where  $\mathcal{L}$  is the aggregate, inelastic, domestic labor supply.

Analogous market clearing conditions hold in the foreign country. The international bond market equilibrium requires that

$$B_t + B_t^* = 0. \quad (16)$$

The last equilibrium condition is the balance-of-payment equilibrium, which requires that:

$$B_{t+1} = (1 + r_t^*)B_t + X_t + (\Xi_t - \hat{\chi}(n_t^f)) - \frac{1}{\eta_t} (\Xi_t^* - \hat{\chi}(n_t^{d*})), \quad (17)$$

where  $X_t$  is the value of *net* real exports of the domestic country expressed in the foreign currency, and real profit flows are given as:

$$\Xi_t = \sum_{\sigma \leq t} (1 - \delta)^{t-\sigma} n_\sigma^f \frac{\tilde{\Pi}_{\sigma,t}^f}{P_t^*},$$

$$\Xi_t^* = \sum_{\sigma \leq t} (1 - \delta)^{t-\sigma} n_\sigma^{d*} \frac{\tilde{\Pi}_{\sigma,t}^{d*}}{P_t}.$$

The definition of the general equilibrium is standard. A more complicated task is to simulate the dynamic path, because the model is effectively a vintage type model. However, the model can be rewritten in the recursive (first-order) form, and the recursive form makes it convenient for application of a variety of efficient numerical methods. It turns out that the notorious domain-truncation approach seems to be the most efficient approach. The full set of equations of the model in the recursive form and a detailed discussion on methods are available in Appendix A.

All formulas derived and summarized in the appendix apply to a dynamic economy. The dynamic model is used to simulate an impact of an exogenous change in parameters. Thus, one can investigate e.g. consequences of a convergence of the domestic total factor productivity to the foreign level:  $A_t^H \rightarrow A^F$  in the consistent framework of the general equilibrium. After the TFP convergence is done  $A_t^H = A^F$ , both economies converge to the steady state.

The steady state is the long-run equilibrium, which is unique in the proposed model, which is obtained when exogenous parameters are constant for a sufficiently long period of time. The steady state is characterized by a number of features. The most important (and intuitive) ones include:

- Zero bond holding  $B_{ss} = 0$ , which is due to adjustment costs  $\psi_B$ .
- Constant endogenous quantities and prices.
- The steady-state effective discount factor reads as  $\frac{1}{\delta(1-\beta)}$  and the steady-state interest rate  $r_{ss} = 1/(\beta - 1)$ .
- If the net asset positions are zero, then the net exports are zero as well.
- In the steady state, the distribution of firms are degenerate over the vintage dimension: thus one can write  $\Gamma_{ss}^d(j)$  instead of  $\Gamma_{ss}^d(j, \tau)$ .



### 3.4 Note on real exchange rate

The prices  $p_{jt}$  and the corresponding price indexes  $P_t$ , and  $P_t^*$  are quality-adjusted prices. Therefore, the real real exchange rate  $\eta_t$  is measured in the terms of qualities. These measures correspond to real-world price indexes only if the latter are quality-adjusted perhaps using a hedonic approach, which is rarely the case for transition countries, see Ahnert and Kenny (2004) for a survey of quality adjustments in prices. It is a fact that price indexes in transition economies are not adjusted for quality changes.

Thus, in order to obtain indexes closer to real-world measures, we have to define aggregate indexes over prices pertaining to physical quantities. Let us denote such indexes as  $\mathcal{P}_t$  and  $\mathcal{P}_t^*$ . Ideally, one can compute these indexes based on theoretical-consistent aggregation. We use a simpler approximation instead and set

$$\mathcal{P}_t = \mathcal{K}_t P_t,$$

where  $\mathcal{K}_t$  is the total amount of quality investment by firms selling its products in the domestic country:

$$\mathcal{K}_t = \sum_{\xi \in \{d, d^*\}} \int_{\Omega^\xi} k_{j\tau} d\Gamma_t^\xi(j, \tau) + \sum_{\xi \in \{f, f^*\}} \int_{\Omega^\xi} k_{j\tau} d\Gamma_t^\xi(j, \tau).$$

Nevertheless,  $\mathcal{P}_t$  might differ from the CPI-based real-world indexes by one more term. The market structure based on the CES aggregation implies the *love-for-variety* effect. This means that the welfare-theoretical price indexes differ from the ‘average’ price by the term  $n^{\frac{1}{\theta-1}}$ , where  $n$  is the number of available varieties and  $\theta$  is the parameter of substitution in the CES function (see Melitz, 2003 for rigorous definition and derivation of the average price).

Hence, quality-unadjusted CPI-based real exchange rate (empirical real exchange rate) is the correct model counterpart of the *measured real exchange rate in reality* and is defined as  $\eta_t^e = \left(\frac{n_t^*}{n_t}\right)^{\frac{1}{\theta-1}} \frac{\mathcal{K}_t^*}{\mathcal{K}_t} \eta_t$ .

The reader is referred to Brůha and Podpiera (2007) for a more detailed discussion on real exchange rate measurements.

## 4 Calibration and projections

When we calibrate the model, we use the iso-elastic production function  $\ell(l, k) \equiv \left(\frac{l}{k}\right)^{1-\alpha}$  for production of physical quantities. This formulation implies the Cobb-Douglas production function  $f(k, l) = k^\alpha l^{1-\alpha}$  for the production of

the quality-quantity bundle. The momentary utility function is parameterized using the common constant-relative-risk-aversion form  $u(C) = (1 - \varepsilon)^{-1} C^{1-\varepsilon}$ , with the parameter of intertemporal substitution  $\varepsilon$ . The distribution  $G$  is calibrated to be uniform<sup>11</sup> on the interval  $[0, 1]$ .

The model is calibrated for the Visegrad-4 countries and Slovenia. The calibration involves primarily a path of the aggregate productivity, mark-ups, and the share of quality input in production. As can be seen from the Table 1 and Figure 2, the productivity was calibrated for three benchmark years, i.e., 1995, 2000, and 2005. The size of mark-ups was chosen as to roughly match the observed range of mark-ups in reality, i.e., between 5 and 30 per cent, for the evidence on Czech export companies, see for instance Podpiera and Raková (2006). The share of quality input in the production was allowed to vary between zero and fifty per cent and the discount factor was set between .95-.98. The probability of exit is calibrated for all countries at .5, thus expected duration of production entity is 2 periods.

The set of costs that are considered in the model are calibrated as follows: the cost for noneligible firms and eligibility costs distinguished by location of the production were chosen from a range of 0-4.5, while the adjustment costs for investments are allowed to take values from the interval 0-10. And finally, for the iceberg transportation cost calibration, we imposed a upper limit of 10 per cent.

The resulting values of the parameters, as they appear in the Table 1, have been estimated by a minimum distance estimator (see Wooldridge, 2001) when the space of possible values have been somewhat restricted to a reasonable intervals<sup>12</sup>. As it can be seen from the Figure 1, the most important differences in the calibration of the model across the five countries are in the size of investment into quality, mark-ups, investment costs (both eligibility and noneligibility), and partially also in productivity development. These differences are responsible for alternative development in the key modeled variables and therefore we will discuss their meaning in more detail.

Investment into quality, export eligibility, and entry (noneligibility) costs as well as market structure affect the development in real exchange rate, however through different channels. The quality investment is the most direct channel

---

<sup>11</sup> Microeconomists usually use other distributions than uniform for modelling the distribution of productivities across firms. The usual choice is the Pareto distribution. However, since we aim at calibrating the long-run trajectories, the uniform distribution is sufficient for that purpose.

<sup>12</sup> The calibration procedure used is responsible for differences in deep parameters, such as the discount factor, across countries. Note that it is possible to impose the common value of  $\beta = 0.95$  without a significant alternation of numerical results reported here.

for, in reality observed, real exchange rate appreciation. The second channel is the new varieties effect, facilitated by number of new entrants (dependence on costs), and finally, market structure plays a role too: the effect of varieties is also intermediated through the size of mark-ups. The higher the mark-ups the higher the incremental relative increase in number of firms in converging economy.<sup>13</sup> At the same time, the calibration of productivity path, market structure, and entry and export eligibility costs are also responsible for the dynamics in output, foreign trade, and financial flows.

The model is calibrated (fixed values of parameters throughout the transition dynamics) as to match the trends in the ratio of GDP to GDP of the EU15, real exchange rate of local currency against euro, trade balance to GDP ratio, and the ratio of consumption to absorption. The financial account balance is just the inverse of the trade balance in our model. The results of calibration are displayed in Figures 3-7. As we can see from the plots, the model succeeds to replicate the respective historical trends quite satisfactorily well. However, if the aim is a tighter fit along the transition dynamics for some reason, the time varying costs would need to be likely considered.

---

<sup>13</sup> At the same time, it is likely that extensive (intensive) economic growth will be correlated with lower (higher) mark-ups, since more of very similar goods (higher variety of similar quantity) is produced, which will result in tougher (looser) competition.

Table 1: Calibrated parameters

	Czech R.	Hungary	Poland	Slovakia	Slovenia
$\phi$	6.64	4.43	5.74	5.58	15.6
$\alpha$	0.44	0.25	0.5	0.29	0.0
$\delta$	0.5	0.5	0.5	0.5	0.5
$\beta$	0.95	0.98	0.95	0.98	0.95
$t$	0.097	0.088	0.094	0.091	0.097
$\psi_d$	0.618	0.61	0.72	0.68	0.618
$\psi_f$	9.97	10	9.99	9.99	9.97
$\psi_B$	0.02	0.02	0.02	0.02	0.02
$c^n$	2.4	2.05	2.16	3.67	2.4
$c_{f^*}^e/c^n$	2.19	2.06	1.95	1.86	2.19
$c_{d^*}^e/c^n$	3.46	4.3	2.9	2.77	3.45
$c_d^e/c^n$	3.62	1.76	3.17	3.03	3.62
$c_f^e/c^n$	3.47	3.41	3.06	2.93	3.47
$A_{2005}^H/A^F$	0.87	0.72	0.76	0.75	0.76
$A_{2000}^H/A^F$	0.83	0.66	0.68	0.66	0.69
$A_{1995}^H/A^F$	0.73	0.59	0.63	0.65	0.66
$A^F$	10	10	10	10	10
$\mathcal{L}/\mathcal{L}^*$	1/4	1/4	1/4	1/4	1/4

Note: the parameter of the relative risk aversion  $\varepsilon$  is set to 2 for all countries. This is a standard choice. The ratio  $\mathcal{L}/\mathcal{L}^*$  might seem to be too low, but – according to our numerical experiments – it is sufficient to model the notion that the foreign country is sufficiently large enough not to be significantly affected by the transition process of the emerging economy. Note also that we set different values of export eligibility  $c^e$  across location and ownership. As we already argued, this probably models the intuitive notion that the relative costs of export eligibility differ for domestically and foreignly owned firms. The subscript convention applied for  $c^e$  is following:

Countries that invested more into quality (intensive economic growth and convergence), the Visegrad-4 countries, exhibit in the model real appreciation of their currencies vis à vis the euro. On the other side, Slovenia appears as an example of extensive economic growth since a model with no investment into quality replicates well the observed historical development including stability of real exchange rate. The extensive vs. intensive growth is also confirmed by the evidence on evolvement of the share of medium-high and high tech products in particular country's export; see Fabrizio et al. (2006). According to the UN Comtrade and IMF staff calculations, the share of medium-high and high tech products in Slovenia hovers quite constantly around fifty percent over the period 1995-2004. In contrast, in all the Visegrad-4 countries they report a significant increase in the share of the medium-high and high tech products over 1995-2004: the Czech Republic by 15 p.p., Hungary by 20 p.p., Poland by 20 p.p., and Slovakia by 15 p.p.

As for the projections of the future, the structural parameters and costs remained unchanged, thus assuming prevailing conditions during 1995-2005 to continue in the future. The productivity of domestically owned firms was derived by interpolation of the benchmark years, 1995, 2000, 2005, and the terminal condition of reaching 100 per cent of the EU15 output between 2025-2030, which anchored the path of productivity in the subsequent projections until 2030. There are two reasons to assume that the productivity of the domestic companies will rise. Firstly, there are improvements in productivity in domestic firms stemming from competition exposure in an open market environment. Secondly, massive inward foreign direct investment speeds up the learning process of domestic companies as there might be taking place various spillovers from foreign firms know-how (the spillover effect has been identified as an important channel through which domestic firms benefit from foreign direct investment, see Fan, 2002). The calibration of the future path in productivity growth is then meant to capture both of these effects. The long run projections of output convergence and real exchange rate development can be seen from the Figures 8-12.

## 5 Policy implications

The provisions of the Treaty (establishing the European Community) and their applications stipulate the convergence criteria for a membership in the European Monetary Union (EMU). There are four criteria in total, i.e., inflation, exchange rate, long-term interest rate, and budget deficit criterion, see for instance the Convergence Report of the ECB (2006). The first two are, however, jointly and directly determined by the endogenous economic convergence of the candidate countries that apply for the EMU. If a country is going through a significant catching-up process through the intensive economic growth, i.e.,

via quality improvements, it is likely to experience pressures on real exchange rate appreciation that is facilitated either through nominal exchange rate appreciation or excessive inflation vis à vis the Euro Area.

Thus, one of the two (nominal exchange rate or relative inflation stability) convergence criteria is likely to be violated while a country exhibits significant real intensive margin convergence towards the EMU countries. In the case of convergence through an extensive growth, the collision of the two criteria does not appear.

While a moderate speed of convergence will be likely acceptable from the point of view of the convergence benchmarks, an expected robust convergence will be incompatible with an entry into a single currency area. However, the timing of the equilibrium convergence processes as well as the terminal point of convergence is hard to gauge and predict.

The assessment of the convergence criteria is also performed by the European Central Bank (ECB) for the participating countries in the ERM II, which is a pre step for adoption of the single currency in an applicant country. In this paper, we propose to assess the potential risks for jointly not satisfactory fulfillment of the benchmark criteria of inflation and nominal exchange rate in a certain point in time. In particular, we assume a hypothetical ERM II country that pegs its national currency to the euro at a set central rate and derive the probability of fulfillment of the inflation criterion, based on projected trajectories of the real exchange rate by the model. We use the example of the five countries for which the model was calibrated and the long run trends of the real exchange rate were obtained.

We decompose the *observed* real exchange rate  $\eta_t^e$

$$\eta_t^e = s_t \frac{\mathcal{P}_t^*}{\mathcal{P}_t}$$

into an inflation differential and nominal exchange rate as follows (after log-differencing):

$$\widehat{\eta}_t^e = \widehat{s}_t + \pi_t^* - \pi_t,$$

where  $\widehat{\eta}_t^e = \Delta \ln \eta_t^e$ ,  $\widehat{s}_t = \Delta \ln s_t$ , and  $\pi_t^* - \pi_t$  is the inflation differential between the converging country and the EMU. Based on the projections of the dynamic equilibrium path of  $\eta_t^e$ , on the condition of stable nominal exchange rate  $\widehat{s}_t = 0$ , and the price stability of the EA,  $\pi_t^* = 0.02$ , we evaluate the dynamic path for inflation of the converging country as follows:

$$\pi_t = \pi_t^* - \widehat{\eta}_t^e.$$

The path can be in turn compared against the benchmark inflation (average inflation in the three best performing Member states plus 1.5 percentage

points), i.e.,  $\pi_t^{**}$  and the sustainability can be judged based on the probability of  $\pi_t^{**} > \pi_t$ . How likely is the inflation criterion fulfilment can be derived conditionally jointly on the past variability of the (detrended) national inflation, variability of the criterion and their covariance. In terms of the probability of fulfilment of the criterion, i.e.,  $P_t(\pi_t^{**} > \pi_t | \sigma, \hat{s}_t = 0, \hat{\eta}_t^e)$ , we can test that  $P(\pi_t^{**} - \pi_t > 0)$ . We construct the following standard test statistics for country  $j$ :

$$t_j = \frac{\pi_t^{**} - \pi_t^j}{\sigma_j} \sim t(n),$$

where  $\sigma_j = \sqrt{\sigma_{**}^2 + \sigma_{j*}^2 - 2\varsigma}$ ,  $\sigma_{**}^2$  stands for the variance of detrended historical benchmark inflation and similarly  $\sigma_{j*}^2$  stands for the variance of detrended historical inflation of country  $j$ , and  $\varsigma$  is the covariance between the two variables.

The empirical evaluation of the probability of inflation criterion fulfilment is performed for all five countries in our sample. In particular,  $\sigma_{**}^2$ ,  $\sigma_{j*}^2$ , and  $\varsigma$  are evaluated using detrended (Hodrick-Prescott filter, 100) inflation (GDP deflator) over period 1995-2005. Also,  $\pi_t^{**} = 0.025$ , which corresponds to the average of the criterion in the past decade.

The Figure 13 presents the resulting probability of fulfilment of the inflation criterion of 2.5 per cent, conditionally on the simulated real exchange rate trajectory, constancy of the nominal exchange rate, and inflation and inflation's criterion variabilities. It follows from the plot that countries, such as the Visegrad-4 countries, that converge through intensive margin (improvements in quality of products) to the EU15 are unlikely to fulfill jointly the inflation and exchange rate criterion until approximately 2025. This conclusion is however conditional on the stability of the nominal exchange rate, which in reality (to the extent it would not be considered in contradiction to the exchange rate stability criterion) could facilitate a part of the real appreciation pressures stemming from real convergence via intensive margin. Thus, the apparent incompatibility of the two criteria when this type of convergence occurs could be alleviated in practise to some extent. However, after the single currency adoption this would lead to a persistent inflation differential vis-à-vis other EA countries, which might represent a policy challenge for the single currency area as well as for the country in question. Another caveat relates to the future projection of the real exchange rate path, namely, the projections hinges on the assumption that the same type of convergence will continue in the future. And finally, the terminal condition for convergence, year 2025-2030 might also be anchoring the stabilization of the real exchange rate and thus also the probability of fulfilment of the inflation criterion. A slower convergence, reaching EU15 beyond 2030 would likely increase the probability of the criterion fulfilment at earlier horizons.

Nevertheless, the simulation does not claim right for precise trajectory evaluation, it should only illustrate and contrast the differences in the type of convergence, since for Slovenia, where we observed the extensive margin convergence, the probability of the inflation criterion fulfilment remains robustly in the save region along the entire horizon considered. This could be best seen from the probability evaluation in the past. The probability of the criterion fulfilment for the Visegrad-4 countries is very low as contrary to the Slovenian case, which stipulates the differences in the type of economic convergence. It follows that the expectations about the type of convergence will predetermine the smoothness of the transition of EMU candidates through ERM II into the EA, especially in the case of the countries pursuing peg exchange rate regime.

## 6 Conclusion

In this paper we seek to explain the transition dynamics of the New EU Member States in a general equilibrium framework. In particular, we design a two-country model with heterogenous firms, explicit investment into quality, export self-selectiveness, and non-trivial cross border asset ownership. We demonstrate on a calibration for five New EU Member States that in particular quality investment (quality improvements) and market structure (creation of new varieties) might be responsible for the observed significant real exchange rate appreciation, that often remains unexplained by prevailing models.

In the next step we simulate the convergence path for output and real exchange rate based on the model's calibration. Our findings suggest that convergence of countries facilitated by an extensive growth might be compatible with stability of real exchange rate, while countries pursuing intensive growth and convergence exhibit significant real exchange rate appreciation. We believe our findings give to rise significant policy implications with regard to the monetary integration of the New EU Member States. Namely, when putting the findings in the context of the monetary union convergence criteria, countries with projected significant real appreciation will be in conflict in jointly fulfilling the exchange rate and inflation stability criteria. Even if the nominal exchange rate appreciation will facilitate the overall convergence in the assessment period, the subsequent period after adoption of the single currency will imply a significant inflation differential vis à vis the rest of the EA countries, which might be again contradictory to sustained price stability.

Even though the projected trajectories of transition dynamics pertain to the calibrated values of parameters, and thus trajectories might be to some extent uncertain, the main conclusion that the expected type of convergence (extensive vs. intensive growth) and the pace of the real appreciation will determine the smoothness of the transition from national currency to the euro remains robust.



## A Detailed Derivation of the Model

### A.1 Model Equation under Particular Functional Forms

In this part of the paper, we derive the main model equation for particular functional forms of the production function, utility function and investment cost functions. In particular, as a benchmark calibration, we use the iso-elastic production function  $\ell(l, k) \equiv \left(\frac{l}{k}\right)^{1-\alpha}$  for production of physical quantities. This formulation implies the Cobb-Douglas production function  $f(k, l) = k^\alpha l^{1-\alpha}$  for the production of the quality-quantity bundle. The momentary utility function is parameterized using the common constant-relative-risk-aversion form  $u(C) = (1 - \varepsilon)^{-1} C^{1-\varepsilon}$ , with the parameter of intertemporal substitution  $\varepsilon$ . As usually, the case of  $\varepsilon = 1$  is interpreted as  $\log(C)$ . The distribution  $G$  of idiosyncratic shocks is uniform on the interval  $[0, 1]$ .

The real cost function associated with the Cobb-Douglas production function is given as follows<sup>14</sup>:

$$\mathbb{C}(q, \mathbb{W}_t, A_t^H, z_j, k_j) = \mathbb{W}_t \left[ \frac{q}{A_t^H z_j k_j^\alpha} \right]^{\frac{1}{1-\alpha}}.$$

First, we derive the optimal investment decision, and the present value of profit flows for a non-eligible firm<sup>15</sup>. Such a firm will supply the following quantity-quality bundle  $q_{jt}^d$  to the domestic market (at time  $t$ ):

$$q_{jt}^d = \left( \left[ \frac{\theta - 1}{\theta} (1 - \alpha) \mathbb{W}_t^{-1} \left[ A_t^H z_j k_j^\alpha \right]^{\frac{1}{1-\alpha}} \right]^\theta Q_t \right)^{\frac{(1-\alpha)}{\alpha\theta + (1-\alpha)}},$$

the real turnover is:

$$\frac{p_{jt}^d}{P_t} q_{jt}^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[ \frac{\theta - 1}{\theta} (1 - \alpha) \mathbb{W}_t^{-1} \left[ A_t^H \right]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

<sup>14</sup> Recall that blackboard fonts, such as  $\mathbb{W}_t$  and  $\mathbb{P}_t$  denote real variables such as real wage and real profits. Following that convention, the blackboard  $\mathbb{C}$  denotes real cost function.

<sup>15</sup> Also, in this part of the paper, we derive expression only for firms located in the domestic country and owned by the domestic agent. The expression for other types of firms are easily derived then.

and the real profit is given by:

$$\begin{aligned}\mathbb{P}_{j\tau t}^d &= \frac{p_{jt}^d}{P_t} q_{jt}^d - \mathbb{C}(q_{jt}^d, \mathbb{W}_t, A_t^H, z_j, k_j) = \\ &= z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \left[ A_t^H \right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}} \mathcal{W}_1,\end{aligned}$$

where we define

$$\begin{aligned}\mathcal{W}_1 &\equiv \left[ \frac{\theta-1}{\theta} (1-\alpha) \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} - \left[ \frac{\theta-1}{\theta} (1-\alpha) \right]^{\frac{\theta}{(1-\alpha)+\alpha\theta}} = \\ &= \frac{\alpha(\theta-1)+1}{(\theta-1)(1-\alpha)} \left[ \frac{\theta-1}{\theta} (1-\alpha) \right]^{\frac{\theta}{(1-\alpha)+\alpha\theta}},\end{aligned}$$

which is obviously positive.

Second, we derive optimal production decisions of eligible firms. The optimal production decision implies that  $q_{jt}^d = \left[ \frac{\theta-1}{\theta} \left( \frac{MC_{jt}}{P_t} \right)^{-1} \right]^\theta Q_t$ , and  $q_{jt}^{m*} = \left[ \frac{\theta-1}{\theta} \frac{\eta_t}{1+\mathbf{t}} \left( \frac{MC_{jt}}{P_t} \right)^{-1} \right]^\theta Q_t^*$ . Some simple, but tedious, algebraic manipulations yield:

$$\kappa_{jt} q_{jt} \equiv q_{jt}^d = \left[ \frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}_t^{-1} \left( A_t^H z_j k_j^\alpha \right)^{\frac{1}{1-\alpha}} \right]^\theta \frac{Q_t}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}},$$

and

$$(1 - \kappa_{jt}) q_{jt} \equiv q_{jt}^{m*} = \left[ \frac{\theta-1}{\theta} (1-\alpha) \frac{\eta_t}{1+\mathbf{t}} \mathbb{W}_t^{-1} \left( A_t^H z_j k_j^\alpha \right)^{\frac{1}{1-\alpha}} \right]^\theta \frac{Q_t^*}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}}.$$

This implies that

$$\kappa_{jt} = \frac{Q_t}{Q_t + Q_t^* \left( \frac{\eta_t}{1+\mathbf{t}} \right)^\theta},$$

observe that  $\kappa_{jt}$  does not depend on individual characteristics of firms:  $z_j$  and  $k_j$ ; it depends only on relative tightness of both markets and on the real exchange rate corrected for transport costs  $\mathbf{t}$ . Therefore, all eligible firms will sell the same share of its products to the domestic resp. foreign markets. Thus henceforth we will simply write  $\kappa_t$  for  $\kappa_{jt}$ . Define

$$\xi_t \equiv Q_t + Q_t^* \left( \frac{\eta_t}{1+\mathbf{t}} \right)^\theta = \frac{Q_t}{\kappa_t}.$$

The total production of eligible firms can be written as follows:

$$q_{jt} = \left( z_j^\theta k_j^{\alpha\theta} \right)^{\frac{1}{(1-\alpha)+\alpha\theta}} \left\{ \left[ \frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}_t^{-1} [A_t^H]^{\frac{1}{1-\alpha}} \right]^\theta \xi_t \right\}^{\frac{(1-\alpha)}{(1-\alpha)+\alpha\theta}},$$

and real turnovers on the domestic and the foreign markets, respectively are given by:

$$\frac{p_{jt}^d}{P_t} q_{jt}^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \kappa_t^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[ \frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}_t^{-1} [A_t^H]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

$$\begin{aligned} \left( \frac{\eta_t}{1+t} \right) \frac{p_{jt}^{m*}}{P_t^*} q_{jt}^{m*} &= z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} (1-\kappa_t)^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left( \frac{\eta_t}{1+t} \right)^{\frac{\theta}{(1-\alpha)+\alpha\theta}} \times \\ &\times \left[ \frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}_t^{-1} [A_t^H]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{*\frac{1}{(1-\alpha)+\alpha\theta}}. \end{aligned}$$

Real production costs of eligible firms read as follows:

$$\mathbb{C}_{jt} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} [A_t^H]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \left\{ \left[ \frac{\theta-1}{\theta} (1-\alpha) \right]^\theta \xi_t \right\}^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

thus, the real profit in a period  $t$  is given as:

$$\mathbb{P}_{j\tau t}^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} [A_t^H]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_1 \xi_t^{\frac{1}{(1-\alpha)+\alpha\theta}}.$$

Now, we are able to derive the expected present value of profit stream. We start with an eligible firm  $\mathbb{P}_{j\tau}^{de}$ , the expected present value satisfies:

$$\mathbb{P}_{j\tau}^{de} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_1 \underbrace{\sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu_\tau^t [A_t^H]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \xi_t^{\frac{1}{(1-\alpha)+\alpha\theta}}}_{\varpi_\tau^e},$$

while the expected present value  $\mathbb{P}_{j\tau}^{dn}$  of a non-eligible firm satisfies:

$$\mathbb{P}_{j\tau}^{dn} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_1 \underbrace{\sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu_\tau^t [A_t^H]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}}}_{\varpi_\tau^n}.$$

The value of an eligible firm located in the domestic country and owned by the domestic household – which enjoys a productivity level  $z_j$  – is determined

by capital investment:

$$\mathbf{V}_\tau^{de}(k_j|z_j) = \mathbb{P}_{j\tau}^{de} - (c^e + k_j) \equiv z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi_\tau^e - (c^e + k_j);$$

and similarly for a non-eligible firm

$$\mathbf{V}_\tau^{dn}(k_j|z_j) = \mathbb{P}_{j\tau}^{dn} - (c^n + k_j) = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi_\tau^n - (c^n + k_j).$$

If firms' managers maximize the value of firms, they choose the following capital level:

$$k_j^{opt,e} = z_j^{\theta-1} \left[ \frac{\alpha(\theta-1)\varpi_\tau^e}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)+1},$$

and the value of an eligible firm is:

$$V_\tau^{de}(z_j) = \max_{k_j \geq 0} \mathbf{V}_\tau^{de}(k_j|z_j) = z_j^{(\theta-1)} [\varpi_\tau^e]^{\alpha(\theta-1)+1} \mathcal{G} - c^e,$$

where

$$\begin{aligned} \mathcal{G} &\equiv \left[ \left( \frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)} - \left( \frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)+1} \right] = \\ &= \frac{1}{\alpha(\theta-1)+1} \left( \frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)}. \end{aligned}$$

Similarly, the value of a non-eligible firm is

$$V_\tau^{dn}(z_j) = \max_{k_j \geq 0} \mathbf{V}_\tau^{dn}(k_j|z_j) = z_j^{(\theta-1)} [\varpi_\tau^n]^{\alpha(\theta-1)+1} \mathcal{G} - c^n,$$

and the optimal capital investment into quality is

$$k_j^{opt,n} = z_j^{\theta-1} \left[ \frac{\alpha(\theta-1)\varpi_\tau^n}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)+1}. \quad (\text{A.1})$$

Value functions  $V_\tau^{dn}(z_j)$ ,  $V_\tau^{de}(z_j)$  implicitly define the cut-off value  $\bar{z}$ , which is the least idiosyncratic shock, which makes the export-eligibility investment profitable. Thus it is defined as

$$\bar{z}_\tau^d = \min_{z_j} (V_\tau^{de}(z_j) \geq V_\tau^{dn}(z_j)).$$

Also for the chosen parameterization of the production function, one can derive the labor demand. The formula is complicated and is given in the next section, since it involves integration over labor demands of firms of various vintages, see (A.8), (A.9) and (A.10) below.

## A.2 Model in the Recursive Form

In this part of the paper, we show how to transform the model into the recursive (first-order) form, which is suitable for numerical evaluation. We do it for parameterization used in Section A.1. Although it is in principle possible to apply selected (but not all) numerical techniques directly to the vintage-formulation of the model, such a strategy would be very inefficient: numerical experiments suggest that the computation time is substantially reduced when the numerical techniques are applied to the recursive formulation of the model.

The first-order form consists of dynamic and static equations. These are listed below.

### A.2.1 Dynamic Equations

#### *Intertemporal Marginal Rate of Substitution*

$$\begin{aligned}\mu_t^{t+1} &= \beta \left( \frac{C_{t+1}}{C_t} \right)^\varepsilon, \\ \mu_t^{*t+1} &= \beta \left( \frac{C_{t+1}^*}{C_t^*} \right)^\varepsilon.\end{aligned}\tag{A.2}$$

#### *Profit Flows*

$$\begin{aligned}\varpi_t^{ndd} &= \mathcal{W}_1 \left( [A_t^H]^{(\theta-1)} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{ndd}, \\ \varpi_t^{edd} &= \mathcal{W}_1 \left( [A_t^H]^{(\theta-1)} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{edd}, \\ \varpi_t^{nfd} &= \mathcal{W}_1 \left( [A_t^F]^{(\theta-1)} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1}\varpi_{t+1}^{nfd}, \\ \varpi_t^{efd} &= \mathcal{W}_1 \left( [A_t^F]^{(\theta-1)} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1}\varpi_{t+1}^{efd}, \\ \varpi_t^{nff} &= \mathcal{W}_1 \left( [A_t^F]^{(\theta-1)} \mathbb{W}_t^{*-(\theta-1)(1-\alpha)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1}\varpi_{t+1}^{nff}, \\ \varpi_t^{eff} &= \mathcal{W}_1 \left( [A_t^F]^{(\theta-1)} \mathbb{W}_t^{*-(\theta-1)(1-\alpha)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1}\varpi_{t+1}^{eff}, \\ \varpi_t^{ndf} &= \mathcal{W}_1 \left( [A_t^H]^{(\theta-1)} \mathbb{W}_t^{*-(\theta-1)(1-\alpha)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{ndf}, \\ \varpi_t^{edf} &= \mathcal{W}_1 \left( [A_t^H]^{(\theta-1)} \mathbb{W}_t^{*-(\theta-1)(1-\alpha)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{edf};\end{aligned}\tag{A.3}$$

where  $\xi_t = Q_t + Q_t^* \left( \frac{\eta_t}{1+t} \right)^\theta$ , and  $\xi_t^* = Q_t^* + Q_t \left( \frac{\eta_t^{-1}}{1+t} \right)^\theta$ .

Expected value of the stream of future profits (from the unit investment now)  $\Omega_t^\circ$  are given as the sum of weighted expected values from eligible and non-eligible profits<sup>16</sup>:  $\Omega_t^{x_1x_2} = \Omega_t^{nx_1x_2} + \Omega_t^{ex_1x_2}$ , (with  $x_i \in \{d, f\}$ ). Also to make the notation as transparent as possible, henceforth the superscript *dd* denotes domestically-owned firms located in the domestic country, *fd* denotes foreignly-owned firms located in the domestic country, *ff* denotes foreignly-owned firms located in the foreign country, and *df* denotes domestically-owned firms located in the foreign country. ) and where:

$$\begin{aligned}
\Omega_t^{edd} &= \tilde{\mathbb{P}}_t^{edd} + \mu_t^{t+1}(1 - \delta)\Omega_{t+1}^{edd} \left( \frac{\varpi_t^{edd}}{\varpi_{t+1}^{edd}} \right) \frac{\int_{\bar{z}_t^{dd}}^{z^U} z^{\theta-1} G(dz)}{\int_{\bar{z}_{t+1}^{dd}}^{z^U} z^{\theta-1} G(dz)}, \\
\Omega_t^{ndd} &= \tilde{\mathbb{P}}_t^{ndd} + \mu_t^{t+1}(1 - \delta)\Omega_{t+1}^{ndd} \left( \frac{\varpi_t^{ndd}}{\varpi_{t+1}^{ndd}} \right) \frac{\int_{z_L}^{\bar{z}_t^{dd}} z^{\theta-1} G(dz)}{\int_{z_L}^{\bar{z}_{t+1}^{dd}} z^{\theta-1} G(dz)}, \\
\Omega_t^{efd} &= \frac{\tilde{\mathbb{P}}_t^{efd}}{\eta_t} + \mu_t^{*t+1}(1 - \delta)\Omega_{t+1}^{efd} \left( \frac{\varpi_t^{efd}}{\varpi_{t+1}^{efd}} \right) \frac{\int_{\bar{z}_t^{fd}}^{z^U} z^{\theta-1} G(dz)}{\int_{\bar{z}_{t+1}^{fd}}^{z^U} z^{\theta-1} G(dz)}, \\
\Omega_t^{nfd} &= \frac{\tilde{\mathbb{P}}_t^{nfd}}{\eta_t} + \mu_t^{*t+1}(1 - \delta)\Omega_{t+1}^{nfd} \left( \frac{\varpi_t^{nfd}}{\varpi_{t+1}^{nfd}} \right) \frac{\int_{z_L}^{\bar{z}_t^{fd}} z^{\theta-1} G(dz)}{\int_{z_L}^{\bar{z}_{t+1}^{fd}} z^{\theta-1} G(dz)}, \\
\Omega_t^{eff} &= \tilde{\mathbb{P}}_t^{eff} + \mu_t^{*t+1}(1 - \delta)\Omega_{t+1}^{eff} \left( \frac{\varpi_t^{eff}}{\varpi_{t+1}^{eff}} \right) \frac{\int_{\bar{z}_t^{ff}}^{z^U} z^{\theta-1} G(dz)}{\int_{\bar{z}_{t+1}^{ff}}^{z^U} z^{\theta-1} G(dz)}, \\
\Omega_t^{nff} &= \tilde{\mathbb{P}}_t^{nff} + \mu_t^{*t+1}(1 - \delta)\Omega_{t+1}^{nff} \left( \frac{\varpi_t^{nff}}{\varpi_{t+1}^{nff}} \right) \frac{\int_{z_L}^{\bar{z}_t^{ff}} z^{\theta-1} G(dz)}{\int_{z_L}^{\bar{z}_{t+1}^{ff}} z^{\theta-1} G(dz)}, \\
\Omega_t^{edf} &= \eta_t \tilde{\mathbb{P}}_t^{edf} + \mu_t^{t+1}(1 - \delta)\Omega_{t+1}^{edf} \left( \frac{\varpi_t^{edf}}{\varpi_{t+1}^{edf}} \right) \frac{\int_{\bar{z}_t^{df}}^{z^U} z^{\theta-1} G(dz)}{\int_{\bar{z}_{t+1}^{df}}^{z^U} z^{\theta-1} G(dz)}, \\
\Omega_t^{ndf} &= \eta_t \tilde{\mathbb{P}}_t^{ndf} + \mu_t^{t+1}(1 - \delta)\Omega_{t+1}^{ndf} \left( \frac{\varpi_t^{ndf}}{\varpi_{t+1}^{ndf}} \right) \frac{\int_{z_L}^{\bar{z}_t^{df}} z^{\theta-1} G(dz)}{\int_{z_L}^{\bar{z}_{t+1}^{df}} z^{\theta-1} G(dz)};
\end{aligned} \tag{A.4}$$

where definitions of expectations of profits  $\Pi_t^{xxx}$  and cut-off values will be given in the next subsection.

To get equations for actual realized profits  $\Xi_t^{x_1x_2}$ ,  $x_i \in \{d, f\}$ , we have to split into two parts (according to eligibility):  $\Xi_t^{x_1x_2} = \Xi_t^{ex_1x_2} + \Xi_t^{nx_1x_2}$ . The first-order

<sup>16</sup> Henceforth, in order to diminish the notational burden, we use  $A^\circ$  in lieu of  $\{A^{ndd}, \dots, A^{eff}\}$ .

equations are then:

$$\begin{aligned}
\Xi_{t+1}^{edd} &= (1 - \delta) \left( \frac{[A_{t+1}^H]^{\theta-1} \mathbb{W}_{t+1}^{-(\theta-1)} \xi_{t+1}}{[A_t^H]^{\theta-1} \mathbb{W}_t^{-(\theta-1)} \xi_t} \right)^{\frac{1}{\alpha(\theta-1)+1}} \Xi_t^{edd} + n_{t+1}^{edd} \tilde{\mathbb{P}}_{t+1}^{edd}, & (A.5) \\
\Xi_{t+1}^{nnd} &= (1 - \delta) \left( \frac{[A_{t+1}^H]^{\theta-1} \mathbb{W}_{t+1}^{-(\theta-1)} Q_{t+1}}{[A_t^H]^{\theta-1} \mathbb{W}_t^{-(\theta-1)} Q_t} \right)^{\frac{1}{\alpha(\theta-1)+1}} \Xi_t^{nnd} + n_{t+1}^{nnd} \tilde{\mathbb{P}}_{t+1}^{nnd}, \\
\Xi_{t+1}^{efd} &= (1 - \delta) \left( \frac{[A_{t+1}^F]^{\theta-1} \mathbb{W}_{t+1}^{-(\theta-1)} \xi_{t+1}}{[A_t^F]^{\theta-1} \mathbb{W}_t^{-(\theta-1)} \xi_t} \right)^{\frac{1}{\alpha(\theta-1)+1}} \Xi_t^{efd} + n_{t+1}^{efd} \tilde{\mathbb{P}}_{t+1}^{efd}, \\
\Xi_{t+1}^{nfd} &= (1 - \delta) \left( \frac{[A_{t+1}^F]^{\theta-1} \mathbb{W}_{t+1}^{-(\theta-1)} Q_{t+1}}{[A_t^F]^{\theta-1} \mathbb{W}_t^{-(\theta-1)} Q_t} \right)^{\frac{1}{\alpha(\theta-1)+1}} \Xi_t^{nfd} + n_{t+1}^{nfd} \tilde{\mathbb{P}}_{t+1}^{nfd}, \\
\Xi_{t+1}^{eff} &= (1 - \delta) \left( \frac{[A_{t+1}^F]^{\theta-1} \mathbb{W}_{t+1}^{*-(\theta-1)} \xi_{t+1}^*}{[A_t^F]^{\theta-1} \mathbb{W}_t^{*-(\theta-1)} \xi_t^*} \right)^{\frac{1}{\alpha(\theta-1)+1}} \Xi_t^{eff} + n_{t+1}^{eff} \tilde{\mathbb{P}}_{t+1}^{eff}, \\
\Xi_{t+1}^{nff} &= (1 - \delta) \left( \frac{[A_{t+1}^F]^{\theta-1} \mathbb{W}_{t+1}^{*-(\theta-1)} Q_{t+1}^*}{[A_t^F]^{\theta-1} \mathbb{W}_t^{*-(\theta-1)} Q_t^*} \right)^{\frac{1}{\alpha(\theta-1)+1}} \Xi_t^{nff} + n_{t+1}^{nff} \tilde{\mathbb{P}}_{t+1}^{nff}, \\
\Xi_{t+1}^{edf} &= (1 - \delta) \left( \frac{[A_{t+1}^H]^{\theta-1} \mathbb{W}_{t+1}^{*-(\theta-1)} \xi_{t+1}^*}{[A_t^H]^{\theta-1} \mathbb{W}_t^{*-(\theta-1)} \xi_t^*} \right)^{\frac{1}{\alpha(\theta-1)+1}} \Xi_t^{edf} + n_{t+1}^{edf} \tilde{\mathbb{P}}_{t+1}^{edf}, \\
\Xi_{t+1}^{ndf} &= (1 - \delta) \left( \frac{[A_{t+1}^H]^{\theta-1} \mathbb{W}_{t+1}^{*-(\theta-1)} Q_{t+1}^*}{[A_t^H]^{\theta-1} \mathbb{W}_t^{*-(\theta-1)} Q_t^*} \right)^{\frac{1}{\alpha(\theta-1)+1}} \Xi_t^{ndf} + n_{t+1}^{ndf} \tilde{\mathbb{P}}_{t+1}^{ndf},
\end{aligned}$$

where the numbers of eligible and non-eligible firms distinguished by location and ownerships (i.e.  $n_t^o$ ) is given in the next subsection.

Exports are given recursively as follows:  $X_t^d = X_t^{dd} + X_t^{fd}$ ,  $X_t^f = X_t^{df} + X_t^{ff}$ , where  $X_t^{dd}$  is the export of firms located in the domestic country and owned by the domestic household to the foreign country (and similarly for  $X_t^{fd}$ ,  $X_t^{df}$ ,  $X_t^{ff}$ ). We use the convention that exports are denominated in the currency of the original market (thus  $X_t^{dd}$ ,  $X_t^{fd}$  are in the domestic currency). Thus, it

holds that:

$$\begin{aligned}
X_t^{dd} &= \widehat{n}_t^{edd} (1 - \kappa_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left( \frac{\eta_t}{1 + \mathbf{t}} \right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[ \frac{\theta-1}{\theta} (1 - \alpha) \mathbb{W}_t^{-1} [A_t^H]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{* \frac{1}{\alpha(\theta-1)+1}}, \\
X_t^{fd} &= \widehat{n}_t^{efd} (1 - \kappa_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left( \frac{\eta_t}{1 + \mathbf{t}} \right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[ \frac{\theta-1}{\theta} (1 - \alpha) \mathbb{W}_t^{-1} [A_t^F]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{* \frac{1}{\alpha(\theta-1)+1}}, \\
X_t^{df} &= \widehat{n}_t^{edf} (1 - \kappa_t^*)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left( \frac{\eta_t^{-1}}{1 + \mathbf{t}} \right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[ \frac{\theta-1}{\theta} (1 - \alpha) \mathbb{W}_t^{*-1} [A_t^H]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}, \\
X_t^{ff} &= \widehat{n}_t^{eff} (1 - \kappa_t^*)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left( \frac{\eta_t^{-1}}{1 + \mathbf{t}} \right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[ \frac{\theta-1}{\theta} (1 - \alpha) \mathbb{W}_t^{*-1} [A_t^F]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},
\end{aligned}$$

where  $\widehat{n}_t^{ex_1x_2}$  are weighted numbers of eligible firms, which obeys the following recursive relation:

$$\widehat{n}_{t+1}^{ex_1x_2} = (1 - \delta) \widehat{n}_t^{ex_1x_2} + n_{t+1}^{ex_1x_2} \left[ \frac{\alpha(\theta-1) \varpi_{t+1}^{ex_1x_2}}{\alpha(\theta-1) + 1} \right]^{\alpha(\theta-1)} \int_{\bar{z}_{t+1}^{x_1x_2}}^{z_U} z^{\theta-1} G(dz).$$

A similar recursive equation holds for non-eligible firms:

$$\widehat{n}_{t+1}^{nx_1x_2} = (1 - \delta) \widehat{n}_t^{nx_1x_2} + n_{t+1}^{nx_1x_2} \left[ \frac{\alpha(\theta-1) \varpi_{t+1}^{nx_1x_2}}{\alpha(\theta-1) + 1} \right]^{\alpha(\theta-1)} \int_{z_L}^{\bar{z}_{t+1}^{x_1x_2}} z^{\theta-1} G(dz).$$

These recursive schemes are used in the next subsection too (when deriving the labor demand).

The rest of model dynamic equations are *balance-of-payment equation* (17), households' *budget constraint* (8), households' *Euler equations* (9), households' *equations*, which determines the *asset holdings*: (11), (12), plus the corresponding equations for the foreign household. Equation describing optimal asset holding are not in the recursive first-order form, but we can easily convert them into it (for sake of clarity, we put the equations for both agents):

$$\begin{aligned}
\widetilde{c}_t^{dd} + \Psi_d n_t^{dd} &= \Omega_t^{dd}, \\
\eta_t \widetilde{c}_t^{df} + \Psi_f n_t^{df} &= \Omega_t^{df}, \\
\widetilde{c}_t^{ff} + \Psi_d n_t^{ff} &= \Omega_t^{ff}, \\
\eta_t^{-1} \widetilde{c}_t^{fd} + \Psi_f n_t^{fd} &= \Omega_t^{fd};
\end{aligned} \tag{A.6}$$



where expected investment costs obey:

$$\begin{aligned} \tilde{c}_t^{x_1x_2} &= G(\bar{z}_t^{x_1x_2})c^n + (1 - G(\bar{z}_t^{x_1x_2}))c^e + \dots \\ &+ \left[ \frac{\alpha(\theta - 1)\varpi_{t+1}^{nx_1x_2}}{\alpha(\theta - 1) + 1} \right]^{\alpha(\theta-1)} \int_{z_L}^{\bar{z}_t^{x_1x_2}} z^{\theta-1} G(dz) + \dots \\ &+ \left[ \frac{\alpha(\theta - 1)\varpi_{t+1}^{ex_1x_2}}{\alpha(\theta - 1) + 1} \right]^{\alpha(\theta-1)} \int_{\bar{z}_t^{x_1x_2}}^{z_U} z^{\theta-1} G(dz). \end{aligned} \quad (\text{A.7})$$

### A.2.2 Static Equations

The model has static equations too. These are mainly market clearing conditions and definitions. The market clearing conditions include the clearing of the goods markets (13), (14), international bond market clearing (16), and labor market clearing conditions. We now show how the labor market conditions look like: define  $\bar{\partial}_t^\circ$  as

$$\begin{aligned} \bar{\partial}_t^{n\text{dd}} &= \left( \frac{\theta - 1}{\theta} (1 - \alpha) [A_t^H]^{\theta-1} \mathbb{W}_t^{-(\theta-1)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}}, \\ \bar{\partial}_t^{e\text{dd}} &= \left( \frac{\theta - 1}{\theta} (1 - \alpha) [A_t^H]^{\theta-1} \mathbb{W}_t^{-(\theta-1)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}}, \\ \bar{\partial}_t^{n\text{fd}} &= \left( \frac{\theta - 1}{\theta} (1 - \alpha) [A_t^F]^{\theta-1} \mathbb{W}_t^{-(\theta-1)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}}, \\ \bar{\partial}_t^{e\text{fd}} &= \left( \frac{\theta - 1}{\theta} (1 - \alpha) [A_t^F]^{\theta-1} \mathbb{W}_t^{-(\theta-1)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}}, \\ \bar{\partial}_t^{n\text{df}} &= \left( \frac{\theta - 1}{\theta} (1 - \alpha) [A_t^H]^{\theta-1} \mathbb{W}_t^{*-(\theta-1)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}}, \\ \bar{\partial}_t^{e\text{df}} &= \left( \frac{\theta - 1}{\theta} (1 - \alpha) [A_t^H]^{\theta-1} \mathbb{W}_t^{*-(\theta-1)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}}, \\ \bar{\partial}_t^{n\text{ff}} &= \left( \frac{\theta - 1}{\theta} (1 - \alpha) [A_t^F]^{\theta-1} \mathbb{W}_t^{*-(\theta-1)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}}, \\ \bar{\partial}_t^{e\text{ff}} &= \left( \frac{\theta - 1}{\theta} (1 - \alpha) [A_t^F]^{\theta-1} \mathbb{W}_t^{*-(\theta-1)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}}. \end{aligned} \quad (\text{A.8})$$

Then the domestic labor demand is given as

$$\mathcal{L}_t = \sum_{\xi \in \{e,n\}} \sum_{x_1 \in \{d,f\}} \bar{\partial}_t^{\xi x_1 d} \hat{n}_t^{\xi x_1 d}, \quad (\text{A.9})$$

and the foreign labor demand is given by

$$\mathcal{L}_t^* = \sum_{\xi \in \{e, n\}} \sum_{x_1 \in \{d, f\}} \delta_t^{\xi x_1 f} \hat{n}_t^{\xi x_1 f}. \quad (\text{A.10})$$

The labor demands should be equal to inelastic labor supply.

The only remaining things are definitions of average profits and expected cut-offs. They follow:

$$\bar{z}_t^{x_1 x_2} = \left( \frac{c^e - c^n}{\mathcal{G} \left[ [\varpi_t^{e x_1 x_2}]^{\alpha(\theta-1)+1} - [\varpi_t^{n x_1 x_2}]^{\alpha(\theta-1)+1} \right]} \right)^{\frac{1}{\theta-1}}, \quad (\text{A.11})$$

for  $x_i \in \{d, f\}$ , and

$$\begin{aligned} \tilde{\mathbb{P}}_t^{ndd} &= \mathcal{W}_1 \int_{z_L}^{\bar{z}_{t+1}^{dd}} z^{\theta-1} G(dz) \left( [A_t^H]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[ \frac{\alpha(\theta-1)\varpi_t^{ndd}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}, \\ \tilde{\mathbb{P}}_t^{edd} &= \mathcal{W}_1 \int_{\bar{z}_{t+1}^{dd}}^{z_U} z^{\theta-1} G(dz) \left( [A_t^H]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[ \frac{\alpha(\theta-1)\varpi_t^{edd}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}, \\ \tilde{\mathbb{P}}_t^{nfd} &= \mathcal{W}_1 \int_{z_L}^{\bar{z}_{t+1}^{fd}} z^{\theta-1} G(dz) \left( [A_t^F]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[ \frac{\alpha(\theta-1)\varpi_t^{nfd}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}, \\ \tilde{\mathbb{P}}_t^{efd} &= \mathcal{W}_1 \int_{\bar{z}_{t+1}^{fd}}^{z_U} z^{\theta-1} G(dz) \left( [A_t^F]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[ \frac{\alpha(\theta-1)\varpi_t^{efd}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}, \\ \tilde{\mathbb{P}}_t^{ndf} &= \mathcal{W}_1 \int_{z_L}^{\bar{z}_{t+1}^{df}} z^{\theta-1} G(dz) \left( [A_t^H]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[ \frac{\alpha(\theta-1)\varpi_t^{ndf}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}, \\ \tilde{\mathbb{P}}_t^{edf} &= \mathcal{W}_1 \int_{\bar{z}_{t+1}^{df}}^{z_U} z^{\theta-1} G(dz) \left( [A_t^H]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[ \frac{\alpha(\theta-1)\varpi_t^{edf}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}, \\ \tilde{\mathbb{P}}_t^{nff} &= \mathcal{W}_1 \int_{z_L}^{\bar{z}_{t+1}^{ff}} z^{\theta-1} G(dz) \left( [A_t^F]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[ \frac{\alpha(\theta-1)\varpi_t^{nff}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}, \\ \tilde{\mathbb{P}}_t^{eff} &= \mathcal{W}_1 \int_{\bar{z}_{t+1}^{ff}}^{z_U} z^{\theta-1} G(dz) \left( [A_t^F]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[ \frac{\alpha(\theta-1)\varpi_t^{eff}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}. \end{aligned}$$

### A.3 Numerical methods

This part of the appendix discusses numerical methods used to simulate the model. Basically, we have experimented with two classes of methods: (i) projection-based methods and (ii) domain-truncation methods.

Before discussing these methods, it is worth to realize a fact, which we use when applying both methods: If one can guess the time profile of the following

six variables: domestic output  $\{Q_t\}_{t=0}^\infty$ , domestic real wage  $\{\mathbb{W}_t\}_{t=0}^\infty$ , domestic consumption  $\{C_t\}_{t=0}^\infty$ , their foreign counterparts:  $\{Q_t^*\}_{t=0}^\infty$ ,  $\{\mathbb{W}_t^*\}_{t=0}^\infty$ ,  $\{C_t^*\}_{t=0}^\infty$  and the real exchange rate  $\{\eta_t\}_{t=0}^\infty$ , one can easily compute the time profile of all other endogenous variables (given exogenous and policy variables). Indeed, the algorithm is the following:

- (1) Given  $\{C_t\}_{t=0}^\infty$ ,  $\{C_t^*\}_{t=0}^\infty$  compute the marginal rate of substitutions  $\{\mu_t^{t+1}\}_{t=0}^\infty$ ,  $\{\mu_t^{*t+1}\}_{t=0}^\infty$  using (A.2).
- (2) Given  $\{Q_t\}_{t=0}^\infty$ ,  $\{\mathbb{W}_t\}_{t=0}^\infty$ ,  $\{Q_t^*\}_{t=0}^\infty$ ,  $\{\mathbb{W}_t^*\}_{t=0}^\infty$  and  $\{\mu_t^{t+1}\}_{t=0}^\infty$ ,  $\{\mu_t^{*t+1}\}_{t=0}^\infty$ , it is possible to solve for  $\{\varpi_t^\circ\}_{t=0}^\infty$ , and therefore for  $\{\bar{z}_t^\circ\}_{t=0}^\infty$ ; use (A.3) and (A.11).
- (3) Then, use backward difference equations (A.4) to compute  $\{\Omega_t^\circ\}_{t=0}^\infty$ , (A.7) to compute expected investment costs  $\{\bar{c}_t^\circ\}_{t=0}^\infty$  and first-order conditions (A.6) to compute the numbers of new entrants.
- (4) Then use the forward difference equation (A.5) to solve for profit flows  $\{\Xi_{t+1}^\circ\}_{t=0}^\infty$  and (A.8), (A.9) and (A.10) to find labor demand in both countries.
- (5) One can use households' Euler equations to derive the optimal bond holding and from the international-bond market clearing condition (16) to derive the equilibrium interest rate  $\{r_t\}_{t=0}^\infty$ ;

Now, one guesses the time profile and verifies the guess. The guess should be verified as follows:

- (1) Budget constraints for both households have to be satisfied: (8) and similarly for the foreign household.
- (2) Labor markets in both countries have to be cleared: (15) and similarly for the foreign country.
- (3) The balance of payment condition has to be satisfied: (17).
- (4) Goods markets have to be cleared as well: (13), (14).

Denote the guess of the seven variables as

$$\vec{\mathcal{H}} = \{\{Q_t\}_{t=0}^\infty, \{\mathbb{W}_t\}_{t=0}^\infty, \{C_t\}_{t=0}^\infty, \{Q_t^*\}_{t=0}^\infty, \{\mathbb{W}_t^*\}_{t=0}^\infty, \{C_t^*\}_{t=0}^\infty, \{\eta_t\}_{t=0}^\infty\},$$

and the seven equilibrium conditions as  $\{\bar{h}_t(\vec{\mathcal{H}})\}_{t=0}^\infty$ , where we interpret  $\bar{h}_t(\vec{\mathcal{H}}^0) = 0$  as the fulfillment of these conditions at time  $t$  for a guess  $\vec{\mathcal{H}}^0$ . Note that the fulfillment of equilibrium condition at time  $t$ ,  $\bar{h}_t = 0$  does not depend on the value of the seven variables at time  $t$  only: it depends on their entire time profiles. It depends on future values because of expectations of profits, e.g. today's investment decisions depend on future streams of profits, cf. (11), (12), and it depends on past values because of predetermined variables in budget constraints.

In any case, the equilibrium candidate  $\vec{\mathcal{H}}$  is an infinite-dimensional object and for practical simulations, we have to approximate it by a finite-dimensional representation. The projection and domain-truncation methods do that in different ways.

The strategy of the projection method is the following: approximate the time profiles using an object parameterized by a low number of parameters (such as polynomials, splines, neural networks, or wavelets). Thus approximate

$$\vec{\mathcal{H}} \approx \widetilde{\mathcal{H}}(\mathcal{U}),$$

where  $\mathcal{U}$  is a finite vector of parameters. Then the problem is to find such a vector of parameters  $\vec{\mathcal{U}}$ , such that the equilibrium conditions  $\tilde{h}_t(\widetilde{\mathcal{H}}(\vec{\mathcal{U}})) = 0$  nearly holds for all  $t$ . Judd (2002) discusses applications of the projection methods in the context of perfect foresight discrete-time models.

Another approach (called domain truncation approach) to reduce dimensionality of  $\vec{\mathcal{H}}$  is to set  $\{Q_t\}_{t=0}^{\infty} \approx \widehat{Q} = \{Q_1, \dots, Q_N, Q_+, Q_+, \dots, Q_+\}$ , where  $Q_+$  is the steady state of the variable  $Q_t$  (and similarly for other variables too) and to set

$$\widehat{\mathcal{H}} = \{\widehat{Q}, \widehat{W}, \widehat{C}, \widehat{Q}^*, \widehat{W}^*, \widehat{C}^*, \widehat{\eta}\},$$

and solve the system

$$\begin{aligned} \tilde{h}_1(\widehat{\mathcal{H}}) &= 0 \\ \tilde{h}_2(\widehat{\mathcal{H}}) &= 0 \\ &\vdots \\ \tilde{h}_M(\widehat{\mathcal{H}}) &= 0 \end{aligned} \tag{A.12}$$

for  $M > N$ . This is a system of  $7M$  unknowns. Lafargue (1990) proposed this approach, and Boucekine (1995) and Juillard et al. (1998) exploited the sparseness of the system to apply an efficient algorithm. Hence, the approach uses to be called as L-B-J approach. The stacked system (A.12) is usually solved using Newton-based iterations. When applied to the model presented in this paper, we cannot use efficient algorithms for sparse systems unless  $\delta = 1$ . The case of  $\delta = 1$  is the only case, when the Jacobian of (A.12) is sparse.

We experimented with both approaches: as projections we chose splines and RBF neural networks. To solve the system (A.12), we apply the quasi-Newton iteration, with the Hessian update via the BFGS formula suggested by Broyden (1970), Fletcher (1970), Goldfarb (1970), and Shanno (1970) implemented in the MATLAB function `fminunc`. The L-B-J approach seems to perform better than the projection methods and therefore simulation results reported in this paper are based on it.

## References

- Ahnert H., G. Kenny, (2004). Quality Adjustment of European Price Statistics and The Role For Hedonics. Occasional ECB WP 15.
- Baldwin, R., Forslid, R. (2000). Trade liberalization and endogenous growth: A  $q$ -theory approach. *Journal of International Economics* 50, pp. 497-517.
- Bayoumi, T., Kumhof, M., Laxton, D., Naknoi, K. (2004). Exchange rate regimes, international linkages, and the macroeconomic performance of the New Member States. Chapter 5 in C. Detken, V. Gaspar, G. Noblet (Eds.) 'The New EU Member States Convergence and Stability', Proceedings of The Third ECB Central Banking Conference, 21-22 October, 2004, pp. 75-119.
- Ben-David, D. (1993). Equalizing Exchange: Trade Liberalization and Income Convergence. *Quarterly Journal of Economics*, vol. 108, no. 3, pp. 653-679.
- Bergin, P., R. Glick (2005). Tradability, Productivity, and Understanding International Economic Integration. NBER Working 11637, September 2005.
- Bergin, P., R. Glick (2006). Endogenous Tradability and Macroeconomic Implications. A revised version of NBER Working Paper No. 9739, February 2006 draft.
- Boucekkine, R. (1995). An alternative methodology for solving nonlinear forward looking models. *Journal of Economic Dynamics and Control* 19, pp. 711-734.
- Broyden, C.G. (1970). The Convergence of a Class of Double-rank Minimization Algorithms. *IMA Journal of Applied Mathematics* 6, pp 76-90.
- Brůha, J., J. Podpiera (2007). Inquiries on Dynamics of Transition Economy Convergence in a Two-Country Model. An ECB manuscript.
- Cincibuch, M., J. Podpiera. (2006). Beyond Balassa-Samuelson: Real Appreciation in Tradables in Transition Countries. *Economics of Transition*, vol. 14(3) 2006, pp. 547-573.
- De Broeck, M., T. Slok, (2006). Interpreting Real Exchange Rate Movements in Transition Countries. *Journal of International Economics* 68, pp.368-383.
- De Melo, M., C. Denizer, A. Gelb (1996). From Plan to Market: Pattern of Transition. *World Bank Economic Review*, vol. 10, no.3, pp.397-424.
- Edison, H., B.D. Pauls. (1993). A Re-assessment of the Relationship between Real Exchange Rates and Real Interest Rates. *Journal of Monetary Economics*, vol 31, no. 2, pp. 165-187.
- Égert, B., K. Lommatzsch (2004). Equilibrium Exchange Rates in the Transition: The Tradable Price-based Real Appreciation and Estimation Uncertainty. William Davidson Institute WP 676.
- European Central Bank (2006). Convergence Report. May 2006.
- Fabrizio S., D. Igan, A. Mody, and N. Tamirisa, (2006). Export Structure and Credit Growth. IMF Country Report No. 06/414, September 2006.
- Fan, Emma Xiaoqin, (2002). Technological Spillovers from Foreign Direct Investment - A Survey. ERD WP 33.

- Fischer, S., R. Sahay, C.A. Végh (1996). Stabilization and Growth in Transition Economies: The Early Experience. *Journal of Economic Perspectives*, vol. 10, Number 2, pp. 46-66.
- Flek, V., J. Podpiera, L. Markova (2003). Sectoral Productivity and Real Exchange Rate Appreciation: Much Ado about Nothing? *Czech Journal of Economics and Finance*, Vol. 53, Issue 3-4, p.130.
- Fletcher, R. (1970). A New Approach to Variable Metric Algorithms. *Computer Journal* 13, pp 317-322.
- Ghironi, F., Melitz M. (2005). International Trade and Macroeconomic Dynamics with Heterogeneous Firms. *Quarterly Journal of Economics*, CXX (August 2005), pp. 865-915.
- Goldfarb, D. (1970). A Family of Variable Metric Updates Derived by Variational Means. *Mathematics of Computing* 24, pp 23-26.
- Hanousek J., R. Filer (2004). Consumers' Opinion of Inflation Bias Due to Quality Improvements. *William Davidson Institute WP* 681.
- Henry, P.B. (2003). Capital-Account Liberalization, the Cost of Capital, and Economic Growth. *American Economic Review*, vol 93, no. 2, pp. 91-96.
- Juillard, M., Laxton, D., McAdam, P., Pioro, H. (1998). An algorithm competition: First-order iterations versus Newton-based techniques. *Journal of Economic Dynamics and Control* 22, pp. 1291-1318.
- Judd, K., 2002. The Parametric Path Method: an Alternative to Fair-Taylor and L-B-J for Solving Perfect Foresight Models. *Journal of Economic Dynamics and Control* 26, pp. 1557-1583.
- Kaminski, B., Z.K. Wang, L.A. Winters (1996). Export Performance in Transition Economies. *Economic Policy*, vol. 11, no. 23, pp. 421-442.
- Laffargue, J.P., 1990. Resolution d'un modèle macroéconomique avec anticipations rationnelles. *Annales d'Economie et Statistique* 17, pp. 97-119.
- Melitz, M. (2003). The Impact of Trade on Intra-Industry Reallocation and Aggregate Industry Productivity. *Econometrica* 71/6, pp. 1695-1725.
- Mihaljek, D., M. Klau (2006). The Balassa-Samuelson effect and the Maastricht criteria: revising the debate in N. Batini (Ed.) *Monetary Policy in Emerging Markets and Other Developing Countries*, IMF, 2006.
- Naknoi, K., Kumhof, M., Laxton, D. (2005). On the Benefits of Exchange Rate Flexibility under Endogenous Tradedness of Goods. *Computing in Economics and Finance* 405, Society for Computational Economics.
- Naknoi, K. (2006). Real Exchange Rate Fluctuations, Endogenous Tradability and Exchange Rate Regime. *International Finance* 0509004, EconWPA, revised July 2006.
- Obstfeld, M., K. Rogoff (1995). Exchange Rate Dynamics Redux. *Journal of Political Economy*, vol. 103, no. 3, pp. 624-660.
- Podpiera J. (2005). Sources of Real Convergence in Emerging Markets. *CERGE-EI Policy Brief* 7.
- Podpiera, J., M. Raková (2006). Degree of competition and export production relative prices when the exchange rate changes: Evidence from a panel of Czech exporting companies. *Czech National Bank Working Paper* 10/2006.
- Roland, G. (2004). After Enlargement: Institutional Achievements and

- Prospects in the New Member States. Chapter 3 in C. Detken, V. Gaspar, G. Noblet (Eds.) 'The New EU Member States Convergence and Stability', Proceedings of The Third ECB Central Banking Conference, 21-22 October, 2004, pp. 35-58.
- Sachs, J.D. (1996). The Transition at Mid Decade. *American Economic Review*, vol. 86, no. 2, pp.128-133.
- Schmitt-Grohe, S., M. Uribe (2003). Closing Small Open Economy Models. *Journal of International Economics* 61, pp. 163-185.
- Shanno, D.F. (1970) Conditioning of Quasi-Newton Methods for Function Minimization. *Mathematics of Computing* 24, pp 647-656.
- Wooldridge J.M., 2001. *Econometric Analysis of Cross Section and Panel Data*, MIT Press.
- Young A. (1998). Growth Without Scale Effects. *Journal of Political Economy* 106-1, 41-63.

Fig. 1. Stylized facts on transition economies

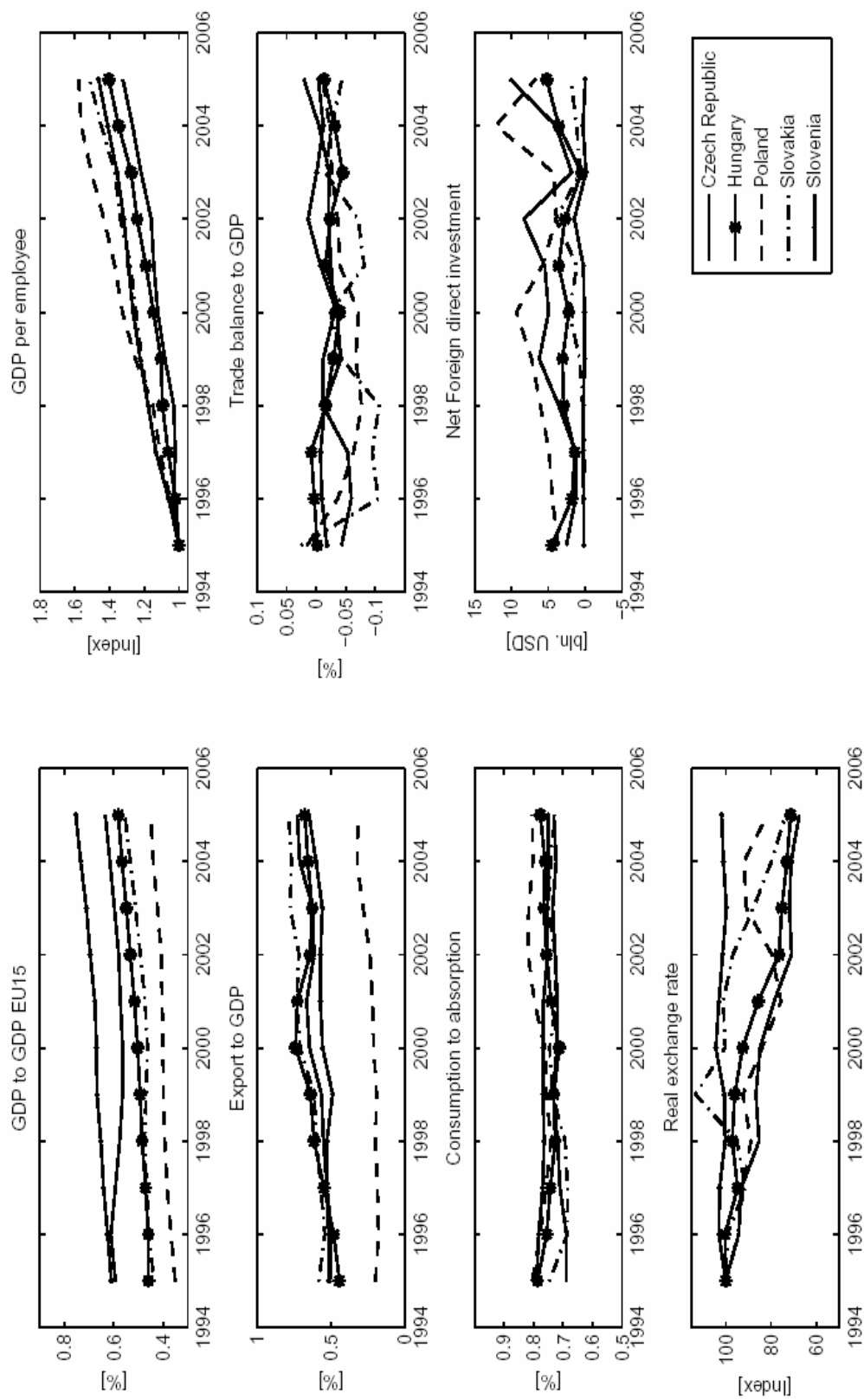




Fig. 2. Parameters calibration

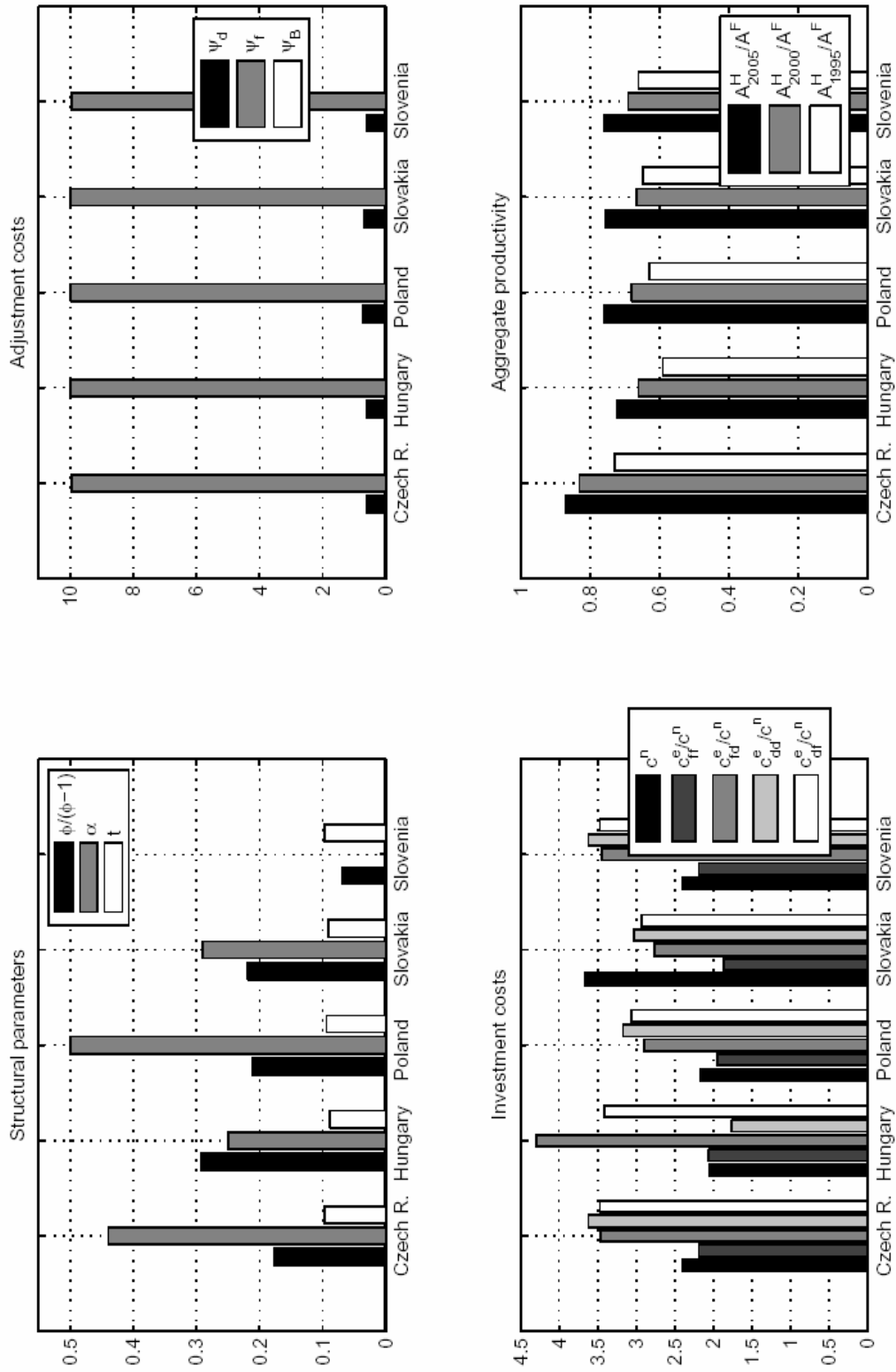


Fig. 3. The Czech economy calibration

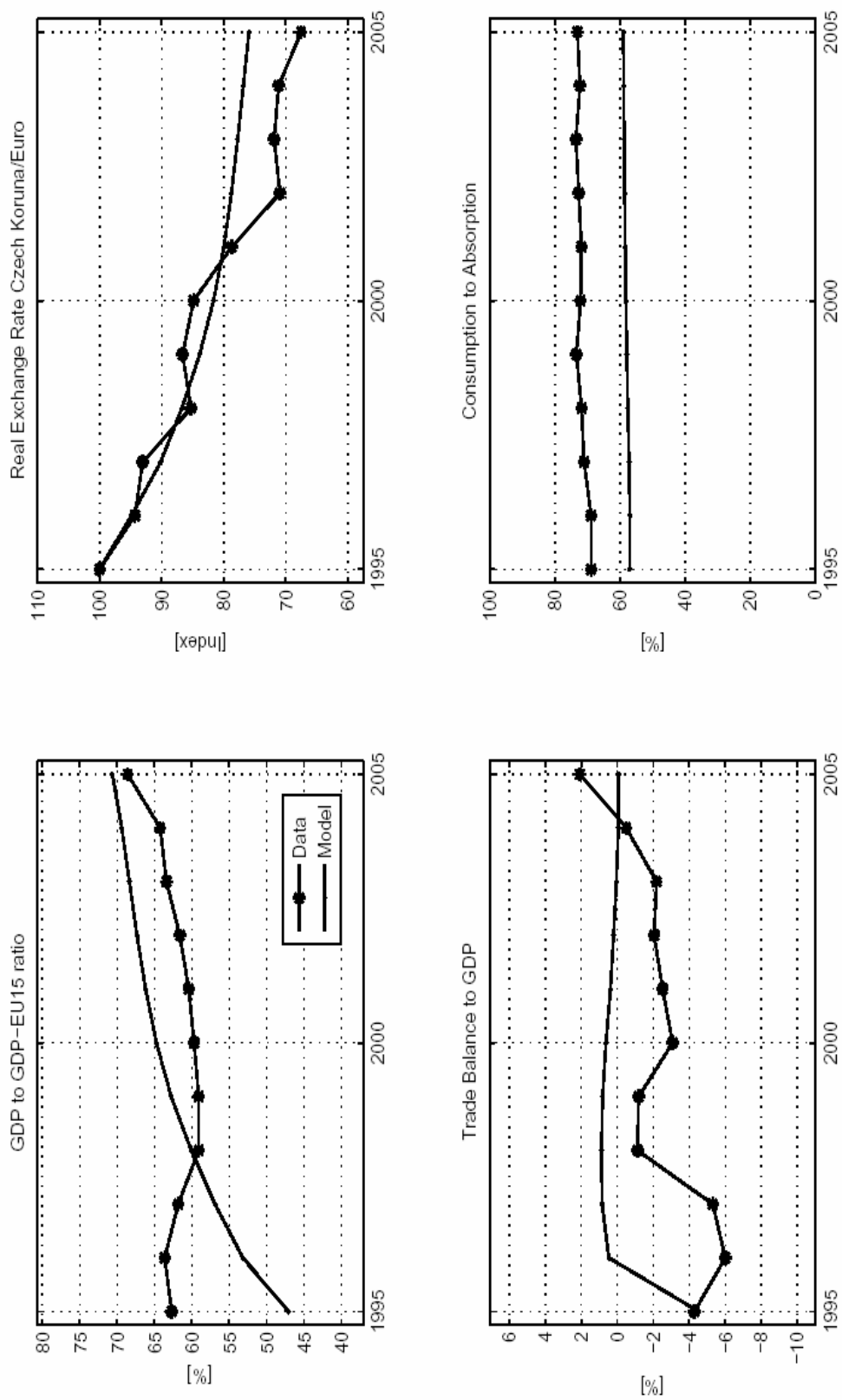


Fig. 4. The Hungarian economy calibration

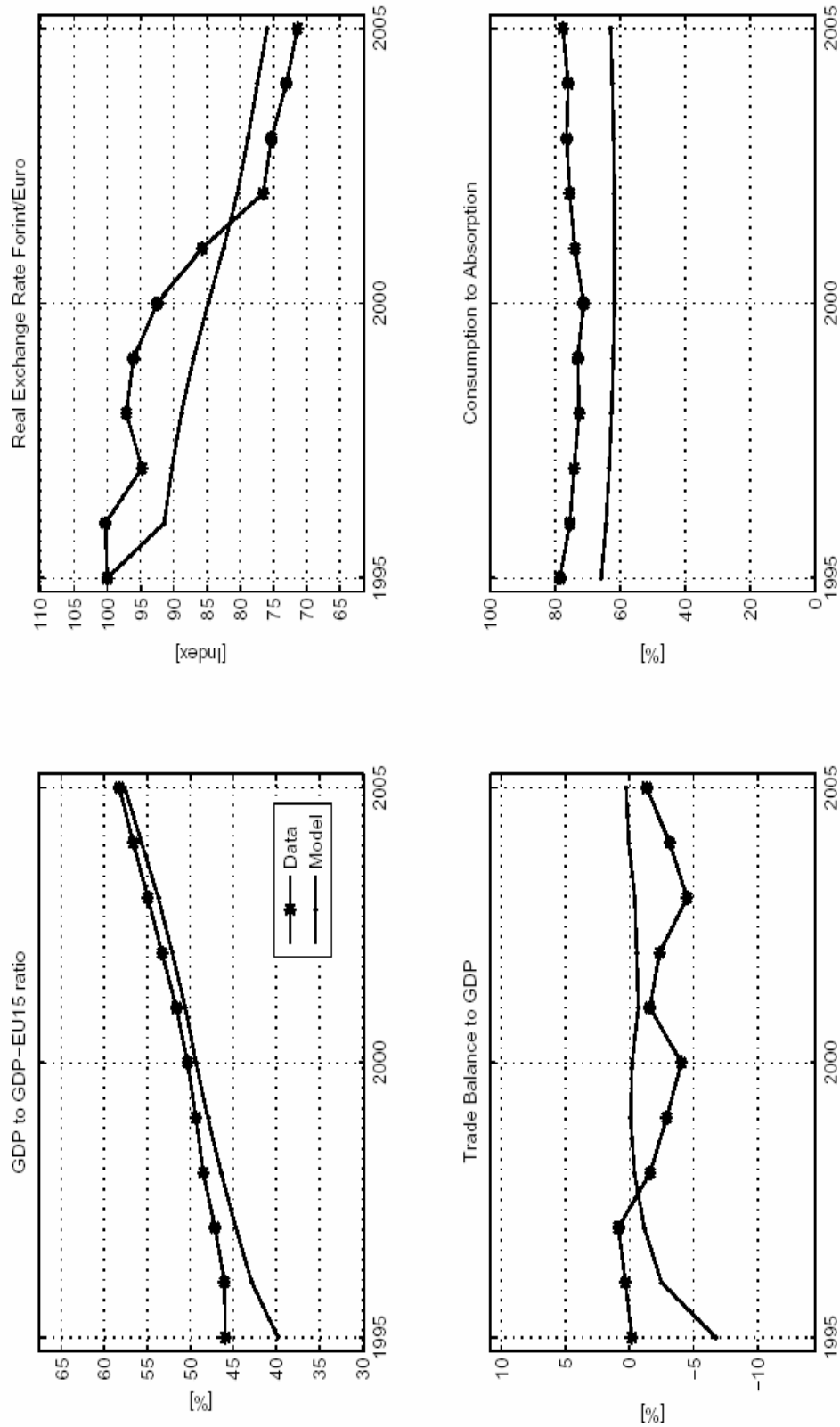


Fig. 5. The Polish economy calibration

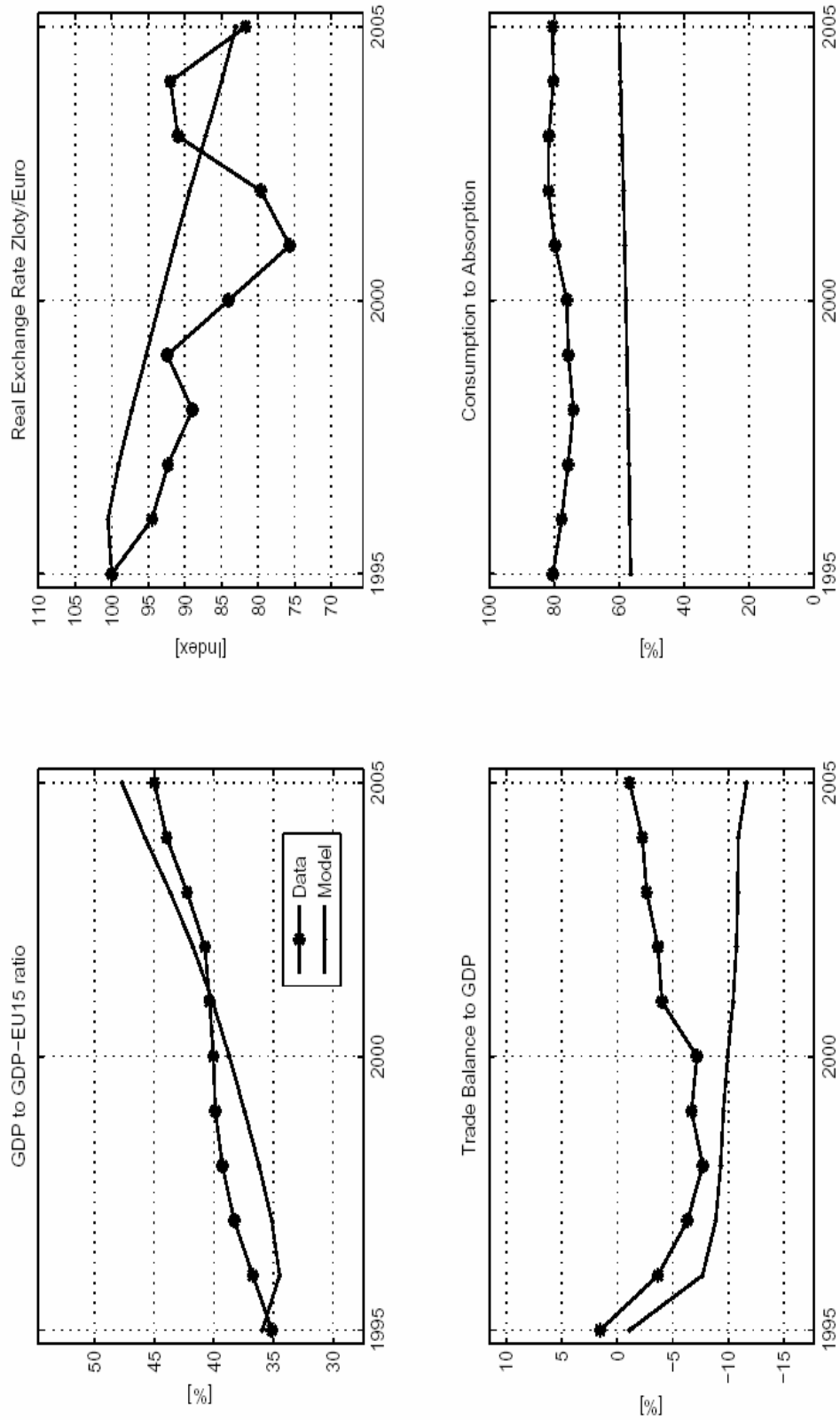


Fig. 6. The Slovak economy calibration

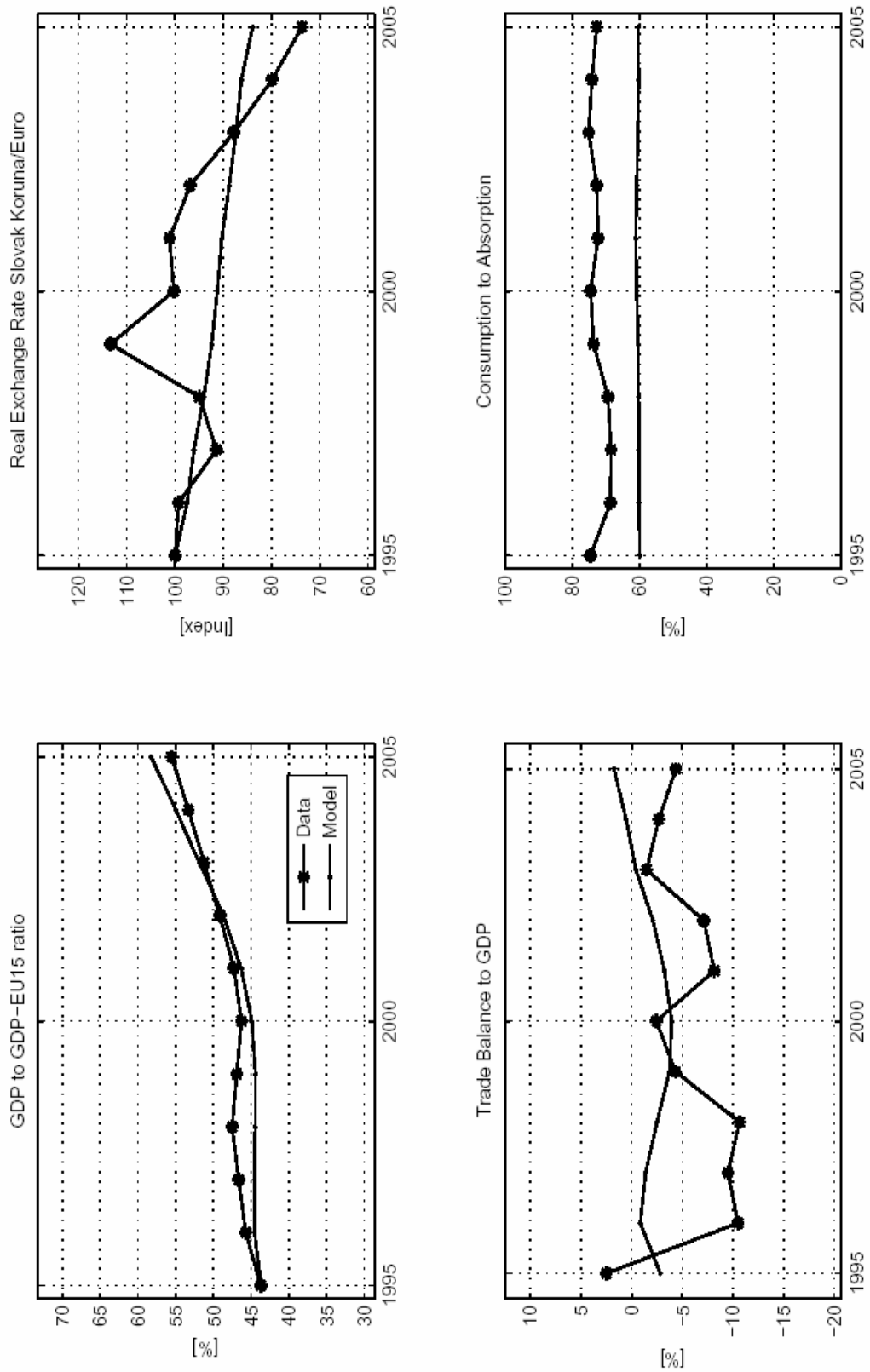


Fig. 7. The Slovenian economy calibration

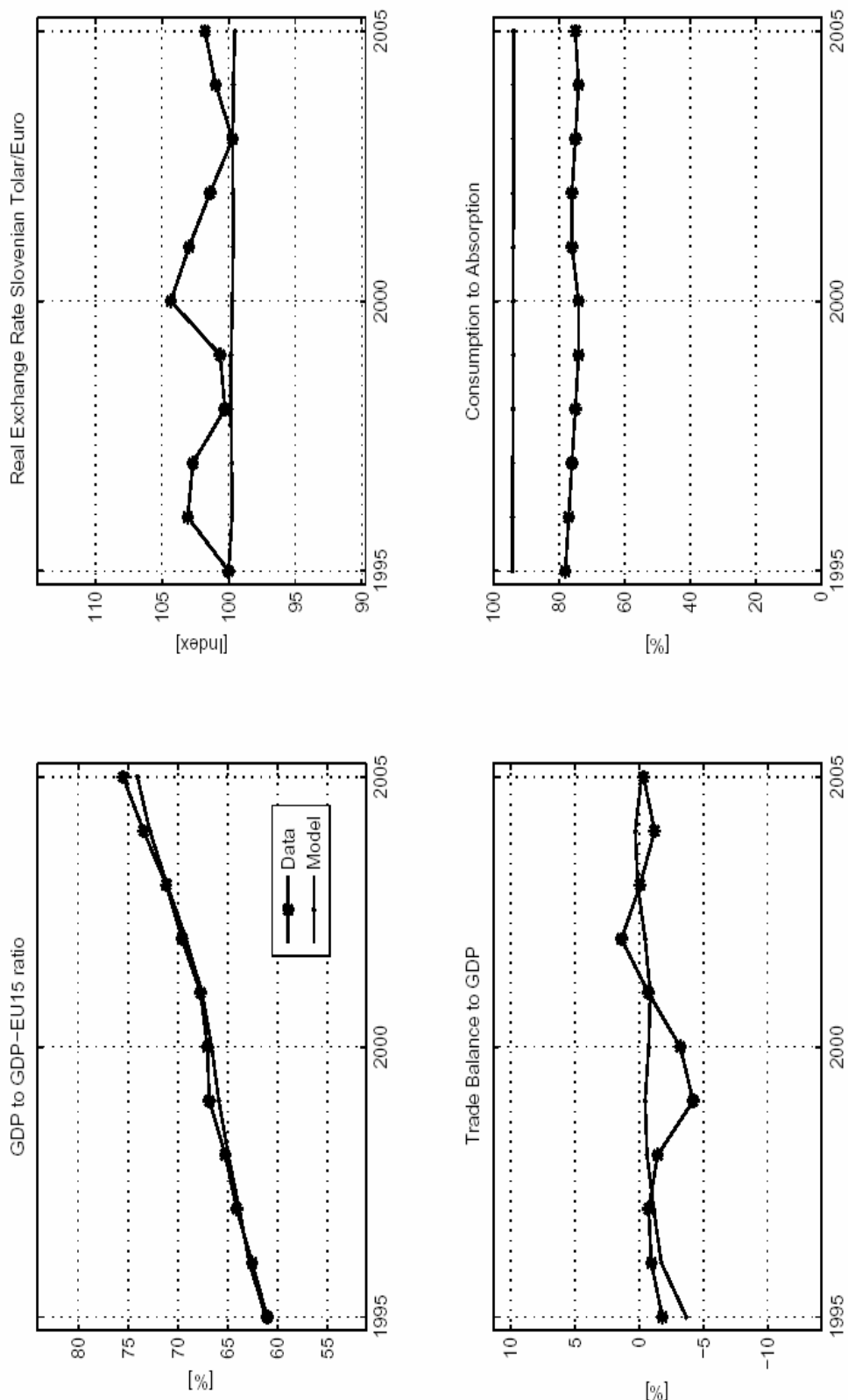


Fig. 8. The Czech economy projection

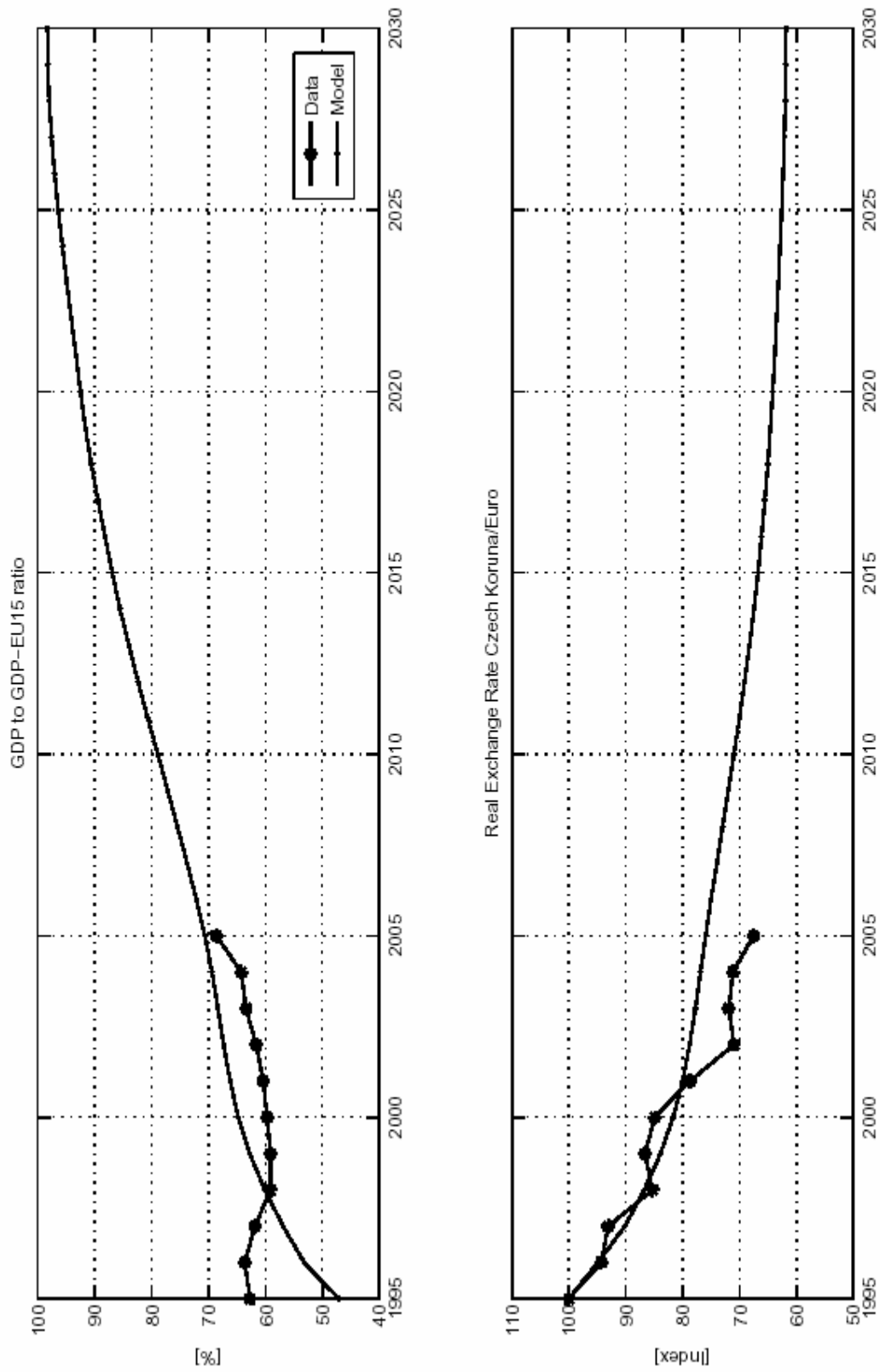


Fig. 9. The Hungarian economy projection

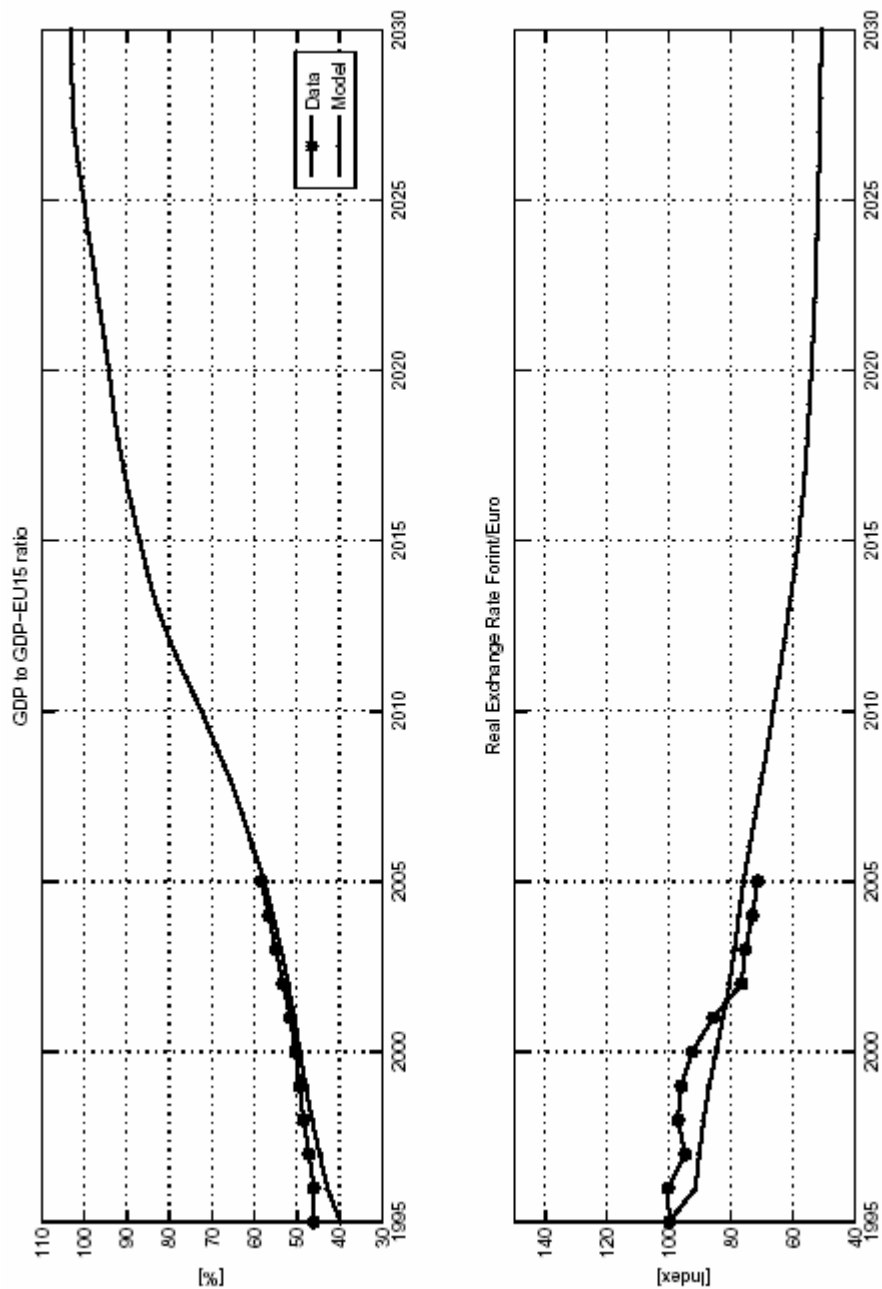




Fig. 10. The Polish economy projection

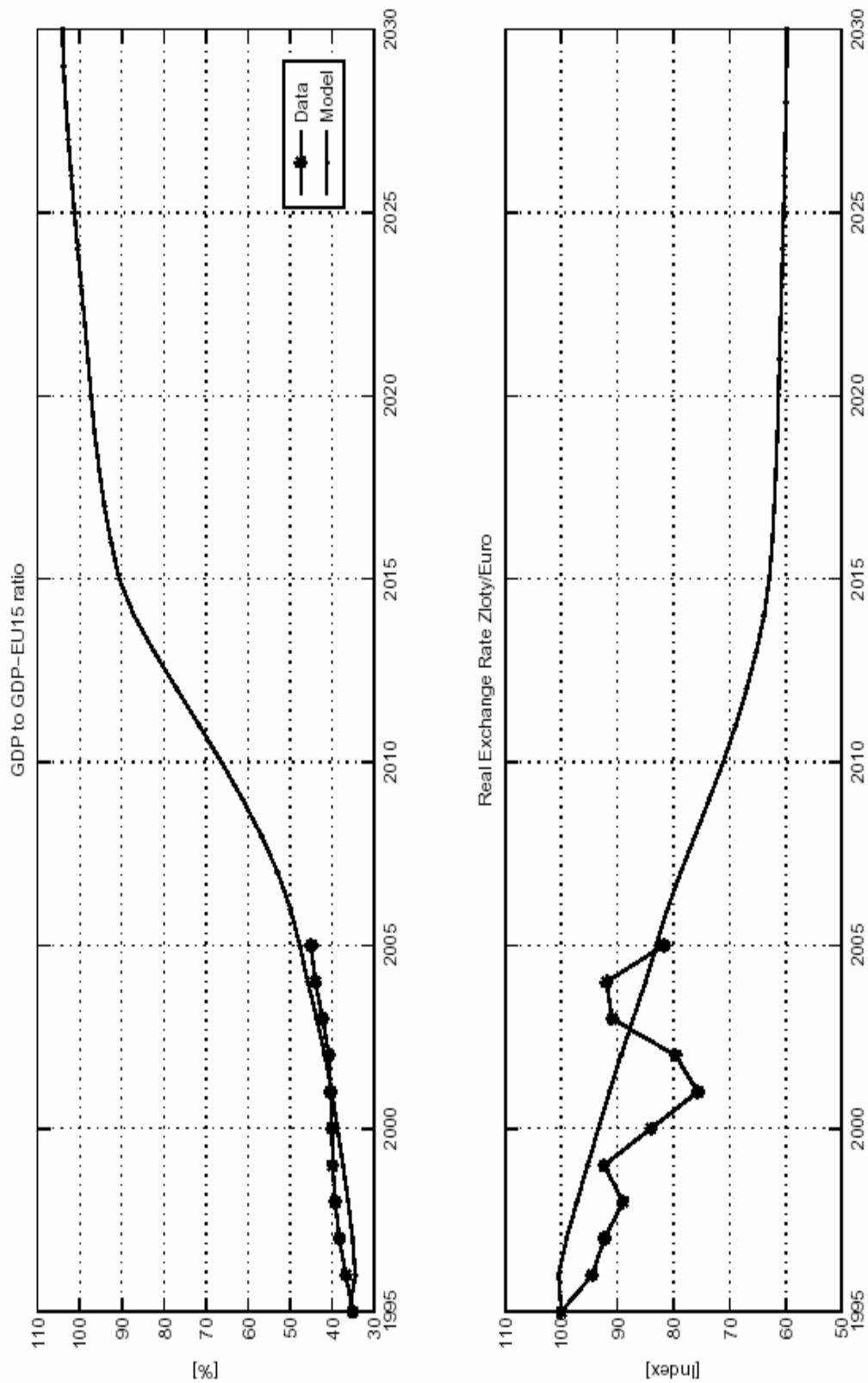


Fig. 11. The Slovak economy projection

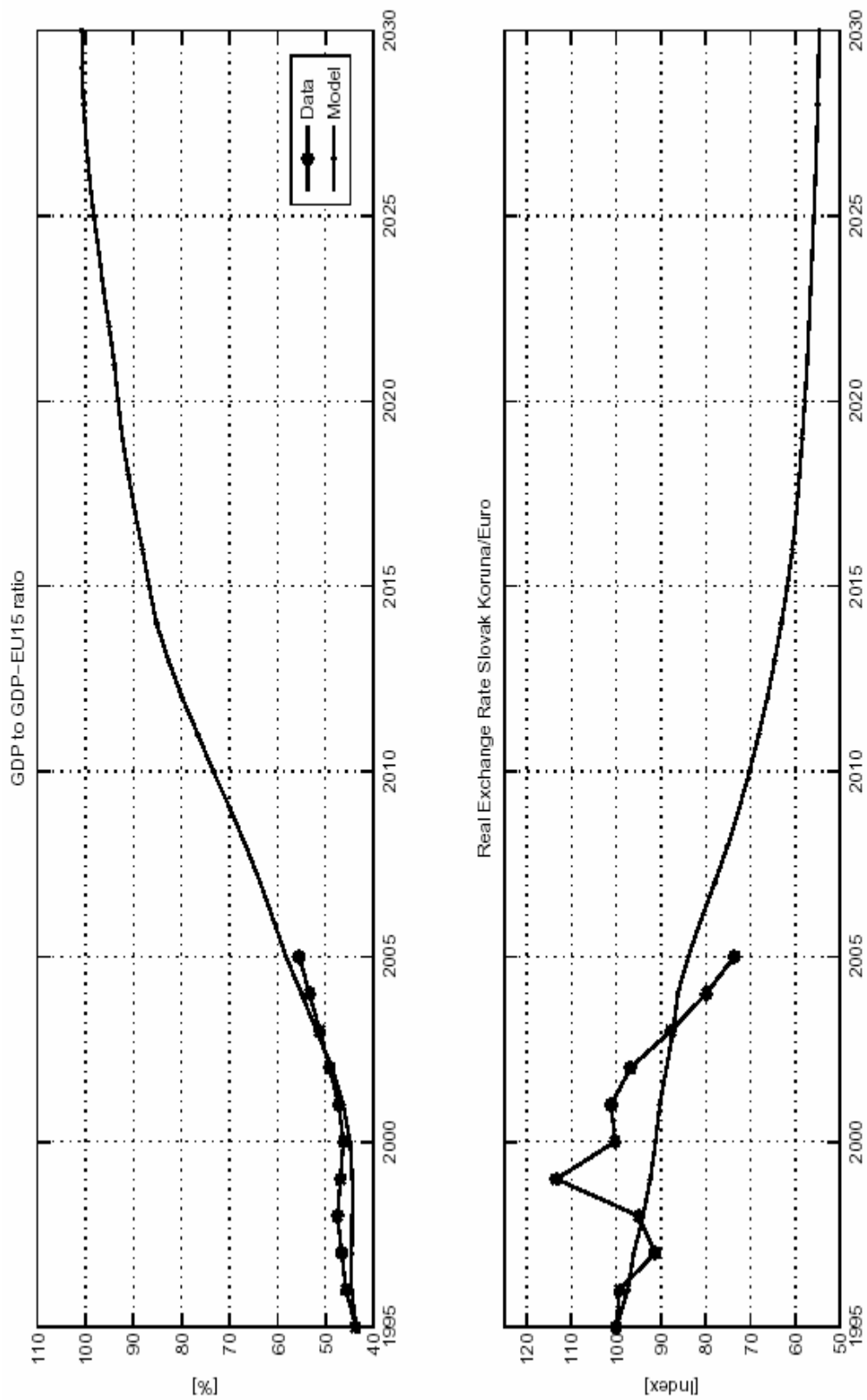


Fig. 12. The Slovenian economy projection

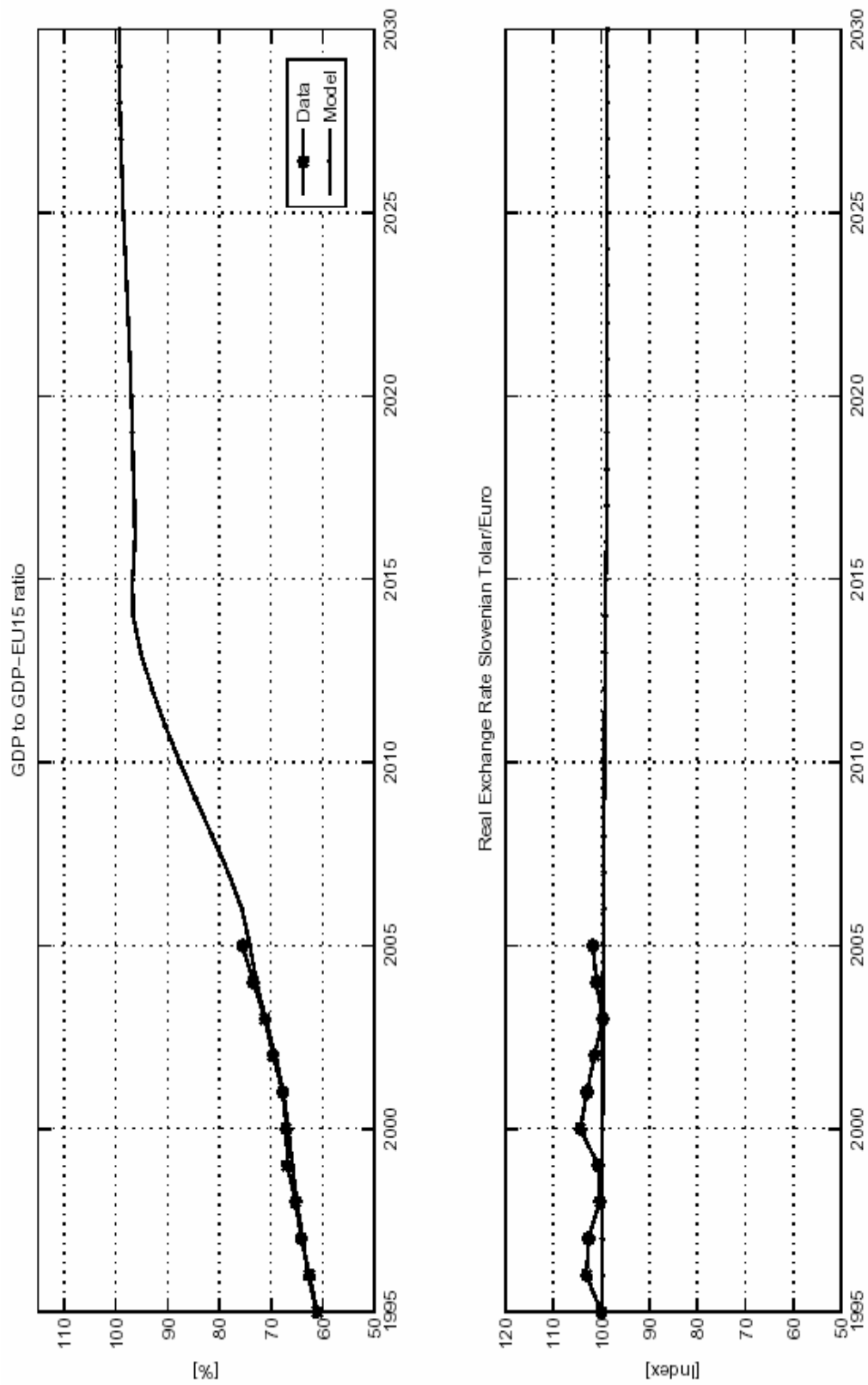
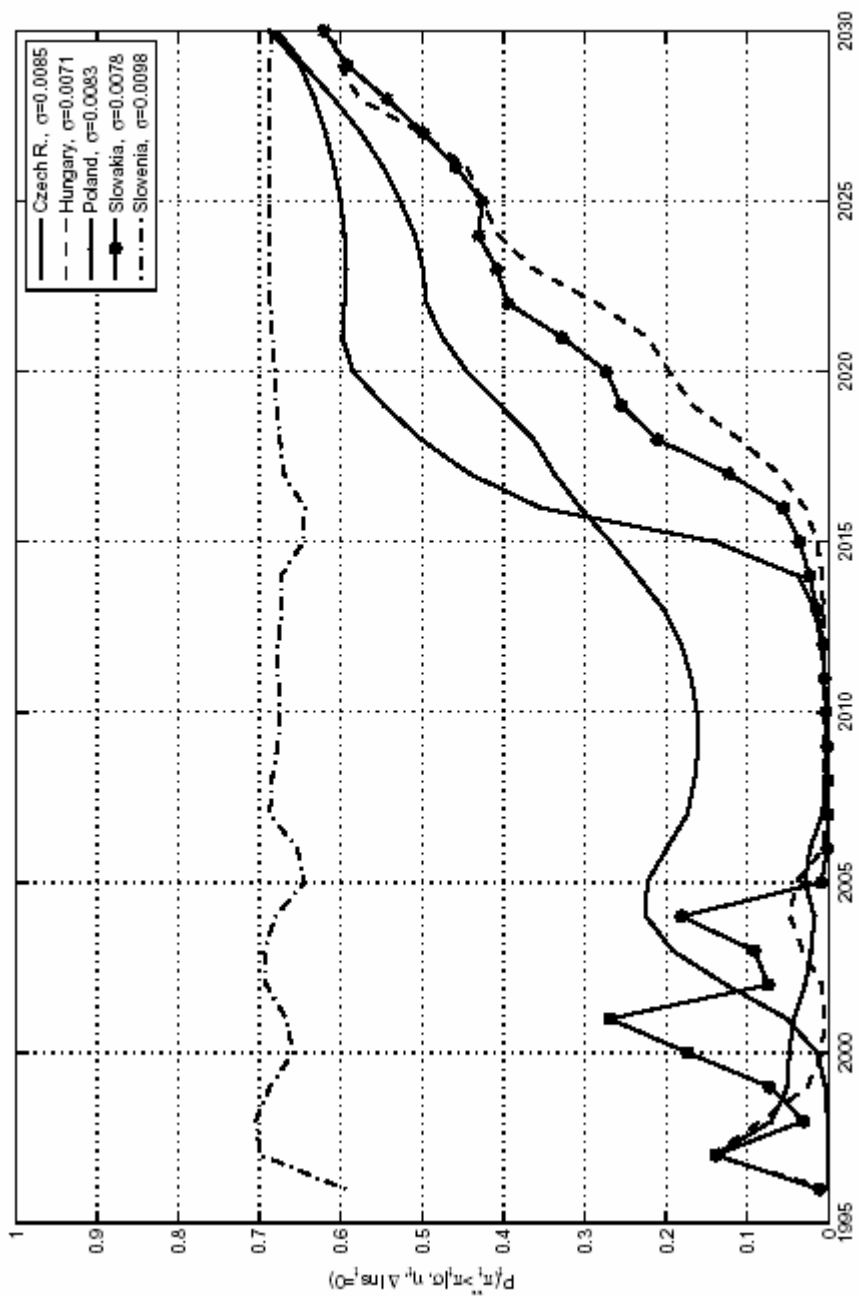


Fig. 13. The probability of inflation criterion fulfillment



## European Central Bank Working Paper Series

For a complete list of Working Papers published by the ECB, please visit the ECB's website (<http://www.ecb.int>)

- 699 "The behaviour of producer prices: some evidence from the French PPI micro data" by E. Gautier, December 2006.
- 700 "Forecasting using a large number of predictors: is Bayesian regression a valid alternative to principal components?" by C. De Mol, D. Giannone and L. Reichlin, December 2006.
- 701 "Is there a single frontier in a single European banking market?" by J.W. B. Bos and H. Schmiedel, December 2006.
- 702 "Comparing financial systems: a structural analysis" by S. Champonnois, December 2006.
- 703 "Comovements in volatility in the euro money market" by N. Cassola and C. Morana, December 2006.
- 704 "Are money and consumption additively separable in the euro area? A non-parametric approach" by B. E. Jones and L. Stracca, December 2006.
- 705 "What does a technology shock do? A VAR analysis with model-based sign restrictions" by L. Dedola and S. Neri, December 2006.
- 706 "What drives investors' behaviour in different FX market segments? A VAR-based return decomposition analysis" by O. Castrén, C. Osbat and M. Sydow, December 2006.
- 707 "Ramsey monetary policy with labour market frictions" by E. Faia, January 2007.
- 708 "Regional housing market spillovers in the US: lessons from regional divergences in a common monetary policy setting" by I. Vansteenkiste, January 2007.
- 709 "Quantifying and sustaining welfare gains from monetary commitment" by P. Levine, P. McAdam and J. Pearlman, January 2007.
- 710 "Pricing of settlement link services and mergers of central securities depositories" by J. Tapking, January 2007.
- 711 "What "hides" behind sovereign debt ratings?" by A. Afonso, P. Gomes and P. Rother, January 2007.
- 712 "Opening the black box: structural factor models with large cross-sections" by M. Forni, D. Giannone, M. Lippi and L. Reichlin, January 2007.
- 713 "Balance of payment crises in emerging markets: how early were the "early" warning signals?" by M. Bussière, January 2007.
- 714 "The dynamics of bank spreads and financial structure" by R. Gropp, C. Kok Sørensen and J.-D. Lichtenberger, January 2007.
- 715 "Emerging Asia's growth and integration: how autonomous are business cycles?" by R. Ruffer, M. Sánchez and J.-G. Shen, January 2007.
- 716 "Adjusting to the euro" by G. Fagan and V. Gaspar, January 2007.
- 717 "Discretion rather than rules? When is discretionary policy-making better than the timeless perspective?" by S. Sauer, January 2007.

- 718 “Drift and breaks in labor productivity” by L. Benati, January 2007.
- 719 “US imbalances: the role of technology and policy” by R. Bems, L. Dedola and F. Smets, January 2007.
- 720 “Real price wage rigidities in a model with matching frictions” by K. Kuester, February 2007.
- 721 “Are survey-based inflation expectations in the euro area informative?” by R. Mestre, February 2007.
- 722 “Shocks and frictions in US business cycles: a Bayesian DSGE approach” by F. Smets and R. Wouters, February 2007.
- 723 “Asset allocation by penalized least squares” by S. Manganelli, February 2007.
- 724 “The transmission of emerging market shocks to global equity markets” by L. Cuadro Sáez, M. Fratzscher and C. Thimann, February 2007.
- 725 “Inflation forecasts, monetary policy and unemployment dynamics: evidence from the US and the euro area” by C. Altavilla and M. Ciccarelli, February 2007.
- 726 “Using intraday data to gauge financial market responses to Fed and ECB monetary policy decisions” by M. Andersson, February 2007.
- 727 “Price setting in the euro area: some stylised facts from individual producer price data” by P. Vermeulen, D. Dias, M. Dossche, E. Gautier, I. Hernando, R. Sabbatini and H. Stahl, February 2007.
- 728 “Price changes in Finland: some evidence from micro CPI data” by S. Kurri, February 2007.
- 729 “Fast micro and slow macro: can aggregation explain the persistence of inflation?” by F. Altissimo, B. Mojon and P. Zaffaroni, February 2007.
- 730 “What drives business cycles and international trade in emerging market economies?” by M. Sánchez, February 2007.
- 731 “International trade, technological shocks and spillovers in the labour market: a GVAR analysis of the US manufacturing sector” by P. Hiebert and I. Vansteenkiste, February 2007.
- 732 “Liquidity shocks and asset price boom/bust cycles” by R. Adalid and C. Detken, February 2007.
- 733 “Mortgage interest rate dispersion in the euro area” by C. Kok Sørensen and J.-D. Lichtenberger, February 2007.
- 734 “Inflation risk premia in the term structure of interest rates” by P. Hördahl and O. Tristani, February 2007.
- 735 “Market based compensation, price informativeness and short-term trading” by R. Calcagno and F. Heider, February 2007.
- 736 “Transaction costs and informational cascades in financial markets: theory and experimental evidence” by M. Cipriani and A. Guarino, February 2007.
- 737 “Structural balances and revenue windfalls: the role of asset prices revisited” by R. Morris and L. Schuknecht, March 2007.
- 738 “Commodity prices, money and inflation” by F. Browne and D. Cronin, March 2007.
- 739 “Exchange rate pass-through in emerging markets” by M. Ca’ Zorzi, E. Hahn and M. Sánchez, March 2007.
- 740 “Transition economy convergence in a two-country model: implications for monetary integration” by J. Brůha and J. Podpiera, March 2007.

ISSN 1561081-0



9 771561 081005