Bank capital, liquid reserves, and insolvency risk[*]

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Abstract

We develop a dynamic model of banking to assess the effects of liquidity and leverage requirements on banks' financing decisions and insolvency risk. In this model, banks face taxation, issuance costs of securities, and default costs and maximize shareholder value by choosing their debt-to-asset ratio, deposits-to-debt ratio, liquid asset holdings, equity issuance and default policies in response to these frictions as well as regulatory requirements. Our analytic characterization of the bank policy choices shows that imposing liquidity requirements leads to lower bank losses in default at the cost of an increased likelihood of default. Combining liquidity and leverage requirements reduces both the likelihood of default and the magnitude of bank losses in default.

Keywords: banks; liquidity buffers; capital structure; insolvency risk; regulation

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1. Introduction

Banks can impose major risks on the economy. Avoidance of these risks and the associated costs is the overwhelming concern of prudential regulation. Given the experience in the global financial crisis of 2007-2009, in which high debt levels and insufficient liquidity buffers led to the collapse of major players of the financial industry such as Bear Sterns or Lehman Brothers, the debate on banking regulation has recently evolved around two main ideas that have been reflected in proposals for regulatory reform. First, because most bank assets are illiquid and raising fresh equity is costly, banks should hold a buffer of liquid reserves to be able to cope with short-term losses. Second, equity capital should be significantly increased, so that if the value of the banks' assets were to decline, this would not automatically lead to distress and the resulting losses would be borne by the bank owners.

While many insightful discussions of liquidity and leverage requirements are available in the literature, financial theory has made little headway in developing models that can provide quantitative guidance for bank capital structure decisions and for the effects of regulatory requirements on these decisions and the resulting insolvency risk. Our objective in this paper is therefore twofold. First, we seek to develop a dynamic model of banks' choices of liquid asset holdings, financing, and default policies in the presence of realistic market frictions. Second, we want to use this model to characterize the endogenous response of banks to the imposition of liquidity and leverage requirements, and to measure the effects of such regulatory requirements on banks' policy choices and insolvency risk. Our focus is thus on micro-prudential regulation and the response of a bank to such regulation rather than on macro-prudential regulation and issues like contagion and fire sale of assets.

We begin our analysis by formulating a dynamic structural model in which banks face taxation, issuance costs of securities, and default costs and may be constrained by a regulator to hold a minimum amount of liquid reserves and/or equity capital. In the model, banks are financed with equity, insured deposits, and risky, subordinated debt. They hold risky, illiquid assets (e.g. risky loans) whose cash flows are subject to small and frequent shocks

as well as large and infrequent negative jumps capturing tail risk. They can also invest in risk-free, liquid assets (e.g. cash reserves) that can be used to absorb losses and save on recapitalization costs. Banks earn revenues from their investments as well as by providing liquidity services to their depositors. They maximize shareholder value by choosing their buffers of liquid assets, their debt-to-asset ratio, their deposits-to-debt ratio, their equity issuance strategy, and their default policy.

We first consider a "laissez-faire" environment in which banks are unregulated and solve for a bank's optimal capital structure, which involves determining how much market debt and deposits to issue and how much liquid reserves to hold. In this environment, we show that the costs of external finance create a wedge between inside and outside equity and generate a precautionary demand for liquidity as well as an optimal capital structure for banks. In our model, banks take high debt ratios to exploit the tax deductibility of interest payments and the potential mispricing of deposits induced by deposit insurance and/or the liquidity premium on deposits. They optimally retain earnings to balance the cost of accessing external liquidity with the cost of holding inside liquidity. Banks absorb small and intermediate losses using their liquid reserves or by raising outside equity at a cost, so that such losses do not trigger default. Large losses due to a realization of tail risk may however lead to default. Notably, we find that bank insolvency risk increases with tail risk, debt levels, and external financing costs and decreases with liquid reserves.

After solving for the policy choices of unregulated banks, we examine the effects of prudential regulation on these policy choices and insolvency risk. We first analyze liquidity requirements that mandate banks to hold a minimum amount of liquid reserves, reflecting the amount of deposits issued by the bank and its expected cash outflows over a given time period. We show that, when facing such a requirement, banks voluntarily choose to hold reserves in excess of the required minimum to reduce the costs associated with breaches of the requirement. In addition, we demonstrate that for any given capital structure, liquidity requirements lead to lower bank losses in default and to an increase in the likelihood of default. Our analysis therefore suggests that while liquidity requirements can reduce capital

injections by the regulator by limiting bank losses in default, they are ineffective at reducing default risk. When endogenizing the bank's capital structure, we also show that by increasing the cost of deposits, liquidity requirements reduce the optimal deposits-to-debt ratio, leading to a drop in bank charter value and to a further increase in insolvency risk.

In addition to liquidity requirements, banks may be subject to leverage requirements. Such requirements do not impose any restriction on the assets that banks should hold, but impose constraints on the way they fund their operations. As argued by Admati and Hellwig (2013), a tightening of leverage requirements transfers a large fraction of the bank's risks to its shareholders, which otherwise might be passed on to creditors or taxpayers. Leverage requirements generally prescribe how much equity capital banks should have relative to their total assets.[1] Using a calibrated version of the model, we show that such requirements have significant effects on insolvency risk but little effect on bank losses in default. For example, we find that imposing a minimum Tier 1 leverage ratio of 9% decreases the one-year default probability of an average U.S. bank by 49.46% while decreasing its losses in default by 0.60%. Importantly, we also show that combining liquidity requirements with leverage requirements reduces both the likelihood of default and bank losses in default. For example, we find that combining a minimum Tier 1 leverage ratio of 9% with a liquidity requirement of 5% of deposits, reduces the one-year default probability of an average U.S. bank by 31.26% while reducing losses in default by 4.03%. There again, liquidity requirements reduce bank losses in default but increase the likelihood of default events.

The model also allows us to investigate how bank capital structure decisions and default risk depend on a bank's economic environment and asset characteristics. For example, we show that an increase in asset risk generally leads to a decrease in the debt-to-asset ratio, an increase in the deposits-to-debt ratio, and an increase in liquid reserves and default risk. We also show that mispriced deposit insurance and/or the existence of a liquidity premium

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[1]For example, banks are expected to maintain a Tier 1 capital to asset ratio of 3% under Basel III. In the U.S., banking regulators require Tier 1 capital to asset ratios ranging from 4% to 9% for Systemically Important Financial Institutions. Similarly, the Vickers Commission in the U.K. has recommended a 4% ratio rather than the 3% global standard set by the Basel III regime.

on deposits makes it optimal for banks to take on substantially more debt, leading to a large increase in insolvency risk.

The literature on default risk in banks has started with the early contributions of Merton (1977, 1978), in which the objective is to determine the cost of deposit insurance and loan guarantees. Although important milestones, these papers suffer from four limitations for our purpose. First, they assume that banks default whenever their assets-to-deposits ratio falls below some *exogenous* barrier that depends neither on the characteristics of the bank nor on the frictions that it faces. Second, the capital structure of banks is set exogenously and does not reflect the frictions that they may face such as taxes, default costs, or issuance costs of securities. Third, the dynamics of the banks' assets are governed by an exogenous process, implying that there is no connection between the banks' asset and capital structures. Fourth, raising outside equity is costless, so that liquid reserves are irrelevant.

Most of the recent quantitative banking models, including Mella-Barral, and Perraudin (1997), Bhattacharya, Planck, Strobl, and Zechner (2002), or Décamps, Rochet, and Roger (2004), examine variants to the first of these assumptions. In these contributions, insolvency is endogenous and triggered by shareholders' decision to cease injecting funds in the bank. While identifying some prime determinants of insolvency risk, these theories assume that asset and liability structures are exogenously given. As a result, they leave open the question of how financing structure and asset structure interact and jointly affect insolvency risk. In addition, these models maintain the assumption that banks can raise outside funds at no cost, thereby leaving no role for liquid reserves.

Our analysis inherits some of the assumptions of this literature. For example, bank shareholders are protected by limited liability and the bank's objective is to maximize shareholder value. However, it differs from these contributions in three important respects. First, we consider that at least part of a bank's assets are illiquid and that it is costly to raise outside equity, thereby providing a role for liquid reserves. Second, we incorporate some of the key market imperfections and regulatory requirements that banks face in practice and relate banks' payout, financing, and default policies to these frictions. Third, in our model,

financing structure and asset structure interact and jointly affect insolvency risk. We show that these unique features have important implications. For example, while in most prior models shareholder value is always increased by making dividend payments, this is not the case in our model with frictions, in which shareholders have incentives to protect the bank's charter value by maintaining adequate liquid reserves.

There exists a large literature analyzing the role of capital in banking regulation (see e.g. Morrison and White (2005) or Repullo and Suarez (2013)). But it is only recently that the question of bank optimal capital structure has begun to be addressed. In our model, banks hold liquid reserves to reduce financing frictions and expected default costs. They take high debt ratios to exploit the mispricing of deposits and the tax deductibility of debt payments. The notion that bank deposits are mispriced and command a liquidity premium is well accepted (see Gorton and Pennacchi (1990) or DeAngelo and Stulz (2015)). By contrast, tax benefits of debt are often overlooked in the banking literature.[2] In our model, banks do not issue market debt when interest payments are not tax deductible and follow conservative financing policies with a ratio of deposits to total assets of about 35%. Tax benefits make it optimal for banks to issue market debt on top of deposits and to follow much more aggressive financing strategies with more than 90% debt financing.

In the banking literature, our work is most closely related to the papers by Froot and Stein (1988), De Nicolo, Gamba, and Lucchetta (2014), Allen, Carletti, and Marquez (2015), and Sundaresan and Wang (2016). Sundaresan and Wang (2016) adapt the framework of Leland (1994) to the study of banks and analyze banks' financing decisions and the effects of deposit insurance and regulatory closure on bank liability structure. Allen, Carletti, and Marquez use a static model to show that banks hold capital to reduce expected default costs and the cost of deposit finance. In these papers, firms can access outside liquidity at no cost, inside and outside equity are perfect substitutes, and there is no role for liquid reserves.

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[2]In a recent study, Schepens (2016) uses a differences-in-differences approach to show that the introduction of an equity tax shield in Belgium in 2006 led banks to significantly increase their capital ratios by increasing their equity capital. See Ashcraft (2008), De Mooij and Keen (2012), De Mooij, Keen, and Orihara (2014), or Horvath (2013) for additional evidence on the effects of tax rates on bank capital ratios.

Froot and Stein (1998) build a two-period model in which capital is initially costless but may become costly in the future. In their model, bank shareholders may reduce future financing costs by raising capital to increase the initial level of liquid reserves. In our model, banks always face the same frictions and choose their payout, financing, and default decisions in response to these frictions as well as regulatory requirements. Another difference is that we consider a dynamic model. In a static model, a regulatory requirement can only affect optimal behavior if it is binding. In practice however, regulatory requirements are binding for a minority of banks and yet to influence the behavior of all banks.

De Nicolo, Gamba, and Lucchetta (2014) build a dynamic, discrete-time model in which banks are financed with equity, deposits, and risk-free debt and can invest in risky loans and risk-free bonds (cash is negative debt). While their analysis provides interesting insights into the benefits and costs of prudential regulation, their framework does not allow banks to issue risky debt and cannot accommodate both liquid reserves and debt on banks' balance sheets. This prevents an analysis of the joint effects of risky debt financing, liquidity choices, and micro-prudential regulation on default risk, which is the main focus of our paper.

From a modeling perspective, our paper relates to the inventory models of Milne and Whalley (2001), Peura and Keppo (2006), Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011, 2014), Asvanunt, Broadie, and Sundaresan (2011), Hugonnier, Malamud, and Morellec (2015), and Décamps, Gryglewicz, Morellec, and Villeneuve (2016) in which financing costs lead banks or corporations to maintain liquidity buffers. In these models, uncertainty is solely driven by Brownian shocks so that there is no risk of default if firms can raise outside equity. Our paper advances this literature in two important dimensions. First, we incorporate jumps in our analysis to account for tail risk and show that in the presence of such risk banks may find it optimal to default following large shocks, even if they can raise outside funds. Second, we endogenize not only banks' payout decisions but also their financing and default policies. This allows us to examine the effects of regulation on financing decisions and insolvency risk. As shown in the paper, this problem is more difficult to solve because it involves two free boundaries (the default and

payout thresholds) instead of one (the payout threshold). Another contribution of this paper is to develop a new method based on fixed-point arguments to solve such problems.

The remainder of the paper is organized as follows. Section 2. presents the model and provides an informal description of the optimal strategy. Section 3. presents our analytic characterization of the value-maximizing payout, financing, and default policies for unregulated banks. Section 4. examines the effects of liquidity and leverage requirements on these policy choices and insolvency risk. Section 5. discusses the model's implications. Section 6. concludes. All proofs are gathered in a separate Appendix.

2. Model

2.1. Assumptions

Throughout the paper, time is continuous and all agents are risk neutral and discount cash flows at a constant rate $\rho > 0$.

The subject of study is a bank held by shareholders that have limited liability. This bank is subject to taxation at rate $\theta \in [0, 1)$ and transforms a fixed (endogenous) volume of deposits into a fixed volume of risky assets (e.g. risky loans or derivatives).[3] The bank's risky assets generate after-tax cumulative cash flows A_t that evolve according to:

$$dA_t = (1 - \theta)\mu dt + \sigma dB_t - Y_{N_t} dN_t. \tag{1}$$

In this equation, B_t is a standard Brownian motion, N_t is a Poisson process, (σ, μ) are positive constants, and $(Y_n)_{n=1}^{\infty}$ is a sequence of independently and identically distributed random variables that are drawn from $(0, \infty)$ according to an exponential distribution with mean $1/\beta > 0$. The increments of the Brownian motion represent small and frequent shocks

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[3]Because the bank's risky assets are fixed, the paper does not explore how regulatory policies may affect the loan portfolio choice of banks. In particular, the choice of the riskiness of the loan portfolio, the possibility of de-leveraging by reducing the size of the loan portfolio, and the option to sell assets to meet regulatory requirements are not included in the model. Given our focus on capital structure, we view these restrictions as a necessary first step.

to the bank cash flows. The jumps of the Poisson process represent large losses that may be due, for example, to defaults across the loan portfolio of the bank (see Acharya, Cooley, Richardson, and Walter (2009) for an analysis of the role of tail risk in the financial crisis).[4] We denote the intensity of the Poisson process by $\lambda \geq 0$, so that over an infinitesimal time interval there is a probability $\lambda \beta e^{-\beta y} dt$ that the bank makes a loss of size $y \geq 0$. In a non-financial firm, dA_t is the total earnings. In a bank, it only represents the earnings from risky assets, not including the income from serving deposit accounts.

In addition to risky assets, the bank can or may be constrained to hold liquid, risk-free reserves (i.e. cash reserves or government bonds). We denote by S_t the liquid reserves of the bank at time $t \geq 0$. Holding liquid reserves generally involves deadweight costs. We capture these costs by assuming that the rate of return on liquid reserves is zero.[5] When optimizing its liquid asset holdings, the bank trades-off the lower returns of these assets with the benefits of liquidity.

As discussed in Sundaresan and Wang (2016), "banks share some common characteristics with non-financial firms: Both have access to the cash flows generated by their assets and both finance their assets with debt and equity. Banks, however, differ from non-financial firms in that they take deposits and provide account services to their depositors [...]" In many countries, deposits are insured by the regulator and the deposit-taking activity comes with heavy regulation. Deposits and the associated account services, deposit insurance, and regulation distinguish the banking business from other businesses and set the capital structure decision of banks apart from that of other firms.

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[4] These jumps may reflect trading losses, such as the \$8.9 billion loss of Morgan Stanley on credit default swaps in 2012 or the \$7.2 billion loss of Société Générale on index futures in 2008. They may also reflect losses on poorly performing acquisitions, such as the \$10.4 billion loss of Crédit Agricole on Emporiki. Lastly, they may reflect the payments made by a number of banks to authorities in the United States for having facilitated tax evasion or for having traded with countries under embargo. For example, on June 30, 2014, BNP Paribas agreed to pay a fine of \$8.9 billion for having violated U.S. sanctions against Cuba, Iran, Sudan, and other countries. For more details, see "Capital Punishment" in *The Economist* July 5, 2014.

[5] A necessary condition for a well-defined payout policy is that there exists a cost of holding liquidity in that the rate of return on liquid reserves is strictly less than the risk-free rate of return ρ.

To capture these important differences, we consider that the liability structure of the bank comprises equity, deposits, and market debt.[6] Deposits are insured against bank failure, have endogenous face value D, and require the bank to make a payment

$$c_D(D) = (\iota + k(D))D \tag{2}$$

per unit of time. The constant ι reflects the interest payment to depositors while $k(D)$ is an increasing and convex function that captures the combined costs of deposit insurance and of servicing depositors, which includes advertising expenses, the cost of building and maintaining branches and ATM machines, employee salaries, and other operating expenses. While banks pay deposit insurance in our model, we do not examine the endogenous determination of the insurance premium, the effect of closure policy regulations, or the objective of the insurance authority. See Sundaresan and Wang (2016) for such an analysis.

Market debt is subordinated, requires the bank to make a payment $c_L \geq 0$ per unit of time, and has face value L, where both c_L and L are endogenously determined. The combined payments of the bank on its market debt and deposits are thus given by $c = c_L + c_D(D) < \mu$ per unit of time and its cumulative earnings C_t evolve according to:

$$dC_t = dA_t - (1-\theta)c\,dt = (1-\theta)(\mu - c)dt + \sigma dB_t - Y_{N_t}dN_t. \tag{3}$$

In most of the model, we take the bank's capital structure as given. Section 4.2. endogenizes the bank's leverage ratio as well as its liability structure (ratio of deposits to total debt).

Because the distribution of the jump magnitudes has unbounded support, the bank will be unable to withstand large losses with positive probability, leading to default. In the following, we use a stock-based definition of default whereby the bank services debt as long as equity value is positive (as in Leland (1994), Duffie and Lando (2001), or Sundaresan and Wang (2015, 2016)). That is, default is the result of the optimizing behavior of shareholders.

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[6]As in many recent contributions (e.g., Allen, Carletti, and Marquez (2014) or Hanson, Shleifer, Stein and Vishny (2015)), we downplay the vulnerability of deposits to runs and emphasize the opposite aspect of deposits: Relative to other forms of private-money creation that occur in the shadow-banking sector, deposits are highly sticky and not prone to run at the first sign of trouble.

We assume that the liquidation value of the bank's risky assets in default is given by

$$\Lambda = (1 - \varphi)V^+ \tag{4}$$

where

$$V = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dA_t\right] = \frac{1}{\rho}\left((1-\theta)\mu - \frac{\lambda}{\beta}\right) \tag{5}$$

denotes the present value of the infinite stream of cash flows generated by these assets, the constant $\varphi \in [0,1]$ captures liquidation costs, and $x^+ = \max\{0, x\}$.

Equity capital and liquid reserves serve as buffers against default risk. The bank can increase its liquid reserves either by retaining earnings or by issuing new equity. There is considerable evidence that firms have to pay significant costs when issuing securities (see Smith (1977) for an early survey and Kim, Palia, and Saunders (2008) for recent evidence). To capture this important feature of capital markets, we consider that the bank has to pay a lump-sum cost ϕ upon raising outside funds. Because of this fixed cost, the bank will retain earnings to build-up liquid reserves in an attempt to save on issuance costs. In addition, it will only raise fresh equity through lumpy and infrequent issues.[7]

A payout and financing strategy is a pair $\pi = (P_t^\pi, R_t^\pi)_{t \geq 0}$ of adapted, left-continuous, and non-decreasing processes with initial value zero, where P_t^π and R_t^π respectively represent the cumulative payouts to shareholders and the cumulative net financing raised from investors up until time $t \geq 0$. The liquid reserves process associated with a strategy π is defined by:

$$S_t^\pi = s + C_t - P_t^\pi + R_t^\pi, \tag{6}$$

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[7]If refinancing costs were purely proportional, it would be optimal for the bank to reflect its cash reserves at zero and either liquidate or raise funds back to zero after a realization of tail risk taking reserves below zero. That is, with purely proportional refinancing costs, the bank would never raise equity to replenish its cash buffer, which would go against the evidence in Kim and Weisbach (2008) and McLean (2011) that decisions to issue equity are essentially driven by a desire to build up cash reserves. With a proportional cost in addition to the fixed cost, the optimal refinancing policy would be qualitatively similar to the one we derive with the exception that the point towards which the bank moves when raising equity would lie below the payout threshold. Otherwise, the economics of the problem would be identical to those described below.

where C_t is defined in Eq.(3) and s is the initial level of liquid reserves. This equation is an accounting identity which shows that liquid reserves increase with earnings and with the funds raised from investors and decrease with payouts and potential operating losses. The liquidation time associated with the strategy π is then defined by:

$$\tau_\pi = \inf\left\{t \geq 0 : S_{t+}^\pi = \lim_{u \downarrow t} S_u^\pi \leq 0\right\}. \tag{7}$$

In the model, bank shareholders make their financing, payout, and default decisions after observing the increment of the cash flow process. As a result, we use left-continuous processes in the definition of strategies and right-hand limits in the definition of the liquidation time. Notably, the occurrence of a cash flow jump

$$\Delta C_t = C_t - C_{t-} = C_t - \lim_{s \uparrow t} C_s \leq -S_{t-}^\pi \tag{8}$$

that depletes the liquid reserves of the bank only results in default if shareholders do not provide sufficient funds for the right hand limit

$$S_{t+}^\pi = S_{t-}^\pi + \Delta C_t + \left(R_{t+}^\pi - R_t^\pi\right) - \left(P_{t+}^\pi - P_t^\pi\right) \tag{9}$$

to be strictly positive. We consider strategies such that $P_{t+}^\pi - P_t^\pi \leq (S_t^\pi + R_{t+}^\pi - R_t^\pi)^+$ implying that the bank cannot distribute cash it does not hold.

Management chooses the payout, financing, and default policies of the bank to maximize the present value of future dividends to shareholders, net of the total cost of capital injections. That is, management solves:[8]

$$v(s) = \sup_{\pi \in \Pi(s)} \mathbb{E}_s\left[\int_0^{\tau_\pi} e^{-\rho t}(dP_t^\pi - d\Phi_t(R^\pi)) + e^{-\rho \tau_\pi} \ell\left(S_{\tau_\pi}^\pi\right)\right] \tag{10}$$

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[8]We assume throughout that the objective of management is shareholder value maximization. Asvanunt, Broadie, and Sundaresan (2011) show that cash accumulation and drainage policies may depend on whether the firm is maximizing equity value or total firm value.

where $\Pi(s)$ denotes the set of admissible strategies, the operator $\mathbb{E}_s[\cdot]$ denotes an expectation conditional on an initial level s of liquid reserves, the function

$$\ell(s) = (s + \Lambda - L - D)^+ \tag{11}$$

represents the liquidation payment to shareholders if the bank holds $s \leq 0$ in liquid reserves, and the non-decreasing process

$$\Phi_t(R^\pi) = R_t^\pi + \sum_{0 \leq u < t} 1_{\left\{R_{u+}^\pi > R_u^\pi\right\}} \phi \tag{12}$$

captures the total contribution of shareholders, including the cumulative cost of financing. The first term on the right-hand side of Eq.(10) captures the present value of dividend payments. The second term gives the present value of the cash flow to shareholders in default.

2.2. Informal description of the optimal strategy

To gain some intuition on the solution to problem Eq.(10) note that, because of the fixed costs of financing, the bank only considers raising equity when cash flow shocks deplete its liquid reserves, i.e. when $s \leq 0$. In such instances, the bank has to either raise new funds to finance the shortfall and continue operating, or default. The bank chooses to default when the shortfall, the cost of refinancing, or the liquidation value of risky assets are large. When defaulting is not optimal, the bank pays the cost ϕ and raises funds to maximize equity value. Taking into account these two possibilities shows that the equity value function satisfies:

$$v(s) = \max\left\{fv(s), \ell(s)\right\} = \left(\max\left\{fv(0), \ell(0)\right\} + s\right)^+, \qquad \text{for } s \leq 0, \tag{13}$$

where the financing operator

$$fv(s) = \sup_{b \geq 0}\left(v(b) - (b - s + \phi)\right) = s + fv(0), \tag{14}$$

gives the maximal value that shareholders can obtain by refinancing the bank at a point where its cash reserves are equal to $s \leq 0$.

Because the likelihood of costly refinancing or inefficient liquidation decreases as liquid reserves increase, we expect the marginal value of liquid reserves to be decreasing and, therefore, the equity value function to be concave on the positive real line. If this conjecture is verified, there should exist some level b_0^* such that $v'(s) \geq 1$ if and only if $s \leq b_0^*$ and the optimal payout policy should consist in distributing dividends to maintain liquid reserves at or below the target level b_0^*. Given the target level of liquid reserves, Eq.(13) shows that the optimal refinancing policy depends on the comparison between

$$fv(0) = \sup_{b \geq 0} \left(v(b) - (b + \phi) \right) = v(b_0^*) - (b_0^* + \phi) \tag{15}$$

and the payoff $\ell(0)$ that shareholders receive if they liquidate the bank at a point where it has no liquid reserves. If $\ell(0) > fv(0)$, as illustrated by the left panel of Fig. 1, it is never optimal for shareholders to refinance and the bank is liquidated the first time that liquid reserves fall below zero. If $fv(0) \geq \ell(0)$, as illustrated by the right panel, shareholders may prefer refinancing over liquidation. In that case, they refinance the bank back to b_0^* whenever liquid reserves become negative with a shortfall smaller than $fv(0)$, and otherwise liquidate. Fig. 2 shows two trajectories for liquid reserves corresponding to these two strategies. In the top panel, $\ell(0) > fv(0)$ and it is never optimal to refinance. In the bottom panel, $fv(0) \geq \ell(0)$ and it is optimal to refinance shortfalls smaller than $fv(0)$.

> Insert Figure 1 about here

The main difficulty in verifying these conjectures is that one needs to simultaneously determine the values of the constants b_0^* and $\alpha_0^* = \max\{\ell(0), fv(0)\}$. To circumvent this difficulty, we proceed in two steps. In the first step, we fix the value of the constant α and solve for the optimal payout policy in an auxiliary problem in which the bank cannot raise funds but produces the payoff $(\alpha + s)^+$ to shareholders when it runs out of liquid reserves and defaults. In the second step, we show that the constant α can be chosen in such a way

that the value of this auxiliary problem coincides with that of problem Eq.(10) and derive the equity value-maximizing payout, financing, and default policies.

$$\boxed{\text{Insert Figure 2 about here}}$$

Before proceeding with to these two steps, we first determine the equity value and the bank's policy choices in a benchmark economy in which there are no refinancing costs. This allows us to derive a *frictionless value* of equity that will be of repeated use when solving shareholders' optimization problem.

3. Value of an unregulated bank

3.1. Frictionless benchmark

When there are no costs of raising funds, in that $\phi = 0$, any loss can be covered by issuing equity at no cost and there is no need to hold liquid reserves inside the bank (as in Leland (1994), Duffie and Lando (2001), or Sundaresan and Wang (2016)). As a result, it is optimal to pay out all positive earnings and the optimization problem of the bank reduces to choosing the default policy that maximizes equity value. That is, the value of the bank's equity is given by

$$v^*(s) = (v^* + s)^+ \tag{16}$$

with the constant v^* defined by

$$v^* = \sup_{\tau \in \mathcal{S}} \mathbb{E}\left[\int_0^{\tau^-} e^{-\rho t} dC_t + e^{-\rho \tau} \ell(\Delta C_\tau)\right], \tag{17}$$

where $\mathcal{S}$ denotes the set of all stopping times and $\Delta C_\tau \leq 0$ represents the loss leading to default. Solving this optimal stopping problem leads to the following result.

Proposition 1 (Frictionless benchmark). *When issuing equity is costless, the value of equity and the optimal default time are given by*

$$v^*(s) = (\max\{v_0^*, \ell(0)\} + s)^+ \tag{18}$$

and

$$\tau^* = 1_{\{v_0^* \geq \ell(0)\}} \inf\{t \geq 0 : |\Delta C_t| \geq v_0^*\}, \tag{19}$$

where the strictly positive constant v_0^* is the unique solution to

$$\rho v_0^* = (1 - \theta)(\mu - c) - \lambda \mathbb{E}[\min\{v_0^*, Y_1\}]. \tag{20}$$

Equity value in the frictionless benchmark is increasing in the cash flow rate μ, and decreasing in the coupon rate c, the jump intensity λ, and the mean jump size $1/\beta$.

Eq.(18) in Proposition 1 demonstrates that absent financing frictions two cases may occur: Either it is optimal to immediately liquidate, in which case shareholders get $\ell(0) + s$, or it is optimal to continuously refinance the bank until the first time that the absolute value of a jump of the cash flow process exceeds equity value. In the later case, the amount $|\Delta C_t| = Y_{N_t}$ plays the role of a cost of investment that shareholders have to pay to keep the bank alive following the occurrence of a large loss.

To better understand this feature, consider the case where $v^*(0) = v_0^* \geq \ell(0)$ so that it is not optimal to immediately liquidate. In this case, Eq.(20) can be written as

$$v^*(0) = p(1 - \theta)\left(\frac{\mu - c}{\rho}\right) + (1 - p)\mathbb{E}\left[(v^*(0) - Y_1)^+\right]. \tag{21}$$

with the constant $p = \frac{\rho}{\rho + \lambda} \in (0, 1]$. This equation shows that, when raising equity is costless, the bank's problem can be interpreted as a discrete-time, infinite horizon problem in which shareholders earn $(1 - \theta)\frac{\mu - c}{\rho}$ each period with probability p and otherwise face a random

liquidity shock that they can decide to pay, in which case the bank continues, or not, in which case the bank is liquidated with a zero payoff to shareholders.

To determine the effect of limited liability on the policy choices of the unconstrained bank, observe that the value of the option to default is $v^* - v_0$ where

$$v_0 = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC_t\right] = \frac{1}{\rho}\left((1-\theta)(\mu - c) - \frac{\lambda}{\beta}\right), \tag{22}$$

gives the equity value in a bank without liquid reserves under the assumption that shareholders never default. Using Eq.(20), we have that when $v_0^* \geq \ell(0)$ this option value satisfies:

$$v^*(0) - v_0 = (\lambda/\rho)\,\mathbb{E}\left[(Y_1 - v^*)^+\right]. \tag{23}$$

This leads to the following result:

Corollary 1. *In a viable bank, limited liability only has value if cash flows are subject to tail risk, i.e. only if $\lambda > 0$, for otherwise the bank would not be subject to default risk.*

Corollary 1 underscores the importance of tail risk by showing that it is the key driver of default risk in the frictionless benchmark. We show below that this is also the case when incorporating financing frictions in the model. Another important result is that in the frictionless benchmark the unconstrained equity value of a bank is strictly positive for all $c < \mu$, irrespective of the frequency and magnitude of jumps. This implies that it may be optimal for the bank to operate even if the present value v_0 of its future cash flows is negative. That is, the option to default associated with limited liability may lead shareholders to invest in risky assets that would otherwise have negative present value.

Lastly, note that the focus of Proposition 1 and Corollary 1 is on shareholders' decision to default following a tail risk event. Because v_0^* decreases with financial leverage, Proposition 1 implies that shareholders may default after a realization of tail risk even if it may be optimal to continue operating the bank's assets from a firm value maximization perspective. In line with Proposition 1, our focus below will be on the default decision of shareholders, rather than on the abandonment decision maximizing firm value, and on the effects of micro-

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prudential regulation on bank default risk (i.e. on shareholders' default decision), assuming that the role of regulation is not to prolong the life of uneconomical banks.

3.2. Equity value and bank policies with no refinancing

Having determined the value of equity when there are no costs of raising funds, we now turn to the solution of the auxiliary problem in which the bank has no access to outside funds. In this case, shareholders cannot refinance, have to default if the bank runs out of liquid reserves, and can only optimize equity value over the bank's payout policy.

Let $\alpha \geq 0$ be a constant and consider a financially constrained bank whose assets produce after-tax net cash flows given by dC_t as long as it is in operation, and a lump sum $(\alpha + s)^+$ to shareholders if liquidation occurs at a point where $S_t = s$. The optimization problem that determines the equity value of such a bank is given by

$$w(s; \alpha) = \sup_{\pi \in \Pi_0(s)} \mathbb{E}_s \left[\int_0^{\tau_\pi} e^{-\rho t} dP_t^\pi + e^{-\rho \tau_\pi} \left(\alpha + S_{\tau_\pi}^\pi \right)^+ \right], \tag{24}$$

where $\Pi_0(s)$ denotes the subset of strategies such that $R^\pi = 0$ (i.e. no refinancing) and the stopping time τ_π denotes the first time that liquid reserves become negative. This auxiliary problem is meant as a building block for the solution of shareholders' optimization problem Eq.(10) and thus should not be interpreted literally. In particular, the payoff $(\alpha + s)^+$ that shareholders receive when cash reserves become negative is not explicitly related to the bank's liability structure. We show in section 3.3. how the constant α can be endogenized to reflect the refinancing and default decisions that are optimal for shareholders. Importantly, for this choice of the constant α the payoff that shareholders receive when cash reserves become negative is consistent with the liquidation function specified in Eq.(11) and thus rules out any violation of absolute priority rules.

Following the literature on optimal dividend policies (see Albrecher and Thonhauser (2009) for a survey), it is natural to expect that the optimal payout strategy for shareholders in the auxiliary problem Eq.(24) should be of barrier type. Specifically, we conjecture that

for any given α, there exists a constant barrier $b^*(\alpha) \geq 0$ such that the optimal policy of the auxiliary bank consists in paying dividends to maintain liquid reserves at or below $b^*(\alpha)$. In addition, it is natural to conjecture that if the value of α is sufficiently large, it should be optimal for shareholders to immediately liquidate and distribute all the available cash. The following result confirms this intuition.

Lemma 1. *If $\alpha \geq v_0^*$, then $b^*(\alpha) = 0$ and it is optimal for shareholders to immediately liquidate the auxiliary bank.*

Given the result in Lemma 1, we consider below the optimization problem Eq.(24) for a bank with $0 \leq \alpha < v_0^*$. To verify our conjecture on the optimal payout policy for such a bank, we start by calculating the equity value associated with a barrier strategy. Let $b > 0$ and consider the strategy π_b that consists in paying dividends to maintain liquid reserves at or below b. The cumulative payout process associated with this strategy is

$$P_t^b = 1_{\{t>0\}} \max_{0 \leq u < t} (X_u - b)^+ \tag{25}$$

where the process $X_t = s + C_t$ denotes the uncontrolled liquid reserves of the bank (i.e. assuming that there are no dividend payments), and the corresponding value is defined by

$$w(s; \alpha, b) = \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} \left(\alpha + S_{\tau_{\pi_b}}^{\pi_b} \right)^+ \right]. \tag{26}$$

Letting ζ_0 denote the first time that X_t becomes negative and using the dividend–penalty identity (see Gerber, Lin, and Yang (2006)) shows that:

$$w(s; \alpha, b) = \begin{cases} (\alpha + s)^+, & \text{for } s \leq 0, \\ \psi(s; \alpha) + \frac{W(s)}{W'(b)} (1 - \psi'(b; \alpha)), & \text{for } 0 < s \leq b, \\ s - b + w(b; \alpha, b), & \text{for } s > b, \end{cases} \tag{27}$$

where the function

$$\psi(s; \alpha) = \mathbb{E}_s \left[e^{-\rho \zeta_0} \left(\alpha + X_{\zeta_0} \right)^+ \right] \tag{28}$$

gives the present value of the cash flow that shareholders receive in liquidation if no dividends are distributed prior to default, and $W(s)$ is the ρ-scale function of the uncontrolled liquid reserves process. Closed-form expressions for these functions are provided in the Appendix.

Equation Eq.(27) shows that in the earnings retention region, the equity value associated with a barrier strategy depends on the barrier level only through

$$H(b; \alpha) = \frac{1 - \psi'(b; \alpha)}{W'(b)}. \tag{29}$$

In the Appendix, we show that for any $0 \leq \alpha < v_0^*$ there exists a unique $b^*(\alpha) > 0$ that maximizes this function over the positive real line and, relying on a verification theorem for the Hamilton-Jacobi-Bellman equation associated with problem Eq.(24), we prove that the corresponding barrier strategy is optimal not only in the class of barrier strategies but among all strategies. This leads to the following result.

Proposition 2 (Auxiliary value function). *Consider a bank with no access to outside funds that produces a cash flow $(\alpha + s)^+$ to shareholders in default. The equity value of such a bank is concave and twice continuously differentiable over $(0, \infty)$ and given by*

$$w(s; \alpha) = w(s; \alpha, b^*(\alpha)), \tag{30}$$

where $b^*(\alpha)$ is the unique solution to $H'(b^*(\alpha); \alpha) = 0$ when $\alpha < v_0^*$ and $b^*(\alpha) = 0$ otherwise. The optimal policy for shareholders in such a bank is to distribute dividends to maintain liquid reserves at or below $b^*(\alpha)$.

3.3. Equity value and optimal bank policies

Having solved the problem of the auxiliary bank, we now show how to obtain the optimal policies for problem Eq.(10) by endogenizing the constant $\alpha = w(0; \alpha)$ that gives the equity value of the auxiliary bank at the point where it runs out of liquid reserves.

Fix $b > 0$ and consider the strategy $\hat{\pi}_b$ that consists in paying dividends to maintain liquid reserves at or below b and either liquidating or raising funds back to b whenever liquid reserves become negative, depending on which is more profitable. Denote by

$$v_b(s) = \mathbb{E}_s \left[\int_0^{\tau_{\hat{\pi}_b}} e^{-\rho t} \left(dP_t^{\hat{\pi}_b} - d\Phi_t(R^{\hat{\pi}_b}) \right) + e^{-\rho \tau_{\hat{\pi}_b}} \ell \left(S_{\tau_{\hat{\pi}_b}}^{\hat{\pi}_b} \right) \right] \tag{31}$$

the corresponding equity value. By definition, this function satisfies

$$v_b(s) = \max \left\{ v_b(b) - b + s - \phi, \ell(s) \right\} = \left(\max \left\{ v_b(b) - b - \phi, \ell(0) \right\} + s \right)^+ \tag{32}$$

for all $s \leq 0$. Since the bank does not raise funds before the first time τ_{π_b} that liquid reserves are negative, we have that $R_t^{\hat{\pi}_b} = P_t^{\hat{\pi}_b} - P_t^b = 0$ for all $0 \leq t \leq \tau_{\pi_b}$. Therefore, using the above equations together with the law of iterated expectations, we get that

$$v_b(s) = \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} v_b \left(S_{\tau_{\pi_b}}^{\pi_b} \right) \right] \tag{33}$$

$$= \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} \left(\max \left\{ v_b(b) - b - \phi, \ell(0) \right\} + S_{\tau_{\pi_b}}^{\pi_b} \right)^+ \right] \tag{34}$$

$$= w(s; v_b(0), b), \tag{35}$$

where the auxiliary equity value function $w(s; \alpha, b)$ is defined as in Eq.(27). Evaluating this equation at $s = b$ and at $s = 0$ gives

$$v_b(b) = w(b; v_b(0); b) \tag{36}$$

$$v_b(0) = \max\{v_b(b) - b - \phi, \ell(0)\} = \max \left\{ w(b; v_b(0); b) - b - \phi, \ell(0) \right\} \tag{37}$$

and it follows that the equity value $\hat{\alpha}(b) = v_b(0)$ of the bank at the point where it runs out of liquid reserves solves

$$\hat{\alpha}(b) = \max\left\{w(b; \hat{\alpha}(b); b) - b - \phi, \ell(0)\right\}. \tag{38}$$

The results of section 3.2. now suggest that to obtain a candidate optimal strategy, we need to look for a dividend barrier b_0^* such that $b_0^* = b^*(\hat{\alpha}(b_0^*))$ or, equivalently, for a fixed point of the function

$$g(\alpha; \ell(0)) = \max\left\{w(b^*(\alpha); \alpha; b^*(\alpha)) - b^*(\alpha) - \phi, \ell(0)\right\}. \tag{39}$$

We show in the Appendix that this fixed point is uniquely given by $\alpha_0^* = \max\{a, \ell(0)\}$ where $a \in [0, v_0^*)$ is the unique fixed point of $g(\alpha; 0)$.

Now, let $b_0^* = b^*(\alpha_0^*)$ and consider the candidate optimal strategy $\hat{\pi}_{b_0^*}$. If the fixed cost of equity financing exceeds the threshold

$$\phi^* = \left(w\left(b^*(\ell(0)); \ell(0); b^*(\ell(0))\right) - b^*(\ell(0)) - \ell(0)\right)^+, \tag{40}$$

then $\alpha_0^* = \ell(0) \geq a$ and the candidate optimal strategy is to distribute dividends to maintain the liquid reserves of the bank at or below the target level $b_0^* = b^*(\alpha_0^*) = b^*(\ell(0))$ and to liquidate the bank the first time that liquid reserves become negative. Otherwise, $\alpha_0^* = a > \ell(0)$ and the candidate optimal strategy is to maintain liquid reserves at or below the level $b_0^* = b^*(a)$, to refinance to this level whenever liquid reserves become negative with a shortfall smaller than a, and to liquidate for larger shortfalls. In this case, a gives both the bank's equity value at the point where it runs out of liquid reserves and the maximum loss that shareholders are willing to refinance.

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The identity Eq.(35) and the definition of the constants α_0^* and b_0^* imply that the equity value associated with the candidate optimal strategy is given by

$$v_{b_0^*}(s) = w(s; \alpha_0^*, b_0^*) = w(s; \alpha_0^*). \tag{41}$$

Using the properties of the auxiliary value function derived in the previous section, we show in the Appendix that for all $s \leq 0$ this function satisfies

$$v_{b_0^*}(s) = (\alpha_0^* + s)^+ = \left(\max \left\{ f v_{b_0^*}(0), \ell(0) \right\} + s \right)^+, \tag{42}$$

where the financing operator $f v_{b_0^*}(s)$ is defined as in Eq.(14) and, relying on a verification theorem for the Hamilton-Jacobi-Bellman equation associated with the bank's optimization problem, we prove that the strategy $\hat{\pi}_{b_0^*}$ is optimal, not only in the class of barrier strategies, but among all strategies. This leads to the following result.

Proposition 3 (Equity value function). *The equity value function is concave and twice continuously differentiable over $(0, \infty)$ and given by*

$$v(s) = v_{b_0^*}(s) = w\left(s; \alpha_0^*\right), \tag{43}$$

with the constant $\alpha_0^* = \max\{a, \ell(0)\}$. The optimal payout strategy is to distribute dividends to maintain liquid reserves at or below the target level $b_0^* = b^*(\alpha_0^*)$. When $\phi < \phi^*$, the bank raises funds to move to b_0^* whenever liquid reserves become negative with a shortfall smaller than $\alpha_0^* = a$, and liquidates otherwise. When $\phi \geq \phi^*$, the bank never raises equity and defaults the first time that liquid reserves become negative.

Lemma 1 and Proposition 3 show that if $\ell(0) > v_0^*$ then $b_0^* = 0$ so that it is optimal to immediately distribute all available liquid reserves and liquidate. Otherwise, the target level of liquid reserves b_0^* is strictly positive and the bank optimally pays dividends to maintain liquid reserves at or below this level. This is illustrated by the smooth pasting and high

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contact conditions (see Dumas (1991)):

$$0 = v'(b_0^*) - 1 = v''(b_0^*), \qquad (44)$$

which show that liquid reserves are optimally reflected down at b_0^*. When liquid reserves exceed b_0^*, the bank is fully capitalized and places no premium on internal funds so that it is optimal distribute the lump sum $s - b_0^*$. The target level of liquid reserves b_0^* results from the trade-off between the cost of accessing outside liquidity and the cost of holding inside liquidity. In particular, we show in the Appendix that the target level of liquid reserves b_0^* increases with refinancing costs ϕ and decreases with the liquidation value Λ of assets.

Another feature that emerges from Proposition 3 is that when $\phi \geq \phi^*$, the bank defaults the first time that liquid reserves become negative. In this case, shareholders can obtain positive payoff in default if the liquidation value of assets net of debt payments is sufficiently high. By contrast, when $\phi < \phi^*$ the bank raises funds whenever liquid reserves become negative with a shortfall smaller than $\alpha_0^* = a > \ell(0)$ and liquidates otherwise, in which case shareholders get a zero payoff due to limited liability.

To conclude this section, note that in our model with tail risk banks are subject to default risk. Notably, we have that:

Corollary 2. *Any realization of tail risk that takes liquid reserves below the critical level*

$$s_0^* = -1_{\{a \geq \ell(0)\}} a \qquad (45)$$

results in default because the present value $v(b_0^*) - (b_0^* - s) - \phi$ of refinancing the bank is negative for all $s \leq s_0^*$.

Corollary 2 shows that our results are in sharp contrast with those of Brownian-driven inventory models, such as Peura and Keppo (2006), Bolton, Chen, and Wang (2011), or Décamps, Mariotti, Rochet, and Villeneuve (2011) among others, in which there is no default if banks are allowed to raise outside equity by paying a moderate fixed cost. Notably,

Corollary 2 demonstrates that in the presence of tail risk banks may find it optimal to default following large shocks, even if they can raise outside funds.

4. Micro-prudential bank regulation

In this section, we examine the effects of prudential regulation on bank policy choices and insolvency risk.[9] To do so, we consider that the regulator can constrain the bank to maintain liquid reserves in excess of a given regulatory level and/or can set a lower bound on the amount of equity capital that the bank must hold at all times. That is, we focus on the instruments of regulation introduced in the Third Basel Accord, namely liquidity and leverage requirements.[10]

4.1. Liquidity requirements

To study the effects of liquidity requirements, we consider a regulation that mandates the bank to hold a minimum amount of liquid reserves at all times. Because in this section the capital structure of the bank is fixed, we may assume without loss of generality that the regulatory amount of liquid reserves is fixed at some level $T \geq 0$. However, when endogenizing the capital structure of the bank in section 4.2., we assume that the regulatory amount of liquid reserves depends on the amount of bank deposits and on expected cash outflows over a given period of time, as proposed in the Basel III framework.

The value of equity in a bank subject to a minimum regulatory level of liquid reserves is defined by the optimization problem

$$v(s;T) = \sup_{\pi \in \Pi(s,T)} \mathbb{E}_s \left[\int_0^{\tau_{\pi,T}} e^{-\rho t} \left(dP_t^\pi - d\Phi_t(R^\pi) \right) + e^{-\rho \tau_{\pi,T}} \ell \left(S_{\tau_{\pi,T}}^\pi \right) \right], \tag{46}$$

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[9]Sundaresan and Wang (2016) also examine the endogenous response of banks to regulation by asking how banks will rearrange their liabilities in response to taxes, deposit insurance, and FDIC closure rules.

[10]An important issue is the effect of regulation on the size and riskiness of the loans extended by banks to individuals and corporations. This issue is beyond the scope of the present paper and left for future research.

where $\tau_{\pi,T} = \inf\{t \geq 0 : S_{t+}^\pi \leq T\}$ denotes the default time associated with the strategy π in the presence of a liquidity requirement at the level T, and $\Pi(s,T)$ denotes the set of payout and financing strategies such that

$$P_{t+}^\pi - P_t^\pi \leq S_t^\pi - T + R_{t+}^\pi - R_t^\pi, \qquad t \geq 0. \tag{47}$$

This constraint reflects the fact that the regulatory amount of liquid reserves T cannot be distributed to shareholders. It implies that the liquidity requirement effectively increases the collateral available to creditors by shifting the refinancing point up from 0 to T, and suggests that the solution to the regulated problem Eq.(46) can be obtained as a translation of the unregulated policy derived in Proposition 3.

The following Proposition derives the value of equity for a bank subject to liquidity requirements and confirms this intuition.

Proposition 4 (Liquidity requirements). *Equity value in a bank subject to a minimum regulatory level $T \geq 0$ of liquid reserves is given by:*

$$v(s;T) = \begin{cases} v(s-T;0), & \text{if } \ell(T) \leq a, \\ w(s-T;\ell(T)), & \text{otherwise,} \end{cases} \tag{48}$$

and the optimal payout strategy for shareholders is to distribute dividends to maintain liquid reserves at or below the target level b_T^* defined by:

$$b_T^* = \begin{cases} T + b_0^*, & \text{if } \ell(T) \leq a, \\ T + b^*(\ell(T)), & \text{otherwise.} \end{cases} \tag{49}$$

When $\ell(T) \leq a$, the bank raises outside equity to move to the target level b_T^* whenever liquid reserves fall below the regulatory level with a shortfall smaller than a, and liquidates otherwise. When $\ell(T) > a$, the bank never raises outside equity, and defaults as soon as liquid reserves fall below the regulatory level.

Proposition 4 shows in the presence of a liquidity requirement the nature of the optimal policy depends on the regulatory amount of liquid reserves. When the regulatory requirement is such that $\ell(T) > a$, it is never optimal for shareholders to refinance since their cash flow in liquidation exceeds the continuation value of equity. In this case, the bank distributes dividends to maintain its liquid reserves at or below the target level and liquidates as soon as they fall short of the regulatory level. By contrast, when $\ell(T) \leq a$ it is optimal for shareholders to refinance small and intermediate losses, and default only occurs following a large loss that takes liquid reserves below $T - a$.

Combining these results with the fact that b^* decreases with the liquidation value of assets (as shown in Lemma A.4 of the Appendix) and the fact that the liquidation cash flow $\ell(T)$ is non-decreasing in T shows that liquidity requirements push the default threshold of the bank up from the unregulated level s_0^* defined in Eq.(45) to the regulated level

$$s_T^* = T - 1_{\{a \geq \ell(T)\}} a \geq T - 1_{\{a \geq \ell(0)\}} a \geq -1_{\{a \geq \ell(0)\}} a = s_0^*, \tag{50}$$

and simultaneously push the optimal level of *excess* liquid reserves down from the unregulated level b_0^* to the regulated level

$$b_T^* - T = b^* \left(\min\{a, \ell(T)\} \right) \leq b^* \left(\min\{a, \ell(0)\} \right) = b_0^*. \tag{51}$$

Now, let τ_T^* denote the default time of a bank subject to a liquidity requirement at T. The increase in the default threshold and the fact liquidity requirement increases the amount of collateral available to creditors suggests that the default loss, which is defined by

$$\mathcal{L}_T^* = \left(L + D - S_{\tau_T^*} - \Lambda \right)^+, \tag{52}$$

should decrease as a result of the liquidity requirement. On the other hand, the decrease in the optimal level of *excess* liquid reserves and the increase in the default threshold imply that banks subject to liquidity requirements are generally closer to their default threshold, which suggests that the imposition of a liquidity requirement will make default events more

frequent. The following Proposition confirms these conjectures by deriving the precise effects of liquidity requirements on default risk and bank losses in default.

Proposition 5. *For any given capital structure (c, L, D), liquidity requirements increase the frequency of default events and decrease bank losses in default in that the probabilities $\mathbb{P}_{T+b}[\tau_T^* \leq h]$ and $\mathbb{P}_{T+b}[\mathcal{L}_T^* \leq k]$ are both non decreasing in T for all $(b, h, k) \in \mathbb{R}_+^3$.*

Proposition 5 shows that for any given liquidity cushion b above the minimum regulatory level T, an increase in T leads to an increase in the likelihood of default and to a decrease in bank losses in default. Thus, while liquidity requirements are effective at reducing the losses that banks can impose on their creditors, they tend to make default events more frequent.

4.2.   Endogenous leverage and leverage requirements

We now turn to the analysis of the privately optimal financing structure of the bank and the effects of regulatory constraints on this financing structure. To do so, we follow the literature on optimal financing decisions (see e.g. Leland (1994) or Duffie and Lando (2001)) and consider a bank that initially raises an amount D of deposits and issues at par a perpetual debt contract with face value L and coupon c_L. In our model, the value-maximizing level of deposits is determined by balancing the benefits of deposits, which include the liquidity premium and the tax deductibility of payments to depositors, with their costs, which include refinancing, default, and servicing costs. The optimal level of market debt is determined by balancing the tax benefits of debt with refinancing and default costs. That is, bank shareholders not only choose the optimal mix between debt and equity but also the optimal mix between deposits and market debt.[11]

When choosing capital structure, shareholders maximize the value of equity after debt has been issued plus the proceeds from the debt issues, net of the cost of providing the required capital. Assuming that the bank is subject to a setup cost $\Psi \geq \phi$ that includes the

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[11]The developments in this section are simplified by the fact that the bank does not have to worry about a depositor run. Sundaresan and Wang (2016) analyze the effects of runs on optimal bank liability structure.

cost of buying assets, we therefore consider the static optimization problem

$$\sup_{(c_L, D) \in \mathbb{R}^2_+} \left(v \left(b_T^*(c_L, D); T | c_L, D \right) + D + L - b_T^*(c_L, D) - \Psi \right), \tag{53}$$

where

$$T = T(c_L, D) \equiv \xi D + \left(c_L + c_D(D) + \frac{\lambda}{\beta} \right) \frac{\Delta}{360}, \tag{54}$$

and

$$L = \mathbb{E}_{b_T^*(c_L, D)} \left[\int_0^{\tau_T^*(c_L, D)} e^{-\rho t} c_L dt + e^{-\rho \tau_T^*(c_L, D)} \min \left\{ L, \left(S_{\tau_T^*(c_L, D)} + \Lambda - D \right)^+ \right\} \right]. \tag{55}$$

In the above expressions, $b_T^*(c_L, D)$, $\tau_T^*(c_L, D)$, and $v(s; T | c_L, D)$ denote respectively the target level of liquid reserves, the default time, and the equity value function of a bank with deposits D and coupon c_L on market debt that is subject to a liquidity requirement at level T. In accordance with the Basel III framework Eq.(54) requires the bank to hold liquidity reserves that represents at least ξ percent of its deposits and the expected cash outflows due to coupon payments or to a realization of tail risk over a period of Δ calendar days. Equation Eq.(55) requires that market debt is issued at par given that the bank starts from the target level of liquid reserves and that creditors correctly anticipate the strategy of shareholders. A closed form solution for the value of the bank's market debt as a function of its liquid reserves is derived in Appendix ??.

The leverage of the bank is measured by the ratio of its tangible equity to the book value its assets, which is referred to as the Tier 1 leverage ratio in banking regulation. Since the tangible equity of the bank equals the difference between the book values of its assets and liabilities, the Tier 1 leverage ratio is given by $1 - \delta(s)$ where

$$\delta(s) = \frac{\text{Book value of Liabilities}}{\text{Book value of Assets}} = \frac{D + L}{s + V} \tag{56}$$

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measures the debt ratio of the bank and V is the book value of assets defined in Eq.(5). When making their capital structure decisions, banks may be constrained to choose a Tier 1 leverage ratio that exceeds some fixed lower bound Ω for any level $s \geq T(c_L, D)$ of liquid reserves. Since the debt ratio is decreasing in liquid reserves, imposing such a leverage requirement is equivalent to requiring that

$$L + D \leq (1 - \Omega)(V + T(c_L, D)). \tag{57}$$

To take into account this leverage requirement, we simply append this constraint to the static optimization problem of shareholders.

The optimization problem defined by Eqs.(53) to (57) is static. However, due to the stationary nature of the bank's cash flows, it can be shown that if deposits and market debt can be repurchased at face value and if liability adjustments are subject to the same fixed cost as equity issuance, then the optimal capital structure prescribed by the solution to this static problem is also dynamically optimal (see Appendix ??). Unfortunately, the optimal amount of deposits and the optimal coupon on market debt cannot be obtained analytically. In the next section, we circumvent this difficulty by first calibrating the parameters of the model to match the observed characteristics of average U.S. banks and then solving numerically for the optimal capital structure of the bank under various regulatory scenarios.

5. Model analysis

This section illustrates the effects of regulatory constraints on bank's policy choices and the induced insolvency risk. To do so we start by calibrating the parameters of the model to match the observed characteristics of average U.S. banks.

5.1. Parameter values and implied variables

In our model, banks optimize their capital structure in response to regulatory constraints and market frictions. The four main frictions that banks face when making these choices

are corporate taxes, refinancing costs, liquidation costs, and servicing costs. Our baseline parametrization sets the corporate tax rate to $\theta = 0.225$, consistent with the estimates of De Mooij, Keen, and Orihara (2014) or Langedijk, Nicodème, Pagano, and Rossi (2014) for the marginal tax rates of banks. We investigate the effects of taxes on the liability structure of banks by considering alternative environments in which there are no corporate taxes, so that $\theta = 0$, or in which the corporate tax rate is equal to the maximum statutory tax rate for U.S. corporations, so that $\theta = 0.35$.

Liquidation costs are set to $\varphi = 20\%$, based on the estimates of James (1991) and Flannery (2011) for risky assets in banks. Boyson, Fahlenbrach, and Stulz (2016) find that the average underwriter spread for bank seasoned equity offerings between 1996 and 2007 is 5.02% (see their Table 1, Panel C). To match this estimate, we set $\phi = 1.55\%$, implying that the cost of financing represents on average 5.08% of the amount raised for an unregulated bank. Finally, we assume that the cost of deposits per dollar of face value is given by

$$\iota + k(D) = 0.012 + 0.003D^4 \tag{58}$$

which yields an average cost of deposits of 1.726% and an optimal deposits-to-debt ratio of 69.78% for an unconstrained bank, close to the estimates of Gropp and Heider (2010).

The values of the other parameters are set follows. The risk-free rate is set to $\rho = 0.035$. The mean after-tax cash flow rate is set to $(1 - \theta)\mu = 0.10$ and the after-tax cash flow volatility to $\sigma = 0.10$, based on the estimates of Sundaresan and Wang (2016). The arrival intensity and the mean size of after-tax cash flow jumps are respectively set to $\lambda = 1/4$ and $1/\beta = 0.165$, implying that on average large losses occur once every four years and represent 9.83% of the book value of the bank's risky assets. In our analysis, we also investigate the effects of asset risk on the optimal capital structure of the bank by varying σ, λ, and $1/\beta$ around their base case values.

For an unregulated bank, the regulatory amount of liquid reserves and the lower bound on the Tier 1 leverage ratio are set to $(\xi, \Delta) = (0, 0)$ and $\Omega = -\infty$. When examining the effects of liquidity requirements on the bank's policy choices and insolvency risk, we consider

that $\xi = 5\%$ of deposits and that $\Delta = 30$ implying that banks need to be able to cover 30 calendar days of cash outflows. The implementation of the Basel III framework by the U.S. federal banking regulators requires U.S. banks is to maintain their Tier 1 leverage ratio in excess of either 4%, 7% or 9% depending their asset base; see Getter (2014). In our analysis we thus consider Tier 1 leverage ratio requirements at $\Omega = 4\%$, 7%, and 9% and implement a leverage ratio requirement at $\Omega = 20\%$ to investigate the effects of the proposal by Admati and Hellwig (2013) to require bank to hold at least 20% of equity capital.

<div style="text-align:center">Insert Table 1 about here</div>

Table 1 summarizes the baseline values of the parameters and reports the corresponding values of the endogenous quantities implied by the model for an unregulated bank and for a bank with a leverage ratio constraint at 4%. In our base case environment, banks find it optimal to issue market debt on top of deposits.[12] For an unregulated bank the optimal amount of subordinated, market debt is $L^* = 0.499$ and the coupon on market debt is $c_L^* = 0.019$. The resulting credit spread on market debt is

$$(c_L^*/L^* - \rho) \times 10^4 = 33.17 \,\text{bps.} \tag{59}$$

and the default probability at a one year horizon is 30.96 basis points, consistent with the values reported by Hamilton, Munves, and Smith (2010) and Crossen and Zhang (2012) for U.S. financial firms. The average debt ratio is 93.26% for an unregulated bank, consistent with the aggregate liability structure of FDIC-insured banks reported by Sundaresan and Wang (2016, Table 3).

The target level of liquid reserves for an unregulated bank is $b_0^* = 0.190$, which represents 10.19% of total asset value. Because the liquid reserves of the bank fluctuate between the minimum regulatory level and the target level b_0^*, the debt ratio of the bank effectively

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[12]Because we assume that the bank is only financed with equity, insured deposits, and subordinated debt, the model cannot capture the rich capital structure of large banks which typically also includes liabilities such as uninsured deposits, repo agreement, contingent bonds, preferred stock. As a result, subordinated debt should be viewed in our model as a proxy for everything that is not insured deposits.

remains within a band, as in the dynamic capital structure models of Fischer, Heinkel, and Zechner (1989), Strebulaev (2007), and Morellec, Nikolov, and Schürhoff (2012) among others. Notably, the debt ratio of the unregulated bank fluctuates between 88.25% at the target level of liquid reserves and 98.26% at the point where it runs out of liquid reserves. Given this optimal capital structure, the charter value of the bank is $\alpha_0^* = a = 0.576$, implying that shareholders are willing refinance the bank up to $s_0^* = -0.576$ which represents 34.31% of the book value of the bank's risky asset. Lastly, given that deposits cost $\iota + k(D^*) = 1.726\%$ per dollar of face value, our model implies that the additional income generated by servicing depositors exceeds $c_L^*/L^* - (\iota + k(D^*)) = 2.105\%$ per dollar of deposit.

5.2. Target liquid reserves and optimal default decisions

We start our analysis by examining the determinants of liquid reserves and default decisions. To do so, Fig. 3 plots the target level of liquid reserves b_T^* and the default threshold s_T^* as functions of the intensity of large losses λ, the volatility of after-tax cash flows σ, the mean cash flow rate μ, and the liquidity requirement T, holding the financing structure of the bank fixed at the benchmark values reported in Table 1.

In the model, the target level of liquid reserves results from the trade-off between the carry cost of liquidity ρ and the cost of raising capital ϕ. Because an increase in volatility raises the likelihood of a costly equity issuance, the target level of liquid reserves increases with σ. An increase in the frequency of large losses λ, or in their expected size $1/\beta$, has two opposite effects. First, it raises the likelihood of a costly refinancing and therefore increases shareholders' incentives to build up liquid reserves. Second, it reduces the expected cash flow from operating the bank's assets and thus decreases shareholders' incentives to contribute capital. As shown by the top-left panel, the first (second) effect dominates for low (large) values of the jump intensity λ and the expected jump size $1/\beta$.

Insert Figure 3 about here

The effect of the mean cash flow rate μ on the target level of liquid reserves also results from two opposite effects. On the one hand, a higher mean cash flow rate increases the

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bank's expected cash flows and thus shareholders' incentives to contribute capital. On the other hand, it increases the bank revenues and reduces the role of liquid reserves as a buffer to absorb losses. As shown by the bottom-left panel, the first (second) effect dominates for low (high) values of μ. Lastly, consistent with Proposition 4, the bottom right panel of the figure shows that both the target level of reserves and the default threshold grow linearly with the regulatory amount of liquid reserves as long as refinancing is optimal. When the regulatory amount of liquid reserves reaches the point where $\ell(T) = a$, refinancing becomes suboptimal. As a result, the default threshold jumps up to the regulatory amount of liquid reserves and the optimal excess level of liquid reserves, $b_T^* - T$, starts to decrease until it reaches zero, at which point shareholders should immediately liquidate the bank.

Consider next the decision to default. When there is no liquidity requirement and refinancing is optimal in that $\alpha_0^* = a$, default occurs when the bank is hit with a negative shock that takes its liquid reserves below the default threshold $s_0^* = -a$. To understand how this threshold is affected by the parameters of the model observe that, in this case, the constant a can be written as the difference between the value of equity at the target, the new provision of capital, and the refinancing cost, i.e.

$$a = v(b_0^*; 0) - b_0^* - \phi. \tag{60}$$

By decreasing the value of the claim of incumbent shareholders at refinancing, an increase in the intensity of large losses λ, in their expected size $1/\beta$, or in the after-tax cash flow volatility σ leads to an increase in the default threshold. By contrast, an increase in the mean cash flow rate μ leads to an increase in the value of equity and, therefore, to a decrease in the default threshold. As shown by the bottom left panel of the figure, when μ falls below c equity becomes worthless and it is optimal to default immediately.

5.3. Optimal bank financing decisions

In our base case environment, the bank takes a high debt ratio to exploit the tax deductibility of interest payments and the liquidity premium on deposits. These benefits are balanced against default, refinancing, and servicing costs. In this section, we investigate the effects of asset volatility, tail risk, and corporate taxes on this tradeoff. Tables 2a and 2b report the book value of risky assets, the default threshold, the total debt, the debt ratio band, the deposits-to-debt ratio, the one-year default probability, the credit spread on market debt, the target level of liquid reserves expressed as a fraction of the book value of assets, and the average financing costs incurred by shareholders in different economic environments. Panel A reports the values obtained in our base case environment.

Insert Table 2a about here

Insert Table 2b about here

Panels B and D of Table 2a show that an increase in the frequency or magnitude of tail risk leads to more conservative debt levels and to higher deposits-to-debt ratios. For example, the total debt issued by the bank decreases from 1.649 to 1.540 when the arrival intensity of negative cash flow jumps increases by 10% to $\lambda = 0.275$, and from 1.649 to 1.510 when their mean size increases by 10%. Despite these more conservative financing strategies, an increase in the frequency or magnitude of tail risk events leads to an increase in the default probability. The table shows that increasing the size of losses has a larger effect than increasing their frequency. Notably, an increase of 10% in the average size of losses increases the default probability by 66.50% to 51.55 basis points whereas a similar increase in the frequency of jumps increases the default probability by 36.12% to 42.14 basis points. Panels B and D also show that when the arrival rate λ of large losses or their average size $1/\beta$ increases, subordinated debt—whose pricing reflects default risk—becomes relatively more expensive so that the optimal liability structure of banks gets tilted towards deposits. In addition, deposits become relatively more mispriced, implying an increase in the income from servicing depositors. By contrast, Panels C and E show that a decrease in the frequency

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or magnitude of tail risk decreases the cost of market debt and leads to a large increase in privately optimal debt levels and to a commensurate decrease in the deposits-to-debt ratio. With less tail risk, privately optimal (book) debt ratios can exceed 100%, implying that unconstrained banks would like to borrow against their charter value.

Volatility risk, which is measured by the parameter σ, has similar but weaker effects. An increase volatility leads to an increase in the expected frequency of refinancing operations, which in turn leads to an increase in the target level of liquid reserves and to wider variations in debt ratios. As shown by Panel F of Table 2b debt ratios decrease and the probability of default increases as volatility increases, but the magnitude is much smaller than that implied by a change in tail risk. Lastly, an increase in volatility makes market debt relatively more expensive and leads the bank to tilt its liability structure towards deposits. There again, the magnitude of the effect is smaller than that associated with a change in tail risk.

To investigate the incentives provided by tax shields Panel H of Table 2b considers an environment without corporate taxes. In this environment, it is optimal for the bank to refrain from issuing market debt whose only benefit is the tax advantage. By contrast, the bank still issues a large amount of debt in the form of deposits, with an average deposits-to-assets ratio of 42.71%. This reduction in the debt ratio implies that cash outflows are lower and, thus, makes it optimal for the bank to increase its payouts to shareholders and to decrease its target level of liquid reserves to 6.42% of total assets. Panel H also reveals that the default probability in this case is close to zero.

Panel J considers an environment in which the corporate tax rate equals the maximum statutory tax rate of 35%. In such an environment, the unregulated optimal capital structure prompts shareholders to liquidate the bank the first time that cash reserves become negative. In the table, we thus report the regulated case where $\Omega = 0$ so that shareholders are prohibited from borrowing against the charter value of the bank and refinancing becomes optimal. Due to the increased tax rate, the operating cash flows are lower and allow the bank to support much lower debt levels. The table shows that the induced adjustment in

debt levels is made via a decrease in the level of market debt, so that the ratio of deposits-to-market debt increases from 69.78% in the base case to 94.80%.

5.4. Regulation, insolvency risk, and value-at-risk

An important question for regulators is whether and how liquidity and leverage require-ments affect insolvency risk and bank losses in default. To quantify the effects of micro-prudential regulation on the probability of default and the magnitude of the default losses loss implied by shareholders' *endogenous* financing and liquidity management decisions, we need to compute the probability

$$f_T(s, y, t) = \mathbb{P}_s \left[\{\tau_T^* \leq t\} \cap \{\mathcal{L}_T^* \geq y\} \right] = \mathbb{P}_s \left[\{\tau_T^* \leq t\} \cap \{S_{\tau_T^*} \leq T - \ell(T + y)\} \right] \qquad (61)$$

that the bank defaults prior to some horizon $t \geq 0$ in a state where the default losses of its creditors exceed some amount $y \geq 0$. Unfortunately, this probability cannot be computed analytically. To circumvent this difficulty, we derive in the Appendix a closed-form solution for its Laplace transform

$$\widehat{f}_T(s, y, k) = \int_0^\infty e^{-kt} f_T(s, y, t) dt = (1/k) \, \mathbb{E}_s \left[e^{-k\tau_T^*} 1_{\left\{\mathcal{L}_T^* \geq y\right\}} \right], \qquad (62)$$

and use the Gaver-Stehfest formula (see Stehfest (1970)) to obtain a numerical approximation of the probability. We then quantify the effects of regulation by computing both the probability of default and the conditional value-at-risk (VaR)

$$\text{VaR}_T(s, q, t) = \inf \left\{ \Phi \geq 0 : \mathbb{P}_s \left[\{\mathcal{L}_T^* \geq \Phi\} \middle| \tau_T^* \leq t \right] \leq q \right\} \qquad (63)$$

$$= \inf \left\{ \Phi \geq 0 : \mathbb{P}_s \left[\{S_{\tau_T^*} \leq T - \ell(T + \Phi)\} \middle| \tau_T^* \leq t \right] \leq q \right\}. \qquad (64)$$

To facilitate the interpretation of our results, we express this VaR as a fraction of the book value V of the bank's risky assets.

Table 3 reports the total amount of debt, the default threshold, the optimal debt ratio band, the deposits to debt ratio, the one-year default probability, the one-year conditional value-at-risk at 1%, and the change in the total bank value due to regulatory requirements. Panel A considers the effects of leverage requirements. Panel B shows the combined effects of liquidity and leverage requirements.

Insert Table 3 about here

Consider first the effects of leverage requirements alone. Panel A shows that leverage requirements lead to a decrease in debt ratios and in the default threshold, leading to a drop in default probabilities. For example, imposing a minimum Tier 1 leverage ratio of 9%, which is the level imposed on large banks in the U.S. implementation of Basel III, reduces the default probability by 49.46% from 30.96 basis points to 15.65 basis points. The table also shows that leverage requirements have a much more modest impact on default losses, as a similar constraint only reduces the conditional value-at-risk by 0.60%. In effect, leverage requirements lead to an increase in the cash flows to shareholders and, hence, to an increase in their willingness to absorb losses, but they do not allow for an effective control of default losses as they have little impact on the collateral available to creditors. Another interesting effect of leverage requirements is that they lead to an increase in the deposits-to-debt ratio of banks. For example, a Tier 1 leverage requirement at 9% decreases the optimal amount of deposits by 1.60% but reduces the optimal amount of market debt by 20.76% and, thus, leads to a 74.14% increase in the deposits-to-debt ratio. This is due to the fact that deposits provide not only tax benefits but also income fee, whereas market debt only provides tax benefits. Lastly, Panel A shows that leverage requirements decrease bank value by increasing the cost of capital. For example, a Tier 1 leverage ratio requirement at 9% reduces total bank value by 0.439%.

Table 3 shows that liquidity requirements increase the cost of debt to the bank (as more debt implies tighter requirements) and, thereby, lead to a decrease in privately optimal debt levels and in the default probability. That is, while leverage requirements constrain the amount of debt that banks *can* issue, liquidity requirements limit the amount of debt that

banks *want* to issue. Comparing the first lines of Panels A and B shows that the optimal amount of debt is 3.67% lower with a liquidity requirement and that this reduction leads to a 4.60% decrease in the default probability from 30.96 basis points to 29.53 basis points. In this case however, the decrease in debt ratios essentially comes from an increase in the implicit cost of deposits to the bank which leads to a decrease in the optimal deposits-to-debt ratio. Since the income fee from deposits represents a significant fraction of the bank charter value, liquidity requirements lead to a significant decrease in bank value. For example, Table 3 shows that liquidity requirements alone reduce bank value by 2.840%. In line with Proposition 5, the table also shows that the loss in default, as measured by the conditional value-at-risk, decreases with liquidity requirements. In our base case environment, this measure of the magnitude of default losses decreases by 3.87% when imposing a minimum liquidity requirement. Another change triggered by the imposition of a liquidity requirement is that the target level of liquid reserves, which now includes both a voluntary part and a regulatory part, increase to 13.12% of total assets. The decrease in debt levels and the increase in liquid assets in turn imply that net debt, defined as $L^* + D^* - b_T^*$, decreases when the bank is subject to a liquidity requirement.

Comparing Panels A and B of Table 3 shows that when combining liquidity requirements with leverage requirements, the regulator can reduce both the likelihood of default and the magnitude of default losses. For example, imposing a minimum Tier 1 capital ratio of 9% together with a liquidity requirement of 5% of deposits and 30 calendar of cash outflows, reduces the default probability of the bank from 30.96 basis points to 21.28 basis points and simultaneously lowers its conditional one-year value-at-risk from 97.85% to 93.91% of the book value of risky assets. This shows that to effectively control both the likelihood of bank defaults and the extent of the losses that banks can impose on their creditors, it is necessary to impose both liquidity and leverage requirements, as mandated in the Basel III accord. However, Panel B also shows that given a leverage requirement, the presence of a liquidity requirement leads to an increase in the default threshold and, therefore, in the likelihood of default. For example combining a liquidity requirement with a minimum Tier 1 leverage

ratio of 9% increases the default probability by 36.00% from 15.65 basis points to 21.28 basis points. This essentially arises because the presence of a liquidity requirement increases the implicit cost of deposits and, thereby, leads to a decrease in both the charter value of the bank and the amount of losses that shareholders are willing to refinance.

Another important aspect of regulation relates to its effects on valuations. To investigate these effects the last column of Table 3 reports the changes in the present value Eq.(53) of the bank creation that are induced by different combinations of regulatory constraints. Consistent with economic intuition the table shows that regulatory constraints decrease the value of the bank and that the magnitude of the effect increases with the stringency of the constraints. The last columns of Tables 2a and 2b complement Table 3 by reporting in different economic environments the value effects of a specific combination of constraints where the bank is subject to a Tier 1 leverage ratio constraint at $\Omega = 7\%$ and to a liquidity requirement at $\xi = 5\%$ of deposits and $\Delta = 30$ calendar days of cash outflows. This table shows that, because higher tax rates push the bank towards more aggressive financing strategies, the value effects increase with the tax rate. For example, the regulatory constraint reduces bank value by 3.957% when $\theta = 0.35$ and only reduces it by 2.078% in the absence of corporate taxes. The last column of Table 2b also shows that banks with riskier assets are more impacted by the presence of the regulatory constraints.

6. Conclusion

We develop and analytically solve a dynamic model of banking in which banks face taxation, issuance costs of securities, and default costs, and are financed with equity, insured deposits, and market debt. In this model, shareholders have limited liability and the bank maximizes shareholder value by choosing its liquidity management, financing, and default decisions. Our analysis delineates how asset risk, corporate taxes, default costs, and the liquidity premium on deposits affect the bank's optimal debt-to-assets ratio, deposits-to-debt ratio, target level of liquid reserves, and default decisions. It also examines the effects of micro-prudential regulation on banks' capital structure decisions and insolvency risk.

The paper delivers three main results regarding the effects of micro-prudential banking regulation. First, we demonstrate that liquidity requirements constraining decrease the magnitude of losses in default at the cost of an increased likelihood of default. Second, we show that leverage requirements increase shareholders' willingness to absorb losses and, thus, reduce default risk. However, such requirements generally have little effect on the magnitude of losses in default. Third, we show that combining liquidity requirements with leverage requirements reduces both the likelihood of default and losses in default. For example, we find that imposing a minimum Tier 1 leverage ratio of 9% together with a liquidity requirement of 5% of deposits and 30 calendar days of cash outflows reduces the one-year default probability by 31.26% while reducing losses in default by 4.03%. Our results therefore show that to effectively control both the likelihood of bank defaults and the magnitude of bank losses in default, it is necessary to impose both liquidity and leverage requirements, as mandated in the Basel III accord.

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Fig. 1. Equity value for an unregulated bank. This figure illustrates the shape of the equity value function and the optimal strategy for an unregulated bank. The unfilled regions on each side correspond respectively to liquidation (left) and dividend payments (right) while the intermediate regions ▨ and ▭ correspond respectively to earnings retention and refinancing.

A. No refinancing: $\ell(0) > fv(0)$

B. Refinancing: $fv(0) \geq \ell(0)$
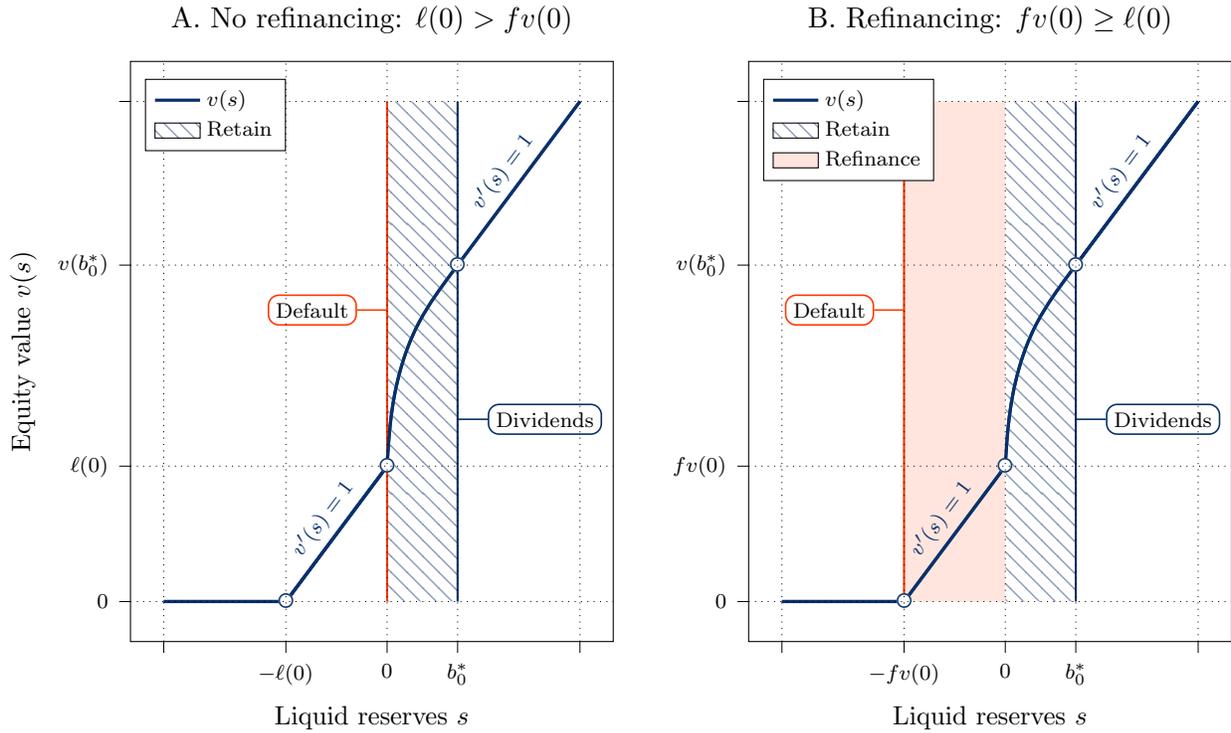
Fig. 2. Optimal policy for an unregulated bank. This figure illustrates the dynamics of the optimal policy for an unregulated bank. In both panels the stopping times $(q_n)_{n=1}^{\infty}$ indicate the occurrence of negative cash flow jumps, the vertical regions ▬ indicate the times at which dividends are paid and ● indicates where the bank is liquidated. In the bottom panel, the horizontal band ▨ represents the shortfalls that shareholders are willing to refinance and ○ indicates where the bank raises outside funds.
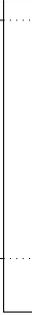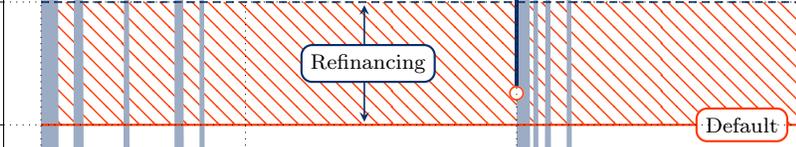
A. No refinancing: $\ell(0) > fv(0)$

B. Refinancing: $fv(0) \geq \ell(0)$
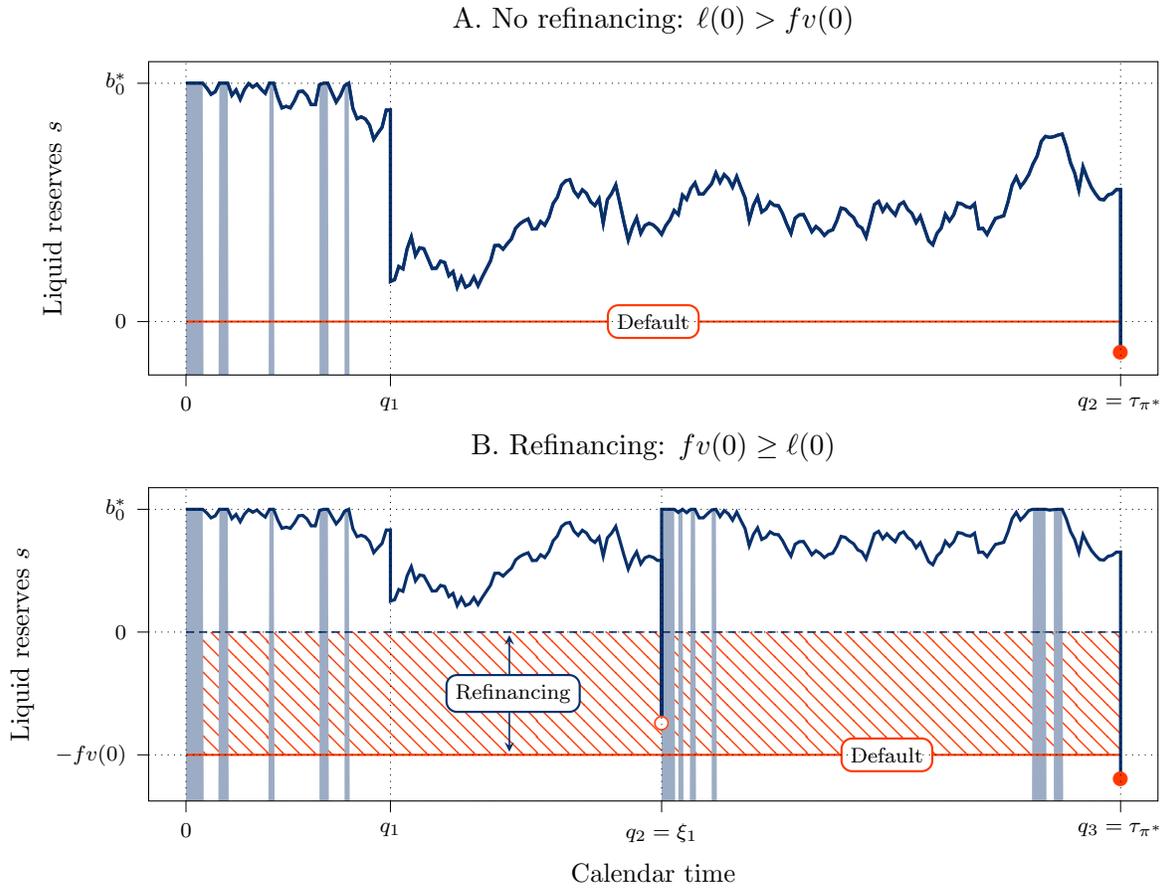
Fig. 3. Effect of the cash-flow parameters. This figure plots the target level of liquid reserves b_T^* (solid line) and the liquidation threshold s_T^* (dashed line) as functions of the jump arrival intensity, the cash flow volatility, the cash flow drift and the liquidity requirement expressed as a fraction of the bank's total debt $F^* = D^* + L^*$. In each panel the regions above the solid line and below the dashed line correspond respectively to dividend payments and liquidation while the intermediate regions ⬜ and ⬜ correspond respectively to earnings retention and refinancing.
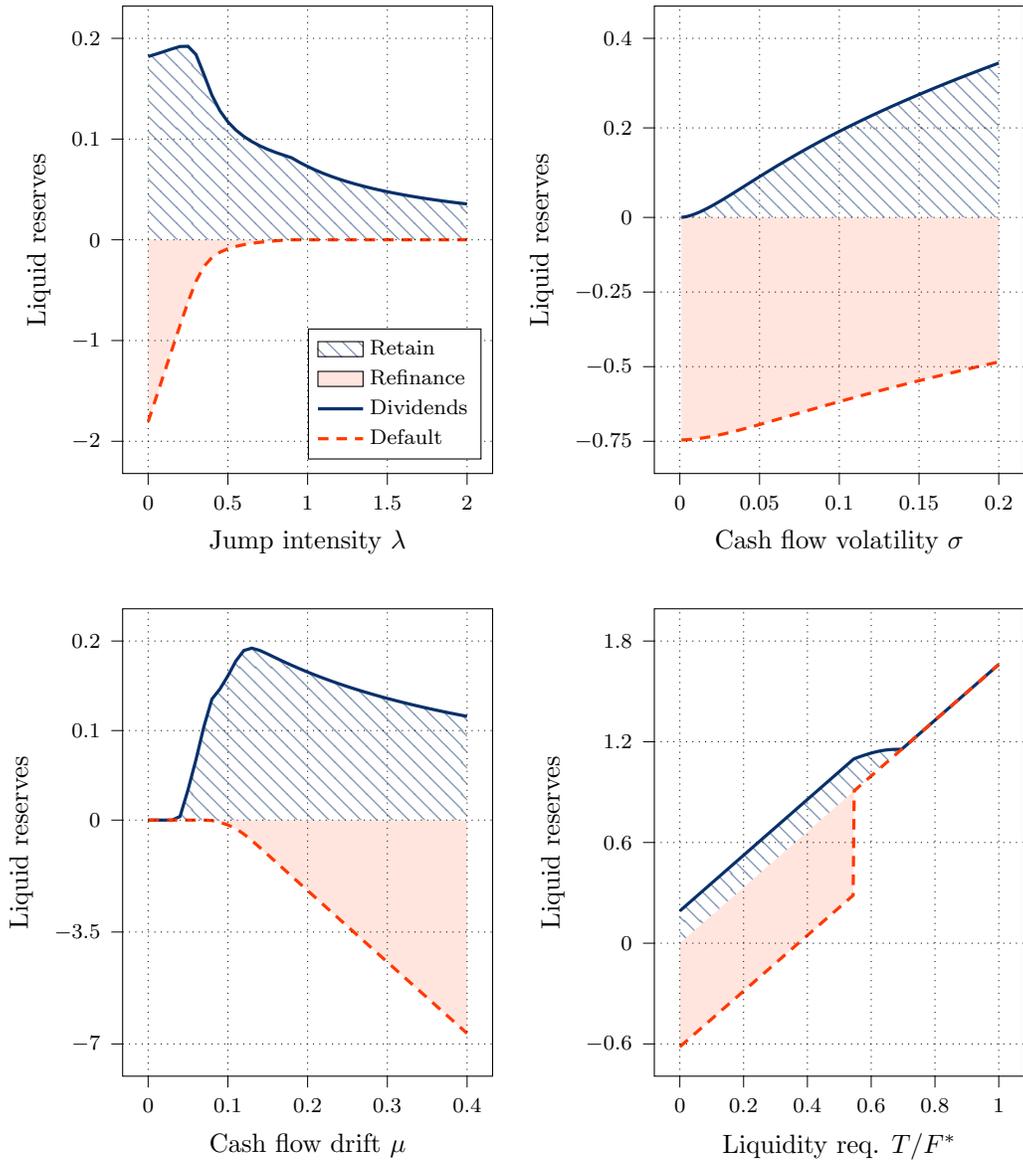
Table 1. Base case parameters and implied variables

|	Symbol	Value
A. Parameter values		
Tax rate	θ	0.225
Liquidation costs	φ	0.20
Financing cost	ϕ	0.0155
Mean cash flow rate	$(1-\theta)\mu$	0.10
Cash flow volatility	σ	0.10
Jump intensity	λ	0.25
Mean jump size	$1/\beta$	0.165
Discount rate	ρ	0.035
Liquidity requirement	(ξ, Δ)	$(0,0)$
B. Implied variables for an unregulated bank		
Book value of risky assets	V	1.679
Target level of liquid reserves	b_0^*	0.190
Optimal coupon on subordinated debt	c_L^*	0.0191
Face value of subordinated debt	L^*	0.499
Credit spread on subordinated debt (bps)		33.17
Face value of deposits	D^*	1.151
Cost of deposits (%)	$\iota + k(D^*)$	1.726
Default threshold	s_0^*	-0.576
Average financing cost (%)		5.08
Debt ratio band (%)		$[88.25, 98.26]$
Deposits-to-debt ratio (%)		69.78
1Y Default probability (bps)		30.96
C. Implied variables for a bank with a leverage requirement at $\Omega = 4\%$		
Book value of risky assets	V	1.679
Target level of liquid reserves	b_0^*	0.191
Optimal coupon on subordinated debt	c_L^*	0.0176
Face value of subordinated debt	L^*	0.468
Credit spread on subordinated debt (bps)		26.47
Face value of deposits	D^*	1.144
Cost of deposits (%)	$\iota + k(D^*)$	1.712
Default threshold	s_0^*	-0.613
Average financing cost (%)		5.03
Debt ratio band (%)		$[86.21, 96.00]$
Deposits-to-debt ratio (%)		70.96
1Y Default probability (bps)		24.73

Table 2a. Optimal bank capital structure and default risk. This table reports the book value of risky assets, the default threshold, the total amount of debt, the optimal debt ratio band, the one year default probability, the credit spread on market debt, the average financing costs as a fraction of the amount raised, the target level of liquid reserves as a fraction of the book value of assets, and the change in the net present value (53) of the bank creation due to liquidity and leverage requirements under different parametric assumptions.

Regulation (ξ, Δ, Ω)	Risky asset value V	Default threshold s_T^*	Total debt $D^* + L^*$	Debt ratio band (%)	Dep. to debt (%)	1Y Def. prob. (bps.)	Credit Sp. (bps.)	Fin. cost (%)	Target level (%)	Ch. in bank value (%)
A. Base case environment										
$(0, 0, -\infty)$	1.679	−0.576	1.649	[88.25, 98.26]	69.78	30.96	33.17	5.08	10.19	—
$(5\%, 30, 7\%)$	1.679	−0.542	1.619	[83.83, 93.00]	69.37	25.97	27.81	3.96	13.11	−2.858
B. Larger average cash-flow jumps ($1/\beta = 0.1815$)										
$(0, 0, -\infty)$	1.561	−0.560	1.522	[87.22, 97.54]	77.10	51.55	54.60	5.24	10.58	—
$(5\%, 30, 7\%)$	1.561	−0.506	1.511	[83.50, 93.00]	76.04	48.56	51.43	4.05	13.76	−3.086
C. Smaller average cash-flow jumps ($1/\beta = 0.1485$)										
$(0, 0, -\infty)$	1.796	−0.567	1.813	[91.01, 100.92]	62.76	19.72	21.46	4.98	9.82	—
$(5\%, 30, 7\%)$	1.796	−0.584	1.728	[84.16, 93.00]	63.99	11.59	12.59	3.87	12.52	−2.827
D. More frequent cash-flow jumps ($\lambda = 0.275$)										
$(0, 0, -\infty)$	1.561	−0.542	1.540	[88.05, 98.67]	75.57	42.14	45.03	5.18	10.77	—
$(5\%, 30, 7\%)$	1.561	−0.507	1.510	[83.32, 93.00]	75.11	35.38	37.79	4.02	13.90	−3.081
E. Less frequent cash-flow jumps ($\lambda = 0.225$)										
$(0, 0, -\infty)$	1.796	−0.599	1.773	[89.17, 98.71]	64.45	24.00	25.80	5.01	9.66	—
$(5\%, 30, 7\%)$	1.796	−0.580	1.729	[84.29, 93.00]	64.47	18.54	19.90	3.91	12.40	−2.695

Table 2b. Optimal bank capital structure and default risk. This table reports the book value of risky assets, the default threshold, the total amount of debt, the optimal debt ratio band, the one year default probability, the credit spread on market debt, the average financing costs as a fraction of the amount raised, the target level of liquid reserves as a fraction of the book value of assets, and the change in the net present value (53) of the bank creation due to liquidity and leverage requirements under different parametric assumptions.

Regulation (ξ, Δ, Ω)	Risky asset value V	Default thr. s_T^*	Total debt $D^* + L^*$	Debt ratio band (%)	Dep. to debt (%)	1Y Def. prob. (bps.)	Credit Sp. (bps.)	Fin. cost (%)	Target level (%)	Ch. in bank value (%)
A. Base case environment										
$(0, 0, -\infty)$	1.679	−0.576	1.649	[88.25, 98.26]	69.78	30.96	33.17	5.08	10.19	—
$(5\%, 30, 7\%)$	1.679	−0.542	1.619	[83.83, 93.00]	69.37	25.97	27.81	3.96	13.11	−2.858
F. Higher cash-flow volatility ($\sigma = 0.12$)										
$(0, 0, -\infty)$	1.679	−0.553	1.641	[86.20, 97.76]	70.17	30.91	33.72	4.44	11.82	—
$(5\%, 30, 7\%)$	1.679	−0.511	1.620	[82.35, 93.00]	69.50	27.25	29.72	3.59	14.65	−2.887
G. Lower cash-flow volatility ($\sigma = 0.08$)										
$(0, 0, -\infty)$	1.679	−0.600	1.657	[90.49, 98.74]	69.42	31.32	32.87	6.09	8.35	—
$(5\%, 30, 7\%)$	1.679	−0.573	1.619	[85.49, 93.00]	69.27	25.13	26.34	4.51	11.38	−2.833
H. No corporate taxes										
$(0, 0, -\infty)$	2.508	−1.750	1.107	[41.29, 44.13]	100	0.027	—	4.91	6.42	—
$(5\%, 30, 7\%)$	2.508	−1.714	1.084	[39.56, 42.21]	100	0.023	—	3.74	8.43	−2.078
J. Tax rate equal to maximum statutory tax rate $\theta = 0.35$										
$(0, 0, 0)$	1.218	−0.562	1.218	[86.48, 100.00]	94.80	33.68	36.09	5.10	13.52	—
$(5\%, 30, 7\%)$	1.218	−0.519	1.190	[80.95, 93.00]	94.77	30.07	32.21	4.01	17.15	−3.957

Table 3. Effect of leverage and liquidity requirements. This table reports the total amount of debt, the default threshold as defined in (50), the optimal debt ratio band, the one year default probability, the one year value at risk at 1% as a fraction of the present value of asset cash flows, and the change in the present value (53) of the bank creation implied by different leverage constraints absent liquidity requirement (Panel A) and in the presence of a liquidity requirement at $\xi = 5\%$ of deposits and $\Delta = 30$ calendar days of cash outflows (Panel B).

Min. Tier 1 lev. ratio Ω	Total debt D^*+L^*	Default threshold s_T^*	Debt ratio band (%)	Deposits to debt ratio	1Y Default prob. (bps.)	1Y1% VaR (%)	Ch. in value (%)
A. Without liquidity requirement							
$-\infty$	1.649	-0.576	[88.25, 98.26]	69.77	30.96	97.85	0.000
4%	1.611	-0.613	[86.21, 96.00]	70.96	24.70	97.77	-0.055
7%	1.561	-0.659	[83.52, 93.00]	72.78	18.70	97.51	-0.253
9%	1.528	-0.688	[81.73, 91.00]	74.14	15.65	97.26	-0.439
20%	1.343	-0.841	[71.91, 80.00]	83.46	6.23	95.36	-1.858
B. With a liquidity requirement at $(\xi, \Delta) = (5\%, 30)$							
$-\infty$	1.641	-0.521	[84.94, 94.23]	68.71	29.53	94.05	-2.840
4%	1.641	-0.521	[84.94, 94.23]	68.71	29.53	94.05	-2.840
7%	1.619	-0.542	[83.83, 93.00]	69.37	25.97	94.03	-2.858
9%	1.584	-0.575	[82.02, 91.00]	70.54	21.28	93.91	-2.951
20%	1.392	-0.740	[72.15, 80.00]	79.07	7.91	92.27	-4.178