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CONTENTS

Abstract

In this paper we propose an extension to New International Macroeconomic framework by introducing the vertical investment margin. The dynamic properties of the extended model are discussed in relation to relevant existing models with particular emphasis on the impact of productivity convergence and effects of timing of trade and financial liberalization on the convergence patterns. We compare the mechanisms behind the three investment margins (horizontal investment to new varieties, vertical investment to quality, and investment to export-eligibility) for the long-run equilibrium. Based on such comparison, the proposed extension proves crucial for consistent explanation of long-term trends in macroeconomic aggregates and the real exchange rate development observed in European transition countries.

Key words: Two-country modeling, Convergence, New International Macroeconomics
J.E.L. Classification: F12, F36, F41.

Non-Technical Summary

This paper analyzes the potential of two-country dynamic general equilibrium modeling initiated by the so-called New International Macroeconomic framework for understanding the convergence processes of emerging market economies. The emphasis is put on long-run trends of the main macroeconomic variables and on the development of the real exchange rate. In particular, proposed explanation of the real exchange rate pattern is an innovative aspect of the paper since the pace of the real exchange rate appreciation, experienced by some central and eastern European countries, remains unaddressed in standard Dynamic Stochastic General Equilibrium models.

The New International Macroeconomic framework is characterized by monopolistic competition, heterogeneity of production entities and microfounded tradeness self-selectiveness. The paper shows that, under suitable extensions of the canonical structure of New International Macroeconomic models, it is possible to explain the long-run development of the main macroeconomic variables together with alternative real exchange rate developments depending on the the type of convergence as recently experienced by transition economies. In this respect, the crucial extension is the introduction of two endogenous dimensions of investments: horizontal (number of varieties) and vertical (quality investment as in quality ladder literature).

The extended model is then adapted to the framework of countries, which are asymmetric in size and in the total factor productivity. First, we inquire on the steady-state impact of an increase in the total factor productivity (TFP) on the real exchange rate and terms-of-trade. We document that the three margins (horizontal, vertical, and export-eligibility) can generate the real exchange rate appreciation after a TFP increase and discuss how the channels behind the three margins differ.

Then we use numerical simulation of the transition dynamics to investigate intertemporal mechanisms implied by the three investment margins. The simulations reveal that the vertical margin reduces consumption smoothing. Moreover, under the realistic calibration of the model, without the vertical investments it is impossible to generate the real-exchange rate appreciation observed in selected Central European transition countries. We further provide a discussion about the implications of different assumptions on the productivity of multinational firms and show what is needed to obtain a realistic pattern of the financial and current account patterns of transition countries. And finally, model properties are also illustrated on simulation experiments with trade-cost declines.

1 Introduction

The purpose of this paper is to introduce an extension to the existing two-country dynamic general equilibrium models with NIM features for understanding the convergence process of emerging market economies and provide an extensive discussion on model properties. This is of special interest, since macroeconomic dynamics of transition economies is even more puzzling from the perspective of standard DSGE models than that of advanced economies.

The following five facts dominate the picture of the economic development in transition countries in Central and Eastern Europe (so called Visegrad-4 countries: the Czech Republic, Hungary, Poland, and Slovakia) during a transition decade 1995-2005, i.e. after the basic institutional foundations of a market economy has been created [1]:

Fact 1: The convergence in GDP per capita of an average Visegrad-4 country to the average of the EU15 attained 1 per cent a year on average over the decade.

Fact 2: Significant trade integration has led to an increase in export to GDP ratio on average over Visegrad-4 countries by 2 per cent a year over the decade. And trade balance, after initial deficit around 5 per cent, has reached balanced position at the end of the decade.

Fact 3: The privatization and economic attractiveness of the region have resulted in a significant inflow of the foreign direct investment; on average the inflow in Visegrad-4 countries reached 5 bln. USD a year over the decade. The foreignly-owned firms have increased their share on the total exports (in some countries even up to 90%).

Fact 4: Real exchange rates – *also in sub-index of tradable goods* – of Visegrad-4 currencies vis à vis the Euro have been appreciating by an average of slightly exceeding 2 per cent a year.

Fact 5: The proportion of medium-high and high tech products in total exports has gained 1.5 to 2 per cent a year, see Fabrizio et al (2006).

The five facts are demonstrated using Figure 1.

In order to consistently explain these facts one needs a quite sophisticated

[1] The following applies to all Visegrad-4 countries: the EBRD index of price liberalization shows that all these countries have liberalized prices to the level 3 that is comparable to advanced industrialized countries by 1993. Similarly, all these countries liberalized trade and foreign exchange until 1995 to the extent (level 4, EBRD index) considered as standard for advanced industrialized countries. And finally, the small as well as large scale privatization has been completed (level 4, EBRD index) by 1995 and 1997, respectively. For more discussion on institutional foundation and reforms implementation, see Roland (2004).

framework that models endogenous heterogeneity of ownerships, composition of aggregate total factor productivity and export self-selectiveness to match the structural changes observed in reality: an increasing foreign ownership of companies, higher total factor productivity in companies under foreign control, increasing share of export facilitated by foreign owned companies, and improvement in trade balance together with appreciating domestic currencies.

Some of the above mentioned mechanisms pertaining to the transition economy are also typical for the New International Macroeconomic (henceforth NIM) framework, i.e., the monopolistic competition, heterogeneity of production entities and trade self-selectiveness, as in Melitz (2003). These models provide a rigorous microfoundation for a bulk of observations, which are puzzling from the perspective of the standard DSGE models (such as persistent deviations from the PPP or low volatility in the relative price of nontraded goods) [2] .

However, the concurrently observed Fact 2 and Fact 4 calls for an extension of the available framework, since relatively more goods (Fact 2 and Fact 3) is sold for relatively higher prices (Fact 4). This trend development in Visegrad-4 countries can be only reconciled by a steady improvement in quality of products (Fact 5). And since the canonical model of NIM operates with homogenous goods, is encounters obvious problems in explaining consistently all the Facts stated above. This could be seen on rather isolated attempt by Bayoumi et al. (2004) to use the NIM framework to explain the macroeconomic dynamics of transition countries. They construct a DSGE model with the NIM features and calibrate it for a transition economy (the Czech Republic). However, their model does not address any specific transition feature and thus its applicability for realistic convergence projections remains limited especially because the model is not able to replicate the significant observed pace of the real exchange rate appreciation (i.e., Fact 4).

Therefore, in this paper we present an extension of the NIM framework, which is essentially aimed at relaxing the homogeneity of products in the NIM models. Our model assumes that investments have two dimensions: investment into new varieties and investment into quality, where the latter is an extension to the canonical NIM structure. Both decisions are taken endogenously and, on aggregate, influence real exchange rate and convergence dynamics. Such a model can then provide a solid basis for policy assessment and transition dynamics projections (including conditional real exchange rate projections, see Brůha and Podpiera, 2007 for such conditional projections and policy implications).

[2] The framework is used, for example, by Ghironi and Melitz (2005) to explain international business-cycle dynamics, by Naknoi (2006) to decompose real exchange rate movements, or by Bergin and Glick (2005) to study the behavior of price dispersion during episodes of international economic integration.

The comprehensive two-country model is formulated with the purpose of capturing long-run trends in main macroeconomic variables of a converging economy. Thus, contrary to a usual practice of applied DSGE models, which attempt to characterize the short-run fluctuations around a steady state or around an exogenously given development trajectory, the proposed model yields a long-run trajectory of convergence of asymmetric countries – the countries differ by total factor productivity (TFP) and size.

Since the stress is in long-run trends rather than short-run fluctuations around these trends, the model is formulated as a dynamic, perfect-foresight model. The model lacks stochastics in aggregates since short run fluctuations are out of the scope: rather the model can provide guidance on how the pace of productivity convergence and policy interventions (such as timing and degree of trade and financial liberalization) affect the long-run convergence and what are the implications for macroeconomic dynamics. The special interest is in the pace of the real exchange rate appreciation and GDP per capita convergence. This is of particular importance in relation to applied model to real data, as provided by Brůha and Podpiera (2007) or Brůha, Podpiera and Polák (2007).

Since the proposed extension is a novel feature, model properties are carefully investigated. Analytically only the symmetric steady state of the model can be characterized. The analysis yields useful insights and enriches the NIM literature by an explicit incorporation of asymmetricity of countries. Thus, we explore the transition dynamics through a set of simulations. We especially concentrate on the role of production factors, the role of export self-selectivity, and productivity diffusion on the transition dynamics of an emerging economy, which converges to its more advanced counterpart. The simulation inquiry on the model properties is as follows.

First, we compare the extended model transition dynamics with those implied by alternative formulations. In particular, we contrast the extended model with (i) a model without quality investments as a specific production factor, (ii) a model under different financial structure and (iii) a model without the export self-selectiveness (thus without a prominent NIM feature). Other aspects and structural parameters of the alternative models are held constant, thus one can sense how various model assumptions are translated into model dynamics.

Second, we provide a set of trade-barriers removals experiments using alternative models since we believe that this set can also yield valuable insights on model properties. This is a popular issue with applied international models (see Bayoumi et al 2004). We are inspired by exercises[3] by Bayoumi et al (2004) and by Baldwin (2005) and stress similarities and differences.

[3] Since the NIM framework seems to be better microfounded than standard open-

The rest of the paper is organized as follows: Section 2 describes the model and Section 3 characterizes the steady state. Section 4 highlights features of the proposed model by contrasting them with alternative formulations. Section 5 simulates policy interventions. Section 6 concludes. Appendix contains detailed derivation of the model equations and discusses numerical methods for model simulation.

2 Description of the Model

This section presents the workhorse model used throughout this paper. The two countries are modeled in discrete time that runs from zero to infinity. The home country is populated by a representative competitive household who has recursive preferences over discounted streams of momentary utilities. The momentary utility is derived from consumption. A similar household inhabits the foreign country. Production takes place in heterogenous production entities called firms [4].

2.1 Firms

In the domestic country, there is a large number of firms, which may be owned by either a domestic or by a foreign household [5]. In each period there is an unbounded mass of new, ex-ante identical, entrants. The entry of new firms is costly because of investment costs and therefore households balance costs and benefits of additional entrants, they will finance and own.

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economy models, it seems to be appropriate for welfare evaluation of policy regimes. Naknoi et al. (2005) use the NIM framework to compare benefits and costs of fixed versus flexible exchange rate regimes and Baldwin and Okubo (2005) integrate the NIM approach to a New Economic Geography model and derive a set of useful normative assessments and positive political-economy predictions of economic integration.

[4] The production entities are called firms. However, since we aim at understanding the development of transition economies, which are likely to experience a significant change in the production structure, it would be more appropriate to think of production entities as of *production projects*. In such an application, it may be reasonable to calibrate the exit rate of production entities relatively high.

[5] Since we aim at investigating the impact of cross-border asset ownership, we will deal with a model version without cross-border asset ownership as well. However, the workhorse version allows for endogenous cross-border asset ownerships.

Firms ex-post entry differ by an idiosyncratic variation of the total factor productivity: when a firm enters, it draws a shock z from a distribution $G(z)$, which has the support on $\langle z_L, z_U \rangle$ with $0 \leq z_L < z_U < \infty$. At the end of each period, there is an exogenous probability that a firm is hit by an exit shock. This probability is δ and is assumed to be independent on aggregate as well as individual states. Hit firms shut down.

The production function maps two inputs into two outputs. The first input is fixed and we label it as 'capital', the second input is variable and is labeled as 'labor'. The variable input – labor – is available in inelastic supply in each country and is immobile between countries.

The first output is quality h and if the firm j uses k_j units of capital, then the quality of its product is given simply as $h_j = k_j$. Capital investment can be thus considered as an improvement in quality. The second output is the physical quantity of produced goods x. The production function is given as follows: $x_{jt} = z_j A_t \ell(l_{jt}, k_j)$. The production function ℓ is strictly increasing in the first argument (labor), but strictly decreasing in the second argument (capital)[6]. This implies that investments into quality increase the needed labor inputs to produce physical quantities. One may think that the production of a more sophisticated good requires more labor or more skilled labor. Thus, quality investment is costly for two reasons: first, it requires fixed input k_j, and second, more labor is required to produce better goods.

The production of the physical quantities is increasing in the level of firm total factor productivity $A_t z_j$, which has two components: (a) idiosyncratic component z_j, which is i.i.d. across firms and which follows distribution $G(z)$ introduced above, and (b) the common component A_t. The total factor productivity A_t pertains to the ownerships: firms owned by the domestic household enjoy at time t the productivity A_t^H, while firms owned by the foreign household enjoy the productivity A_t^F. The productivity does not depend on the location of production or on the time of entry (the time of entry is henceforth called *vintage*) of firms.

We assume that the final output of the firm is given by the product of quality and quantity: $q_{jt} = h_j x_{jt}$ and that this final quality-quantity bundle is what is sold at the market. This assumption reflects the nowadays standard approach of growth theoreticians, for example Young (1998). Thus, the production of the final bundle can be described as $q_{jt} = z_j A_t f(k_j, l_{jt})$, where f is given as $f(k_j, l_{jt}) \equiv k_j \ell(l_{jt}, k_j)$. We assume that the final bundle production function is increasing in both arguments and is homogenous of degree one. This places some restrictions on the quantity production function ℓ; the most important restriction is that ℓ should be homogenous of degree zero.

[6] We require that the function ℓ is strictly decreasing in the capital. If the function ℓ were not decreasing in capital, the linearity of h_j in k_j would imply endogenous growth, as in Young (1998) or Baldwin, Forslid (2000).

The quality investment is a fixed factor, set at the time of entry, while labor can be freely adjusted[7]. Given a realization of the productivity shock z_j, the probability of the exit shock δ, and a chosen production plan, the value of a firm is determined by the stream of discounted profits. Given a realization of the productivity shock z_j, the probability of the exit shock δ, and a chosen production plan, the value of a firm is determined by the expected present value of the stream of profits.

Since the presented model involves several kinds of goods and firms, we have to use indexes to distinguish them. To make reading the paper easier, we introduce the following convention. Firms differ by location, ownership, and vintage. Location of firms is distinguished by superscript d - for the *domestic* country and f - for the *foreign* country. Firms owned by the foreign household are denoted by the $*$ superscript, while the domestic ownership is given no special superscript. Vintage is denoted by Greek letters τ, σ, while the real time is denoted by Latin character t, v.

Firms produce differentiated goods. The good produced by the firm located in the destination market is denoted by the d superscript, while goods imported are denoted by the m superscript. The destination market is denoted by the * superscript again: goods consumed by the domestic household are without superscript, while goods consumed by the foreign household do have it. Thus p^d_{jt} will denote the price of a good produced by a firm j located in the domestic country at time t sold to the domestic market, p^m_{jt} is the price of a good j imported to the domestic market from the foreign country, while p^{m*}_{jt} would be a price of a good from the domestic country to the foreign household. We further assume that prices are denominated in the currency of the market.

According to the introduced convention, $\mathbb{P}^d_{j\tau t}$ denotes a t-period *real operating profit* of the firm located in the domestic country of vintage τ and owned by the domestic household. The real operating profit $\mathbb{P}^d_{j\tau t}$ is given as follows:

$$\mathbb{P}^d_{j\tau t} = \left[\kappa_{jt}\frac{p^d_{jt}}{P_t} + (1-\kappa_{jt})\,\frac{\eta_t}{1+\mathbf{t}}\frac{p^{m*}_{jt}}{P^*_t}\right] A^H_t z_j f(k_j, l_{jt}) - \mathbb{W}_t l_{jt},$$

where $0 \leq \kappa_{jt} \leq 1$ is the share of product sold in the domestic markets, P_t is the domestic price level, P^*_t is the foreign price level, η_t is the *real exchange rate*, which is linked to the nominal exchange rate s_t as $\eta_t = s_t P^*_t / P_t$, $\mathbf{t} \geq 0$ represents unit iceberg exporting costs and $\mathbb{W}_t$ is the *real wage*. Firms of different vintage and different ownership have different levels of invested capital,

[7] The capital is firm specific and the model lacks the usual one-lag time-to-build assumption. The time-to-build is not needed in our model since we aim at long-run dynamics, not at short-run fluctuations.

that is why $\mathbb{P}^d_{j\tau t}$ will be naturally different along these dimensions. Similar definitions apply to the remaining types of firms as well.

Note that prices such as p^d_{jt} are prices of the final quantity-quality bundles and therefore derived indexes P_t, P^*_t, η_t are related to aggregations of these final bundles. The prices related to physical quantities are then given by $\wp^d_{jt} \equiv k_j p^d_{jt}$. The discussion about distinct roles of prices of quality-quantity bundles and of prices defined on physical quantities is left to subsection 2.4.

Firms may export only if special fixed costs are sunk. If a firm at the time of entry decides to sunk the fixed export costs, then it becomes eligible to export in all subsequent periods, otherwise it is for all periods not eligible to export. Exporting decisions of *eligible* firms are taken on a period-by-period basis. Thus an eligible firm may decide not to export in a given period. However, in equilibrium all eligible firms will find profitable to export in any period; even in the case of very unfavorable prices, eligible firms will export at least a small amount of production (see Lemma 1 in 2.1.2). Unit iceberg exporting costs $\mathbf{t}$ represents transportation costs, policy barriers such as tariffs, while the fixed costs may represent expenditures associated with acquiring necessary expertise such as legal, business, or accounting issues of the foreign markets. Obviously, non-eligible firms have $\kappa_{jt} \equiv 1$ regardless of the state of the world.

Capital is the fixed factor and each firm decides how much capital to acquire at the time of entry: this means that the firm decides the quality of its product at the entry time, while produced quantities are variable during its lifetime. We assume that real investment costs take the following form: $(k + c^{\xi})$, $\xi \in \{e, n\}$. We assume that:

$$c^e > c^n > 0,$$

where the superscript refers to eligibility, i.e. $e - \textit{eligible}$ or $n - \textit{noneligible}$: eligible firms pay larger fixed costs.

The cost structure implies – as in Melitz (2003) – that in equilibrium there is a cut-off productivity value $\overline{z}$, such that firms with lower idiosyncratic productivity $z_j < \overline{z}$ will not invest to become eligible, while firms with a sufficiently high productivity level $z_j \geq \overline{z}$ will do[8].

Three types of entry costs are usually dealt with in the NIM framework (Baldwin 2005): the first one is the invention cost, i.e. a cost of inventing a new variety. After a variety is invented and its productivity is revealed, there is a fixed set-up cost of production and finally there is also a fixed cost of export eligibility. Not in all models, all types of costs are necessarily present, but the

[8] We assume that if a firm is indifferent whether to become export eligible, it will decide to become. This is completely an innocent assumption provided that the distribution function G is absolutely continuous with respect to the Lebesgue measure: in such a case the probability of indifference is zero.

distinguishing feature of the NIM is the non-trivial export decision cost. Under the model specification employed here, the fixed cost c^n is actually a cost of inventing a new variety (an action, which is not necessarily profitable ex-post [9]), while the difference $c^e - c^n$ would correspond to the export-eligibility costs.

The fixed cost of the production set-up is not present in this model. Nevertheless the formulation of the model implies that all firms find profitable to spend strictly positive investment into quality ($k_j > 0$), since otherwise its products would be worthless. Therefore, there are set-up costs given by k_j, but these are variable, since capital is invested *after* learning z_j and therefore invested capital will be different for firms with different z_j. Note that the invested capital as a function of the idiosyncratic shock is not likely to be continuous: in fact it exhibits a jump discontinuity at $\overline{z}$: the invested capital jumps at the margin when a firm decides to become export-eligible. Figure 2 illustrates this fact. Subfigures show the optimal quality investment k_j as a function of idiosyncratic productivity z_j for firms located in the domestic country under both ownerships. The upper subfigure shows this for a symmetric steady state, while the lower subfigure shows asymmetric steady state with $A^H = 0.7A^F$. The rest of parametrization corresponds to the baseline of Section 4.1. The jumps occur at the cut-off level. As can be expected, the investment into quality is higher for a foreignly owned firm than for a domestically owned firm under the symmetric steady state. The reason is the higher productivity A^F of foreignly owned firms. Naturally, this asymmetry collapses in the case of symmetric steady state.

There are two parameters related to the degree of trade frictions and trade openness: the iceberg costs $\mathbf{t}$ and the ratio c^e/c^n. The fall in the former is related to a fall in iceberg (ad valorem) costs, which are for example transportation costs, while the latter is fixed in nature. Section 5 simulates consequences of declines in both types of costs and discusses the differences. This is in our view an important, but sometimes neglected, issue. The study by Bayoumi et al. (2004) considers only a decrease in iceberg costs, which is insufficient for the comprehensive modeling of macroeconomic consequences of the EU integration. Garganas (2004) argues that the integration to the EU is probably more appropriately seen as a fall in fixed trade costs because of integration of legal and institutional environments and therefore we provide both kinds of simulations. Nevertheless, the EU accession is not only about the trade barriers removal – which happened in earlier years of transition (Roland, 2004) – but also about financial integration. Indeed, the institutional accession close to the EU structures may diminish fears of political reversals, and implementation of *acquis communautaire* has improved legal environments. Both of which

[9] This will be the case, if the invented variety experiences a low enough idiosyncratic shock z_j.

have probably contributed to a decrease in the perceived country risk. A possible way of modeling the degree of financial integration and openness will be discussed in Subsection 2.3.

We assume that firm's manager maximizes the expected discounted stream of profits. The discounting respects the ownerships. Thus, the value of the profit stream of the firm of vintage τ, enjoying the idiosyncratic productivity level z_j and owned by the domestic household is (in real terms):

$$V_\tau^d(z_j) = \max_{\xi,k,\{l_\tau\},} \sum_{t=\tau}^{\infty}(1-\delta)^{t-\tau}\mu_\tau^t \mathbb{P}_{j\tau t}^d - (c^\xi + k), \tag{1}$$

where $\mathbb{P}_{j\tau t}^d$ is the t-time real operating profit of a firm of vintage τ, enjoying the productivity level z_j under the optimal production plan (derived later in Subsubsection 2.1.2), and the effective discount factor is given as $(1-\delta)^{\tau-t}\mu_\tau^t$, where μ_τ^t is the marginal rate of intertemporal substitution between dates τ and t. The rate of the intertemporal substitution is defined in more details in Subsection 2.2. The value of the firm owned by the foreign household is defined analogously, with the exception that the marginal rate of the intertemporal substitution is taken from the perspective of the foreign household.

To summarize the sequencing, the timing proceeds first with the domestic and foreign households' decision about a number of new entrants in both countries. Then, each new entrant draws a productivity level from the distribution G and the owner decides the amount of invested capital and whether to invest for export eligibility. Then, labor demand and production (of both entrants and incumbents) take place. At the end of the period, some firms experience the exit shock and shut down.

Even firms located in the same country and owned by the same household differ along two dimensions: idiosyncratic productivity level z_j and vintage τ. Ownership within each country affects the amount of invested capital since both households have different rates of the intertemporal substitution along the transition path. Likewise, the vintage affects incentives to invest. This implies that firms of different vintage and ownership will invest different amounts of capital, even if they experience the same idiosyncratic productivity level. Therefore we shall define the time-varying distribution measure over firms as $\Gamma_t^d(j,\tau)$ for the firms in the home country owned by the domestic household; the star version $\Gamma_t^{d*}(j,\tau)$ will denote the analogous measure for the firms owned by the foreign household. The counterparts of firms located in the foreign country are denoted by $\Gamma_t^f(j,\tau)$ and $\Gamma_t^{f*}(j,\tau)$. The superscript convention applied to the distributions follows the one applied to firms.

For the sake of reader's convenience, we stress that the distribution $\Gamma_t^d(j,\tau)$

For the sake of reader's convenience, we stress that the distribution $\Gamma_t^d(j,\tau)$ should be understand as follows:

$$\int \lambda_{\tau t}\,\mathrm{d}\Gamma_t^d(j,\tau) \equiv \sum_{\tau \leq t} n_\tau^e (1-\delta)^{t-\tau} \int \lambda_{\tau t}\, G(\mathrm{d}z_j),$$

where $\lambda_{\tau t}$ represents suitable integrands (such as prices or quantities) of vintage τ at time t and n_τ^e is the number of d-type entrants of vintage τ. Therefore $n_\tau^e(1-\delta)^{t-\tau}$ is the number of firms, which has entered at time τ and are still alive at time t. Analogous definitions hold for distributions $\Gamma_t^{d*}(j,\tau)$, $\Gamma_t^f(j,\tau)$, $\Gamma_t^{f*}(j,\tau)$ as well.

2.1.1 Market Structure

The final good[10] Q in domestic country is composed of a continuum of quality-quantity bundles (goods), some of them are produced in the domestic country and some are imported. There is an imperfect substitution among these goods. The parameter $\theta > 1$ measures substitution among goods. The limit case $\theta \to \infty$ implies perfect substitution and hence perfect competition. The aggregate good in the domestic country is defined as:

$$Q_t = \left(\sum_{\xi \in \{d,d*\}} \int_{\Omega^\xi} \left(q_{jt}^d\right)^{\frac{\theta-1}{\theta}} \mathrm{d}\Gamma_t^\xi(j,\tau) + \sum_{\xi \in \{f,f*\}} \int_{\Omega_e^\xi} \left(q_{jt}^m\right)^{\frac{\theta-1}{\theta}} \mathrm{d}\Gamma_t^\xi(j,\tau) \right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

where, q_j is the output of the firm j, Ω^d denotes the set of products of firms located in the domestic country and owned by the domestic household, Ω^{d*} denotes the set of products of firms located in the domestic country and owned by the foreign household. The analogous convention holds for sets of firms located in the foreign country: Ω^f, Ω^{f*}. If a set is labeled by the subscript e, it reads as a subset of eligible firms. Thus, $\Omega_e^{f*} \subset \Omega^{f*}$ is the subset of goods produced by *eligible* firms owned by the foreign household located in the foreign country[11]. The final good in the foreign country is defined similarly. The market structure implies the following definition of the aggregate price index:

$$P_t = \left(\sum_{\xi \in \{d,d*\}} \int_{\Omega^\xi} \left(p_{jt}^d\right)^{1-\theta} \mathrm{d}\Gamma_t^\xi(j,\tau) + \sum_{\xi \in \{f,f*\}} \int_{\Omega_e^\xi} \left(p_{jt}^m\right)^{1-\theta} \mathrm{d}\Gamma_t^\xi(j,\tau) \right)^{\frac{1}{1-\theta}}, \tag{3}$$

where p_{jt} is the price of products of firm j at time t. Note that the final good Q_t represents both physical quantities as well as qualities and that the price

[10] The final good is consumption as well as investment good, so that Q can be interpreted as domestic absorption.

[11] Therefore it holds that $q_j^d \in \Omega^d$ or $q_j^d \in \Omega^{d*}$ and $q_j^{m*} \in \Omega_e^d$, $q_j^{m*} \in \Omega_e^{d*}$, but $q_j^{m*} \notin \Omega^d \setminus \Omega_e^d$ nor $q_j^{m*} \notin \Omega^{d*} \setminus \Omega_e^{d*}$.

indexes P_t, P_t^* aggregate both: available quantities and qualities. In that sense, these are quality-adjusted price indexes. If one wants to construct counterparts of empirical price indexes, one has to aggregate price $\wp_{jt}^d$, rather than p_{jt}^d.

The CES market structure implies that the residual demand at the domestic market satisfies:

$$q_{jt}^d = \left(\frac{p_{jt}^d}{P_t}\right)^{-\theta} Q,$$
$$q_{jt}^m = \left(\frac{p_{jt}^m}{P_t}\right)^{-\theta} Q_t.$$

Analogous formulae apply to the residual demands at the foreign market as well.

2.1.2 Optimal Plans

In this part, we derive optimal production and investment plans using the backward induction. We derive it for a firm located in the domestic country, which is owned by the domestic household. The reader can then easily derive optimal plans for other types of firms. This part of the paper shows the backward induction for general neoclassical production function satisfying the Inada condition at zero (thus we rule out the constant-elasticity-of-substitution production functions, with the elasticity of substitution less than one [12]). The parametric example of model equations for the Cobb-Douglas production function is given in Appendix A.1.

Thus, assume the problem of maximizing the value of a firm, under given location, ownership, and sunk investments. Since there are no labor adjustment costs, labor decisions are made on a period-by-period basis. Standard results of monopolistically competitive pricing suggest that prices are set as a mark-up over marginal costs. Marginal costs differ by idiosyncratic productivity and invested capital, thus firms enjoying identical productivity levels z_j and identical capital levels k_j are supposed to price identically, but firms with different characteristics charge different prices $\left\{p_{jt}^d, p_{jt}^{m*}\right\}$, and obviously produce different outputs. Simultaneously with prices, firms also decide κ_j.

[12] This requirement is imposed to rule out the corner solution, which would complicate the algebra of the model. The corner solution will be ruled out if the factor price is lower than the marginal product of the factor at zero (which is always the case if the production function obeys the Inada condition at zero). Anyhow, if one is willing to undergo complications induced by possibility that some firms will not find profitable to produce anything, then one can work with the general neoclassical production function.

Under the CES market structure, Lemma 1 shows that eligible firms would produce goods for both markets (if they produce at all).

Lemma 1 *Under the monopolistically competitive CES market structures with $\theta < \infty$, eligible firms produce for both markets.*

PROOF. Define $q_{jt} = q^d_{jt} + q^{m*}_{jt}$ and consider an eligible firm. We will show that $q^d_{jt} q^{m*}_{jt} > 0$. The production decision problem can be rewritten as

$$\mathbb{P}^d_{j\tau t} = \frac{p^d_{jt}}{P_t}(q^d_{jt})q^d_{jt} + \frac{\eta_t}{(1+\mathbf{t})} p^m_{jt} * (q^{m*}_{jt})q^{m*}_{jt} - \mathbb{C}(q^d_{jt} + q^{m*}_{jt}) \rightarrow \max,$$
$$\text{s.t.} \quad q^d_{jt} \geq 0, q^m_{jt} * \geq 0. \tag{4}$$

where $\mathbb{C}$ is a cost function, associated with the production function f. First order conditions dictate:

$$\frac{\mathrm{d}\mathbb{P}^d_{j\tau t}}{\mathrm{d}q^d_{jt}} = -\lambda^d_{jt} \leq 0, \quad , \frac{\mathrm{d}\mathbb{P}^d_{j\tau t}}{\mathrm{d}q^{*m}_{jt}} = -\lambda^{m*}_{jt} \leq 0,$$

where λ^ξ_{jt} are Kuhn-Tucker multipliers associated with the non-negativity constraints. These multipliers satisfy: $\lambda^\xi_{jt} \geq 0$ and $q^\xi_{jt}\lambda^\xi_{jt} = 0$, $\xi \in \{d, m*\}$ Expanding F.O.C. yields

$$\lambda^d_{jt} + \left(\frac{q^d_{jt}}{Q_t}\right)^{-\frac{1}{\theta}} \frac{\theta - 1}{\theta} = \lambda^{m*}_{jt} + \frac{\eta_t}{(1+\mathbf{t})} \left(\frac{q^{m*}_{jt}}{Q^*_t}\right)^{-\frac{1}{\theta}} \frac{\theta - 1}{\theta} = MC_{jt} > 0,$$

which implies that $\lambda^d_{jt} = \lambda^{m*}_{jt} = 0$ because $\lim_{q^\xi_{jt} \rightarrow 0^+} \left(q^\xi_{jt}\right)^{\frac{-1}{\theta}} \rightarrow +\infty$, thus proving the claim.

Since it is impossible that eligible firms would experience strictly lower expected present value of its operating profit stream than non-eligible firms (eligible firms can always secure as large expected present value of operating profit streams as non-eligible firms by selling the total output at the domestic market), and since they sell at least some amount at the foreign market, we immediately get the following corollary:

Corollary: *Lemma 1 implies that eligible firms experience a strictly higher expected present value of the operating profit stream than non-eligible firms.*

Now, let us take the perspective of a non-eligible firm of vintage τ and productivity level A^H_t. Its real operating profit $\mathbb{P}^{dn}_{j\tau t}$ in a period t is given – conditional on non-eligibility status, aggregate productivity, idiosyncratic productivity z_j,

– as a solution to the following program:

$$\mathbb{P}^{dn}_{j\tau t} = \max_{l_{jt}} \left\{ \frac{p_{jt}}{P_t} A^H_t z_j f(k_j, l_{jt}) - \mathbb{W}_t l_{jt} \right\} = \tag{5}$$
$$= \max_{l_{jt}} \left\{ \left[A^H_t z_j f(k_j, l_{jt}) \right]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathbb{W}_t l_{jt} \right\}.$$

The second row of expression (5) (and the subsequent expression) follows from the CES market structure. Similarly, the real operating profit of an eligible firm $\mathbb{P}^{de}_{j\tau t}$ of vintage τ in a period t is given by:

$$\mathbb{P}^{de}_{j\tau t} = \max_{l_{jt}} \left\{ \left(\kappa_{jt} \frac{p_{jt}}{P_t} + (1-\kappa_{jt}) \frac{\eta_t}{1+t} \frac{p^*_{jt}}{P^*_t} \right) A^H_t z_j f(k_j, l_{jt}) - \mathbb{W}_t l_{jt} \right\} = \tag{6}$$
$$= \max_{l_{jt}} \left\{ \left(\kappa_{jt} Q_t^{\frac{1}{\theta}} + (1-\kappa_{jt}) \frac{\eta_t}{1+t} Q_t^{*\frac{1}{\theta}} \right) \left[A^H_t z_j f(k_j, l_{jt}) \right]^{\frac{\theta-1}{\theta}} - \mathbb{W}_t l_{jt} \right\}.$$

By comparing (5) with (6), it is obvious that if $\kappa_{jt} < 1$, then for given capital and idiosyncratic shocks z_j, $\mathbb{P}^{de}_{j\tau t} > \mathbb{P}^{dn}_{j\tau t}$. Indeed, Lemma 1 shows that $\kappa_{jt} < 1$ is the optimal choice of an eligible firm.

Then the expected present value of operating profit streams is given as follows

$$\mathbb{P}^{d\xi}_{j\tau} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \mathbb{P}^{d\xi}_{j\tau t}$$

with $\xi \in \{n, e\}$. It is obvious that $\mathbb{P}^{de}_{j\tau} > \mathbb{P}^{dn}_{j\tau}$. The expected present values depend on idiosyncratic productivity z_j, invested capital k_j, and the future path of productivities, real wages and demands.

The optimal investment decision of an eligible firm located in the domestic country and owned by the domestic household, which enjoys a productivity level z_j, maximizes the value of the firm, which is given as

$$\mathbf{V}^{de}_\tau(k_j|z_j) = \mathbb{P}^{de}_{j\tau} \left(z_j, k_j, \left\{ \mathbb{W}_{t+\tau}, Q_{\tau+t}, Q^*_{\tau+t}, A^H_{\tau+t}, \eta_{\tau+t} \right\}_{t=0}^{\infty} \right) - (c^e + k_j) \tag{7}$$

and similarly for a non-eligible firm:

$$\mathbf{V}^{dn}_\tau(k_j|z_j) = \mathbb{P}^{dn}_{j\tau} \left(z_j, k_j, \left\{ \mathbb{W}_{t+\tau}, Q_{\tau+t}, A^H_{\tau+t} \right\}_{t=0}^{\infty} \right) - (c^n + k_j). \tag{8}$$

Maximization of $\mathbf{V}^{de}_\tau(k_j|z_j)$ (resp. $\mathbf{V}^{dn}_\tau(k_j|z_j)$) yields the optimal demand for quality investment (capital) for eligible (resp. non-eligible) firms, and the value of a firm is:

$$V^{d\xi}_\tau(z_j) = \max_{k_j \geq 0} \mathbf{V}^{d\xi}_\tau(k_j|z_j),$$

where $\xi \in \{e, n\}$. Value functions $V_\tau^{dn}(z_j)$, $V_\tau^{de}(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the least idiosyncratic shock, which makes the the export-eligibility investment profitable[13]. Thus it is defined as

$$\overline{z}_\tau^d = \min_{z_j}(V_\tau^{de}(z_j) \geq V_\tau^{dn}(z_j)).$$

The value of a firm is given by

$$V_\tau^d(z_j) = \max_{\xi \in \{n,e\}} V_\tau^{d\xi}(z_j) = \begin{cases} V_\tau^{de}(z_j) & \text{if } z_j \geq \overline{z}_\tau^d \\ V_\tau^{dn}(z_j) & \text{if } z_j < \overline{z}_\tau^d \end{cases},$$

and the expected value of a new entrant, owned by the domestic household, of vintage τ, $\mathcal{V}_\tau^d$ is:

$$\mathcal{V}_\tau^d = \int_{z_L}^{z_u} V_\tau^d(z)\, G(\mathrm{d}z), \tag{9}$$

This completes the backward induction.

The, just derived, optimal production plan induces a measure over firms. Denote $\widetilde{\mathbb{P}}_{\tau,t}^d$ the t-time expected real operating profit of a domestically-owned firm, which enters in time τ, expectation being taken with respect to that measure $\widetilde{\mathbb{P}}_{\tau,t}^d = \int_{z_L}^{z_u} \mathbb{P}_{j\tau t}^d\, G(\mathrm{d}z_j)$, and $\widetilde{c}_\tau^d$ the expected real investment costs under such measure. Then:

$$\mathcal{V}_\tau^d = \sum_{\sigma \geq 0} \mu_\tau^{\tau+\sigma}(1-\delta)^\sigma \widetilde{\mathbb{P}}_{\tau,\tau+\sigma}^d - \widetilde{c}_\tau^d.$$

Similarly, one can express the expected real investment costs as

$$\widetilde{c}_\tau^d = G(\overline{z}_\tau^d)c^n + (1 - G(\overline{z}_\tau^d))c^e + \int_{z_L}^{\overline{z}_\tau^d} k_j^{opt,n}\, G(\mathrm{d}z) + \int_{\overline{z}_\tau^d}^{z_U} k_j^{opt,e}\, G(\mathrm{d}z).$$

The first two terms correspond to the expected fixed costs, while the last two terms correspond to the expected costs of capital investment. The expected investment costs differ across locations, vintages, and ownerships and this is because (i) the cut-off values differ across these dimensions too (as was already described) and (ii) these dimensions also change the optimal amount of invested capital $k_j^{opt,e}$ and $k_j^{opt,n}$. Therefore – in accordance with the convention introduced above – we will denote expected real investment costs in the domestic country from the perspective of the domestic household by $\widetilde{c}_t^d$

[13] It is worth to mention that the cut-off value differs across locations and vintages (since firms located in different location and / or firms appeared in different times face different relative prices) and across ownership (because the marginal rate of substitution is – in general – different).

and from the perspective of the foreign household by $\tilde{c}_t^{d*}$. The counterpart of these costs in the foreign country will be denoted as $\tilde{c}_t^{f}$ (from the perspective of the domestic household) and as $\tilde{c}_t^{f*}$ (when foreign household's perspective is taken).

2.2 Household behavior

The home country is populated by a representative competitive household who has recursive preferences over discounted stochastic streams of period utilities. The period utilities are derived from consumption of the aggregate good. Leisure does not enter the utility, so labor is supplied inelastically. The aggregate labor supply in the domestic country is $\mathcal{L}$, while $\mathcal{L}^*$ is the aggregate labor supply in the foreign country. Households can trade bonds denominated in the foreign currency.

The domestic household maximizes

$$\max U = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

subject to

$$B_t = (1 + r^*_{t-1}) B_{t-1} + \frac{1}{\eta_t} (-C_t + \mathbb{W}_t \mathcal{L}) + \tag{10}$$

$$+ \frac{1}{\eta_t} \left(\Xi_t^d - \widehat{\chi}(n_t^d) \right) + (\Xi_t^f - -\widehat{\chi}(n_t^f)) - \frac{\Psi_B}{2} B_t^2 + \mathcal{T}_t, \tag{11}$$

where B_t is the real bond holding of the domestic household, C_t is consumption, r^*_{t-1} is the real interest rate of the internationally traded bond, Ψ_B presents adjustment portfolio costs, as in Schmitt-Grohe, Uribe (2003) to stabilize the model[14], and $\mathcal{T}_t$ is the rebate of these costs in a lump-sum fashion to the household. The flow of real operating profits from the ownerships of firms of all vintages owned by the domestic household located in the domestic country is denoted as Ξ_t^d and is given by

$$\Xi_t^d = \sum_{\sigma \le t} (1 - \delta)^{t-\sigma} \, n_\sigma^f \widetilde{\mathbb{P}}_{\sigma,t}^d,$$

[14] In a strict sense, the model is stable even without portfolio adjustment costs (i.e. under $\Psi_B = 0$). The model is deterministic and therefore it would not exhibit the unit-root behavior even under $\Psi_B = 0$. On the other hand, if $\Psi_B = 0$, then the model would exhibit the steady state dependence on the initial asset holding. Therefore we use the nontrivial adjustment costs $\Psi_B > 0$ to give up the dependence of the steady state on the initial asset holding.

while, the flow of real operating profits from firms, which are owned by the domestic household and located in the foreign country is denoted as Ξ_t^f, and is given by

$$\Xi_t^f = \sum_{\sigma \le t} (1-\delta)^{t-\sigma} \, n_\sigma^f \widetilde{\mathbb{P}}_{\sigma,t}^f.$$

Because of the law of large numbers and of perfect foresight, the *ex-ante* expected values of the key variables for household decisions (such as investment costs or profit flows) coincide with *ex-post* realizations.

The number of new domestically located entrants owned by the domestic household in time t is n_t^d, while $\widetilde{\chi}(n^d)$ presents the real investment cost associated with entry of n^d entrants. These costs are given as follows:

$$\widetilde{\chi}(n_t^d) = \widetilde{c}_t^d n_t^d + \frac{\Psi_d}{2}\left(n_t^d\right)^2. \tag{12}$$

The first term is obvious – it is the expected investment cost (where the expectation is taken with respect to the measure induced by the optimal production plan). The second term may be interpreted as adjustment costs (e.g. due to limited supply of skills needed to run firms, such as legal expertise), and its purpose is to mitigate knife-edge conditions on household investments. These adjustment costs are assumed to be rebated by the lump-sum fashion to households (e.g. they are included in $\mathcal{T}_t$). Similarly, n_t^f denotes number of new entrants in the foreign country owned by the domestic household. The associated costs are given as

$$\widehat{\chi}(n_t^f) = \widetilde{c}_t^f n_t^f + \frac{\Psi_f}{2}\left(n_t^f\right)^2. \tag{13}$$

The two functions $\widetilde{\chi}$, $\widehat{\chi}$ differ by terms Ψ_d, Ψ_f only. The parameter Ψ_d is the adjustment cost of investing in the resident country (i.e. in the domestic country for the domestic household and in the foreign country for the foreign household), while the parameter Ψ_f is the adjustment cost of investing in the non-resident country.

The first order conditions for the domestic household are standard ones:

$$u'(C_t)\left(1+\Psi_B B_t\right) = \frac{\eta_{t+1}}{\eta_t}(1+r_t^*)\beta u'(C_{t+1}), \tag{14}$$

$$\lim_{t\to\infty} B_{t+1} = 0, \tag{15}$$

$$\widetilde{\chi}'(n_t^d)u'(C_t) = \sum_{v\ge 0} (1-\delta)^v \, \beta^v u'(C_{t+v}) \widetilde{\mathbb{P}}_{t,t+v}^d, \tag{16}$$

$$\eta_t \widehat{\chi}'(n_t^f)u'(C_t) = \sum_{v\ge 0} (1-\delta)^v \, \beta^v u'(C_{t+v}) \eta_{t+v} \widetilde{\mathbb{P}}_{t,t+v}^f. \tag{17}$$

The last two conditions read as:

$$\widetilde{c}_t^d + \Psi_d n_t^d = \sum_{v \geq 0} (1-\delta)^v \mu_t^{t+v} \widetilde{\mathbb{P}}_{t,t+v}^d, \tag{18}$$

$$\eta_t(\widetilde{c}_t^f + \Psi_f n_t^f) = \sum_{v \geq 0} (1-\delta)^v \eta_{t+v} \mu_t^{t+v} \widetilde{\mathbb{P}}_{t,t+v}^f. \tag{19}$$

The marginal rate of substitution between times t_1 and t_2 is defined as usually as:

$$\mu_{t_1}^{t_2} \equiv \beta^{t_2-t_1} \frac{u'(C_{t_2})}{u'(C_{t_1})}.$$

It is worth to note that although there is an idiosyncratic variance at the firm level, the model is deterministic at the aggregate level, thus the dynasty problem is deterministic too. Therefore the marginal rate of substitution does not involve the expectation operator. The Euler equation (14) can be then restated as:

$$(1 + \Psi_B B_t) = \frac{\eta_{t+1}}{\eta_t}(1 + r_t^*)\mu_t^{t+1}.$$

The household problem in the foreign country is defined symmetrically. Thus, the budget constraint reads as follows:

$$B_t^* = (1 + r_{t-1})B_{t-1}^* - C_t^* + \mathbb{W}_t^* \mathcal{L}^* + \left(\Xi_t^{f*} - \widetilde{\chi}(n_t^{f*})\right) + \tag{20}$$

$$+\frac{1}{\eta_t}\left(\Xi_t^{d*} - \widehat{\chi}(n_t^{d*})\right) - \frac{\Psi_B}{2} B_t^{*2} + \mathcal{T}_t^*.$$

The Euler equation reads as:

$$(1 + \Psi_B B_t^*)\, u'(C_t^*) = (1 + r_t^*)\beta u'(C_{t+1}^*), \tag{21}$$

and first-order conditions for investments are given by:

$$\eta_t^{-1}\left(\widetilde{c}_t^{d*} + \Psi_f n_t^{d*}\right) = \sum_{v \geq 0} (1-\delta)^v \eta_{t+v}^{-1} \mu_t^{*t+v} \widetilde{\mathbb{P}}_{t,t+v}^{d*}, \tag{22}$$

$$\widetilde{c}_t^{f*} + \Psi_d n_t^{f*} = \sum_{v \geq 0} (1-\delta)^v \mu_t^{*t+v} \widetilde{\mathbb{P}}_{t,t+v}^{f*}. \tag{23}$$

The Euler equations of both households imply a consumption-based version of the uncovered interest rate parity:

$$\frac{\eta_{t+1}}{\eta_t} = \frac{\mu_t^{*t+1}}{\mu_t^{t+1}} \frac{(1 + \Psi_B B_t)}{(1 + \Psi_B B_t^*)}.$$

Bonds are denominated in the foreign currency and since the model is deterministic, this is a completely innocent assumption. The international bond market equilibrium requires that $B_t + B_t^* = 0$.

As described above, the parameter $\mathbf{t}$ and the ratio c^e/c^n model the degree of trade openness. Changes in the former might be more appropriate for modeling changes in transport technology, while changes in the latter are more relevant for modeling the accession to the EU institutional structures. The accession diminishes the trade costs above all by uniforming legal, business or cultural environments and translates into a decrease in fixed costs of exporting.

But the convergence towards the EU structures has probably even more important aspect, which is financial openness and integration [15] . There are two ways of modeling financial openness in this framework, the ratio Ψ_f/Ψ_d and the parameter Ψ_B. The parameter Ψ_B models costs of consumption-smoothing by debt accumulation. The real-world counterpart of this debt may be represented by other capital flows than foreign direct investment (FDI). If this parameter is huge, households of the converging economy have little possibilities of consumption smoothing and the consumption path and the output growth will closely follow the productivity growth. On the other hand, if the portfolio-adjustment cost parameter Ψ_B is low, the transition economy can accumulate debt to smooth the consumption and even the output growth may be more rapid than the productivity growth, since transition economy can borrow also for investments. Nevertheless, the impact on the consumption is more significant.

The ratio of Ψ_f/Ψ_d is related to FDI. If the ratio is not too high, the advanced-economy household has the incentive to invest in the converging country. Parameters Ψ_f and Ψ_d can be seen as reduced-form modeling devices for agency costs. An alternative story behind the parameter Ψ_f is the perceived country-specific risk. It is well intuitive that a fall in agency costs or a fall in perceived risk will increase incentives for foreign direct investments. It is worth to mention that the assumption that the quadratic adjustment costs are returned to the households in a lump-sum fashion is done for analytical convenience only – the assumption avoids unnecessary complications due to the income effect.

Both channels of financial integration enable the transition economy to smooth consumption, which can be shown to be beneficial for both countries. But there is an important distinction between the two mechanisms: a fall in Ψ_B only smoothed consumption along the convergence trajectory, but it does not change the steady state. On the other hand, a fall in Ψ_f not only increases the consumption smoothing during the transition, but also affects the steady state. Macroeconomists are puzzled by the fact that model-based estimation

[15] Surprisingly, this important aspect of the accession to the monetary union is not considered in the elaborate model by Bayoumi et al. (2004), which aims at assessments of costs and benefits of monetary-union integration.

welfare consequences of financial liberalization seems to be low, see Gourinchas and Jeanne (2006). The reason is that speeding capital accumulation in the standard Ramsey framework has a temporary effect only. A prominent setting, which yields permanent effects of financial liberalization, identifies benefits from risk diversification as a source of large welfare gains of financial liberalization, see Obstfled (1994), Henry (2006). Although the presented model is deterministic – and therefore risk diversification plays no role – it can mimic a similar pattern: financial integration has positive permanent welfare gains for both countries. From the formal point of view, this results is achieved by quadratic adjustment costs (12), (13). Indeed, if we accept the view that quadratic adjustment costs represent a reduced form of agency costs, then a fall in Ψ_d can be considered as an improvement in the domestic financial technology, which is widely believed to affect economic performance, see Levine (1997). Similarly, a fall in Ψ_f can be viewed as international financial integration (perhaps because of adopting the common legal system, which decreases agency costs for foreigners). In any case, a fall in Ψ_f speeds transition (in terms of output and real wage convergence) and increases output [16] in both countries with larger welfare gains for a smaller and less advanced country (where the agency costs are more important because of resource constraints bind tighter).

2.4 Notes on Price Indexes

As mentioned above, prices p_{jt} and the corresponding price indexes P_t, and P_t^* are quality-adjusted prices. Therefore, the real wages $\mathbb{W}_t$ and $\mathbb{W}_t^*$ and the real exchange rate η_t are measured in the terms of qualities. These measures correspond to real-world price indexes only if the latter are quality-adjusted perhaps using a hedonic approach, which is rarely a case for transition countries (Ahnert and Kenny, 2004, p. 28). To get indexes closer to real-world measures, we have to define aggregate indexes over $\wp_{jt}$. Denote such indexes as $\mathcal{P}_t$ and $\mathcal{P}_t^*$.

Brůha and Podpiera (2007) use a simple approximation to $\mathcal{P}$ and set

$$\mathcal{P}_t = \mathcal{K}_t P_t,$$

where $\mathcal{K}_t$ is the total amount of invested capital by firms selling its products

[16] Under the present setting, a fall in Ψ increases the steady-state *output level*. One can conjecture that under endogenous growth, it will increase the *growth rate* as well. To confirm or reject this conjecture is left for future research.

in the domestic country:

$$\mathcal{K}_t = \sum_{\xi \in \{d,d*\}} \int_{\Omega^\xi} k_{j\tau} \, \mathrm{d}\Gamma_t^\xi(j,\tau) + \sum_{\xi \in \{f,f*\}} \int_{\Omega_e^\xi} k_{j\tau} \, \mathrm{d}\Gamma_t^\xi(j,\tau).$$

Alternatively, one can instead use a theoretical consistent index; let $\mathcal{P}$ to be defined:

$$\mathcal{P}_t = \frac{\sum_{\xi \in \{d,d*\}} \int_{\Omega^\xi} q_{jt}^d \wp_{jt}^d \, \mathrm{d}\Gamma_t^\xi(j,\tau) + \sum_{\xi \in \{f,f*\}} \int_{\Omega_e^\xi} q_{jt}^m \wp_{jt}^m \, \mathrm{d}\Gamma_t^\xi(j,\tau)}{Q_t}. \tag{24}$$

Nevertheless, $\mathcal{P}_t$ might differ from the CPI-based real-world indexes by one more term. The market structure based on the CES aggregation implies the *love-for-variety* effect, which means that the welfare-theoretical price index differs from the 'average' price by the term $\nu^{\frac{1}{\theta-1}}$, where ν is the number of available varieties and θ is the parameter of substitution in the CES function (see Melitz, 2003 for rigorous definition and derivation of the average price). Therefore, we distinguish the following definitions of the real exchange rate:

Quality-adjusted theoretically-consistent RER η_t is the real exchange rate, which enters the decisions of agents in the model.

Quality-unadjusted theoretically-consistent RER is the real exchange rate defined over physical quantities and is related to the quality-adjusted theoretically-consistent RER as $\frac{\mathcal{P}_t^*/P_t^*}{\mathcal{P}_t/P_t}\eta_t$.

Quality-adjusted CPI-based RER is related to its theoretically consistent counterpart as $\left(\frac{\nu_t^*}{\nu_t}\right)^{\frac{1}{\theta-1}}\eta_t$, where ν_t and ν_t^* is the number of varieties available at time t in the domestic and foreign country, respectively.

Quality-unadjusted CPI-based RER is probably the correct counterpart of the *measured real exchange rate* and is defined as $\left(\frac{\nu_t^*}{\nu_t}\right)^{\frac{1}{\theta-1}} \frac{\mathcal{P}_t^*/P_t^*}{\mathcal{P}_t/P_t}\eta_t$.

As discussed below in Section 3, the quality-adjusted theoretically consistent real exchange rate η_t depreciates during the transition and the reason is that the quality-quantity bundles produced in the transition country becomes less and less scarce. On the other hand, the three remaining indexes appreciate because both investments to quality and the love-for-variety effect outweigh the depreciation of η_t.

The distinction among various definitions of real exchange rate is reflected also in comparison of the economic performance of countries. If one wants to compute a model counterpart of the ratio of GDP per capita in PPP, one has to use $\frac{Y_t}{\eta_t Y_t^*}\frac{\mathcal{L}^*}{\mathcal{L}}$, where $Y_t = Q_t + \eta_t X_t$, $Y_t^* = Q_t^* - X_t$ are the model counterparts of real GDP (in the currency of the respective country) and X_t is the value of *net* real exports of the domestic country expressed in the foreign currency. On the other hand, if one wants to compute a model counterpart of the ratio of the nominal GDP using the nominal exchange rate (which is the same as a ratio of real GDP using the measured real exchange rate), one has to use $\frac{Y_t}{\eta_t Y_t^*} \frac{\mathcal{L}^*}{\mathcal{L}} \left(\frac{\nu_t}{\nu_t^*} \right)^{\frac{1}{\theta-1}} \frac{\mathcal{P}_t/P_t}{\mathcal{P}_t^*/P_t^*}$.

2.5 General Equilibrium

As usual, the general equilibrium is defined as a time profile of prices such that all households optimize and all markets clear. Since there are no price rigidities, nominal prices are indeterminate. Therefore, only the relative prices $\{\mathbb{W}_t, \mathbb{W}_t^*, \eta_t, r_t^*\}_{t=0}^{\infty}$ matter. The general equilibrium requires that the market-clearing conditions hold.

The aggregate resources constraints are given as follows:

$$C_t + n_t^d \tilde{c}_t^d + n_t^{d*} \tilde{c}_t^{d*} = Q_t, \tag{25}$$

$$C_t^* + n_t^f \tilde{c}_t^f + n_t^{f*} \tilde{c}_t^{f*} = Q_t^*. \tag{26}$$

Similarly, the labor market equilibrium requires:

$$\int_{z_L}^{z_U} l_{jt} \, \mathrm{d}\Gamma_t^d(j,\tau) + \int_{z_L}^{z_U} l_{jt} \, \mathrm{d}\Gamma_t^{d*}(j,\tau) = \mathcal{L}, \tag{27}$$

where l_{jt} is the labor demand by individual firms, and $\mathcal{L}$ is the aggregate, inelastic, labor supply.

Analogous market clearing conditions hold in the foreign country. The international bond market equilibrium requires that

$$B_t + B_t^* = 0. \tag{28}$$

The last equilibrium condition is the balance-of-payment equilibrium, which requires that:

$$B_{t+1} = (1 + r_t^*) B_t + X_t + \left(\Xi_t^f - \widehat{\chi}(n_t^f) \right) - \frac{1}{\eta_t} \left(\Xi_t^{d*} - \widehat{\chi}(n_t^{d*}) \right), \tag{29}$$

where X_t is the value of *net* real exports of the domestic country expressed in the foreign currency.

The definition of the general equilibrium is again very standard. A more involved task is to simulate the transition dynamics, because the model is effectively a vintage type model. However, if one realizes that the model can be rewritten in the first-order form of $f(\mathbf{x}_t, \mathbf{x}_{t-1}) = 0$, – the full set of equations of the model in the first-order form are available in Appendix A.2 – then the variety of methods can be used to simulate the model. The method actually used is described in Appendix A.3.

3 Steady state

The steady state is the long-run equilibrium and it is obtained when exogenous parameters (particulary productivity parameters A^F and A^H and financial and trade costs) are constant for a sufficiently long period of time. The speed of convergence to the steady state is influenced mainly by parameters β and δ.

The steady state is characterized by a number of features. The most important (and intuitive) ones include:

- Zero bond holding $B_{ss} = 0$, which is due to adjustment costs ψ_B.
- Constant endogenous quantities and prices.
- The steady-state effective discount rate reads as $\frac{1}{1-\beta(1-\delta)}$ and the steady-state interest rate $r_{ss} = \beta^{-1} - 1$.
- If the net asset positions are zero, then the net exports are zero as well.
- In the steady state, the distribution of firms degenerate over the vintage dimension: thus one can write $\Gamma^d_{ss}(j)$ instead of $\Gamma^d_{ss}(j, \tau)$.

In this section, we discuss how properties of the steady state are influenced by the productivity parameters A^H and A^F. We concentrates especially on the real exchange rate.

Consider a canonical two-country model with differentiated goods (to allow for finite price elasticity) without the vertical and horizontal investments (new varieties and quality) and without the extensive export margin. Such model would predict that an increase in the productivity (uniform across all sectors) in one country would cause the real-exchange rate of that country to depreciate. The intuition is straightforward: the output expansion can be sustained as an equilibrium only if the corresponding prices decline.

This does not fit well with the observation that more advanced countries tend to have higher price levels, nor it fits the experience of transition countries, which are becoming able to sell more for higher prices (see Fact 4 in Introduction).

To explain the fact that more advanced countries tend to have higher price level, it is commonly assumed that the productivity growth is biased towards the sector of tradable goods. Then, it is possible to obtain a real-exchange rate appreciation because of a rise in price of non-traded goods (this is the notorious Harrod-Balassa-Samuelson story, henceforth HBS). Nevertheless, such a story implies that terms-of-trade of a country experiencing productivity gains in tradable sector declines [17]: export prices of such country will decline relative to import prices. This implication of the HBS story is in contradiction with empirical finding for transition countries, see for instance Cincibuch and Podpiera (2006) and Podpiera (2005).

There are several possibilities how to generate real-exchange rate appreciation after a *uniform* productivity increase. First, if one works with a model, where the number of varieties is endogenous (such as Krugman, 1979), then any market structure featuring love-for-variety, which is *inter alia* case for the CES, implies divergence between the welfare-theoretical price index and the price index based on 'average' prices. Therefore an increase in productivity may cause the expansion of the number of varieties, which would mean that although the exchange rate defined per unit of utility or welfare depreciates, the exchange rate defined per unit of physical products may appreciate (see Section 2.4). The intuition is that the variety expansion may increase the welfare derived from the country export basket so that the consumers in both countries are willing to pay higher **average** prices for the same or even higher quantity of purchased goods. Thus, the love-for-variety effect of the CES market structure may represent a reduced-form modeling for the final good quality [18].

Another possibility is to explicitly introduce quality improvements. For example, Dury and Oomen (2007) present such a model. They show that a quality improvement leads to the appreciation of the real exchange rate defined in quality-unadjusted prices, provided that the quality improvement does not decrease unit production costs. Dury and Oomen (2007) thus do not require the new-varieties effect. Note that the mechanism behind the model by Dury and Oomen (2007) will work without any non-traded goods: it is simply based on dichotomy between quality-adjusted and quality-unadjusted prices.

The third possibility is to introduce the extensive-export margin. For example, Bergin et al (2006) present such a model and using numerical simulation they show that it is possible to replicate the observation that the faster-growing country tends to have a higher price level. The mechanism is based on self-selection of high productive firms into the exporting sector. High productive exporting firms push wages up even for non-exporting firms and this mechanism increases the price level of such country (thus, this is the standard HBS mechanism). The self-selection mechanism suggests why the productivity gains are likely to be biased towards exporting sectors. Unfortunately, Bergin et al (2006) do not report simulation of terms-of-trade thus it is not

[17] Provided, of course, that tradable goods are not internationally homogeneous. If they are, then terms-of-trade is trivially unity.

[18] Nevertheless, it is unlikely that this effect alone would generate the real-exchange rate appreciation strong enough to replicate the experience of transition countries.

clear whether their model would be consistent with empirical regularities of transition countries that terms-of-trade and real exchange rate both improve. But it is unlikely that it is the case, since their model implies that more productive firms charge lower prices (because of the downward sloping demand curve derived from the CES assumption).

Thus, there are at least several independent possibilities how to generate a real-exchange rate appreciation after an increase in productivity. See table 2 for an overview.

Models can hybridize these approaches: for example Ghironi and Melitz (2005) use in their model two mechanisms: investments into new varieties and export-eligibility margins. Thus Ghironi and Melitz (2005) are able to isolate three mechanisms of the real exchange rate appreciation:

- an increase in domestic wages caused by an increase in the number of domestic entrants,
- expenditures switching due to the love-for-variety;
- a decline in the share of the domestic traded goods (and an increase in the foreign share), which means that the relatively less productive domestic firms exit the tradable sector (and vice versa in the other country).

Nevertheless, a careful calibration suggests that the three mechanisms are not enough to replicate the observed pace of the real exchange rate appreciation in the CEE countries.

Therefore, we introduce the vertical investments (investments into quality improvements) into the NIM framework. Contrary to Dury and Oomen (2007), we consider endogenous quality improvements: firms decide to invest into quality improvements only if they expect that the investment will be profitable. The straightforward application of the envelope theorem to (5) and (6) reveals that the present value of real operating profits $\mathbb{P}_j^{d\xi}$ is increasing in A. By (7) and (8), the first-order condition for optimal investment is given by $\frac{\mathrm{d}\mathbb{P}_j^{d\xi}}{\mathrm{d}k} = 1$, so that the increase in the TFP boosts the quality investments. Therefore, the vertical margin adds an additional force for the long-run exchange rate appreciation after an increase in the average productivity.

4 An Inquiry on Model Dynamics

This section inquires about model dynamics and explains the role of model components. Since the proposed model extends the canonical framework and is solved for transition dynamics, the model properties ought to be primarily investigated with respect to the extension. In addition, the standard model blocks should be also analyzed in the framework of dynamic simulations. Therefore, we have focused on three major building blocks, namely on the role of production factors, the role of the export self-selectivity, and on the meaning of the productivity diffusion for the transition dynamics of an emerging economy that is converging to its developed counterpart. In particular, the most important features of the proposed model consist of the following:

Several dimensions of investments: the present model allows for horizontal investments to new varieties, vertical investments to quality, and investment to export-eligibility. Ghironi, Melitz (2005) model possesses horizontal investments and export-eligibility investments; Bergin et al (2006) considers export eligibility investments only, while Dury and Oomen (2007) ignore horizontal and export eligibility investments and concentrates on vertical (quality) investments only. Thus various models introduce various investment dimensions and thus it is important to learn their exact implications.

Asymmetric countries: most models are solved for a steady state of symmetric countries. We allow for asymmetricity both in the level of development and in size measured by labor forces. The crucial issue is whether and how the transition dynamics can differ when a small country converges to a bigger one (such as in the case of CEE country convergence to the EU) from the case where the converging country is comparable in the size to the developed world (this will be a relevant case if one wishes to investigate other emerging economies such as India).

Cross-border asset ownerships: in order to replicate the pace and the structure of the financial account, the present model introduces the cross-border asset ownerships. It is important to understand the value added of such a feature and how it influences the model properties.

Trade frictions: the present model has two kinds of trade barriers (iceberg costs and export-eligibility costs) and thus it is investigated what these barriers imply for the model properties. This issue is left to Section 5.

4.1 Calibration of the Model

Under the proposed modeling framework, one can investigate a number of changes in model parameters in the consistent framework of the general equilibrium; experiments of main interest are with an exogenous convergence of the domestic total factor productivity to the foreign level: $A_t^H \to A^F$. After the TFP convergence is reached $A_t^H = A^F$, both economies converge to the steady state. As mentioned earlier, the speed of convergence to the steady state is influenced mainly by parameters β and δ.

The calibration of the model for simulations is close to the calibration of the model that were used for successful replication of the macroeconomic dynamics in selected CEE by Brůha and Podpiera (2007) and for the Czech Republic alone by Brůha, Podpiera, and Polák (2007). Nevertheless, the difference is that here we do not aim at replicating the dynamics, but rather at highlighting the most important features of the model with realistic calibration using a set of computational experiments.

In the parametrization we assume two countries that have liberalized current and financial account of the balance of payments: free debt securities trading on which is levied a portfolio adjustment cost of $\Psi_B = 0.01$ (identical calibration to Ghironi and Melitz, 2005), further permitted acquiring of national assets only with associated portfolio adjustment costs of the size $\Psi_d = 0.7$. In both cases, the adjustment costs are relatively small (they are between 1% and 3% of GDP). The only purpose of the portfolio-adjustment cost Ψ_B is to avoid the initial-conditions dependence.

The trade liberalization is represented by a low value of transaction costs (0.05), and the export eligibility costs are twice higher than costs for non-eligibility to export (domestic market entry). The calibration is higher than in Ghironi and Melitz (2005), but the reader has to have in mind that we consider once-and-for-all export-eligibility costs while Ghironi and Melitz (2005) have period-by-period export-eligibility costs. The present value of these costs for a firm deciding to export in all periods of its expected lifetime is actually higher in their model than in our.

The values c^n and c^e are calibrated to reflect the consumption-to-absorbtion and investment-to-absorbtion ratio observed in data. These ratios (both in data[19] and in the model) are about 70% and 30%.

The convergence of a less developed to a more developed country, in terms of total factor productivity, starts at 70 percent of the developed country, which is motivated by the initial position of a typical transition country from CEE, such as the Czech Republic, Slovakia, Hungary, and Poland.

The exit rate for companies is fifty per cent, which means that the average duration of a project is two years and reflects higher frequency of closures and entries of companies in a transition economy. In comparison to a steady state exit rate in developed countries, such as in Ghironi and Melitz (2005), where the job destruction rate is 10 percent a year (U.S. evidence), our five times higher calibration also with respect to the processes taking place in the converging country might be quite realistic. At the same time, a higher exit rate should not significantly influence the developed country.

[19] Note that when dealing with the absorption in data, we divide the government consumption into consumption and investments. This is necessary for comparison of the model and data, since the model lacks the public sector.

Next, in both countries there is an average mark-up over marginal cost of twenty eight percent, which falls into the conventional calibration range in the literature. Standard macro models such as Rotemberg and Woodford (1992) use $\theta = 6$, while Ghironi amd Melitz (2005) opt for a value of 3.8 (based on empirically found mark-ups for the U.S. by Bernard et al, 2003). Since the difference in the two mentioned models is in the presence or absence of entry costs, the interpretation of the average vs. marginal costs is crucial. While the mark-ups over average vs. marginal costs are equal in the model without entry costs, the model with entry costs has different mark-ups over marginal and average costs. Consequently, a model with entry costs and lower θ would correspond to the same mark-ups over average costs in a model without entry costs and higher θ. Based on the evidence of mark-ups over average costs in the Czech Republic, provided by Podpiera and Raková (2006), in the range of 15-20 percent, we calibrate the model parameter of elasticity of substitution at the value of 4.5.

The calibration of the extent to which quality investment influences the production of quality-quantity basket (α) is set to 0.35. This value is based on the calibration experiments with regard to the pace of real exchange rate development.

And finally, the choice of the elasticity of intertemporal substitution and the discount factor are based on conventional calibration in the literature, i.e. 2 and 95 percent, respectively, which is identical to yearly frequency calibration in Ghironi and Melitz (2005). Table 1 provides an overview of model's parameters calibration.

4.2 Setup of Simulations

There are two sets of simulations. The first set concentrates on the role of specific types of investments, while the second set deals with the impact of active cross-border asset ownership in the present model. The present model nests all variants considered and thus by appropriately adjusting the calibration we can reach all four distinct models, distinguished by the symbol $\mathbb{B}$. Namely, we distinguish:

Benchmark model $\mathbb{B}_0$ is the model, which is described in Section 2, but without the cross-border assets. Formally, one sets $\Psi_f \to \infty$.

Alternative model $\mathbb{B}_1$ is a version of the benchmark model $\mathbb{B}_0$, namely without investment into quality. This can be formally achieved by setting $\alpha = 0$ and taking the relevant limit where necessary. Thus the core of the model $\mathbb{B}_1$ roughly corresponds to the steady state of the model by Ghironi, Melitz (2005).

Alternative model $\mathbb{B}_2$ is a variant without investment into export eligibility: simply all firms are eligible, but they invest into quality. This will be an equilibrium outcome if $c^e - c^n = 0$.

Alternative model $\mathbb{B}_3$ is a model without both features, i.e. without export eligibility and without investment to quality. Thus this variant can correspond to international economics models with monopolistic competition, Krugman (1979).

The assumption on the country size affects the model dynamics. Thus we simulate each model for two cases:

(1) converging economy is smaller than the developed one;
(2) both economies have the same size.

In the first set of simulations we study the differences in convergence dynamics across all four models. In the second set of simulations, we use the present model and highlight the role of cross-border assets ownership and by the location vs. ownership determined productivity. We compare three models: the model without cross-border asset ownership, with active cross-border assets ownership and productivity of investment by location (hence the productivity of a firm owned by the foreign agent and located in the domestic country is A^H – labeled henceforth as A^H case), and finally with active cross-border assets ownership and productivity of investment by ownership (hence the productivity of such a firm is A^F – this case is labeled A^F case).

In addition, while we assume unequally developed countries (convergence issue) throughout the simulations, we permit for relaxing the asymmetricity in country sizes. Thus we perform both sets of simulations of all four models under two size-scenarios: First, the converging economy is smaller than the developed one (we chose that the foreign country is six time larger since in such a situation, the larger country variables are almost uninfluenced by the convergence of the smaller country: $\mathcal{L}^* = 6\mathcal{L}$), and second, both economies have the same size ($\mathcal{L}^* = \mathcal{L}$).

Simulation experiments are done under the Cobb-Douglas production function for production of the quality-quantity basket $f(k,l) = k^\alpha l^{(1-\alpha)}$, the constant-relative-risk-aversion momentary utility function u, and the uniform distribution for $G(z)$; more details about functional forms and their implications are given in the Appendix A.1. Calibration of the parameters is given in Table 1.

The simulations are performed for a hypothetical economy under the calibration given in Table 1 and simulations run from 1995 to 2170. It is assumed that by 2040 the convergence is completed. We assume that the productivity

A^H growths according to the logistic curve:

$$A_t^H = A^F \frac{1 + m \exp(-(t - 1995)/\varsigma)}{1 + n \exp(-(t - 1995)/\varsigma)},$$

and use the following numerical values: $m = 8$, $n = 11$, $\varsigma = 5$. These values imply that the initial factor productivity of the converging economy reaches a slightly more than 75% of the value of the advanced country. By 2040 the factor productivity difference is negligible. The next years are simulated in order to settle the model in the steady state. In most cases, the model is settled after 30 years. However, for some version of the model, one needs at least 60 years to settle the model in the steady state. This occurs when one assumes that the converging economy has the same size as its more developed counterpart.

4.3 Results of Simulations

The output of the simulations is represented by a set of five variables: the ratio of *per capita* GDP in both countries, an index of the welfare-theoretical real exchange rate η_t, the real empirical exchange rate (an index of the quality-unadjusted CPI-based real exchange rate), trade balance (as a percentage of the domestic-country GDP), and debt (international bold holding), also as a percentage of the domestic-country GDP.

Figures 3-6 display results for the first set of simulations, while Figures 7-

Table 1
Parametrization of models

Parameter	*Workhorse model* $\mathbb{B}_0$	*Model* $\mathbb{B}_1$	*Model* $\mathbb{B}_2$	*Model* $\mathbb{B}_3$
α	0.35	0	0.35	0
θ	4.50	4.50	4.50	4.50
β	0.95	0.95	0.95	0.95
δ	0.50	0.50	0.50	0.50
$\mathbf{t}$	0.05	0.05	0.05	0.05
ε	2	2	2	2
c^e	9.0	9.0	NA	NA
c^n	4.5	4.5	4.5	4.5
Ψ_d	0.7	0.7	0.7	0.7
Ψ_B	0.01	0.01	0.01	0.01
A^F	10	10	10	10

8 display display results for the second set of simulations. The second set of simulations contain in addition a plot of net FDI inflows to the domestic country.

In the benchmark model $\mathbb{B}_0$ (see Figure 3), under unequally sized countries, the convergence of the less developed to the more developed country is characterized by halving the gap between GDP per capita within 15 years, empirical exchange rate appreciation by 28% by the end of convergence, real exchange rate depreciation by 8 percent, initial trade balance deficit of 1.5% turning into surplus of roughly 1% in 15 years, and finally a temporary debt to GDP ratio of the size of 8%.

In comparison of the variant $\mathbb{B}_1$ to $\mathbb{B}_0$ (see Figure 3), the absence of the quality investments causes a faster closure of the convergence gap; the half of the gap is reached in roughly 10 years. However, the empirical exchange rate appreciates very negligibly and similarly the real exchange rate depreciates also only by 5%. Even though the dynamics of the trade balance exhibits similar pattern, the extremes are 4-5 times greater than in the benchmark model. Also, the debt to GDP is recorded at the level of 30% (approximately three times the size of the debt under benchmark model).

The model without self-selection to export, i.e., $\mathbb{B}_2$, departs from the benchmark model in real exchange rate (see Figure 4), which remains constant, but due to presence of quality investment the empirical exchange rate appreciates, albeit a third of the size in the benchmark. In other variables, the $\mathbb{B}_2$ variant is close to the benchmark: trade balance, debt to GDP or convergence of GDP per capita. Overall, the model $\mathbb{B}_2$ (i.e. without export self-selection) has a similar dynamics to the benchmark model with the exception of the real exchange rate. The explanation is that tradeness self-selection increases the ex-ante value of new entrants, since the selection mechanism respects entrants' productivity, and therefore the increase in the number of firms during convergence is *ceteris paribus* higher under the benchmark than under $\mathbb{B}_2$.

The model without quality investment and without self-selection to export, i.e. $\mathbb{B}_3$, exhibits very the same dynamics as model $\mathbb{B}_1$ (see Figure 3) in trade balance, debt to GDP, real exchange rate, or convergence ratio of GDP per capita.

In summary: the models without investment into quality ($\mathbb{B}_1$ and $\mathbb{B}_3$) exhibits a faster convergence and their debt-to-GDP ratios and trade balances reach more extreme values than models with investment to quality ($\mathbb{B}_0$ and $\mathbb{B}_2$). In that respect, the models $\mathbb{B}_1$ and $\mathbb{B}_3$ are more similar to the standard small open economy models: the expectation of future wealth leads agents to borrow heavily in the presence (initial large trade balance deficits), which is repaid later (later trade balances surpluses).

Thus, the standard consumption-smoothing effect is somehow alleviated for the models with quality investments. The intuition behind is the following: in these models consumption today is too expensive not only because of the future need of debt repayments, but also because of the depreciation of the welfare theoretical real exchange rate. This channel is stronger for model with vertical investments. It can be considered as a virtue of these models, given that international macro models usually introduce consumption habits or some kind of frictions or adjustment costs to mitigate otherwise significative consumption smoothing.

The impact of the size on the macroeconomic dynamics can be seen from the comparison of the Figures 3 and 4 (unequally sized countries: converging country is 1/6 of the developed one) against Figures 5 and 6 (equally sized countries). It follows that under all four models, smaller converging countries exhibits greater appreciation of the empirical exchange rate (by roughly 10%) and greater depreciation of the real exchange rate, while the rest of the trajectories remain roughly equal for different sizes of the converging country. This can be expected, since if the two countries are similar then the relative expansions in both vertical and horizontal investments is lower than if the converging country is smaller. The mechanism behind differences in consumption smoothing between models with and without quality investment is present in both scenarios.

The simulations of the benchmark model under unequally sized countries with and without cross-border asset ownership are featured on Figure 7. By allowing for cross-border assets ownership, the convergence of GDP per capita, real exchange rate depreciation, and debt to GDP ratio remain roughly unchanged, while larger differences can be observed in the case of the rest of variables.

For the model dynamics under cross-border assets ownership the productivity of investment is the crucial aspect. In particular, in the case that the investment carries productivity of the source country (labeled above as the A^F case), the empirical real exchange rate appreciation is significantly greater than in the case that the productivity of investment is the local one (i.e., that of converging country - the A^H case). Similarly, the net FDI inflow in the former case is positive and large, while in the latter case it is negative. The intuition is that in the A^H case both agents find more profitable to invest in the more advanced countries: benefits of lower wages in the domestic country do not outweigh costs in terms of lower productivity and a smaller domestic market [20]. Therefore under the A^H case, the first years of convergence are

[20] The market-size effect is crucial: if we assume that multinational firms must export all its production to the origin country, then the A^H case would exhibit zero FDI along the transition, but under the A^F case there will be positive FDI inflows in the first years of transition and zero FDI after the converging country gets its productivity close enough to the advanced country.

characterized by depressed domestic production and that the domestic agents finance their consumption from repatriated profits from the advanced country.

This mechanism is responsible for rest of patterns of the A^H case: such economy initially accumulates a large debt to finance its investment abroad, the debt is gradually paid by a part of repatriated profits, and its trade-balance remain negative through the whole period (the converging economy is a net investor through the whole period!). Its production is depressed initially (they invest initially only abroad) and starts to rise later on when its productivity reaches a certain threshold. This also explains the pattern of the observed exchange rate and its initial depreciation.

On the other hand, the A^F case is closer to the usually-told story of the converging economy. There is initial net investment inflow to the converging economy and the converging economy exhibits a large trade-balance surplus since foreign companies are more likely to be exporters (the cut-off $\overline{z}$ is lower for such firms: They have more funds for investment into quality and thus their operating profits from the market in the more advanced country is higher) [21] .

The profile of the debt is different between the two cases too. In the A^H case, the economy accumulate the debt almost instantaneously, the reason is that this debt serves as a financial source of investments to the advanced country. The fact that the productivity differential makes the advanced country relatively more favorable place for investments in the beginning of the transition implies that the transition country has the biggest incentives to borrow at the beginning too. On the other hand, in the A^F case, the debt profile is standard one: it serves as the standard consumption smoothing channel mainly and thus its profile resemble the profile for the economy without cross-asset ownerships.

[21] In a typical CEE country, the trade balance was in deficit during the first years of the transition, and it has started to improve since 2000. Likewise, the beginning of FDI inflows dates around the year 2000, see Figure 1. The logic of the A^F model – on contrary – implies that FDI inflows should be most important at the beginning of the transition and this results in the trade surplus from the beginning. The likely explanation of the difference is that during first years of the transition, foreign investors faced significant uncertainty about business and legal environment, or feared possibility of political reversals and thus they started to invest later than predicted by the A^F model. This implies that if Brůha, Podpiera and Polák (2007) in their case study wanted to replicate not only the trade balance, but the structure of the financial account as well, they would need to introduce a fall in the adjustment cost parameter for foreign investment by 2000. The initial high value of the parameter may represent these initial fears of foreign investors, which were diminished after they learnt something more about transition countries.

The simulation with countries of equal size, see Figure 8, suggest that for the A^F case, the convergence in GDP per capita is twice faster, empirical real exchange rate appreciation slightly stronger, the debt to GDP is deeper, and the net FDI inflow is higher compared to the simulation with countries of unequal size. In other words, the convergence process appears to be faster and through higher dynamics of the convergence processes. In the A^H case, the differences are smaller, nevertheless important. Although, the same size of countries does not imply different pattern of GDP per capita convergence nor empirical real exchange rate and real exchange rate, it however suggests that the net inflow of FDI and the trade balance will be both slightly positive. Also the debt to GDP ratio will be deeper compared to unequally sized countries.

The reader may notice one issue: for symmetric countries both approaches yield zero net FDI in the steady state, which is not true when the countries have different sizes. In the latter situation if the countries enjoy the same TFP, the smaller country will be a net exporter of FDI. This is caused by the above mentioned home-market effect: it is more profitable to locate in the larger market; only the quadratic adjustment costs ensure that there are some entrants in the smaller country; a reader may want to compare this issue with the footlose capital model with asymmetric countries (cf. chapter 14 and appendix to chapter 3 in Baldwin et al, 2003). If one wants to cancel out the home-market effect, one has to calibrate investment costs for $f*$ firms higher than for $d*$ firms.

5 Macroeconomic implications of trade cost declines

5.1 Motivation and overview of the literature

The effect of declining trade costs is a prominent topic of the New International Macroeconomics. For instance, Baldwin (2005) discusses testable properties of trade liberalization in the NIM framework, while Baldwin and Forslid (2000) and Baldwin and Robert-Nicoud (2004) explore trade-growth linkage in a set-up of symmetric countries (in size and development). Bayoumi et al (2004) simulate removal of trade barriers as an integral feature of the economic integration. Although the model by Bayoumi et al (2004) is calibrated to a transition country, the model does not address some important stylized facts related to transition countries, such as the pace of the real exchange rate appreciation.

Therefore, an investigation of the macroeconomic implications of a decline in trade costs between asymmetric countries in the NIM framework together with the comparison to other selected models will shed more light into the inquiries on trade costs effects. Especially, as emphasized by Willenbockel (1999) models abstracting from cross-border asset ownership (as all mentioned above) might lead to false (inaccurate) predictions. The intuition behind his results is that when nontrivial cross-border asset ownership is introduced, the otherwise favorable effect of real exchange rate appreciation on import costs may be either boosted or dominated by a change in valuation of international profit flows. Therefore, the cross-border asset ownership may have a non-negligible effect on macroeconomic implications of trade barriers removals. Although Willenbockel (1999) finds that these effects are potentially important, he imposes an exogenous structure of cross-border asset ownership, which suffers from obvious limitations.

Since the present model allows for non-trivial cross-border asset ownership, which arises endogenously in the intertemporal general equilibrium, it can be used to assess how important are caveats raised by Willenbockel (1999) in the consistent intertemporal framework under endogenous cross-border asset ownership. By examining the implications in the model versions, which were introduced above (Section 4), we expose the model properties from a different perspective.

5.2 Setup of Simulations

A helpful contribution of Baldwin (2005) derives a set of testable predictions how trade cost removals influence the profitability of new entry, distribution of income, consumer welfare and trade patterns. These are important issues, nevertheless we concentrate rather on macroeconomic issues and less on welfare and income distribution issues.

We explore the effects of trade cost declines on (i) trade balance, (ii) openness, (iii) empirical exchange rate, (iv) asset positions, (v) GDP per capita convergence, and (vi) GDP levels in both countries.

We run the following set of experiments:

Iceberg costs: we compare implied transition dynamics of economies with quality investment for the case that initial iceberg costs $\mathbf{t}$ are two-times higher than the benchmark calibration. The experiment considers a fall after 60 years of transition (i.e. when the transition in productivity levels is finished).

Export-eligibility costs: we compare implied transition dynamics of these economies for the case that the ratio of c^e/c^n is of 20% higher than in the benchmark calibration (we fix c^n and increase initial c^e). Again it is assumed that c^e falls after 60 years of transition. A 20% fall in c^e roughly corresponds to the same steady state change in the eligibility cut-off $\overline{z}_d$ as the above introduced fall in $\mathbf{t}$.

Asset influence: the two experiments above are repeated for an economy with cross-border asset ownership.

In all the simulated cases, we assume that the change in the trade costs are correctly anticipated by agents. The productivity growth of the domestic economy A_t^H is calibrated exactly as in the preceding section.

5.3 Results of Simulation

Figure 9 shows the impact of a fall in iceberg costs from 0.1 to 0.05 at 2050. The simulations are run for three types of models: the benchmark model $\mathbb{B}_0$, the model without non-traded goods $\mathbb{B}_2$ and for the model with exogenous non-traded goods[22]. A number of features – which are confirmed by simulations – can be predicted:

- under larger iceberg costs, the export-market effect is smaller and thus the ratio of per capita GDP is initially smaller; it rises after the change. This is due to the export-market effect, which is more important for the smaller country as explained in Section 3.
- the smaller country borrows to finance entry of additional entrants, relatively more entrants enters in the smaller country because of the export-market effect;
- indeed if the two countries were similar in size, then the real interest rate would rise, but the trade balance would remain balanced;
- for models with non-traded goods, the empirical exchange rate appreciate; this effect is more important for the model with endogenous non-tradeness. The intuition is that more entrants enter under endogenous tradeness, since the cut-off $\overline{z}$ endogenously accommodates (it declines) and the expected profit from entry rises more than in the case that the tradeness is decided based on a random mechanism.

A similar impact can have a drop in export eligibility cost c^e.This can be seen from Figure 10, which displays simulation for a 20% decline in c^e. The difference between the two cases is that the smaller economy accumulates a lower debt under the latter scenario. This is quite intuitive: a drop in c^e *ceteris paribus* translates to lower investment requirements and this mitigates the effect of the increase in investments.

[22] This model is calibrated so that the steady state ratio of the numbers of traded and non-traded goods equal to the steady state number implied by the benchmark model. The difference is that this alternative does not exhibit the export self-selectiveness based on high productivity – the export eligibility is based on a random mechanism.

Figure 11 then displays a fall in iceberg costs under the presence of the cross-border asset ownership. The dynamics is very similar under all three cases. Thus, we do not confirm Willenbockel (1999) concerns in a model with endogenous and determinate cross-border asset ownerships. This holds also for a fall in the eligibility cost c^e.

6 Conclusion

In this paper we introduced a two-country model that goes in its structure beyond the existing models in the literature, and as such it allows for consistent explanation of greater range of key macroeconomic variables of a transition converging economy.

The major conceptual differences stem from modeling unequally developed countries that are potentially distinct in their relative sizes and the introduction of explicit investment into quality. From the technical point of view, the difference in the modeling approach compared to the literature is in using dynamic simulations for solving the model.

The paper presents an analysis of properties of the proposed model in contrast to similar models in the literature in the framework of dynamic simulations. In particular, it explores the conceptual differences in terms of horizontal and vertical (explicit quality) investment and export-eligibility investment. In addition, the analysis of effects of trade costs and cross-border asset ownership on the model properties is also presented.

As it follows from the results, various models in the literature, that were considered alternatively to the proposed model, proved to be unsatisfactory for consistent explanation of stylized facts of macroeconomic dynamics in transition countries of the Central and Eastern Europe (CCE). In particular, the scale of real exchange rate appreciation observed in the CEE can be explained only if one accepts the proposed extension (vertical investment) introduced in the present model. In addition, allowing for cross-border asset ownership in the present model helps to motivate the faster speed of real convergence and greater dynamics in macroeconomic variables observed in the CEE countries – i.e., the speed-up of the entire convergence process, than would be predicted by models without this feature.

Consequently, the proposed model in this paper appears as an important explanation and forecasting tool of the convergence process of a transition economy. For his potential for policy related applications using particular country calibrations, see Brůha and Podpiera (2007) and Brůha, Polák, and Podpiera (2007).

A Detailed Derivation of the Model

A.1 Model Equation under Particular Functional Forms

In this part of the paper, we derive the main model equation for particular functional forms of the production function, utility function and investment cost functions. In particular, as a benchmark calibration, we use the iso-elastic production function $\ell(l,k) \equiv \left(\frac{l}{k}\right)^{1-\alpha}$ for production of physical quantities. This formulation implies the Cobb-Douglas production function $f(k,l) = k^{\alpha} l^{1-\alpha}$ for the production of the quality-quantity bundle. The momentary utility function is parameterized using the common constant-relative-risk-aversion form $u(C) = (1-\varepsilon)^{-1} C^{1-\varepsilon}$, with the parameter of intertemporal substitution ε. As usually, the case of $\varepsilon = 1$ is interpreted as $\log(C)$. The distribution G of idiosyncratic shocks is uniform on the interval $[0,1]$.

The cost function associated with the Cobb-Douglas production function is given as follows:

$$\mathbb{C}(q, \mathbb{W}_t, A_t^H, z_j, k_j) = \mathbb{W}_t \left[\frac{q}{A_t^H z_j k_j^{\alpha}} \right]^{\frac{1}{1-\alpha}}.$$

First, we derive the optimal investment decision, and the present value of profit flows for a non-eligible firm [23]. Such a firm will supply the following quantity-quality bundle q_{jt}^d to the domestic market (at time t):

$$q_{jt}^d = \left(\left[\frac{\theta - 1}{\theta} (1-\alpha) \mathbb{W}_t^{-1} \left[A_t^H z_j k_j^{\alpha} \right]^{\frac{1}{1-\alpha}} \right]^{\theta} Q_t \right)^{\frac{(1-\alpha)}{\alpha\theta + (1-\alpha)}},$$

the real turnover is:

$$\frac{p_{jt}^d}{P_t} q_{jt}^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[\frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}_t^{-1} \left[A_t^H \right]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

and the real operating profit is given by:

$$\mathbb{P}_{j\tau t}^d = \frac{p_{jt}^d}{P_t} q_{jt}^d - \mathbb{C}(q_{jt}^d, \mathbb{W}_t, A_t^H, z_j, k_j) =$$
$$= z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \left[A_t^H \right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}} \mathcal{W}_1,$$

[23] Also, in this part of the paper, we derive expression only for firms located in the domestic country and owned by the domestic agent. The expression for other types of firms are easily derived then.

where we define

$$\mathcal{W}_1 \equiv \left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} - \left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{(1-\alpha)+\alpha\theta}} =$$

$$\frac{\alpha(\theta-1)+1}{(\theta-1)(1-\alpha)}\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{(1-\alpha)+\alpha\theta}},$$

which is obviously positive.

Second, we derive optimal production decisions of eligible firms is derived. Lemma 1 implies that $q^d_{jt} = \left[\frac{\theta-1}{\theta}\left(\frac{MC_{jt}}{P_t}\right)^{-1}\right]^\theta Q_t$, and $q^{*m}_{jt} = \left[\frac{\theta-1}{\theta}\frac{\eta_t}{1+\mathbf{t}}\left(\frac{MC_{jt}}{P_t}\right)^{-1}\right]^\theta Q^*_t$. Some simple, but tedious, algebraic manipulations yield:

$$\kappa_{jt}q_{jt} \equiv q^d_{jt} = \left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}\left(A^H_t z_j k^\alpha_j\right)^{\frac{1}{1-\alpha}}\right]^\theta \frac{Q_t}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}},$$

and

$$(1-\kappa_{jt})q_{jt} \equiv q^{m*}_{jt} = \left[\frac{\theta-1}{\theta}(1-\alpha)\frac{\eta_t}{1+\mathbf{t}}\mathbb{W}_t^{-1}\left(A^H_t z_j k^\alpha_j\right)^{\frac{1}{1-\alpha}}\right]^\theta \frac{Q^*_t}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}}.$$

This implies that

$$\kappa_{jt} = \frac{Q_t}{Q_t + Q^*_t\left(\frac{\eta_t}{1+\mathbf{t}}\right)^\theta},$$

observe that κ_{jt} does not depend on individual characteristics of firms: z_j and k_j; it depends only on relative tightness of both markets and on the real exchange rate corrected for transport costs $\mathbf{t}$. Therefore, all eligible firms will sell the same share of its products to the domestic resp. foreign markets. Thus henceforth we will simply write κ_t for κ_{jt}. Define

$$\xi_t \equiv Q_t + Q^*_t\left(\frac{\eta_t}{1+\mathbf{t}}\right)^\theta = \frac{Q_t}{\kappa_t}.$$

The total production of eligible firms can be written as follows:

$$q_{jt} = \left(z^\theta_j k^{\alpha\theta}_j\right)^{\frac{1}{(1-\alpha)+\alpha\theta}}\left\{\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}\left[A^H_t\right]^{\frac{1}{1-\alpha}}\right]^\theta \xi_t\right\}^{\frac{(1-\alpha)}{(1-\alpha)+\alpha\theta}},$$

and real turnovers on the domestic and the foreign markets, respectively is given by:

$$\frac{p^d_{jt}}{P_t}q^d_{jt} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \kappa_t^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}}\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}\left[A^H_t\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

$$\left(\frac{\eta_t}{1+t}\right)\frac{p_{jt}^{m*}}{P_t^*}q_{jt}^{m*} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} (1-\kappa_t)^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left(\frac{\eta_t}{1+\mathbf{t}}\right)^{\frac{\theta}{(1-\alpha)+\alpha\theta}} \times$$
$$\times \left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}\left[A_t^H\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{*\frac{1}{(1-\alpha)+\alpha\theta}}.$$

Real production costs of eligible firms read as follows:

$$\mathbb{C}_{jt} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[A_t^H\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \left\{\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\theta}\xi_t\right\}^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

thus, the real operating profit in a period t is given as:

$$\mathbb{P}_{j\tau t}^{d} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[A_t^H\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_1 \xi_t^{\frac{1}{(1-\alpha)+\alpha\theta}}.$$

Now, we are able to derive the expected present value of profit stream. We start with an eligible firm $\mathbb{P}_{j\tau}^{de}$, the expected present value satisfies:

$$\mathbb{P}_{j\tau}^{de} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \underbrace{\mathcal{W}_1 \sum_{t=\tau}^{\infty}(1-\delta)^{t-\tau}\mu_\tau^t \left[A_t^H\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \xi_t^{\frac{1}{(1-\alpha)+\alpha\theta}}}_{\varpi_\tau^e},$$

while the expected present value $\mathbb{P}_{j\tau}^{dn}$ of a non-eligible firm satisfies:

$$\mathbb{P}_{j\tau}^{dn} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \underbrace{\mathcal{W}_1 \sum_{t=\tau}^{\infty}(1-\delta)^{t-\tau}\mu_\tau^t \left[A_t^H\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}}}_{\varpi_\tau^n}.$$

The value of an eligible firm located in the domestic country and owned by the domestic household – which enjoys a productivity level z_j – is determined by capital investment:

$$\mathbf{V}_\tau^{de}(k_j|z_j) = \mathbb{P}_{j\tau}^{de} - (c^e + k_j) \equiv z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi_\tau^e - (c^e + k_j)\,;$$

and similarly for a non-eligible firm

$$\mathbf{V}_\tau^{dn}(k_j|z_j) = \mathbb{P}_{j\tau}^{dn} - (c^n + c^n k_j) = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi_\tau^n - (c^n + k_j)\,.$$

If firms ' managers maximize the value of firms, they chose the following capital level:

$$k_j^{opt,e} = z_j^{\theta-1}\left[\frac{\alpha(\theta-1)\varpi_\tau^e}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1},$$

and the value of an eligible firm is:

$$V_\tau^{de}(z_j) = \max_{k_j \geq 0} \mathbf{V}_\tau^{de}(k_j|z_j) = z_j^{(\theta-1)} \left[\varpi_\tau^e\right]^{\alpha(\theta-1)+1} \mathcal{G} - c^e,$$

where

$$\mathcal{G} \equiv \left[\left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)} - \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)+1} \right] =$$
$$= \frac{1}{\alpha(\theta-1)+1} \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)}.$$

Similarly, the value of a non-eligible firm is

$$V_\tau^{dn}(z_j) = \max_{k_j \geq 0} \mathbf{V}_\tau^{dn}(k_j|z_j) = z_j^{(\theta-1)} \left[\varpi_\tau^n\right]^{\alpha(\theta-1)+1} \mathcal{G} - c^n,$$

and the optimal capital investment to quality is

$$k_j^{opt,n} = z_j^{\theta-1} \left[\frac{\alpha(\theta-1)\varpi_\tau^n}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)+1}. \tag{A.1}$$

Value functions $V_\tau^{dn}(z_j)$, $V_\tau^{de}(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the least idiosyncratic shock, which makes the the export-eligibility investment profitable. Thus it is defined as

$$\overline{z}_\tau^d = \min_{z_j}(V_\tau^{de}(z_j) \geq V_\tau^{dn}(z_j)).$$

Also for the chosen parametrization of the production function, one can derive the labor demand. The formula is complicated and is given in the next section, since it involves integration over labor demands of firms of various vintages, see (A.8), (A.9) and (A.10) below.

A.2 Model in the First-order Form

In this part of the paper, we transform the model into the first-order form, which is suitable for numerical evaluation. We do it for parametrization used in A.1. The first-order form consists of dynamic and static equations. These are listed below.

A.2.1 Dynamic Equations

Intertemporal Marginal Rate of Substitution

$$\mu_t^{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\varepsilon}, \tag{A.2}$$
$$\mu_t^{*t+1} = \beta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{\varepsilon}.$$

Profit Flows

$$\varpi_t^{ndd} = \mathcal{W}_1 \left(\left[A_t^H \right]^{(\theta-1)} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1} \varpi_{t+1}^{ndd}, \tag{A.3}$$
$$\varpi_t^{edd} = \mathcal{W}_1 \left(\left[A_t^H \right]^{(\theta-1)} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1} \varpi_{t+1}^{edd},$$
$$\varpi_t^{nfd} = \mathcal{W}_1 \left(\left[A_t^F \right]^{(\theta-1)} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1} \varpi_{t+1}^{nfd},$$
$$\varpi_t^{efd} = \mathcal{W}_1 \left(\left[A_t^F \right]^{(\theta-1)} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1} \varpi_{t+1}^{efd},$$
$$\varpi_t^{nff} = \mathcal{W}_1 \left(\left[A_t^F \right]^{(\theta-1)} \mathbb{W}_t^{*-(\theta-1)(1-\alpha)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1} \varpi_{t+1}^{nff},$$
$$\varpi_t^{eff} = \mathcal{W}_1 \left(\left[A_t^F \right]^{(\theta-1)} \mathbb{W}_t^{*-(\theta-1)(1-\alpha)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1} \varpi_{t+1}^{eff},$$
$$\varpi_t^{ndf} = \mathcal{W}_1 \left(\left[A_t^H \right]^{(\theta-1)} \mathbb{W}_t^{*-(\theta-1)(1-\alpha)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1} \varpi_{t+1}^{ndf},$$
$$\varpi_t^{edf} = \mathcal{W}_1 \left(\left[A_t^H \right]^{(\theta-1)} \mathbb{W}_t^{*-(\theta-1)(1-\alpha)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1} \varpi_{t+1}^{edf};$$

where $\xi_t = Q_t + Q_t^* \left(\frac{\eta_t}{1+\mathbf{t}} \right)^{\theta}$, and $\xi_t^* = Q_t^* + Q_t \left(\frac{\eta_t^{-1}}{1+\mathbf{t}} \right)^{\theta}$.

Expected value of the stream of future profits (from the unit investment now) Ω_t° are given as the sum of weighted expected values from eligible and non-eligible profits: $\Omega_t^{x_1x_2} = \Omega_t^{nx_1x_2} + \Omega_t^{ex_1x_2}$, (with $x_i \in \{d, f\}$) and where:

$$\Omega_t^{edd} = \widetilde{\mathbb{P}}_t^{edd} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{edd}\left(\frac{\varpi_t^{edd}}{\varpi_{t+1}^{edd}}\right)\frac{\int_{\bar{z}_t^{dd}}^{z_U} z^{\theta-1}\,G(\mathrm{d}z)}{\int_{\bar{z}_{t+1}^{dd}}^{z_U} z^{\theta-1}\,G(\mathrm{d}z)}, \quad \text{(A.4)}$$

$$\Omega_t^{ndd} = \widetilde{\mathbb{P}}_t^{ndd} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{ndd}\left(\frac{\varpi_t^{ndd}}{\varpi_{t+1}^{ndd}}\right)\frac{\int_{z_L}^{\bar{z}_t^{dd}} z^{\theta-1}\,G(\mathrm{d}z)}{\int_{z_L}^{\bar{z}_{t+1}^{dd}} z^{\theta-1}\,G(\mathrm{d}z)},$$

$$\Omega_t^{efd} = \frac{\widetilde{\mathbb{P}}_t^{efd}}{\eta_t} + \mu_t^{*t+1}(1-\delta)\Omega_{t+1}^{efd}\left(\frac{\varpi_t^{efd}}{\varpi_{t+1}^{efd}}\right)\frac{\int_{\bar{z}_t^{fd}}^{z_U} z^{\theta-1}\,G(\mathrm{d}z)}{\int_{\bar{z}_{t+1}^{fd}}^{z_U} z^{\theta-1}\,G(\mathrm{d}z)},$$

$$\Omega_t^{nfd} = \frac{\widetilde{\mathbb{P}}_t^{nfd}}{\eta_t} + \mu_t^{*t+1}(1-\delta)\Omega_{t+1}^{nfd}\left(\frac{\varpi_t^{nfd}}{\varpi_{t+1}^{nfd}}\right)\frac{\int_{z_L}^{\bar{z}_t^{fd}} z^{\theta-1}\,G(\mathrm{d}z)}{\int_{z_L}^{\bar{z}_{t+1}^{fd}} z^{\theta-1}\,G(\mathrm{d}z)},$$

$$\Omega_t^{eff} = \widetilde{\mathbb{P}}_t^{eff} + \mu_t^{*t+1}(1-\delta)\Omega_{t+1}^{eff}\left(\frac{\varpi_t^{eff}}{\varpi_{t+1}^{eff}}\right)\frac{\int_{\bar{z}_t^{ff}}^{z_U} z^{\theta-1}\,G(\mathrm{d}z)}{\int_{\bar{z}_{t+1}^{ff}}^{z_U} z^{\theta-1}\,G(\mathrm{d}z)},$$

$$\Omega_t^{nff} = \widetilde{\mathbb{P}}_t^{nff} + \mu_t^{*t+1}(1-\delta)\Omega_{t+1}^{nff}\left(\frac{\varpi_t^{nff}}{\varpi_{t+1}^{nff}}\right)\frac{\int_{z_L}^{\bar{z}_t^{ff}} z^{\theta-1}\,G(\mathrm{d}z)}{\int_{z_L}^{\bar{z}_{t+1}^{ff}} z^{\theta-1}\,G(\mathrm{d}z)},$$

$$\Omega_t^{edf} = \eta_t\widetilde{\mathbb{P}}_t^{edf} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{edf}\left(\frac{\varpi_t^{edf}}{\varpi_{t+1}^{edf}}\right)\frac{\int_{\bar{z}_t^{df}}^{z_U} z^{\theta-1}\,G(\mathrm{d}z)}{\int_{\bar{z}_{t+1}^{df}}^{z_U} z^{\theta-1}\,G(\mathrm{d}z)},$$

$$\Omega_t^{ndf} = \eta_t\widetilde{\mathbb{P}}_t^{ndf} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{ndf}\left(\frac{\varpi_t^{ndf}}{\varpi_{t+1}^{ndf}}\right)\frac{\int_{z_L}^{\bar{z}_t^{df}} z^{\theta-1}\,G(\mathrm{d}z)}{\int_{z_L}^{\bar{z}_{t+1}^{df}} z^{\theta-1}\,G(\mathrm{d}z)};$$

where definitions of expectations of profits Π_t^{xxx} and cut-off values will be given in the next subsubsection.

To get equations for actual realized profits $\Xi_t^{x_1x_2}$, $x_i \in \{d, f\}$, we have to split into two parts (according to eligibility): $\Xi_t^{x_1x_2} = \Xi_t^{ex_1x_2} + \Xi_t^{nx_1x_2}$. The first-order equations are then:

$$\Xi^{edd}_{t+1} = (1-\delta)\left(\frac{\left[A^H_{t+1}\right]^{\theta-1}\mathbb{W}^{-(\theta-1)}_{t+1}\xi_{t+1}}{\left[A^H_t\right]^{\theta-1}\mathbb{W}^{-(\theta-1)}_t\xi_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\Xi^{edd}_t + n^{edd}_{t+1}\widetilde{\mathbb{P}}^{edd}_{t+1}, \qquad \text{(A.5)}$$

$$\Xi^{ndd}_{t+1} = (1-\delta)\left(\frac{\left[A^H_{t+1}\right]^{\theta-1}\mathbb{W}^{-(\theta-1)}_{t+1}Q_{t+1}}{\left[A^H_t\right]^{\theta-1}\mathbb{W}^{-(\theta-1)}_tQ_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\Xi^{ndd}_t + n^{ndd}_{t+1}\widetilde{\mathbb{P}}^{ndd}_{t+1},$$

$$\Xi^{efd}_{t+1} = (1-\delta)\left(\frac{\left[A^F_{t+1}\right]^{\theta-1}\mathbb{W}^{-(\theta-1)}_{t+1}\xi_{t+1}}{\left[A^F_t\right]^{\theta-1}\mathbb{W}^{-(\theta-1)}_t\xi_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\Xi^{efd}_t + n^{efd}_{t+1}\widetilde{\mathbb{P}}^{efd}_{t+1},$$

$$\Xi^{nfd}_{t+1} = (1-\delta)\left(\frac{\left[A^F_{t+1}\right]^{\theta-1}\mathbb{W}^{-(\theta-1)}_{t+1}Q_{t+1}}{\left[A^F_t\right]^{\theta-1}\mathbb{W}^{-(\theta-1)}_tQ_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\Xi^{nfd}_t + n^{nfd}_{t+1}\widetilde{\mathbb{P}}^{nfd}_{t+1},$$

$$\Xi^{eff}_{t+1} = (1-\delta)\left(\frac{\left[A^F_{t+1}\right]^{\theta-1}\mathbb{W}^{*-(\theta-1)}_{t+1}\xi^*_{t+1}}{\left[A^F_t\right]^{\theta-1}\mathbb{W}^{*-(\theta-1)}_t\xi^*_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\Xi^{eff}_t + n^{eff}_{t+1}\widetilde{\mathbb{P}}^{eff}_{t+1},$$

$$\Xi^{nff}_{t+1} = (1-\delta)\left(\frac{\left[A^F_{t+1}\right]^{\theta-1}\mathbb{W}^{*-(\theta-1)}_{t+1}Q^*_{t+1}}{\left[A^F_t\right]^{\theta-1}\mathbb{W}^{*-(\theta-1)}_tQ^*_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\Xi^{nff}_t + n^{nff}_{t+1}\widetilde{\mathbb{P}}^{nff}_{t+1},$$

$$\Xi^{edf}_{t+1} = (1-\delta)\left(\frac{\left[A^H_{t+1}\right]^{\theta-1}\mathbb{W}^{*-(\theta-1)}_{t+1}\xi^*_{t+1}}{\left[A^H_t\right]^{\theta-1}\mathbb{W}^{*-(\theta-1)}_t\xi^*_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\Xi^{edf}_t + n^{edf}_{t+1}\widetilde{\mathbb{P}}^{edf}_{t+1},$$

$$\Xi^{ndf}_{t+1} = (1-\delta)\left(\frac{\left[A^H_{t+1}\right]^{\theta-1}\mathbb{W}^{*-(\theta-1)}_{t+1}Q^*_{t+1}}{\left[A^H_t\right]^{\theta-1}\mathbb{W}^{*-(\theta-1)}_tQ^*_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\Xi^{ndf}_t + n^{ndf}_{t+1}\widetilde{\mathbb{P}}^{ndf}_{t+1},$$

where the numbers of eligible / non-eligible firms distinguished by location and ownerships (i.e. $n^{\xi x_1 x_2}_t$, $\xi \in \{e,n\}$, $x_i \in \{d,f\}$) is given in the next sub-subsection.

Exports are given recursively as follows: $X^d_t = X^{dd}_t + X^{fd}_t$, $X^f_t = X^{df}_t + X^{ff}_t$, where X^{dd}_t is the export of firms located in the domestic country and owned by the domestic household to the foreign country (and similarly for X^{fd}_t, X^{df}_t, X^{ff}_t). We have the convention that exports are denominated in the currency of the original market (thus X^{dd}_t, X^{fd}_t are in the domestic currency), cf. Eq.

29. Now, back to the system:

$$X_t^{dd} = \widehat{n}_t^{edd}(1-\kappa_t)^{\frac{a(\theta-1)}{a(\theta-1)+1}} \left(\frac{\eta_t}{1+\mathbf{t}}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}\left[A_t^H\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{*\frac{1}{\alpha(\theta-1)+1}},$$

$$X_t^{fd} = \widehat{n}_t^{efd}(1-\kappa_t)^{\frac{a(\theta-1)}{a(\theta-1)+1}} \left(\frac{\eta_t}{1+\mathbf{t}}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}\left[A_t^F\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{*\frac{1}{\alpha(\theta-1)+1}},$$

$$X_t^{df} = \widehat{n}_t^{edf}(1-\kappa_t^*)^{\frac{a(\theta-1)}{a(\theta-1)+1}} \left(\frac{\eta_t^{-1}}{1+\mathbf{t}}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{*-1}\left[A_t^H\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

$$X_t^{ff} = \widehat{n}_t^{eff}(1-\kappa_t^*)^{\frac{a(\theta-1)}{a(\theta-1)+1}} \left(\frac{\eta_t^{-1}}{1+\mathbf{t}}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{*-1}\left[A_t^F\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

where $\widehat{n}_t^{ex_1x_2}$ are weighted numbers of eligible firms, which obeys the following recursive relation:

$$\widehat{n}_{t+1}^{ex_1x_2} = (1-\delta)\widehat{n}_t^{ex_1x_2} + n_{t+1}^{ex_1x_2}\left[\frac{\alpha(\theta-1)\varpi_{t+1}^{ex_1x_2}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{\overline{z}_{t+1}^{x_1x_2}}^{z_U} z^{\theta-1}\, G(\mathrm{d}z).$$

A similar recursive equation holds for non-eligible firms:

$$\widehat{n}_{t+1}^{nx_1x_2} = (1-\delta)\widehat{n}_t^{nx_1x_2} + n_{t+1}^{nx_1x_2}\left[\frac{\alpha(\theta-1)\varpi_{t+1}^{nx_1x_2}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{z_L}^{\overline{z}_{t+1}^{x_1x_2}} z^{\theta-1}\, G(\mathrm{d}z).$$

These recursive schemes are used in the next subsubsection too (when deriving the labor demand).

The rest of model dynamic equations are *balance-of-payment equation* (29), households' *budget constraints* (10), (20), households' *Euler equations* (14), (21) and households' *equations*, which determines the *asset holdings*: (18), (19), (22), (23). These last 4 equations are not in the first-order form, but we will convert them into it:

$$\begin{aligned} \widetilde{c}_t^{dd} + \Psi_d n_t^{dd} &= \Omega_t^{dd}, \\ \eta_t \widetilde{c}_t^{df} + \Psi_f n_t^{df} &= \Omega_t^{df}, \\ \widetilde{c}_t^{ff} + \Psi_d n_t^{ff} &= \Omega_t^{ff}, \\ \eta_t^{-1}\widetilde{c}_t^{fd} + \Psi_f n_t^{fd} &= \Omega_t^{fd}; \end{aligned} \tag{A.6}$$

where expected investment costs obey:

$$\begin{aligned} \widetilde{c}_t^{x_1x_2} &= G(\overline{z}_t^{x_1x_2})c^n + (1-G(\overline{z}_t^{x_1x_2}))c^e + ... \\ &+ \left[\frac{\alpha(\theta-1)\varpi_{t+1}^{nx_1x_2}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{z_L}^{\overline{z}_{t+1}^{x_1x_2}} z^{\theta-1}\, G(\mathrm{d}z) + ... \\ &+ \left[\frac{\alpha(\theta-1)\varpi_{t+1}^{ex_1x_2}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{\overline{z}_{t+1}^{x_1x_2}}^{z_U} z^{\theta-1}\, G(\mathrm{d}z). \end{aligned} \tag{A.7}$$

A.2.2 Static Equations

The model has static equations too. These are mainly market clearing conditions and definitions. The market clearing conditions include the clearing of the goods markets (25), (26), international bond market clearing (28) and labor market clearing conditions. We now show how the labor market conditions look like: define $\eth_t^{\xi x_1 x_2}$ as

$$\eth_t^{ndd} = \left(\frac{\theta-1}{\theta}(1-\alpha)\left[A_t^H\right]^{\theta-1}\mathbb{W}_t^{-(\theta-1)}Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \tag{A.8}$$

$$\eth_t^{edd} = \left(\frac{\theta-1}{\theta}(1-\alpha)\left[A_t^H\right]^{\theta-1}\mathbb{W}_t^{-(\theta-1)}\xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\eth_t^{nfd} = \left(\frac{\theta-1}{\theta}(1-\alpha)\left[A_t^F\right]^{\theta-1}\mathbb{W}_t^{-(\theta-1)}Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\eth_t^{efd} = \left(\frac{\theta-1}{\theta}(1-\alpha)\left[A_t^F\right]^{\theta-1}\mathbb{W}_t^{-(\theta-1)}\xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\eth_t^{ndf} = \left(\frac{\theta-1}{\theta}(1-\alpha)\left[A_t^H\right]^{\theta-1}\mathbb{W}_t^{-*(\theta-1)}Q_t^*\right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\eth_t^{edf} = \left(\frac{\theta-1}{\theta}(1-\alpha)\left[A_t^H\right]^{\theta-1}\mathbb{W}_t^{-*(\theta-1)}\xi_t^*\right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\eth_t^{nff} = \left(\frac{\theta-1}{\theta}(1-\alpha)\left[A_t^F\right]^{\theta-1}\mathbb{W}_t^{-*(\theta-1)}Q_t^*\right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\eth_t^{eff} = \left(\frac{\theta-1}{\theta}(1-\alpha)\left[A_t^F\right]^{\theta-1}\mathbb{W}_t^{-*(\theta-1)}\xi_t^*\right)^{\frac{1}{\alpha(\theta-1)+1}}.$$

Then the domestic labor demand is given as

$$\mathcal{L}_t = \sum_{\xi\in\{e,n\}}\sum_{x_1\in\{d,f\}}\eth_t^{\xi x_1 d}\widehat{n}_t^{\xi x_1 d}, \tag{A.9}$$

and the foreign labor demand is given by

$$\mathcal{L}_t^* = \sum_{\xi\in\{e,n\}}\sum_{x_1\in\{d,f\}}\eth_t^{\xi x_1 f}\widehat{n}_t^{\xi x_1 f}. \tag{A.10}$$

The labor demands should be equal to inelastic labor supply.

The only remaining things are definitions of average profits and expected cutoffs. They follow:

$$\overline{z}_t^{x_1 x_2} = \left(\frac{c^e - c^n}{\mathcal{G}\left[\left[\varpi_t^{e x_1 x_2}\right]^{\alpha(\theta-1)+1} - \left[\varpi_t^{n x_1 x_2}\right]^{\alpha(\theta-1)+1}\right]}\right)^{\frac{1}{\theta-1}}, \tag{A.11}$$

for $x_i \in \{d, f\}$, and

$$\widetilde{\mathbb{P}}_t^{ndd} = \mathcal{W}_1 \int_{z_L}^{\overline{z}_{t+1}^{dd}} z^{\theta-1} G(\mathrm{d}z) \left(\left[A_t^H \right]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi^{ndd}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\widetilde{\mathbb{P}}_t^{edd} = \mathcal{W}_1 \int_{\overline{z}_{t+1}^{dd}}^{z_U} z^{\theta-1} G(\mathrm{d}z) \left(\left[A_t^H \right]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi^{edd}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\widetilde{\mathbb{P}}_t^{nfd} = \mathcal{W}_1 \int_{z_L}^{\overline{z}_{t+1}^{fd}} z^{\theta-1} G(\mathrm{d}z) \left(\left[A_t^F \right]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi^{nfd}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\widetilde{\mathbb{P}}_t^{efd} = \mathcal{W}_1 \int_{\overline{z}_{t+1}^{fd}}^{z_U} z^{\theta-1} G(\mathrm{d}z) \left(\left[A_t^F \right]^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi^{efd}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\widetilde{\mathbb{P}}_t^{ndf} = \mathcal{W}_1 \int_{z_L}^{\overline{z}_{t+1}^{df}} z^{\theta-1} G(\mathrm{d}z) \left(\left[A_t^H \right]^{\theta-1} \mathbb{W}_t^{-*(\theta-1)(1-\alpha)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi^{ndf}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\widetilde{\mathbb{P}}_t^{edf} = \mathcal{W}_1 \int_{\overline{z}_{t+1}^{df}}^{z_U} z^{\theta-1} G(\mathrm{d}z) \left(\left[A_t^H \right]^{\theta-1} \mathbb{W}_t^{-*(\theta-1)(1-\alpha)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi^{edf}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\widetilde{\mathbb{P}}_t^{nff} = \mathcal{W}_1 \int_{z_L}^{\overline{z}_{t+1}^{ff}} z^{\theta-1} G(\mathrm{d}z) \left(\left[A_t^F \right]^{\theta-1} \mathbb{W}_t^{-*(\theta-1)(1-\alpha)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi^{nff}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\widetilde{\mathbb{P}}_t^{eff} = \mathcal{W}_1 \int_{\overline{z}_{t+1}^{ff}}^{z_U} z^{\theta-1} G(\mathrm{d}z) \left(\left[A_t^F \right]^{\theta-1} \mathbb{W}_t^{-*(\theta-1)(1-\alpha)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi^{eff}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}.$$

A.3 Numerical methods

This part of the appendix discusses numerical methods used to simulate the model. Basically, we have experimented with two classes of methods: (i) projection-based methods and (ii) domain-truncation (such as L-B-J approach) methods.

Before discussing these methods, it is worth to realize a fact, which we use when applying both methods: If one can guess the time profile of the following six variables: domestic output $\{Q_t\}_{t=0}^{\infty}$, domestic real wage $\{\mathbb{W}_t\}_{t=0}^{\infty}$, domestic consumption $\{C_t\}_{t=0}^{\infty}$, their foreign counterparts: $\{Q_t^*\}_{t=0}^{\infty}$, $\{\mathbb{W}_t^*\}_{t=0}^{\infty}$, $\{C_t^*\}_{t=0}^{\infty}$ and the real exchange rate $\{\eta_t\}_{t=0}^{\infty}$, one can easily compute the time profile of all other endogenous variables (given exogenous and policy variables). Indeed, the algorithm is following:

(1) Given $\{C_t\}_{t=0}^{\infty}$, $\{C_t^*\}_{t=0}^{\infty}$ compute the marginal rate of substitutions $\left\{\mu_t^{t+1}\right\}_{t=0}^{\infty}$, $\left\{\mu_t^{*t+1}\right\}_{t=0}^{\infty}$ using (A.2).

(2) Given $\{Q_t\}_{t=0}^{\infty}$, $\{\mathbb{W}_t\}_{t=0}^{\infty}$, $\{Q_t^*\}_{t=0}^{\infty}$, $\{\mathbb{W}_t^*\}_{t=0}^{\infty}$ and $\left\{\mu_t^{t+1}\right\}_{t=0}^{\infty}$, $\left\{\mu_t^{*t+1}\right\}_{t=0}^{\infty}$, it

is possible to solve for[24] $\{\varpi_t^\circ\}_{t=0}^{\infty}$, and therefore for $\{\overline{z}_t^\circ\}_{t=0}^{\infty}$; use (A.3) and (A.11).

(3) Then, use backward difference equations (A.4) to compute $\{\Omega_t^\circ\}_{t=0}^{\infty}$, (A.7) to compute expected investment costs $\{\widetilde{c}_t^\circ\}_{t=0}^{\infty}$ and first-order conditions (A.6) to compute the numbers of new entrants.

(4) Then use the forward difference equation (A.5) to solve for profits flows $\left\{\Xi_{t+1}^\circ\right\}_{t=0}^{\infty}$ and (A.8), (A.9) and (A.10) to find labor demand in both countries.

(5) One can use households' Euler equations to derive the optimal bond holding and from the international-bond market clearing condition to derive the interest rate $\{r_t\}_{t=0}^{\infty}$;

Now, the idea is clear: to guess the time profile and verify the guess. The guess should be verified as follows:

(1) Budget constraints for both households have to be satisfied: (10), (20).
(2) Labor markets in both countries have to be cleared: (27) and similarly for the foreign country.
(3) The balance of payment condition has to be satisfied: (29).
(4) Goods markets have to be cleared as well: (25), (26).

Denote the guess of the seven variables as

$$\overrightarrow{\mathcal{H}} = \{\{Q_t\}_{t=0}^{\infty}, \{\mathbb{W}_t\}_{t=0}^{\infty}, \{C_t\}_{t=0}^{\infty}, \{Q_t^*\}_{t=0}^{\infty}, \{\mathbb{W}_t^*\}_{t=0}^{\infty}, \{C_t^*\}_{t=0}^{\infty}, \{\eta_t\}_{t=0}^{\infty}\},$$

and the seven equilibrium conditions as $\left\{\hbar_t(\overrightarrow{\mathcal{H}})\right\}_{t=0}^{\infty}$, where we interpret $\hbar_t(\overrightarrow{\mathcal{H}^0}) = 0$ as the fulfilment of these conditions at time t for a guess $\overrightarrow{\mathcal{H}^0}$. Note that the fulfillment of equilibrium condition at time t, $\hbar_t = 0$ does not depend on the value of the seven variables at time t only: it depends on their whole time profiles. It depends on future values because of expectations of profits, e..g todays' investment decisions depend on future streams of profits, cf. (7), (8), and the it depends on past values because of predetermined variables in budget constraints.

In any case, the equilibrium candidate $\overrightarrow{\mathcal{H}}$ is an infinite-dimensional object and for practical simulations, we have to approximate it by a finite-dimensional representation. The projection and L-B-J -based methods do that in different ways.

The strategy of the projection method is following: approximate the time profiles using a object parameterized by a low number of parameters (such as

[24] The circle $^\circ$ in the superscript henceforth stands for any of these superscripts edd, ndd, nfd, efd, eff, nff, ndf, edf.

polynomials, splines, neutral networks, or wavelets). Thus approximate

$$\overrightarrow{\mathcal{H}} \approx \widetilde{\mathcal{H}}(\mho),$$

where $\mho$ is a finite vector of parameters. Then the problem is to find such a vector of parameters $\overrightarrow{\mho}$, such that the equilibrium conditions $\hbar_t(\widetilde{\mathcal{H}}(\overrightarrow{\mho})) = 0$ nearly holds for all t. Judd (2002) discusses applications of the projection methods in the context of perfect foresight discrete-time models.

Another approach (called domain truncation approach) to reduce dimensionality of $\overrightarrow{\mathcal{H}}$ is to set $\{Q_t\}_{t=0}^{\infty} \approx \widehat{Q} = \{Q_1, ..., Q_N, Q_+, Q_+, ..., Q_+\}$, where Q_+ is the steady state of the variable Q_t (and similarly for other variables too) and to set

$$\widehat{\mathcal{H}} = \left\{\widehat{Q}, \widehat{\mathbb{W}}, \widehat{C}, \widehat{Q^*}, \widehat{\mathbb{W}^*}, \widehat{C^*}, \widehat{\eta}\right\},$$

and solve the system

$$\begin{aligned} \hbar_1(\widehat{\mathcal{H}}) &= 0 \\ \hbar_2(\widehat{\mathcal{H}}) &= 0 \\ &\vdots \\ \hbar_M(\widehat{\mathcal{H}}) &= 0 \end{aligned} \tag{A.12}$$

for $M \gg N$. This is a system of $7M$ unknowns. Lafargue (1990) proposed this approach, and Boucekkine (1995) and Juillard et al. (1998) exploited the sparseness of the system to apply an efficient algorithm. Hence, the approach uses to be called as L-B-J approach (see also Gilli and Pauletto 1998 or Armstrong et al. 1998 for further discussions about efficient implementation). The stacked system (A.12) is usually solved using Newton-based iterations. When applied to the model presented in this paper, we cannot use efficient algorithms for sparse systems unless $\delta = 1$. The case of $\delta = 1$ is the only case, when the Jacobian of (A.12) is sparse.

We experimented with both approaches: as projections we chose splines and RBC neural networks. To solve the system (A.12), we apply the quasi-Newton iteration, with the Hessian update via the BFGS method suggested by Broyden (1970), Fletcher (1970), Goldfarb (1970), and Shanno (1970). Our numerical experiments seems to suggest that for our problem the BFGS formula outperforms the Hessian update formula of Davidson (1959) and Fletcher and Powell (1963) and the steepest-descent approach[25]. Likewise, numerical experiments suggest that quasi-Newton iterations outperform the Nelder-Mead simplex algorithm by Lagarias et al. (1998) implemented in MATLAB function `fminsearch`.

[25] These methods are implemented in the MATLAB function `fminunc`, which is used.

Surprisingly, the L-B-J approach seems to perform better than the projection methods. We plan to investigate why this is so in future. Therefore, simulation results reported in this paper are based on quasi-Newton iterations on (A.12) with the BFGS Hessian update formula.

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Table 2. The Bestiary of Models

Horizontal Investments (endogenous varieties)	Vertical Investments (endogenous quality)	Export-eligibility Investments	**Model**	**Notes**
+	+	+	Benchmark model, $\mathcal{B}0$	Need to distinguish quality adjusted from quality unadjusted prices. Even for quality adjusted prices, the distinction of welfare-theoretical and average price indexes remains. It seems that both ingredients needed to match the pace of the RER in Central European transition countries.
+	–	+	Ghironi, Melitz (2005), $\mathcal{B}1$	Need to distinguish welfare theoretical price and average price indexes. These differ since the numbers of varieties and of traded varieties are both endogenous. Welfare-theoretical and average indexes can move in the opposite directions.
+	+	–	$\mathcal{B}2$	Similar to the benchmark without the extensive export margin. This margin is important close to the steady state, but it may be dominated by vertical investments along the transition convergence.
+	–	–	Small Open Economy Model (as in Krugman, 1979), $\mathcal{B}3$	When using welfare theoretical price indexes, terms of trade are equal to one; PPP holds (up to icebergs and home bias). This can break down for "average" indexes. Close to the steady state the difference is relatively small, since the extensive export margin is missing.
–	+	+		We are not aware of such a model.
–	–	+	Bergin, Glick, Taylor (2006)	Contrary to Ghironi, Melitz (2005), they concentrate on the extensive margin on the export market only. They do not use the dichotomy between theoretical and empirical price indexes. Closer to the original HBS ideas.
–	+	–	Dury, Oomen (2007)	Quality adjusted and quality-unadjusted prices may move in the opposite directions, which is an alternative explanation to Ghironi, Melitz (2005) and Bergin et al (2006) why after a positive productivity shock the RER appreciates. PPP holds for quality-adjusted prices (up to iceberg costs and home bias).
–	–	–	Standard Small Open Economy Model	Terms of trade are equal to one; PPP holds (up to iceberg costs and home bias).

Figure 1. Stylized Facts

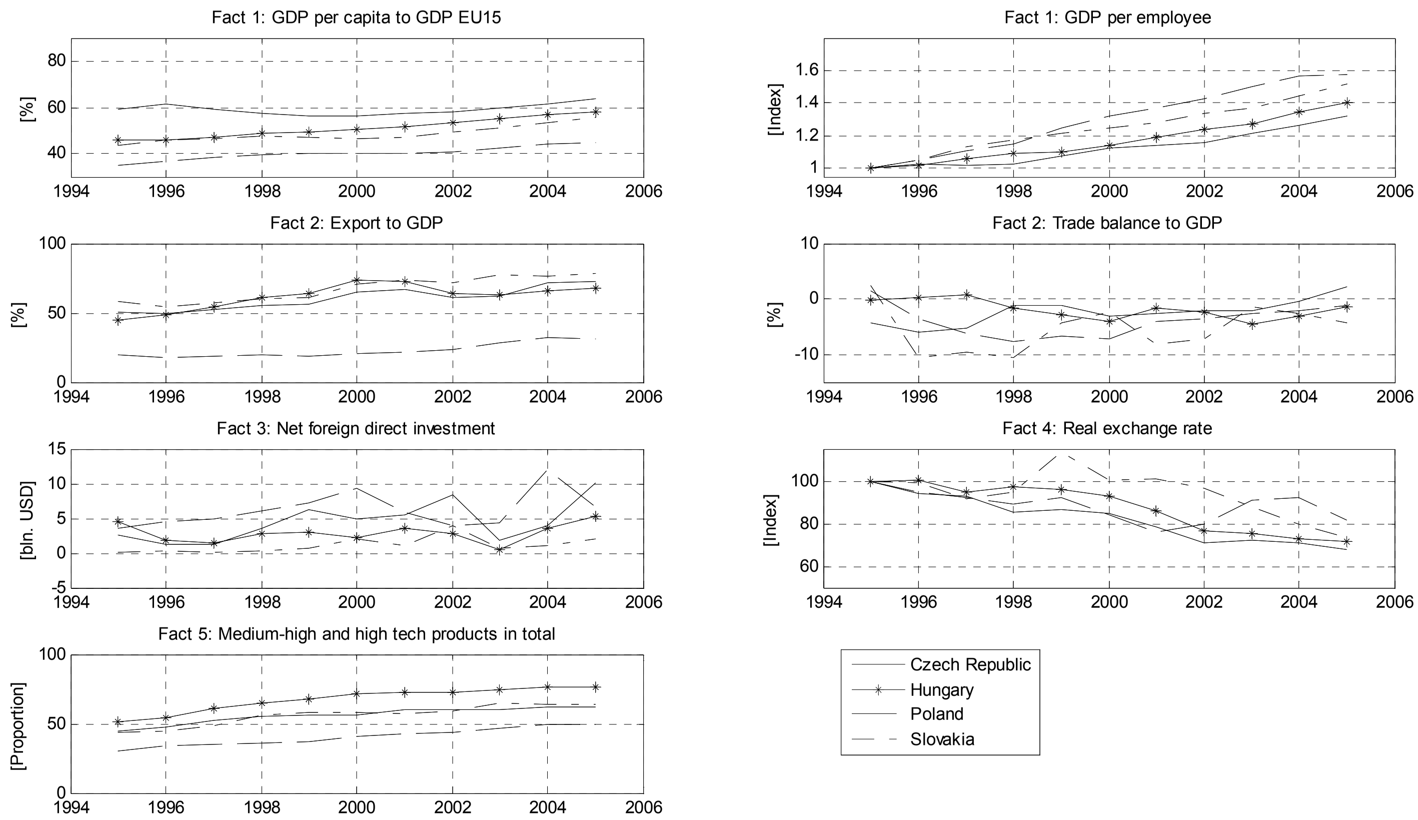


Figure 2. Investment to quality as a function of idiosyncratic productivity

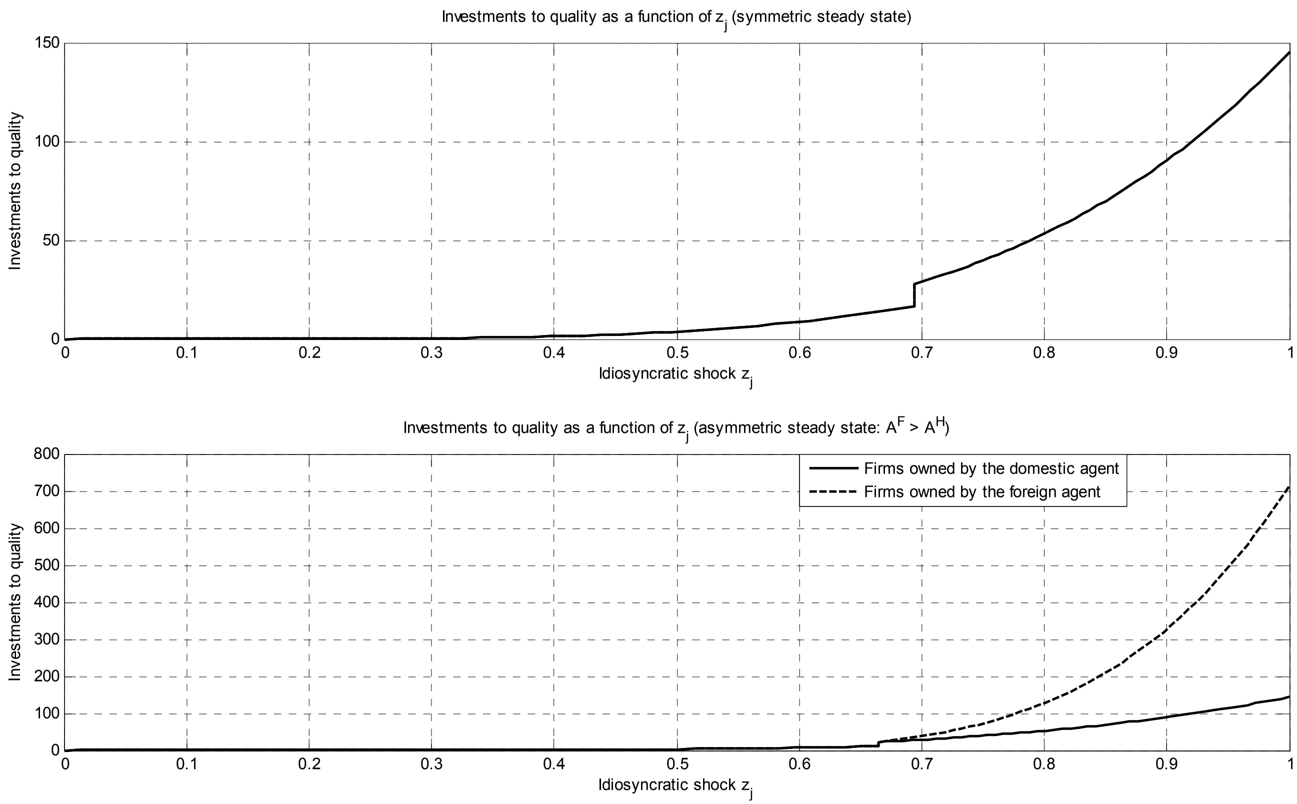


Figure 3. Scenarios $\mathcal{B}0$, $\mathcal{B}1$ and $\mathcal{B}3$ (converging country smaller than the developed one)

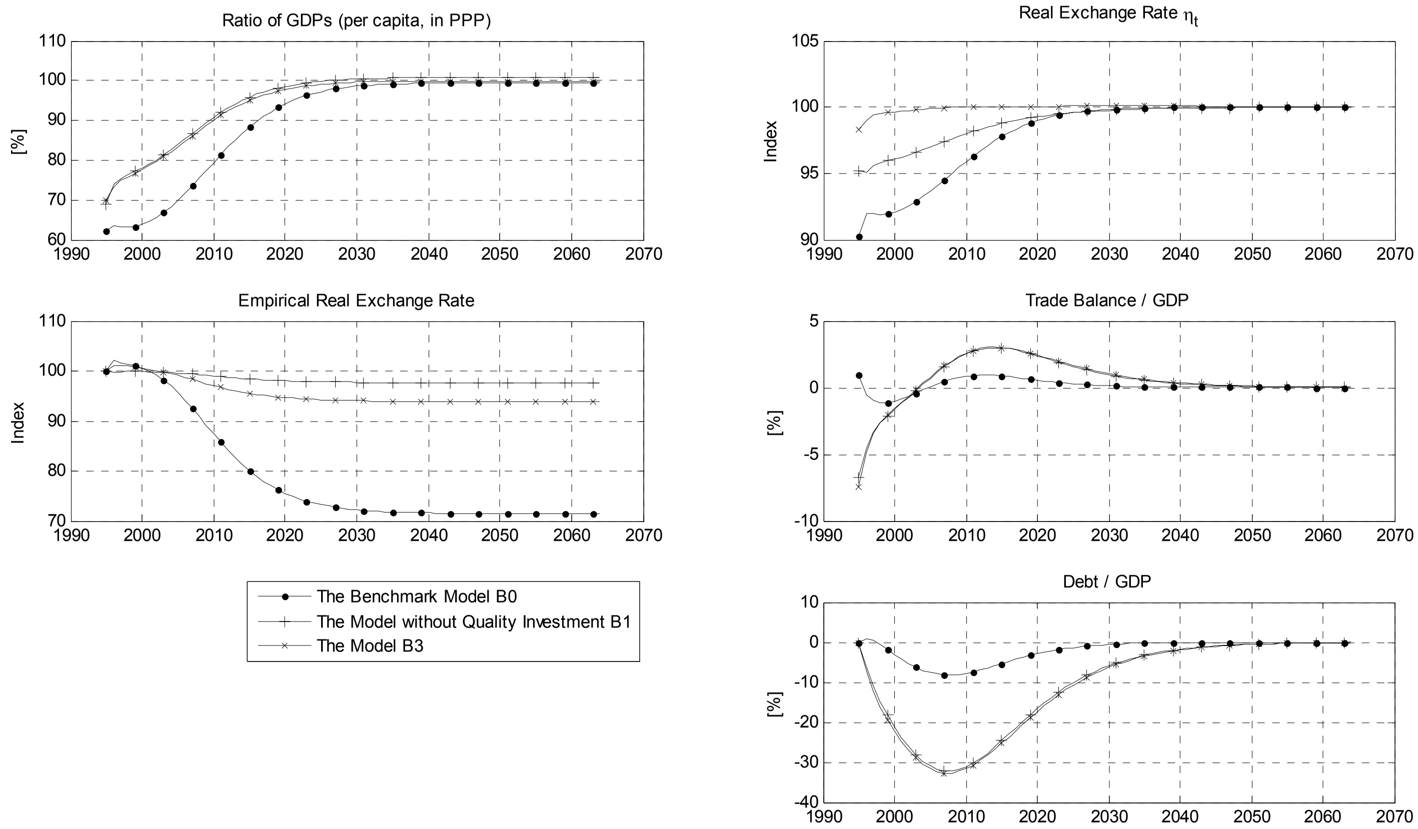


Figure 4. Scenarios $\mathcal{B}0$, $\mathcal{B}2$ and $\mathcal{B}3$ (converging country smaller than the developed one)

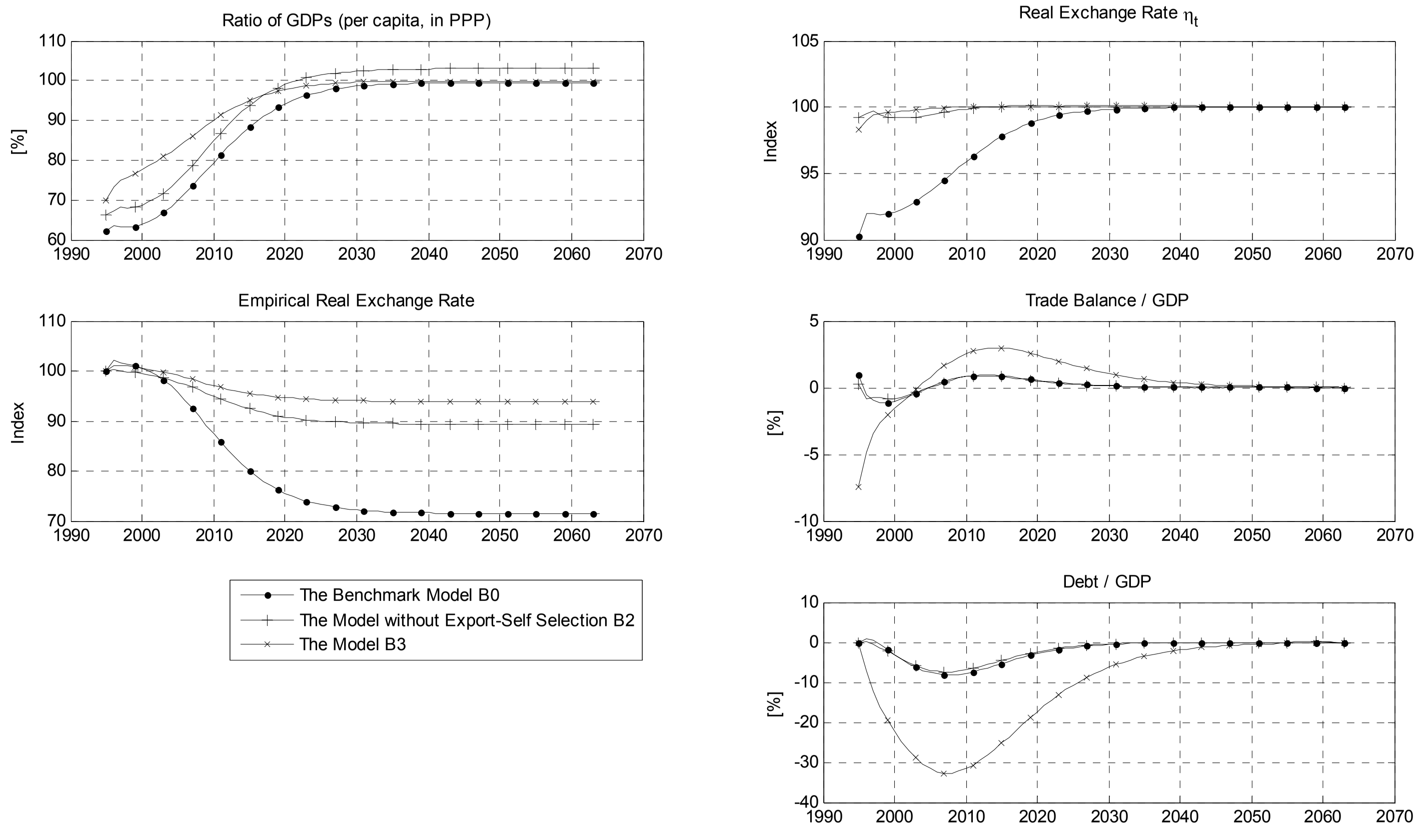


Figure 5. Scenarios $\mathcal{B}0$, $\mathcal{B}1$ and $\mathcal{B}3$ (converging countries have the same size)

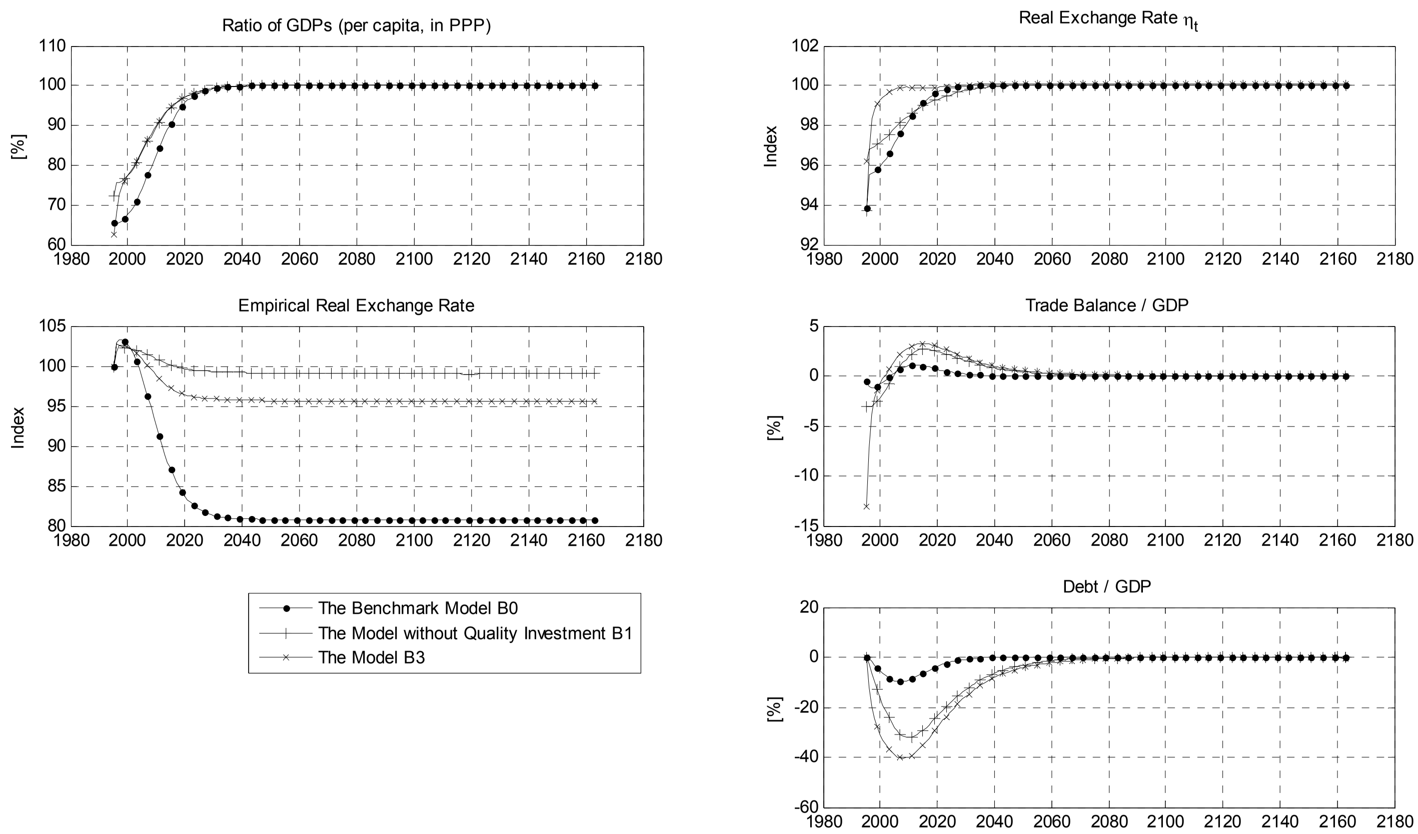


Figure 6. , $\mathcal{B}0$, $\mathcal{B}2$ and $\mathcal{B}3$ (converging countries have the same size)

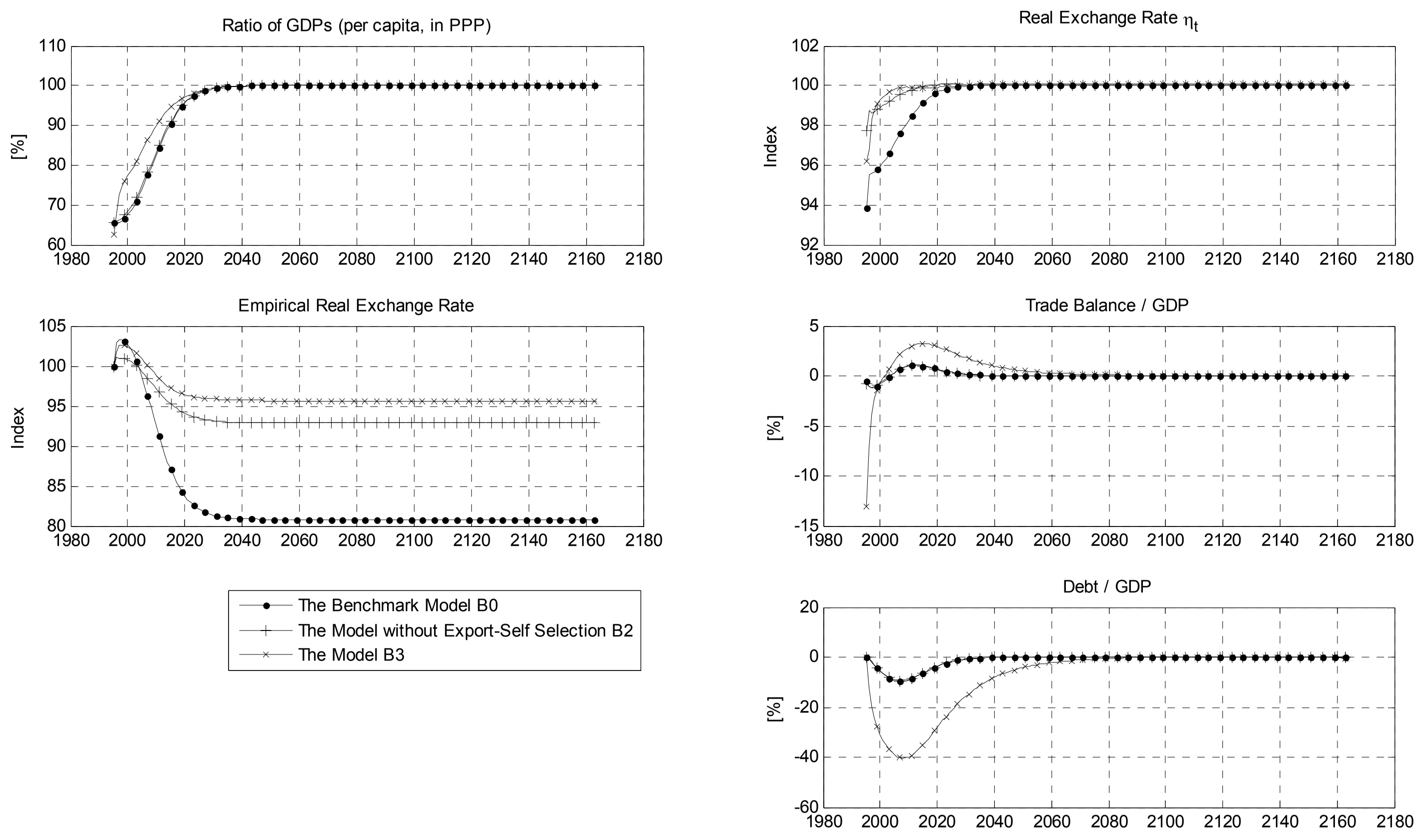


Figure 7. Transitory dynamics, cross-border asset ownerships (converging country smaller than the developed one)

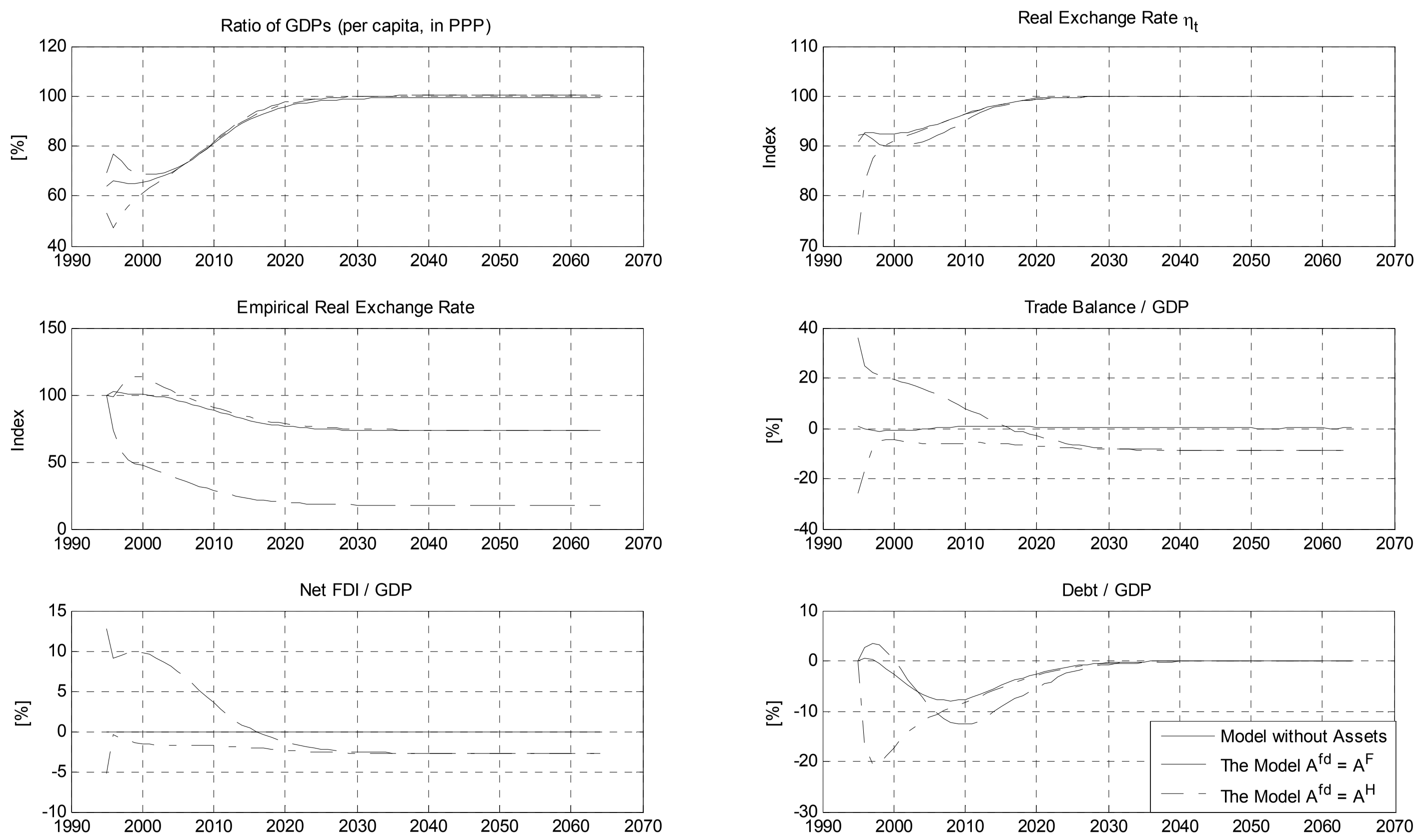


Figure 8. Transitory dynamics, cross-border asset ownerships (both countries have the same size)

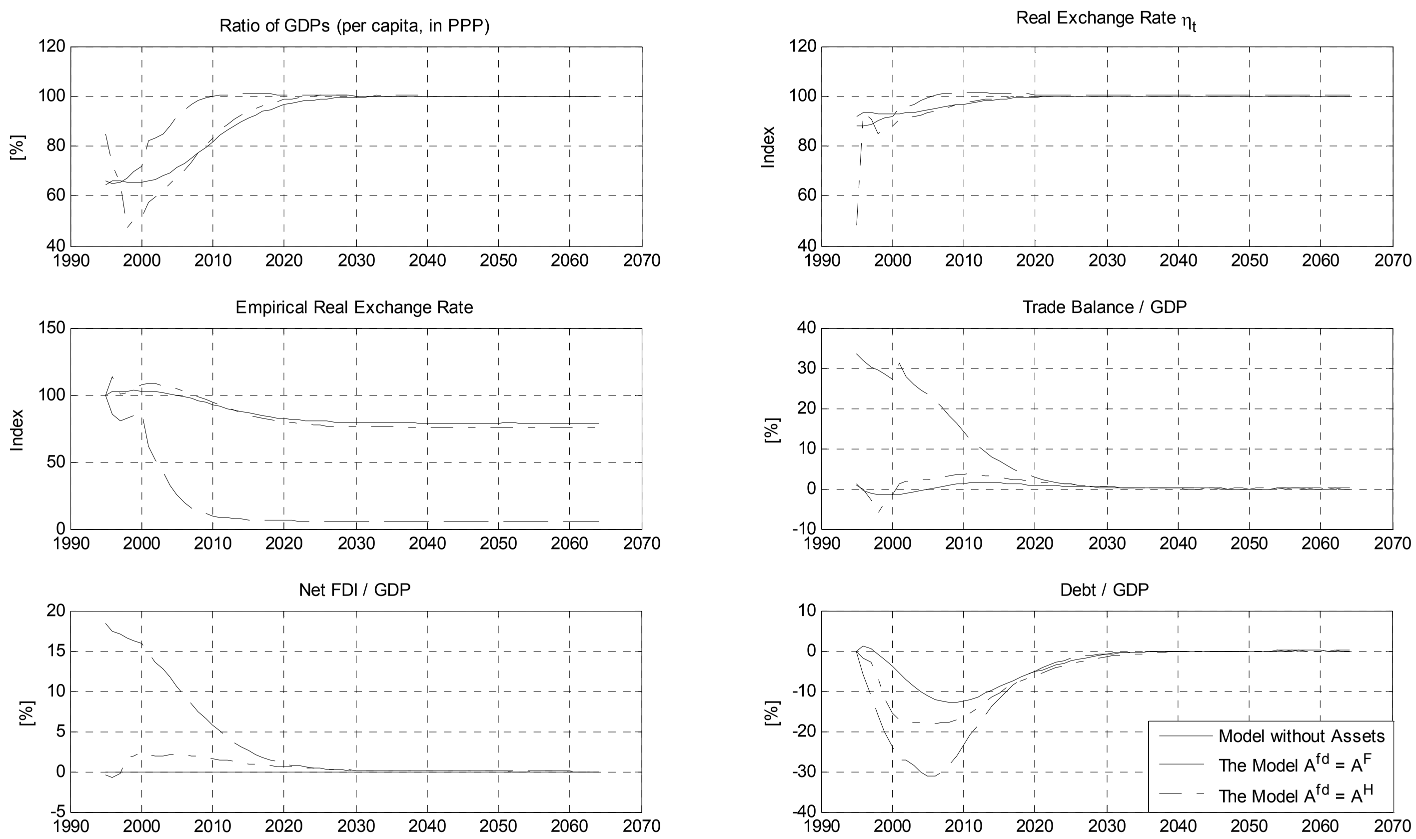


Figure 9. A fall in iceberg costs at year 2050 (converging country smaller than the developed one)

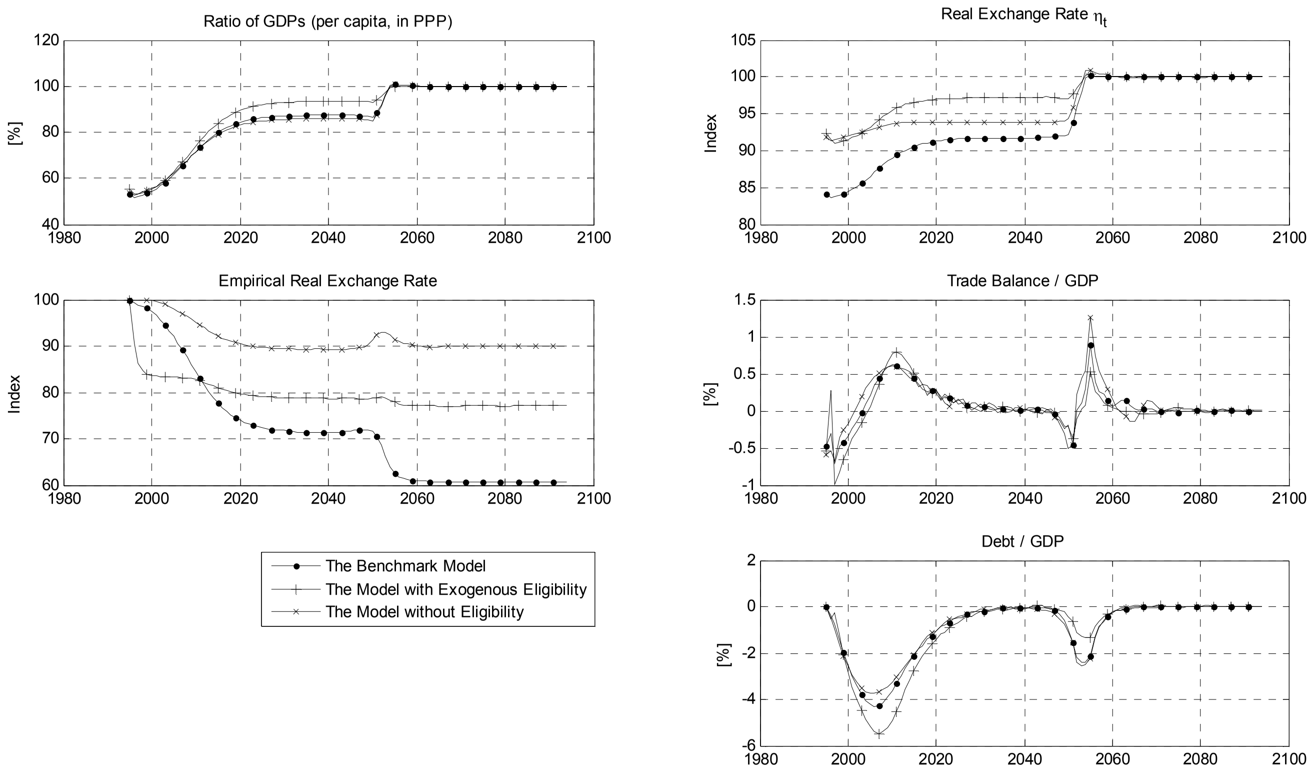


Figure 10. A fall in export-eligibility costs at year 2050 (converging country smaller than the developed one)

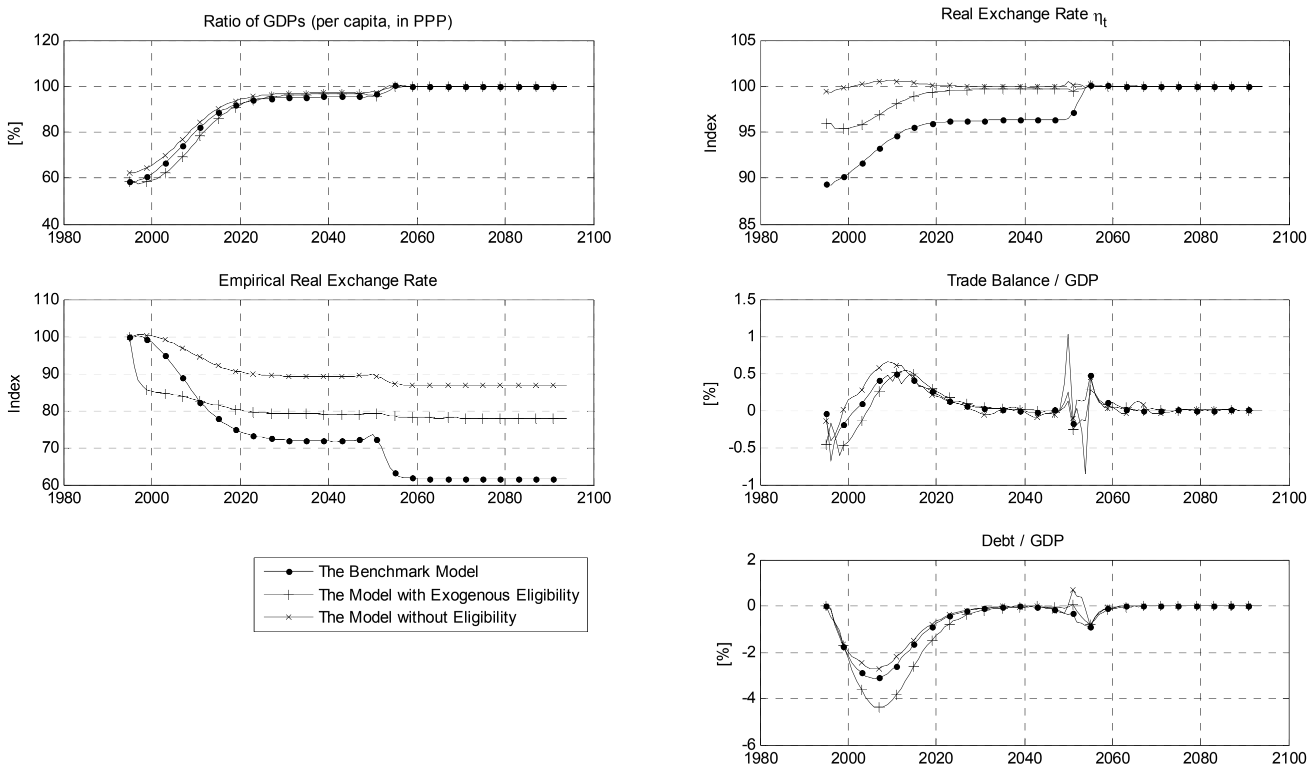
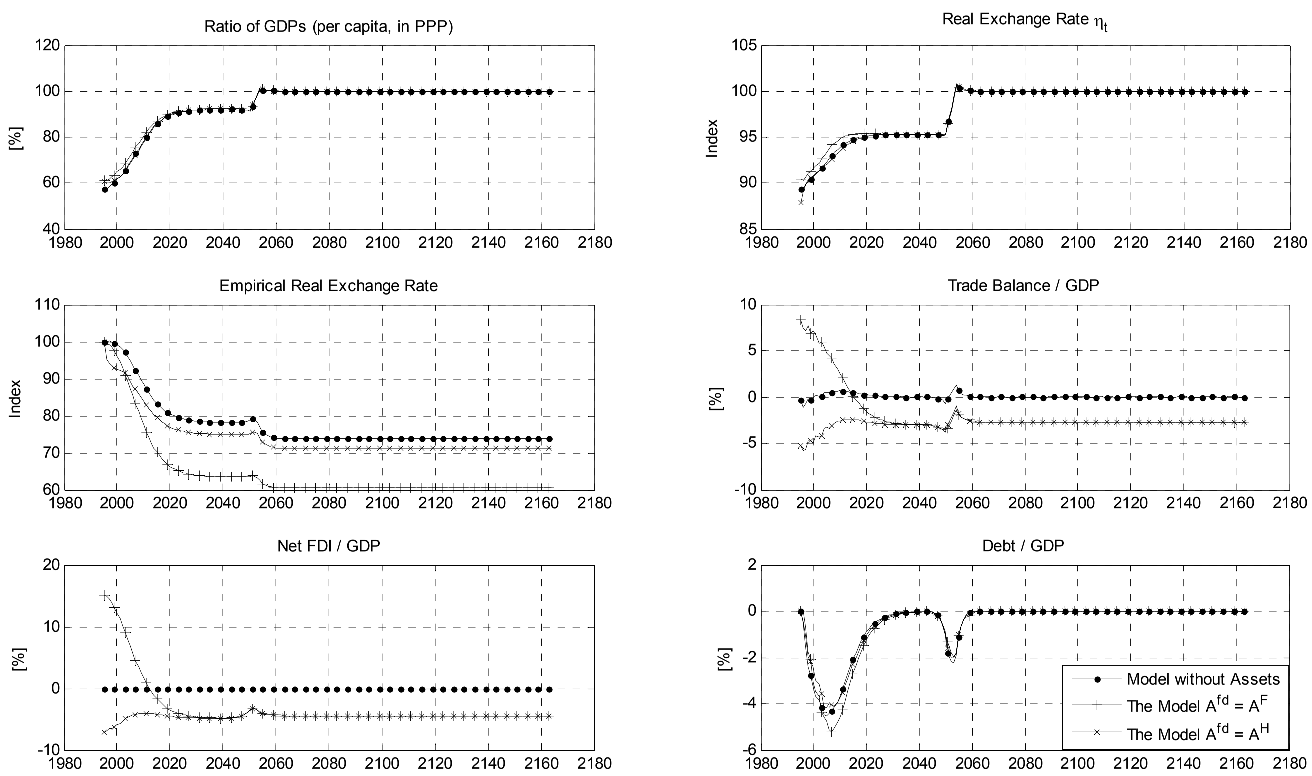


Figure 11. A fall in iceberg costs at year 2050 (converging country smaller than the developed one, with cross-asset ownership)



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