Online supplement to

Frictional intermediation in over-the-counter markets

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C Proof of Proposition 2

We fix some $k = (k_0, k_1) \in K$, and we seek to find the solutions (m, μ, Φ) to the following system of equations:

$$m_0 = \Phi_0(x_h) - k_0 \tag{C.1}$$

$$m_1 = \Phi_1(x_h) - k_1 \tag{C.2}$$

$$m_0 + k_0 + m_1 + k_1 = m \tag{C.3}$$

$$\mu_{h0} + \mu_{h1} = \pi_h \tag{C.4}$$

$$\mu_{\ell 0} + \mu_{\ell 1} = \pi_\ell \tag{C.5}$$

$$\mu_{h1} + \mu_{\ell 1} + m_1 + k_1 = s \tag{C.6}$$

$$\gamma \pi_h \mu_{\ell 0} = \gamma \pi_\ell \mu_{h0} + \rho \mu_{h0} m_1 \tag{C.7}$$

$$\gamma \pi_\ell \mu_{h1} = \gamma \pi_h \mu_{\ell 1} + \rho \mu_{\ell 1} m_0 \tag{C.8}$$

$$\rho \mu_{\ell 1} \max \{0, \Phi_0(x) - k_0\} = \rho \mu_{h0} \min \{\Phi_1(x), m_1\} + \frac{\lambda}{m} \Phi_1(x) \left(m_0 + k_0 - \Phi_0(x)\right) \tag{C.9}$$

$$\Phi_0(x) + \Phi_1(x) = mF(x). \tag{C.10}$$

This is the same system as shown in the text, with the addition of (C.3). This equation is redundant (it can be obtained by adding up (C.1), (C.2) and using (C.10) evaluated at $x = x_h$) but will prove convenient. This system has ten equations and eight unknowns, which suggests that one more equation is redundant. Hence, our solution strategy below is to relax the system by dropping the first two equations, (C.1) and (C.2), show that the relaxed system of eight equations (C.3) through (C.10) has a unique solution, and verify that this solution satisfies the two dropped equations.

Notice as well that the system given by (C.3) through (C.10) is block diagonal. The first six equations, (C.3) through (C.8), only involve m and μ, the measures of active dealers and customers. The distributions across dealers, Φ, only appear in the last two equations, (C.9) and (C.10). Thus, we solve the system in two steps: we first solve for (m, μ) using (C.3) through (C.8), and then for Φ using (C.9) and (C.10).

C.1 Solving for (m, μ) given k

In this subsection, we fix an arbitrary $k = (k_0, k_1) \in K$ and use the first six equations, (C.3) through (C.8), to solve for (m, μ). To construct this solution, we distinguish two cases.

Assume first that k is such that $k_0 + k_1 = m$. Since (m_0, m_1) must be nonnegative, it follows from (C.3) that $m_0 = m_1 = 0$, and it is then easy to verify that the solution is

$$\mu_{\ell 0} = \pi_\ell - \mu_{\ell 1} = \pi_\ell (s - k_1), \tag{C.11a}$$

$$\mu_{h0} = \pi_h - \mu_{h1} = \pi_h (1 + m - s - k_0). \tag{C.11b}$$

Assume next that $k_0 + k_1 < m$. Substituting $\mu_{\ell 0} = \pi_\ell - \mu_{\ell 1}$, from (C.5), into (C.7) and multiplying both sides of the equation by m_0, we obtain that

$$\gamma \pi_h \pi_\ell m_0 = \gamma \pi_h \mu_{\ell 1} m_0 + \gamma \pi_\ell \mu_{h0} m_0 + \rho \mu_{h0} m_0 m_1. \tag{C.12}$$

On the other hand, subtracting (C.8) from (C.9) and using (C.4) and (C.5), we obtain that

$$\rho \mu_{h0} m_1 = \rho \mu_{\ell 1} m_0. \tag{C.13}$$

Substituting (C.13) into (C.12) and solving for μ_{h0}, we obtain the formula for μ_{h0} shown in Proposition 2. In doing so, we are using that $k_0 + k_1 < m$ which, together with (C.3), implies that either $m_1 > 0$ or $m_0 > 0$, and ensures that the denominator is not zero. If $m_0 > 0$, then the formula for $\mu_{\ell 1}$ in Proposition 2 follows from the just derived formula for μ_{h0} and from (C.13). If $m_0 = 0$, then the just-derived formula for μ_{h0} implies that $\mu_{h0} = 0$, (C.7) implies that $\mu_{\ell 0} = 0$ and so from (C.5) $\mu_{\ell 1} = \pi_\ell$, meaning that the formula for $\mu_{\ell 1}$ in Proposition 2 holds as well. Now, substituting these formulas into (C.6), and using (C.3), we obtain the following equation for m_1:

$$\begin{aligned} s = {} & \pi_h + m_1 + k_1 \\ & + \frac{\gamma \pi_h \pi_\ell \left(2m_1 + k_1 + k_0 - m\right)}{\gamma \pi_h m_1 + \gamma \pi_\ell \left(m - k_0 - k_1 - m_1\right) + \rho m_1 \left(m - k_0 - k_1 - m_1\right)} \end{aligned} \tag{C.14}$$

The derivative of the right-hand side with respect to m_1 is

$$1 + \frac{\gamma \pi_\ell \pi_h \left(\rho m_1^2 + \gamma (m - k_0 - k_1) + \rho (m - k_0 - k_1 - m_1)^2\right)}{\left(\rho m_1 (m - k_0 - k_1 - m_1) + \gamma (\pi_\ell (m - k_0 - k_1 - m_1) + \pi_h m_1)\right)^2} > 1,$$

hence the right-hand side of (C.14) is strictly increasing in m_1. Moreover at $m_1 = 0$, the right-hand side is $k_1 \leq s$ by definition of the set K, while at $m_1 = m - k_0 - k_1$ it is equal to $1 + m - k_0 > s$ by definition of the set K. Therefore, it follows from the intermediate value theorem that this equation has a unique solution, and it is now straightforward to verify that this construction leads to a solution of (C.3) through (C.8).

To complete the proof, it remains to establish continuity (which is not completely obvious at points such that $k_0 + k_1 = m$ where $m_0 = m_1 = 0$). To do so, rewrite (C.3)-(C.10) as

$$\mathbf{0} = f\left(\mu, m_0, m_1, k\right)$$

for some function $f : [0,1]^4 \times [0,m]^2 \times K \to \mathbb{R}^6$. Fix an arbitrary $k \in K$, consider a sequence $(k^n)_{n=1}^{\infty} \subset K$ converging to $k \in K$ and denote by $\left(\mu^n, m_0^n, m_1^n\right)_{n=1}^{\infty}$ such that the associated sequence of measures of customers and active dealers. Since solutions are uniformly bounded, we can extract a subsequence $\left(\mu^\alpha, m_0^\alpha, m_1^\alpha\right)_{m=1}^{\infty}$ converging to some (μ, m_0, m_1). Now, since f is clearly jointly continuous we obtain that

$$\mathbf{0}_6 = \lim_{\alpha \to \infty} f\left(\mu^\alpha, m_0^\alpha, m_1^\alpha, k^\alpha\right) = f\left(\mu, m_0, m_1, k\right).$$

But we have already shown that the system (C.3)-(C.10) has a unique solution. This means that all subsequences have the same limit, equal to the unique solution of the system given k. Therefore, the original sequence converge to that limit as well, and continuity is established.

C.2 Solving for Φ given k

We now turn to the last two equations, (C.9) and (C.10), given the tuple (m, μ) solving the first six equations (C.3) through (C.8). As stated in the main body of the text, we substitute (C.10) into (C.9), and we obtain that for each each $x \in [x_\ell, x_h]$ the measure $\phi = \Phi_1(x)$ of dealer owners with utility type below x solves

$$\begin{aligned} 0 = {} & \phi(\lambda/m)\left(m_0 + k_0 + \phi - mF(x)\right) \\ & + \rho\mu_{h0} \min\left\{\phi, m_1\right\} + \rho\mu_{\ell 1} \min\left\{\phi - mF(x) + k_0, 0\right\}. \end{aligned} \tag{C.15}$$

Existence. It is straightforward to check that the solution reported in Proposition 2 indeed solves equation (C.15).

Uniqueness. Suppose first that $\mu_{\ell 1} = 0$. In this case, it follows from (C.8) that $\mu_{h1} = 0$, and from (C.4) and (C.5) that $\mu_{h0} = \pi_h$ and $\mu_{\ell 0} = \pi_\ell$. Since $\mu_{h0} m_1 = \mu_{\ell 1} m_0$, we thus obtain that $m_1 = 0$. The market clearing equation (C.6) then implies that $k_1 = s$, and (C.3) implies that $m_0 + k_0 = m - s$. Using these results and plugging (C.10) into (C.9), we obtain that

$$\Phi_1(x) \left(m - s - mF(x) + \Phi_1(x) \right) = 0.$$

for each $x \in [x_\ell, x_h]$. Using the fact that $\Phi_1(x)$ is increasing (because it is a cumulative distribution function), continuous (because it is absolutely continuous with respect to $F(x)$), and such that $\Phi_1(x_h) = s$ we then deduce that the unique solution is

$$\Phi_1(x) = \left(s - m(1 - F(x))\right)^+.$$

Assume next that the given masses of dormant dealers are such that $\mu_{\ell 1} > 0$. In this case, we rewrite equation (C.15) as

$$0 = \phi(\lambda/m) \left(\phi - mF(x) + m_0 + k_0 \right) + \rho \mu_{h0} m_1 + \rho \mu_{h0} \min \left\{ \phi - m_1, 0 \right\} \\ + \rho \mu_{\ell 1} \left(\phi - mF(x) + k_0 \right) + \rho \mu_{\ell 1} \min \left\{ -\phi + mF(x) - k_0, 0 \right\}.$$

Using $\rho \mu_{h0} m_1 = \rho \mu_{\ell 1} m_0$ and factoring terms, we obtain the equivalent equation:

$$0 = \left(\frac{\lambda}{m} \phi + \rho \mu_{\ell 1} \right) \left(\phi - mF(x) + m_0 + k_0 \right) \\ + \rho \mu_{h0} \min \left\{ \phi - m_1, 0 \right\} + \rho \mu_{\ell 1} \min \left\{ -\phi + mF(x) - k_0, 0 \right\}. \tag{C.16}$$

Since $\mu_{\ell 1} > 0$ we have that the right hand side is strictly negative for $0 \leq \phi < (mF(x) - m_0 - k_0)^+$, and it follows that any solution must lie above that threshold. Now, the derivative of the right hand side of (C.16) with respect to ϕ is greater than:

$$\frac{\lambda}{m} \phi + \rho \mu_{\ell 1} + \frac{\lambda}{m} \left(\phi - mF(x) + m_0 + k_0 \right) - \rho \mu_{\ell 1},$$

where we took derivative of the first term and we used the fact that the derivatives of the second term and third terms are, respectively, greater than zero and $-\rho\mu_{\ell 1}$. Clearly, this lower bound is strictly positive for all $\phi > (mF(x) - m_0 - k_0)^+$. Therefore, the right hand side of (C.16) is strictly increasing in ϕ, and the existence of a unique solution now follows from an application of the intermediate value theorem.

C.3 Verifying that the dropped equations hold

We need to verify that the two equations we dropped at the beginning of this construction, (C.1) and (C.2), hold. Given (C.3) and (C.10) evaluated at x_h, this is equivalent to verifying that (C.2) holds, that is, $m_1 + k_1 = \Phi_1(x_h)$. If $k_0 = m$, then $m_1 + k_1 = 0$, and it thus follows from (C.11) and the formula of Proposition 2 that $\Phi_1(x_h) = 0$. Otherwise, it follows from (C.3) that $m_0 + k_0 + k_1 \leq mF(x_h) = x_h$, and from the formula of Proposition 2 that $\Phi_1(x) = m - k_0 - m_0 = m_1 + k_1$.

D Appendix of Section 5

D.1 Proof that $\{s, \pi_h, \gamma, m, \rho, \lambda\}$ are uniquely identified

In this section, we formally state the system of equations that we use to identify the demographic parameters $\{s, \pi_h, \gamma, m, \rho, \lambda\}$ and establish that this system admits a unique solution.

D.1.1 The system of equations

First equation. The first equation is for the supply per capita s. In the model, agents hold and trade asset "blocks" of identical size. To map the data to our model, then, we first normalize the total supply of municipal securities in circulation, A by the average size of a block, Q. We set $Q = \$206,989$, which is the average inter-dealer trade size of seasoned securities reported by Green, Hollifield, and Schürhoff (2006) for the 2000-2004 period.[1]

[1] Determining the appropriate measure of Q is non-trivial for (at least) two reasons: the average trade size of newly issued securities tends to be different than those of seasoned securities (i.e., more than 90 days after issuance); and the average size of prearranged trades tends to be different than trades in which dealers hold the asset as inventory for some period of time. For these reasons, we choose to look at seasoned securities that are traded between dealers.

Focusing on the same time period, we use data from the Flow of Funds to calculate the average par value of municipal securities held directly or indirectly by households, or held by broker dealers, which yields an estimate for A of just over \$2.3 trillion.[2] Finally, to express the number of asset blocks in per capita terms, we need to estimate the number of customers, N. We assume that half of the household population, as measured by the U.S. Census, is a potential direct or indirect participant in the municipal bond market.[3] This implies a value of $N = 54,187,500$. Using these figures, we obtain

$$s = \frac{A}{N \times Q} = 0.2058. \tag{D.1}$$

Second equation. The second equation imposes that the mass of high-valuation investor is equal to the asset supply:

$$\pi_h = s. \tag{D.2}$$

Third equation. The third equation is for turnover, which we estimate to be

$$\frac{\rho \mu_{h0} m_1}{s} = 0.411. \tag{D.3}$$

Fourth equation. The fourth equation is obtained by imposing that it takes on average 5 days for a customer to sell an asset to a dealer:

$$\rho m_0 = \frac{1}{5/250} = 50. \tag{D.4}$$

Fifth and sixith equation. For our last two equations, we first obtain an empirical estimate of the parameter $\chi = \left(\frac{\lambda m_0}{m}\right) / (\rho \mu_{h0})$. Li and Schürhoff (2018) measure that the

[2]To estimate the supply, we follow the methodology of the U.S. Securities and Exchange Commission (2012) and focus on the bonds that are either held by broker dealers, directly held by households, or indirectly held by households (via mutual, money market, closed-end, or exchange traded funds). We obtain the total from the Flow of Funds Account of the United States, Table L.211 and L.212 (see federalreserve.gov/releases/Z1). Importantly, starting with its 2011-Q3 release, the Flow of Funds adjusted up its estimate of the bonds held by households by a factor of about two, from 2005 onwards. We make the same adjustment for the 1998-2004 period.

[3]This estimate of financial market participation is motivated by data from the Survey of Consumer Finance (SCF). In particular, Bricker et al. (2017) show that, during the 2010-2016 period, about half of U.S. households had direct or indirect holding of publicly traded stocks.

average chain length is 1.34. On the other hand, Lemma 4 implies that the model-implied average chain length is:

$$\left(1+\frac{1}{\chi}\right)\log\left(1+\chi\right)=1.34.$$

It is straightforward to verify that the left-hand side is strictly increasing in χ, that it goes to 1 when $\chi \to 0$, and it goes to infinity when χ goes to infinity. Hence, the equation has a unique solution which is easily calculated numerically to be $\chi = 0.8737$. Next, we use the average inventory duration, which Li and Schürhoff (2018) measure to be equal $D = 3.3$ days. Assuming 250 trading days per year, this gives $D = 0.0132$ years. The equation is thus

$$0.0132=\frac{1}{\rho\mu_{h0}}\left(1-\frac{\chi}{2(1+\chi)}\right).$$

Using our estimate for χ, we obtain our second identification equation:

$$\rho\mu_{h0}=58.09. \tag{D.5}$$

Using the definition of χ, we obtain our third identification equation:

$$\frac{\lambda m_0}{m}=50.75. \tag{D.6}$$

D.1.2 The solution to the system of equation

Evidently, equations (D.1) and (D.5) directly pin down values for s and π_h. To identify the other parameters, we combine the identification equations (D.1) through (D.6) with the equations for a steady state distribution, stated in Section 3.3.

Consider first the market-clearing condition, $\mu_{\ell 1}+\mu_{h1}+m_1=s$. Since the distribution of preference types is stationary, we have that $\mu_{h1}=\pi_h-\mu_{h0}$. Since the inflow and outflow of assets in the dealer sector are equal, we have that $\mu_{h0}m_1=\mu_{\ell 1}m_0$. Using that the measures of active dealers add up to the total measure of dealers, we have $m_1+m_0=m$. Substituting these relationships in the market clearing condition, and using the identification equation (D.2), we obtain:

$$\mu_{h0}\frac{2m_0 - m}{m_0} + m - m_0 = 0 \Leftrightarrow \frac{\rho\mu_{h0}}{\rho m_0}(2m_0 - m) + m - m_0 = 0.$$

This implies that:

$$\frac{m_0}{m} = \frac{1 + \frac{\rho\mu_{h0}}{\rho m_0}}{1 + 2\frac{\rho\mu_{h0}}{\rho m_0}} = 0.6504,$$

where we used identification equation (D.4) and (D.5) to calculate the ratio $\frac{\rho\mu_{h0}}{\rho m_0} = 1.1619$. Combining $m_0/m = 0.6504$ with equation (D.6), we obtain:

$$\lambda = 78.03.$$

Next, we combine equations (D.1), (D.3) and (D.5) to obtain that:

$$m_1 = 0.411 \times s\frac{1}{\rho\mu_{h0}} = 0.0015.$$

This estimate of m_1 with our estimate of m_0/m, and keeping in mind that $m_0 + m_1 = m$, we obtain:

$$m = \frac{m_1}{1 - m_0/m} = 0.0042.$$

Combining $m_0 = m - m_1 = 0.0027$ with equation (D.4), we obtain

$$\rho = 18,440.$$

The last parameter is the rate γ at which customers are subject to preference shock. We obtain the value of this parameter by using the inflow-outflow equation for μ_{h0}. This gives

$$\gamma = \frac{\rho\mu_{h0}m_1m_0}{\pi_h\pi_\ell m_0 - \mu_{h0}(\pi_h m_1 + \pi_\ell m_0)}$$

which is readily calculated as $\gamma = 0.5267$ given that we now have found estimates for all the terms on the right-hand side.

D.2 Separate identification of (θ, y_ℓ)

In this section we discuss how bargaining power, θ, and customer valuation, y_ℓ, are separately identified by the average markup and liquidity yield spread.

D.2.1 Numerical calculations

It is not immediately obvious how (θ, y_ℓ) are separately identified. Indeed, one might expect that an increase in dealers' bargaining power (a higher θ) or an increase in customers' distress cost (a lower y_ℓ) would reduce measures of market liquidity, and hence increase both the liquidity yield spread and the markup at the same time. What delivers identification is the observation that the bargaining power has a greater impact on the markup than on the yield spread. This is seen most clearly in the following case: if all dealers have the same utility flow, and if the bargaining power is zero, then the markup is also zero. Yet, the yield spread is positive, because high-valuation customers who purchase the asset have to be compensated for not being able to immediately re-sell when they switch to a low flow valuation.

While we are not able to formally establish an identification result, we can check local identification numerically, as in Figure 4. The figure shows, as suggested by our intuition, that the locus of pairs (θ, y_ℓ) that match the target markup level is steeper than the locus of pairs (θ, y_ℓ) that match the observed yield spread. In other word, a given in increase in θ must be compensated by a larger increase in y_ℓ to keep the markup constant, than to keep the liquidity yield spread constant.

D.2.2 Identification in a simpler case

In the previous section we used some numerical calculations to offer an intuitive identification argument. To strengthen the case for this argument, we now study it analytically, but in the context of the simpler model of Duffie, Gârleanu, and Pedersen (2005), in which the inter-dealer market is competitive instead of frictional.

Namely, we consider the same preference structure as in our main model but assume for simplicity that dealers have identical utility flow $x = y_\ell$. Differently from the main model, we assume that the inter-dealer market is frictionless: When contacted by a high type customer non-owner a dealer can immediately locate an asset to purchase in the inter-dealer market and when contacted by a low type customer owner a dealer can

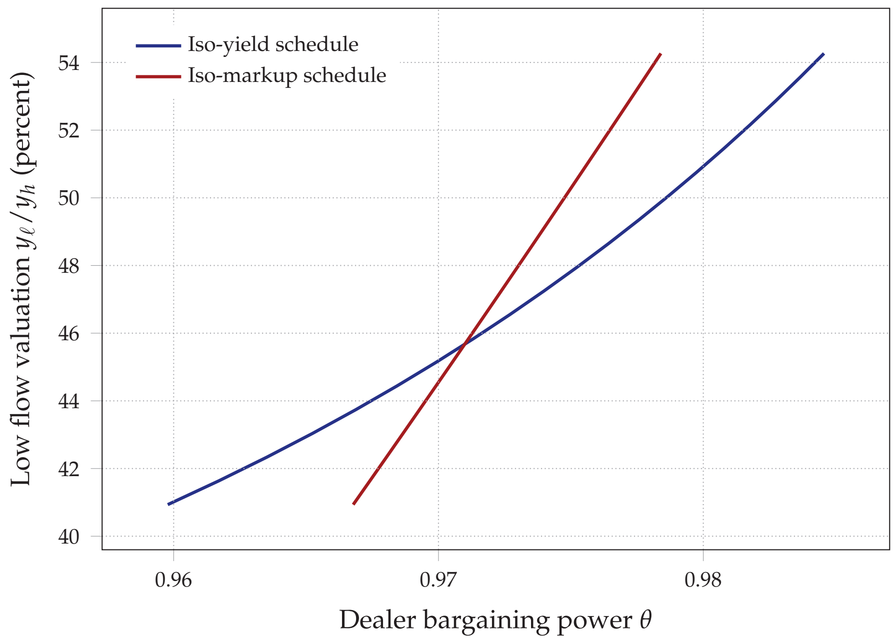

Figure 4: The iso-yield and iso-markup schedules under the assumption that all dealers have identical utility types.

immediately sell the asset in the inter-dealer market. Finally, as in Duffie, Gârleanu, and Pedersen (2005), we impose the restriction $\pi_h > s$ which implies that, in a frictionless market, high type customers are marginal.

Reservation values, and prices. We first state standard results about equilibrium. These results are easily derived, based for example on the calculations of Duffie, Gârleanu, and Pedersen (2005), or Lagos and Rocheteau (2007). First, the reservation value of high type customers can be written:

$$r\Delta W(y_h) = y_h - \gamma\pi_\ell\Sigma,$$

where

$$\Sigma \equiv \Delta W(y_h) - \Delta W(y_\ell) = \frac{y_h - y_\ell}{r + \gamma + \rho m(1-\theta)} > 0$$

is the trade surplus between a high and a low-type customer. Second, given our maintained assumption that $\pi_h > s$, the inter-dealer price is

$$P = \Delta W(y_h).$$

Third, the ask and the bid prices are:

$$\begin{aligned} A &= \theta \Delta W(y_h) + (1-\theta) P = \theta \Delta W(y_h), \\ B &= \theta \Delta W(y_\ell) + (1-\theta) P = \theta \Delta W(y_\ell) + (1-\theta) \Delta W(y_h). \end{aligned}$$

Yield spread and markup. Based on the above we obtain the following expressions for the yield spread and the markup. First, using that $P = \Delta W(y_h)$ and substituting in the formula for the reservation value of high type customers we find the yield spread $\mathbf{s} = y_h/P - r$ satisfies:

$$\frac{\mathbf{s} y_h}{r + \mathbf{s}} = \gamma \pi_\ell \Sigma. \tag{D.7}$$

Since Σ is decreasing in y_ℓ and increasing in θ, this equation defines an upward-slopping locus of pairs (θ, y_ℓ) that are consistent with the same spread level. By an application of the implicit function theorem, the slope of this locus is

$$-\left(\frac{\partial \Sigma}{\partial \theta}\right) / \left(\frac{\partial \Sigma}{\partial y_\ell}\right).$$

Using the above expressions for the bid and the ask price shows that the markup $M = A/B - 1$ satisfies

$$\frac{M y_h}{1+M} = \left(r\theta + \gamma \pi_\ell \frac{M}{1+M}\right) \Sigma. \tag{D.8}$$

This time, the equation defines an upward slopping locus of pairs (θ, y_ℓ) that consistent with the same markup level, and the slope of this locus is given by:

$$-\left(\frac{r\Sigma}{r\theta + \gamma \pi_\ell \frac{M}{1+M}} + \frac{\partial \Sigma}{\partial \theta}\right) / \left(\frac{\partial \Sigma}{\partial y_\ell}\right).$$

Keeping in mind that $\partial\Sigma/\partial\theta > 0$ and $\partial\Sigma/\partial y_\ell < 0$, one clearly sees that the iso-markup schedule has a larger slope than the iso-yield schedule, as in Figure 4. The intuition is that the yield spread depends on (θ, y_ℓ) only through the surplus: this is because the yield spread capitalizes the loss that an investor experiences when switching to low. The markup, on the other hand, depends on (θ, y_ℓ) through both the surplus Σ and through the bargaining power θ. For example, the markup can be very small because of small bargaining power, even if the surplus is large. This means that bargaining power has a stronger impact on the markup than on the yield spread, leading to the identification result.

Finally, taking the ratio of (D.8) and (D.7) we obtain that

$$\theta = \frac{M}{1+M}\frac{\gamma\pi_\ell}{\mathbf{s}}.$$

where M is the markup, $\mathbf{s}$ is the yield spread, and $\gamma\pi_\ell$ is approximately equal to turnover. Beside providing a simple formula for bargaining power as a function of observables, this formula also shows that, in the model, the yield spread cannot be too small relative to the markup because otherwise the bargaining power of dealers would exceed one.

D.3 Computations

D.3.1 Reservation values in the main model

In this section we explain how to efficiently calculate the equilibrium reservation values of all market participants under the assumption that

$$\Delta W(y_\ell) \leq \Delta V(x_\ell) < \Delta V(x_h) \leq \Delta W(y_h). \tag{D.9}$$

This assumption is straightforward to verify numerically once reservation values have been calculated, and holds in all of our calibrated examples. Assuming (D.9) we have that the reservation value of customers solve:

$$
\begin{aligned}
r\Delta W(y_\ell) &= y_\ell + \gamma\pi_h \left(\Delta W(y_h) - \Delta W(y_\ell)\right) \\
&\qquad + \rho(1-\theta) \int_{x_\ell}^{x_h} \left(\Delta V(x') - \Delta W(y_\ell)\right) d\Phi_0(x'), \\
r\Delta W(y_h) &= y_h + \gamma\pi_\ell \left(\Delta W(y_\ell) - \Delta W(y_h)\right) \\
&\qquad - \rho(1-\theta) \int_{x_\ell}^{x_h} \left(\Delta W(y_h) - \Delta V(x')\right) d\Phi_1(x').
\end{aligned}
$$

On the other hand, the reservation value function of dealers solves:

$$
\begin{aligned}
r\Delta V(x) = x &+ \rho\mu_{h0}\theta \left(\Delta W(y_h) - \Delta V(x)\right) - \rho\mu_{\ell 1}\theta \left(\Delta V(x) - \Delta W(y_\ell)\right) \\
&+ \lambda\theta_1 \int_{x}^{x_h} \left(\Delta V(x') - \Delta V(x)\right) \frac{d\Phi_0(x')}{m} \\
&- \lambda\theta_0 \int_{x_\ell}^{x} \left(\Delta V(x) - \Delta V(x')\right) \frac{d\Phi_1(x')}{m}.
\end{aligned}
$$

Since the distributions are continuous this equation implies that the reservation value function of dealers is absolutely continuous with a derivative given by

$$
\Delta V'(x) = \sigma(x) \equiv \frac{1}{r + \rho\theta\left(\mu_{h0} + \mu_{\ell 1}\right) + \frac{\lambda}{m}\left(\theta_1\left(m_0 - \Phi_0(x)\right) + \theta_0\Phi_1(x)\right)}.
$$

The derivative has a natural economic interpretation. Indeed, $\sigma(x)\,dx$ represents the "local surplus", that is, the total gains from trades between a dealer of type x and a dealer of type $x + dx$. Computationally, working with the derivative turns out to be very convenient because it can be computed before the reservation values, given only the knowledge of the distributions. Moreover, the absolute continuity of reservation values and the fundamental theorem of calculus imply that

$$
\Delta V(x) = \Delta V\left(x_\ell\right) + \int_{x_\ell}^{x} \sigma(x')dx'.
$$

This observation considerably simplifies the computations: Instead of calculating the entire function it is sufficient to calculate $\Delta V(x_\ell)$ first, and then obtain the reservation values of all other dealers by direct integration. Precisely, substituting the integral equation above for $\Delta V(x)$ into the HJB equations shows that the reservation values $\Delta W(y_\ell)$,

$\Delta W(y_h)$ and $\Delta V(x_\ell)$ solve the linear system given by

$$
\begin{aligned}
r\Delta W(y_\ell) &= y_\ell + \gamma\pi_h\left(\Delta W(y_h) - \Delta W(y_\ell)\right) && \text{(D.10a)}\\
&\quad + \rho m_0(1-\theta)\left(\Delta V(x_\ell) - \Delta W(y_\ell)\right) + \rho(1-\theta)\int_{x_\ell}^{x_h}\left(m_0 - \Phi_0(x')\right)\sigma(x')dx' \\
r\Delta W(y_h) &= y_h + \gamma\pi_\ell\left(\Delta W(y_\ell) - \Delta W(y_h)\right) && \text{(D.10b)}\\
&\quad + \rho m_1(1-\theta)\left(\Delta V(x_\ell) - \Delta W(y_h)\right) + \rho(1-\theta)\int_{x_\ell}^{x_h}\left(m_1 - \Phi_1(x')\right)\sigma(x')dx' \\
r\Delta V(x_\ell) &= x_\ell + \rho\mu_{h0}\theta\left(\Delta W(y_h) - \Delta V(x_\ell)\right) && \text{(D.10c)}\\
&\quad - \rho\mu_{\ell 1}\theta\left(\Delta V(x_\ell) - \Delta W(y_\ell)\right) + \lambda\theta_1\int_{x_\ell}^{x_h}\left(\frac{m_0 - \Phi_0(x')}{m}\right)\sigma(x')dx',
\end{aligned}
$$

where, e.g., the last equation is derived by noting that

$$\int_{x_\ell}^{x_h}\left(\Delta V(x') - \Delta V(x_\ell)\right)\frac{d\Phi_0(x')}{m} = \int_{x_\ell}^{x_h}\left(\int_{x_\ell}^{x'}\sigma(z)dz\right)\frac{d\Phi_0(x')}{m}$$

and changing the order of integration.

D.3.2 Reservation values in the extended model

Let us index each high type customer by the utility type $x \in [x_\ell, x_h]$ of the dealers it matches with. Hence, a high type customer who matches with dealers of type x derives the flow utility $y_h + \varepsilon(x)$ whenever he holds the asset. Let us assume that as in the main model the reservation value of dealers is strictly increasing and such that

$$\Delta W(y_\ell) \leq \Delta V(x_\ell) < \Delta V(x_h) \leq \Delta W(y_h, x).$$

This assumption is straightforward to verify numerically once reservation values have been calculated, and implies that the trading pattern of our model with $k_0 = k_1 = 0$ remains optimal. As a result, the equilibrium distributions solve the exact same equations as before. Only the HJB equations for the reservation values change. Specifically, the reservation value of a high type customer who matches with dealers of type x solves

$$r\Delta W(y_h, x) = y_h + \varepsilon(x) \\ + \gamma\pi_\ell \left(\Delta W(y_\ell) - \Delta W(y_h, x)\right) - \rho m_1(1-\theta)\left(\Delta W(y_h, x) - \Delta V(x)\right).$$

This equation differs from its counterpart in the main model in two ways. First, the utility flow is different, reflecting heterogeneity among high type customers. Second, the last term is different, reflecting the fact that the type-x customers only match with dealers of type x. Next, the reservation value of low-valuation customer solves:

$$r\Delta W(y_\ell) = y_\ell + \rho(1-\theta)\int_{x_\ell}^{x_h} \left(\Delta V(x') - \Delta W(y_\ell)\right) d\Phi_0(x') \\ + \gamma\pi_h \int_{x_\ell}^{x_h} \left(\Delta W(y_h, x) - \Delta W(y_\ell)\right) \varepsilon'(x) dF\left(\varepsilon(x)\right).$$

This equation differs from its counterpart in the main model in only one way. The second term is different because, upon switching to the high type, customers draw their extra utility at random according to the distribution $F(e)$. After making the change of variable $e = \varepsilon(x)$, one obtains that the average reservation value of high type customers is equal to $\int_{x_\ell}^{x_h} \Delta W(y_h, x) dF\left(\varepsilon(x)\right) \varepsilon'(x)$, which explains the formula for the second term on the right-hand side of the equation. Finally, the reservation value function of dealers solves:

$$r\Delta V(x) = y_\ell + \rho\mu_{h0}\theta\left(\Delta W(y_h, x) - \Delta V(x)\right) - \rho\mu_{\ell 1}\theta\left(\Delta V(x) - \Delta W(y_\ell)\right) \\ + \lambda\theta_1 \int_x^{x_h} \left(\Delta V(x') - \Delta V(x)\right) \frac{d\Phi_0(x')}{m} \\ - \lambda\theta_0 \int_{x_\ell}^{x} \left(\Delta V(x) - \Delta V(x')\right) \frac{d\Phi_1(x')}{m},$$

where we assumed as in the text that the utility flow of a dealer is the same as that of a low type customer (this can be relaxed, for example by assuming that the utility flow is an increasing and differentiable function of the dealer's type, x). Following the same logic as in Section D.3.1 we have that the derivatives

$$\sigma_V(x) \equiv \frac{d}{dx}\Delta V(x), \\ \sigma_W(x) \equiv \frac{\partial}{\partial x}\Delta W(y_h, x),$$

satisfy the linear system given by

$$\sigma_W(x) = \frac{\varepsilon'(x)}{r+\gamma\pi_\ell+\rho m_1(1-\theta)} + \frac{\rho m_1(1-\theta)\sigma_V(x)}{r+\gamma\pi_\ell+\rho m_1(1-\theta)},$$
$$\sigma_V(x) = \frac{\rho\mu_{h0}\theta\sigma_W(x)}{r+\rho\mu_{h0}\theta+\rho\mu_{\ell 1}\theta+\frac{\lambda}{m}\left(\theta_1\left(m_0-\Phi_0(x)\right)+\theta_0\Phi_1(x)\right)}.$$

Solving this system provides formulas for $\sigma_W(x)$ and $\sigma_V(x)$ that only depend on the equilibrium distributions, and combining these formulas with the fundamental theorem of calculus finally shows that $\Delta V(x_\ell)$, $\Delta W(y_\ell)$, and $\Delta W(y_h, x_\ell)$ solve the linear system given by

$$\begin{aligned}
r\Delta W(y_h, x_\ell) &= y_h + \varepsilon(x_\ell) + \gamma\pi_\ell\left(\Delta W(y_\ell) - \Delta W(y_h, x_\ell)\right) \\
&\quad + \rho m_1(1-\theta)\left(\Delta V(x_\ell) - \Delta W(y_h, x_\ell)\right) \\
r\Delta W(y_\ell) &= y_\ell + \gamma\pi_h\left(\Delta W(y_h, x_\ell) - \Delta W(y_\ell)\right) \\
&\quad + \gamma\pi_h \int_{x_\ell}^{x_h} \sigma_W(x)\left(1 - \frac{\Phi_1(x)}{m_1}\right) dx \\
&\quad + \rho m_0(1-\theta)\left(\Delta V(x_\ell) - \Delta W(y_\ell)\right) \\
&\quad + \rho(1-\theta)\int_{x_\ell}^{x_h} \sigma_V(x)\left(m_0 - \Phi_0(x)\right) dx \\
r\Delta V(x_\ell) &= y_\ell + \rho\mu_{h0}\theta\left(\Delta W(y_h, x_\ell) - \Delta V(x_\ell)\right) \\
&\quad - \rho\mu_{\ell 1}\theta\left(\Delta V(y_\ell) - \Delta W(x_\ell)\right) + \lambda\theta_1\int_{x_\ell}^{x_h}\sigma_V(x)\left(\frac{m_0 - \Phi_0(x)}{m}\right) dx
\end{aligned}$$

where, in the second equation, we used the assortative matching condition (5.3).

D.4 Distribution of markups

Definitions. Let $x^{(1)} \leq x^{(2)} \leq \ldots \leq x^{(k)}$ denote the utility types of successive dealers in a chain of length $\mathbf{n} = k$ and denote by $P^{(j)}$ the price at which the j^{th} dealer resells the asset. With this notation, the bid and ask prices correspond to $j = 0$ and $j = k$:

$$P^{(0)} = \text{Bid} = \theta\Delta W(y_\ell) + (1-\theta)\Delta V\left(x^{(1)}\right),$$
$$P^{(k)} = \text{Ask} = \theta\Delta W\left(y_h, x^{(k)}\right) + (1-\theta)\Delta V\left(x^{(k)}\right),$$

while the successive inter-dealer prices correspond to $j \in \{1, 2, \ldots, k-1\}$:

$$P^{(j)} = \theta_0 \Delta V \left(x^{(j)}\right) + \theta_1 \Delta V \left(x^{(j+1)}\right).$$

The total markup along the intermediation chain is then defined as:

$$M = \frac{P^{(k)} - P^{(0)}}{P^{(0)}} = \sum_{j=1}^{k} M^{(j)}$$

where

$$M^{(j)} \equiv \frac{P^{(j)} - P^{(j-1)}}{P^{(0)}},$$

is the markup of the j^{th} dealer in the chain.

The calculation. To reproduce the model-implied equivalent of (Li and Schürhoff, 2018, Table 7) we need to calculate the ratio

$$\frac{1}{E\left[M \middle| \{\mathbf{n} = k\}\right]} E\left[M^{(j)} \middle| \{\mathbf{n} = k\}\right], \tag{D.11}$$

of the expected markup of dealer j conditional on chain length to the expected total markup. This can be a complicated multidimensional integral if we integrate against the joint distribution of all types in the chain, conditional on $\mathbf{n} = k$. However, the calculation can be simplified because we have closed form solution for all the relevant marginal distributions. Specifically, since

$$E\left[M^{(j)} \middle| \{\mathbf{n} = k\}\right] = E\left[\frac{P^{(j)}}{P^{(0)}} \middle| \{\mathbf{n} = k\}\right] - E\left[\frac{P^{(j-1)}}{P^{(0)}} \middle| \{\mathbf{n} = k\}\right].$$

and the prices are convex combinations of reservation values, we have that the elementary integral needed to compute (D.11) is given by

$$E\left[\frac{\Delta V\left(x^{(j)}\right)}{\theta \Delta W(y_\ell) + (1-\theta)\Delta V\left(x^{(1)}\right)} \middle| \{\mathbf{n} = k\}\right].$$

This observation reduces the calculations to that of a several *double* integrals against the joint distribution of $x^{(1)}$ and $x^{(j)}$) conditional on $\mathbf{n} = k$ that we compute next.

The joint distribution of $x^{(1)}$ and $x^{(j)}$ conditional on $\mathbf{n} = k$. To calculate the distribution of chain length, first and j-th dealer, we integrate (17) over the other dealer types, that is over the set $(x_2, \ldots, x_{j-1})$ and $(x_{j+1}, \ldots x_k)$ such that $x_1 \leq x_2 \leq \ldots x_j$, and $x_j \leq x_{j+1} \leq x_k$. Using (B.4), we obtain:

$$\begin{aligned}
&\mathbf{P}\left(\{\mathbf{n} = k\} \cap \{x^{(1)} \in dx_1\} \cap \{x^{(j)} \in dx_j\}\right) \\
&\qquad = \frac{1}{\chi}\left(\frac{-d\lambda_1(x_1)}{\rho\mu_{h0} + \lambda_1(x_1)} \frac{\Lambda(x_1, x_j)^{j-2}}{(j-2)!}\right)\left(\frac{-d\lambda_1(x_j)}{\rho\mu_{h0} + \lambda_1(x_j)} \frac{\Lambda(x_j, x_h)^{k-j}}{(k-j)!}\right)
\end{aligned}$$

and combining this identity with (18) finally shows that the joint distribution of $x^{(1)}$ and $x^{(j)}$ conditional on $\mathbf{n} = k$ is explicitly given by

$$\begin{aligned}
&\mathbf{P}\left(\{x^{(1)} \in dx_1\} \cap \{x^{(j)} \in dx_j\} \,|\{\mathbf{n} = k\}\right) \\
&\quad = \frac{\mathbf{P}\left(\{\mathbf{n} = k\} \cap \{x^{(1)} \in dx_1\} \cap \{x^{(j)} \in dx_j\}\right)}{\mathbf{P}\left(\{\mathbf{n} = k\}\right)} \\
&\quad = \frac{k!}{\Lambda(x_\ell, x_h)^k}\left(\frac{-d\lambda_1(x_1)}{\rho\mu_{h0} + \lambda_1(x_1)} \frac{\Lambda(x_1, x_j)^{j-2}}{(j-2)!}\right)\left(\frac{-d\lambda_1(x_j)}{\rho\mu_{h0} + \lambda_1(x_j)} \frac{\Lambda(x_j, x_h)^{k-j}}{(k-j)!}\right).
\end{aligned}$$

E Auxiliary results

This section provides a proof of Lemma A.4 and gathers technical results that were used in the proofs of our main results.

Proof of Lemma A.4. Let $\mathcal{X}_0 \subseteq \mathcal{X}$ denote the set of functions $f : \{c, d\} \times \mathcal{D} \to \mathbb{R}$ that are non decreasing in utility type and such that

$$\sup_{\alpha \in \{c,d\}} \left(f(\alpha, \delta') - f(\alpha, \delta)\right) \leq \frac{\delta' - \delta}{r}, \qquad \delta \leq \delta' \in \mathcal{D}^2. \tag{E.1}$$

Because the unique solution to (A.3) satisfies (A.2), we have that $\Delta U_k \in \mathcal{X}_0$, and continuity in $\delta \in \mathcal{D}$ for each fixed $k \in K$ follows immediately. To prove continuity in k we argue as follows. Consider the operator defined by

$$P_k[f](\alpha,\delta) \equiv \frac{r}{r+a} R_k[f](\alpha,\delta) + \frac{a}{r+a} f(\alpha,a),$$

with a as in (A.3) and observe that $f = R_k[f]$ if and only if $f = P_k[f]$. The same arguments as in the proof of Lemma A.2 show that for each $k \in K$, the operator P_k satisfies Blackwell's conditions for a contraction on $\mathcal{X}$ with modulus $\frac{a}{r+a}$. Since $\mathcal{X}_0 + \mathbb{R}_+ \subseteq \mathcal{X}_0$, the only thing required to conclude that the same properties also hold on the closed subset $\mathcal{X}_0$ is to show that P_k maps $\mathcal{X}_0$ into itself. Fix an arbitrary $f \in \mathcal{X}_0$. From (A.1), we have that the evaluation $(r+a)P_k[f](\alpha,\delta) - \delta$ is increasing in $f(\alpha,\delta)$, and since the latter is increasing in δ we have that $P_k[f](\alpha,\delta)$ inherits this property. On the other hand, using (A.1) and the assumed increase of $f \in \mathcal{X}_0$ in conjunction with the fact that

$$(\max,\min)\{a,b\} - (\max,\min)\{a,c\} \leq b - c, \qquad \text{for } b \geq c$$

we deduce that

$$\begin{aligned}
&(r+a)\left(P_k[f](\alpha,\delta') - P_k[f](\alpha,\delta)\right) - (\delta' - \delta) \\
&\qquad \leq \mathbf{1}_{\{\alpha=c\}}\left(a-\gamma\right)\left(f(c,\delta') - f(c,\delta)\right) \\
&\qquad + \mathbf{1}_{\{\alpha=d\}} a \left(f(d,\delta') - f(d,\delta)\right) \leq (a/r)\left(\delta' - \delta\right)
\end{aligned}$$

for all $\delta \leq \delta'$, and it follows $P_k[f]$ satisfies (E.1). Next, we claim that the map $k \mapsto P_k[f]$ is continuous from K into $\mathcal{X}$ for any given function $f \in \mathcal{X}_0$. Indeed, using (A.1) and

$$0 = mF(\delta) - \sum_{q=0}^{1} \Phi_q(\delta,k) = mF_c(\delta) - \sum_{q=0}^{1} \mu_q(\delta,k), \qquad (\delta,k) \in \mathcal{D} \times K,$$

we deduce that for any $(\delta, k, k') \in \mathcal{D} \times K^2$, we have

$$
\begin{aligned}
&P_{k'}[f](\alpha,\delta) - P_k[f](\alpha,\delta) \qquad \text{(E.2)}\\
&\quad = \mathbf{1}_{\{\alpha=c\}} \frac{\rho(1-\theta)}{r+a} \int_{\mathcal{D}} |f(d,\delta') - f(c,\delta)| \left(d\Phi_1(\delta',k) - d\Phi_1(\delta',k')\right)\\
&\quad + \mathbf{1}_{\{\alpha=d\}} \frac{\rho\theta}{r+a} \int_{\mathcal{D}} |f(c,\delta') - f(d,\delta)| \left(d\mu_1(\delta',k) - \mu_1(\delta',k')\right)\\
&\quad + \mathbf{1}_{\{\alpha=d\}} \frac{\lambda\theta_1}{m(r+a)} \int_{\mathcal{D}} \max\{f(d,\delta'), f(d,\delta)\} \left(d\Phi_1(\delta',k) - d\Phi_1(\delta',k')\right)\\
&\quad - \mathbf{1}_{\{\alpha=d\}} \frac{\lambda\theta_0}{m(r+a)} \int_{\mathcal{D}} \min\{f(d,\delta'), f(d,\delta)\} \left(d\Phi_1(\delta',k) - d\Phi_1(\delta',k')\right).
\end{aligned}
$$

If $f \in \mathcal{X}_0$, then (E.1) and the fact that the composition of Lipschitz functions is itself Lipschitz imply that, for every fixed $\delta \in \mathcal{D}$, there are functions $(\phi_{i,\delta})_{i=1}^3$ such that

$$
\sup_{\delta' \in \mathcal{D}} |\phi_{i,\delta}(\delta')| \le 1/r \qquad \text{(E.3)}
$$

and

$$
\begin{aligned}
q_1(\delta,\delta') &\equiv |f(d,\delta') - f(c,\delta)| = q_1(\delta,\delta_h) - \int_{\delta'}^{\delta_h} \phi_{1,\delta}(x)dx,\\
q_2(\delta,\delta') &\equiv |f(c,\delta') - f(d,\delta)| = q_2(\delta,\delta_h) - \int_{\delta'}^{\delta_h} \phi_{2,\delta}(x)dx,\\
q_3(\delta,\delta') &\equiv (\theta_1 \max - \theta_0 \min)\,\{f(d,\delta'), f(d,\delta)\} = q_3(\delta,\delta_h) - \int_{\delta'}^{\delta_h} \phi_{3,\delta}(x)dx
\end{aligned}
$$

for all $\delta' \in \mathcal{D}$. Substituting these identities into (E.2) and changing the order of integration shows that, for any $(\delta, k, k') \in \mathcal{D} \times K^2$, we have

$$
\begin{aligned}
&P_{k'}[f](\alpha,\delta) - P_k[f](\alpha,\delta)\\
&\quad = \mathbf{1}_{\{\alpha=c\}} \frac{\rho(1-\theta)}{r+a} \left\{ q_1(\delta,\delta_h)\Delta\Phi_1(\delta_h,k,k') - \int_{\mathcal{D}} \phi_{1,\delta}(x)\Delta\Phi_1(x,k,k')dx \right\}\\
&\quad + \mathbf{1}_{\{\alpha=d\}} \frac{\rho\theta}{r+a} \left\{ q_2(\delta,\delta_h)\Delta\mu_1(\delta_h,k,k') - \int_{\mathcal{D}} \phi_{2,\delta}(x)\Delta\mu_1(x,k,k')dx \right\}\\
&\quad + \mathbf{1}_{\{\alpha=d\}} \frac{\lambda}{m(r+a)} \left\{ q_3(\delta,\delta_h)\Delta\Phi_1(\delta_h,k,k') - \int_{\mathcal{D}} \phi_{3,\delta}(x)\Delta\Phi_1(x,k,k')dx \right\},
\end{aligned}
$$

where

$$(\Delta\mu_1, \Delta\Phi_1)(\delta, k, k') \equiv \left(\mu_1(\delta, k) - \mu_1(\delta, k'), \Phi_1(\delta, k) - \Phi_1(\delta, k')\right)$$

denotes the changes in the distributions when moving from k' to k. It now follows from (E.3) and the boundedness of $f \in \mathcal{X}_0$ that

$$\sup_{(\alpha,\delta)} |(P_k - P_{k'})[f](\alpha, \delta)| \leq B \left(\sup_{\delta \in \mathcal{D}} |\Delta\Phi_1(\delta, k, k')| + \max_{j \in \{\ell, h\}} |\mu_{j1}(k) - \mu_{j1}(k')| \right) \tag{E.4}$$

for some $B > 0$. Since the functions $(\mu_{j1}(k))_{j=\ell}^{h}$ are continuous on K, the second term on the right hand side converges to zero when $k' \to k$. On the other hand, because the function $\Phi_1(\delta, k)$ is continuous on $\mathcal{D} \times K$ and this set is compact, we have that it is uniformly continuous on that set. Therefore, for every $\epsilon > 0$ there exists $\beta > 0$ such that

$$\|(\delta, k) - (\delta', k')\| < \beta \Longrightarrow |\Phi_1(\delta, k) - \Phi_1(\delta', k')| < \epsilon.$$

Observing that $|k - k'| < \beta$ if and only if $\|(\delta, k) - (\delta, k')\| < \beta$, we conclude that for every $\epsilon > 0$ there exists $\beta > 0$ such that

$$|k - k'| < \beta \Longrightarrow \sup_{\delta \in \mathcal{D}} |\Delta\Phi_1(\delta, k, k')| = \sup_{\delta \in \mathcal{D}} |\Phi_1(\delta, k) - \Phi_1(\delta, k')| < \epsilon.$$

This in turn implies that the first term on right hand side of (E.4) tends to zero whenever $k' \to k$ and continuity follows. Combining the above results shows that $P[k, f] \equiv P_k[f]$ is continuous in $k \in K$ for each given $f \in \mathcal{X}_0$ and such that

$$\sup_{(\alpha,\delta)} |(P[k, f] - P[k, g])(\alpha, \delta)| \leq \frac{a}{r + a} \sup_{(\alpha,\delta)} |(f - g)(\alpha, \delta)|.$$

Therefore, it follows from Lemma E.3 that $k \mapsto \Delta U_k$ is continuous from K into $\mathcal{X}$. This in turn implies that $\Delta U_k(\alpha, \delta)$ is equicontinuous in k, and the required joint continuity on $\mathcal{D} \times K$ now follows from the result of Lemma E.2. ■

Lemma E.1 *Assume that the operator $T : \mathcal{X} \to \mathcal{X}$ satisfies Blackwell's conditions and let $a \in \mathbb{R}$ be given. Then $a(G - T[G]) \geq 0$ implies that $a(G - G^*) \geq 0$ where G^* is the unique fixed point of T in $\mathcal{X}$.*

Proof. Iterating the given condition shows that $a(G - T^n[G]) \geq 0$ for all $n \geq 1$ and the result now follows from the assumption that T is a contraction. ■

Lemma E.2 *Assume that the function $f : \mathcal{D} \times K \to \mathbb{R}$ is continuous in δ for each fixed $k \in K$ and equicontinuous in k. Then it is jointly continuous in (δ, k).*

Proof. Fix a point $(\delta_0, k_0) \in \mathcal{D} \times K$ and let $\epsilon > 0$. Since $f(\delta, k)$ is continuous in δ for each fixed k, there exists a constant $\alpha > 0$ such that

$$|\delta - \delta_0| < \alpha \Longrightarrow |f(\delta, k_0) - f(\delta_0, k_0)| < \epsilon/2.$$

On the other hand, because $f(\delta, k)$ is equicontinuous in k we know that there exists a constant $\beta > 0$ such that

$$|k - k_0| < \beta \Longrightarrow \sup_{\delta \in \mathcal{D}} |f(\delta, k) - f(\delta, k_0)| < \epsilon/2$$

and the desired result now follows by combining the two estimates. ■

Lemma E.3 *Assume that the operator $\mathcal{O} : K \times \mathcal{X}_0 \to \mathcal{X}_0$ is continuous in $k \in K$ for each fixed $f \in \mathcal{X}_0$ and such that*

$$\sup_{(\alpha,\delta,k)} |(\mathcal{O}[k, f] - \mathcal{O}[k, g])(\alpha, \delta)| \leq \beta \sup_{(\alpha,\delta)} |(f - g)(\alpha, \delta)|, \qquad (f, g) \in \mathcal{X}_0^2,$$

for some $\beta < 1$. Then for each $k \in K$ there exists a unique $f_k \in \mathcal{X}_0$ such that $f_k = \mathcal{O}[k, f_k]$ and the mapping $k \mapsto f_k$ is continuous from K into $\mathcal{X}$.

Proof. Fix a point $k \in K$ and let $\epsilon > 0$ be arbitrary. Using the assumed continuity of the operator $\mathcal{O}$ we can pick a constant $\varphi > 0$ such that

$$|k - k'| < \varphi \Longrightarrow \sup_{(\alpha,\delta)} |\left(\mathcal{O}[k, f_k] - \mathcal{O}[k', f_k]\right)(\alpha, \delta)| < (1 - \beta)\epsilon$$

where $\beta < 1$ is the constant given in the statement. Combining this with the triangle inequality then shows that

$$\begin{aligned}\sup_{(\alpha,\delta)} |(f_k - f_{k'})(\alpha,\delta)| &= \sup_{(\alpha,\delta)} \left| \left(\mathcal{O}[k,f_k] - \mathcal{O}[k',f_{k'}]\right)(\alpha,\delta)\right| \\ &\le \sup_{(\alpha,\delta)} \left| \left(\mathcal{O}[k,f_k] - \mathcal{O}[k',f_k]\right)(\alpha,\delta)\right| + \sup_{(\alpha,\delta)} \left| \left(\mathcal{O}[k',f_k] - \mathcal{O}[k',f_{k'}]\right)(\alpha,\delta)\right| \\ &< (1-\beta)\epsilon + \beta \sup_{(\alpha,\delta)} \left| \left(f_k - f_{k'}\right)(\alpha,\delta)\right|\end{aligned}$$

for all $|k-k'| < \varphi$ and the desired result follows. ■

Lemma E.4 *Assume that $f : \mathcal{D} \times K \to \mathbb{R}$ is continuous and such that*

$$c \le \frac{f(\delta',k) - f(\delta,k)}{\delta' - \delta} \le C, \qquad (k,\delta,\delta') \in K \times \mathcal{D}^2, \tag{E.5}$$

for some constants $0 < c \le C$. Then there exists a unique $\hat{g} : K \to \mathbb{R}$ such that $f(\hat{g}(k),k) = 0$ for all $k \in K$ and this function is continuous.

Proof. Consider the family of functions $(\sigma_k)_{k\in K}$ defined by

$$\sigma_k(\delta) \equiv \delta - f(\delta,k)/C.$$

As is easily seen we have that $\hat{g}(k) \in \mathbb{R}$ solves $f(\hat{g}(k),k) = 0$ if and only if it is a fixed point of σ_k. Therefore, the first part will follow if we show that $\sigma_k(\delta)$ is a contraction for each fixed $k \in K$. To this end it suffices to observe that we have

$$\frac{\sigma_k(\delta') - \sigma_k(\delta)}{\delta' - \delta} = 1 - \frac{f(\delta',k) - f(\delta,k)}{C(\delta' - \delta)}$$

and therefore

$$\left|\frac{\sigma_k(\delta') - \sigma_k(\delta)}{\delta' - \delta}\right| = \left|1 - \frac{f(\delta',k) - f(\delta,k)}{C(\delta' - \delta)}\right| \le \left(1 - \frac{c}{C}\right) < 1$$

as a result of (E.5). Let now $C(K)$ denote the set of continuous functions on K and consider the operator defined by

$$\Sigma[G](k) \equiv G(k) - f(G(k),k)/C.$$

Since $f(\delta, k)$ is by assumption continuous we have that Σ maps $C(K)$ into itself. On the other hand, using (E.5) in conjunction with the same arguments as in the first part of the proof we deduce that

$$\sup_{k \in K} |\Sigma[G](k) - \Sigma[H](k)| \leq \left(1 - \frac{c}{C}\right) \sup_{k \in K} |G(k) - H(k)|$$

and it follows that Σ admits a unique fixed point $\hat{G} \in C(K)$. Since this fixed point satisfies $f(\hat{G}(k), k) = 0$ for all $k \in K$ it now follows from the uniqueness established in the first part that the function $\hat{g}(k) = \hat{G}(k)$ is continuous. ■

F Overview of Matlab routines

In this section we provide a brief description of the different matlab routines needed to solve and calibrate the main model and the extended model.

F.1 Main model

Main routine.

- `MainModel_main.m` : calculate an equilibrium given a user-provided set of parameters.

Sub routines.

- `MainModel_backsolvedemographics.m` : this program calculate demographic parameters $(s, m, \rho, \lambda, \gamma, \pi_h)$ to match the six targets described in the text.
- `MainModel_datamoments.m` : define targets for the calibration of parameters as well as other data moments.
- `MainModel_display.m` : display the results.
- `MainModel_distribution.m` : calculate distributions.
- `MainModel_functions.m` : define functions used in various calculations.

- MainModel_markup.m : calculate the distribution of markup along intermediation chains.
- MainModel_paramset.m : sets parameters for a one-off calculation of equilibrium.
- MainModel_price_and_welfare.m : calculate from the price distribution and welfare statistics in the main model.
- MainModel_values.m : calculate reservation values.
- MainModel_yieldspread.m : calculate the yield spread.

F.2 Extended model

Main routine.

- ExtendedModel_main.m : calculate an equilibrium given a user-provided set of parameters.

Sub routines.

- ExtendedModel_backsolvedemographics.m : this program calculate demographic parameters $(s, m, \rho, \lambda, \gamma, \pi_h)$ to match the six targets described in the text.
- ExtendedModel_datamoments.m : define targets for the calibration of parameters as well as other data moments.
- ExtendedModel_display.m : display the results.
- ExtendedModel_distribution.m : calculate distributions.
- ExtendedModel_functions.m : define functions used in various calculations.
- ExtendedModel_globalparameters.m : define global parameters for the calculations.
- ExtendedModel_markup.m : calculate the distribution of markup along intermediation chains.
- ExtendedModel_optim_objective.m : the function to be optimized over when searching for parameters.

- `ExtendedModel_paramset.m` : sets parameters for a one-off calculation of equilibrium.
- `ExtendedModel_price_and_welfare.m` : calculate from the price distribution and welfare statistics in the main model.
- `ExtendedModel_values.m` : calculate reservation values.
- `ExtendedModel_yieldspread.m` : calculate the yield spread.

References

Jesse Bricker, Lisa J. Dettling, Alice Henriques, Joanne W. Hsu, Lindsay Jacobs, Kevin B. Moore, Sarah Pack, John Sabelhaus, Jeffrey Thompson, and Richard A. Windle. Changes in u.s. family finance from 2013 to 2016: Evidences from the survey of consumer finances. *Federal Reserve Bulletin*, 2017.

Darrell Duffie, Nicolae Gârleanu, Nicolae, and Lasse H. Pedersen. Over-the-Counter Markets. *Econometrica*, 73(6):1815–1847, 2005.

Richard Green, Burton Hollifield, and Norman Schürhoff. Financial intermediation and the costs of trading in an opaque market. *Review of Financial Studies*, 20:275–314, 2006.

Ricardo Lagos and Guillaume Rocheteau. Search in asset markets: Market structure, liquidity, and welfare. *The American Economic Review*, 97(2):198–202, 2007.

Dan Li and Norman Schürhoff. Dealer networks. *Journal of Finance*, 74:91–144, 2018.

U.S. Securities and Exchange Commission. Report on the municipal securities market. Technical report, 2012.