Incomplete information, idiosyncratic volatility and stock returns*

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July 2012

Forthcoming: *Journal of Banking and Finance*

Abstract

When investors have incomplete information, expected returns, as measured by an econometrician, deviate from those predicted by standard asset pricing models by including a term that is the product of the stock's idiosyncratic volatility and the investors' aggregated forecast errors. If investors are biased this term generates a relation between idiosyncratic volatility and expected stocks returns. Relying on forecast revisions from IBES, we construct a new variable that proxies for this term and show that it explains a significant part of the empirical relation between idiosyncratic volatility and stock returns.

Keywords: Idiosyncratic volatility, incomplete information, cross-section of stock returns.

JEL Classification. G12, D83, D92.

*We would like to thank Martin Boyer, Jerôme Detemple, Bernard Dumas, Giovanni Favara, Amit Goyal, Erwan Morellec, Paolo Porchia, Rodolfo Prieto, Marcel Rindisbacher, Mickaël Rockinger, Norman Schürhoff, Pascal Saint Amour, Dimitri Vayanos and seminar participants at HEC Montréal, VU Amsterdam, Boston University and the 2009 EFA meetings for comments and discussions on the topic of this paper. Financial support by the Swiss Finance Institute and the Swiss National Center of Competence in Research "Financial Valuation and Risk Management" (NCCR FinRisk, Project A5 and B2) is gratefully acknowledged.

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1 Introduction

According to textbook asset pricing theory, investors are only rewarded for bearing aggregate risk and, consequently, idiosyncratic volatility should not be priced in the cross section of stock returns. However, numerous recent empirical studies have documented a relation between stock returns and idiosyncratic volatility. In particular, Ang, Hodrick, Xing, and Zhang (2006), Jiang, Xu, and Yao (2007), Brockman and Yan (2008) and Guo and Savickas (2010) provide evidence of a negative relation for the US stock market, and Ang, Hodrick, Xing, and Zhang (2008) confirm that a similar relation also holds in other markets. There is however no consensus as to the direction of this effect. Indeed, Malkiel and Xu (2001), Spiegel and Wang (2005) and Fu (2005) find positive relations between idiosyncratic volatility and expected returns, while Longstaff (1989) finds a weakly negative relation.

We propose a simple model of firm valuation under incomplete information that sheds some light on the ambiguous link between idiosyncratic volatility and stock returns. Specifically, we assume that investors observe aggregate shocks as well as the cash flows of all firms but have incomplete information about idiosyncratic shocks and therefore have to estimate the growth rates of cash flows. Rather than modeling the learning mechanism at the individual level, we assume that the investors' perceptions can be summarized by a single subjective probability measure that is equivalent to the objective or "true" probability measure. Because investors behave rationally the CAPM holds under their subjective probability measure in the sense that expected returns under this measure only reflect exposure to aggregate risk. However, it fails under the objective measure as expected returns under that measure also depend on the investors' forecast errors. Indeed, the idiosyncratic shocks perceived by investors are a combination of the true idiosyncratic shocks and forecast errors that cannot be disentangled given the available information. Since idiosyncratic volatility is defined as the loading of the firm's stock returns on the perceived idiosyncratic shocks, this implies that expected returns under the objective probability measure contain an additional term that is given by the product of the firm's idiosyncratic volatility and the investors' aggregated forecast error. This additional term, which we refer to as the idiosyncratic volatility effect, is the basis for our explanation of the relation between idiosyncratic volatility and stock returns.

As explained by Timmermann (1993; 1996) and Leuwellen and Shanken (2002) among others, unconditional tests do not capture the expected returns as perceived by investors. Rather, they measure a combination of these perceived expected returns that are solely due to aggregate risk exposure, and forecast errors that are due to incomplete information. Since the weight of the forecast errors in this combination is given by the firm's idiosyncratic volatility it follows that idiosyncratic volatility can have an impact on expected returns as measured by regressions. It is important to observe that the deviation from the CAPM which is implied by our model under the objective measure is not due to a missing factor. The idiosyncratic volatility effect that we identify is generated by the investors' aggregated forecast errors and, hence, does not represent a reward for exposure to a systematic risk factor. The presence of such a component in expected returns is entirely due to incomplete information and cannot be generated by introducing additional state variables into an otherwise standard model.

If investors are unbiased in aggregated terms, that is if they consider the correct underlying model and use Bayes'rule to update their beliefs, or equivalently if there exists a representative agent with unbiased beliefs, then their aggregated forecast errors are zero on average. In this case the idiosyncratic volatility effect predicted by our model is by construction equal to zero on average and, therefore, does not affect unconditional estimates of expected returns. While it may be natural to assume that investors are Bayesian at the individual level, this assumption does not necessarily imply that the aggregation of their beliefs is itself Bayesian (see e.g. Detemple and Murthy (1997), Berrada (2006) and Jouini and Napp (2007)) and one should therefore expect that their perceptions are biased in aggregate terms. If that is indeed the case then the idiosyncratic volatility effect is different from zero on average and, hence, affects unconditional estimates of expected returns. The existence and direction of this bias, and whether or not it can help us understand the empirical relation between idiosyncratic volatility and stock returns are the main questions we address in the empirical part of the paper.

We focus our empirical investigation on two important implications of the model. First, firms with high idiosyncratic volatility should underperform when news are bad, and overperform when news are good. In the context of our model, where the growth rates of cash flows are unobserved, bad news correspond to situations where realized earning growth is smaller than expected and induce negative perceived shocks on returns through the mechanism highlighted above. Since stocks with high idiosyncratic volatility are more exposed to such shocks, they should underperform following bad news. An identical reasoning suggests that high idiosyncratic volatility stocks should overperform following good news. This implication of the model relates to the vast literature on the post-earning announcement drift, see e.g. Ball and Brown (1968), Watts (1978), Foster, Olsen, and Shevlin (1984) and Bernard and Thomas (1990). We contribute to this literature by proposing a model that not only explains the response of returns to news but also predicts a stronger effect on the return of high idiosyncratic volatility stocks.

Second, our model predicts that if there appears to be a relation between idiosyncratic volatility and stock returns in the data, then this relation should not remain significant when controlling for the idiosyncratic volatility effect. This implication of our model provides a potential explanation for the empirical results documenting a cross sectional relation between idiosyncratic volatility and stock returns. Note that while our model predicts the existence of such a relation it is silent about its direction and, therefore, can be consistent with both a negative relation (e.g. Ang et al. (2006; 2008), Jiang et al. (2007) and Brockman and Yan (2008)) and a positive relation (e.g. Malkiel and Xu (2001), Spiegel and Wang (2005) and Fu (2005)).

To test the predictions of our model, the first step is to construct a proxy for the idiosyncratic volatility effect. Since this effect is defined as the product of a stock's idiosyncratic volatility and the investors' aggregated forecast errors, we need proxies for both quantities. Following standard practice we measure a stock's idiosyncratic volatility in a given month by the standard deviation of the residuals from the 3 factor model of Fama and French (1993) run at a daily frequency. To approximate the investors' aggregated forecast errors about the growth rates of cash flows we use the average of analyst forecast revisions for end-of-year earning growth obtained from the I/B/E/S database and standardize this measure to obtain comparable quantities across firms. Our proxy for the idiosyncratic volatility effect is computed for each firm at a monthly frequency using all analyst forecasts from January 1982 to December 2007.

When we split the sample in good and bad news groups, the first implication of the model is remarkably well verified. Indeed, we find that portfolios of high and low idiosyncratic volatility stocks behave very differently following good and bad news and that the risk-adjusted effect goes in the direction predicted by the model. In particular, portfolios of high idiosyncratic volatility stocks have a significant and largely positive alpha after good news and a significant and largely negative alpha after bad news. Furthermore, the magnitude of the average idiosyncratic volatility effect for ten idiosyncratic volatility-sorted portfolios is very close to the magnitude of the alphas. In particular, the difference between alpha and our proxy for the idiosyncratic volatility effect is not statistically significant for eighteen out of the twenty portfolios we construct.

In the split sample, the evidence is mixed relative to the second implication of the model. Controlling for the idiosyncratic volatility effect reduces the magnitude and statistical significance of the alphas of the decile portfolios but fails to completely explain the cross-sectional relation between idiosyncratic volatility and stock returns. As this may be due to the presence of outliers we repeat the same regressions after applying a monthly filter that eliminates 1% of most extreme idiosyncratic volatility effects as well as those firms which are followed by less than five analysts in the given month. The results for the filtered sample are more in line with the predictions of the model. In particular, controlling for the idiosyncratic volatility effect now makes the alpha insignificant on

5 of the decile portfolios in the bad news group. Unfortunately, controlling for the idiosyncratic volatility effect still has a marginal impact on the alphas in the good news group and we therefore cannot conclude that the second implication of the model holds in the split sample. These results are confirmed by a detailed analysis of the returns on a portfolio that is long in high idiosyncratic volatility stocks and short in low idiosyncratic volatility stocks. In particular, we show that controlling for the idiosyncratic volatility effect completely eliminates the alpha of the long/short portfolio in the bad news group but has little effect on that of the good news group.

To investigate the validity of the model's second implication we perform a number of tests on the whole sample. Specifically, we follow Ang et al. (2006; 2008) in constructing ten portfolios sorted on the previous month's idiosyncratic volatility and compare the alphas of these portfolios to the idiosyncratic volatility effects implied by the model. As in Ang et al. (2006) we find a negative relation between idiosyncratic volatility and stock returns in the whole sample. In particular, a portfolio that is long in high idiosyncratic volatility stocks and short in low idiosyncratic volatility stocks produces a negative and significant alpha of −66 basis points per month. When comparing the alphas of the decile portfolios to the predictions of our model we find that the idiosyncratic volatility effect decreases as idiosyncratic volatility increases, and that its magnitude explains about half of the negative abnormal risk-adjusted return on the long/short portfolio. To confirm this finding we conduct a regression analysis which shows that controlling for the idiosyncratic volatility effect eliminates the alpha of the long/short portfolio. Remarkably, the estimated coefficient on the idiosyncratic volatility effect is not significantly different from the value predicted by the model.

In summary, we find significant empirical support for the predictions of our model. In particular, the idiosyncratic volatility effect that we identify explains the different behavior of high and low idiosyncratic volatility stocks following good and bad news, and can account for a sizable part of the cross-sectional relation between idiosyncratic volatility and stock returns. To ascertain the robustness of these empirical findings, we repeat the statistical tests with alternative proxies for the idiosyncratic volatility effect as well as alternative sampling frequencies and show that the results remain qualitatively unchanged. In the final section of the paper, we also perform further tests to successfully verify that our explanation of the cross-sectional relation between idiosyncratic volatility and stock returns is different from both the dispersion of analyst forecasts effect of Diether, Malloy, and Scherbina (2002), and the return reversal effect of Huang, Liu, Rhee, and Zhang (2010).

This paper belongs to the growing literature that relies on incomplete information models to explain properties of asset returns and/or corporate policies. For example, Pastor and Veronesi (2003) show that the need to learn about firms' profitability explains the seemingly high valuation of young firms; Brennan and Xia (2001) show that incomplete information can generate excess volatility; Alti (2003) shows that incomplete information can explain the investment/cash flow sensitivity puzzle even in the absence of financing frictions; and Décamps, Mariotti, and Villeneuve (2005) and Grenadier and Malenko (2010) study the impact of incomplete information on the exercise of real options and show that it can help to explain some features of the investment policies observed in practice. An extensive and up-to-date survey of the applications of incomplete information and learning in finance can be found in Pastor and Veronesi (2009). The papers that are perhaps the most closely related to ours are Timmermann (1993; 1996) and Leuwellen and Shanken (2002) who show that, when investors must learn about future expected cash-flows, empirical tests can find patterns in the data that significantly differ from those perceived by investors. We add to the contribution of these papers by showing that such a mechanism can create a relation between idiosyncratic volatility and stock returns even if only systematic risk is priced in the market, and by providing a way to empirically measure the corresponding idiosyncratic volatility effect.

The remainder of the paper is organized as follows. In Section 2 we present the valuation model. In Section 3 we identify the idiosyncratic volatility effect and present testable implications. In Section 4 we present the data used in our tests, discuss the construction of our proxy for the idiosyncratic volatility effect and present our empirical results. We conclude in Section 5. The appendix contains the proofs omitted from the main text and provides the details on the computation of firm values.

2 The model

In this section we build a simple continuous-time model of firm valuation under incomplete information. As in Berk, Green, and Naik (1999), we work in a partial equilibrium setting in the sense that we take the pricing kernel as given. This gives us the tractability we need in order to focus on the relation between idiosyncratic volatility, incomplete information and stock returns.

2.1 Information structure

We consider an economy in which many firms are active. The instantaneous cash flow of firm i evolves according to

$$dX_{it} = X_{it}\theta_{it}dt + X_{it}\sigma_{ia}^{\top}dB_{at} + X_{it}\sigma_{ii}dW_{it}$$

where the process $B_{at} \in \mathbb{R}^n$ is a Brownian motion that affects all firms, W_{it} is a firm specific Brownian motion that is independent from both aggregate shocks and the specific shock of firm $j \neq i$, and the constants $\sigma_{ia} \in \mathbb{R}^n$ and $\sigma_{ii} \neq 0$ represent the sensitivity of the firm's cash flows to aggregate and idiosyncratic shocks. The growth rate of the firm's cash flows, θ_{it}, is a stochastic process but its precise dynamic does not affect our results and therefore is left unspecified.

A key feature of our model is that investors have incomplete information about the growth rates of the firms' cash flows. More precisely, we assume that investors observe the aggregate shock B_a as well as the cash flows of all firms X_i but do not observe the firm specific shocks W_i and therefore have to estimate the growth rates θ_i before they can value firms. We denote $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$ the information set available to investors. At time t, it contains the realization of all cash flows X_{is} for $s \leq t$, the aggregate shock B_{as} for $s \leq t$, the constant vectors σ_{ia} for all i and the constants σ_{ii} for all i.

Instead of modeling the investors' individual perceptions, we assume that their views can be summarized by a probability measure P_s that is equivalent to the objective probability measure P_o, and under which the cash flow evolves according to

$$dX_{it} = X_{it} m_{it} dt + X_{it} \sigma_{ia}^{\top} dB_{at} + X_{it} \sigma_{ii} dB_{it}. \tag{1}$$

In this equation, the process m_{it} represents the investors' perception of the growth rate of the cash flows of firm i, and the process

$$B_{it} = W_{it} + \int_0^t \sigma_{ii}^{-1} \left(\theta_{i\tau} - m_{i\tau} \right) d\tau = \int_0^t \sigma_{ii}^{-1} (dX_{i\tau}/X_{i\tau} - \sigma_{ia}^{\top} dB_{a\tau} - m_{i\tau} d\tau) \tag{2}$$

is a Brownian motion under the equivalent probability measure P_s with respect to the information set $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$ available to investors. As was the case for the growth rates, the precise dynamics of the investors' perceived growth rates are not crucial for the purpose of this paper and therefore are left unspecified.

If investors are in aggregated terms Bayesian, or equivalently if there exists a representative agent with Bayesian beliefs, then m_{it} is an unbiased estimator of the true growth rate and its dynamics can be obtained by application of standard filtering results, see Liptser and Shiryaev (2001). While it may be natural to assume that investors are Bayesian at the individual level, this assumption does not necessarily imply that the aggregation of their beliefs is itself Bayesian. In particular, Detemple and Murthy (1997), Berrada (2006) and Jouini and Napp (2007) among others have shown that in standard equilibrium models with heterogenous Bayesian investors the beliefs of the social planner who determines prices are not Bayesian except in very special cases. This implies that one should expect the aggregate perception, m_{it}, to be biased in the sense that $m_{it} \neq \hat{\theta}_{it} \equiv E_o[\theta_{it}|\mathcal{F}_t]$ and we show in the next sections that such a bias is key in generating a relation between idiosyncratic volatility and stock returns.

2.2 Firm valuation

To compute stock prices we assume that any security can be valued by discounting its future cash flows using a stochastic discount factor that evolves according to

$$d\xi_t = -r_t \xi_t dt - \xi_t \kappa_t^\top dB_{at}, \qquad \xi_0 = 1. \tag{3}$$

In this equation, the processes r_t and $\kappa_t \in \mathbb{R}^n$ are measurable with respect to the information set $\mathbb{F}$ available to investors and represent, respectively, the risk free rate and the market prices associated with each of the aggregate sources of risk.

This specification is quite natural in the context of our model. In particular, only the aggregate shocks which are common to all firms are priced. Furthermore, the fact that the dynamics of the SDF only depends on observable quantities implies that investors have the same perception of what is the stochastic discount factor and therefore guarantees that they agree on the prices of traded securities.

Putting together the various pieces of the model, we can now formally define the market value of firm i at time t as

$$V_{it} = E_s\left[\int_t^{T_i} \xi_{t,\tau} X_{i\tau} d\tau \,\middle|\, \mathcal{F}_t \right] \tag{4}$$

where the subscript s denotes an expectation under the investors' subjective probability measure, $T_i \leq \infty$ denotes the lifetime of firm i and $\xi_{t,\tau} = \xi_\tau / \xi_t$ is the SDF that applies at time t to cash flows paid at time $\tau \geq t$.

3 Idiosyncratic volatility and stock returns

This section derives the relation between idiosyncratic volatility and stock returns implied by our model. We first discuss our theoretical findings in Section 3.1 and then present testable implications in Section 3.2.

3.1 Theoretical findings

Using the definition of the firm value in equation (4) together with the dynamics of the state price density and the martingale representation theorem it can be shown (see

Appendix A for details) that the value of firm i satisfies

$$dR_{it} \equiv \frac{dV_{it} + X_{it}dt}{V_{it}} = (r_t + a_{it}^\top \kappa_t)dt + a_{it}^\top dB_{at} + \iota_{it} dB_{it} \tag{5}$$

for some observed processes $a_{it} \in \mathbb{R}^n$ and ι_{it} that represent the firm's systematic and idiosyncratic volatility.

Conditional on the information set $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$ available to investors and with respect to their subjective probability P_s, expected returns depend only on exposure to aggregate risk as measured by a_{it}. It follows that the intertemporal CAPM holds from the point of view of investors' in the sense that

$$E_s\left[dR_{it}^e \middle| \mathcal{F}_t\right] = E_s\left[dR_{it} - r_t dt \middle| \mathcal{F}_t\right] = a_{it}^\top \kappa_t dt.$$

On the other hand, using the relation between the true firm-specific shock W_{it} and the Brownian motion B_{it} perceived by investors (see equation (2)) we can rewrite the dynamics of the firm value process as

$$dR_{it} = \left(r_t + a_{it}^\top \kappa_t + \iota_{it}\eta_{it}\right) dt + a_{it}^\top dB_{at} + \iota_{it} dW_{it}, \tag{6}$$

where

$$\eta_{it} = \sigma_{ii}^{-1}(\theta_{it} - m_{it}).$$

Relative to equation (5), the drift now contains an additional component that depends on the idiosyncratic volatility of the firm ι_{it} and the normalized estimation error η_{it}. Comparing the two dynamics shows that this additional term does not modify expected returns from the perspective of investors. Indeed, we have

$$E_s\left[dW_{it} + \iota_{it}\eta_{it} dt \middle| \mathcal{F}_t\right] = E_s\left[dB_{it} \middle| \mathcal{F}_t\right] = 0$$

by definition of the investors' subjective probability measure. However, if investors are biased at the aggregate level in the sense that m_{it} is different from the Bayesian estimate $\hat{\theta}_{it} = E_o[\theta_{it} | \mathcal{F}_t]$ then this additional term will influence expected returns under the objective probability as demonstrated by the following:

Proposition 1: *Under the objective probability P_o the instantaneous expected excess return of firm i conditional on the observed filtration is*

$$E_o\left[dR_{it} - r_t dt \middle| \mathcal{F}_t\right] = \left(a_{it}^\top \kappa_t + \iota_{it}\hat{\eta}_{it}\right) dt, \tag{7}$$

where $\hat{\eta}_{it} = \sigma_{ii}^{-1}(\hat{\theta}_{it} - m_{it})$ is a normalized measure of the aggregate estimation bias.

It is important to observe that equation (7) is not a multi-factor specification of the intertemporal CAPM. The first term on the right hand side is a reward for bearing aggregate risk. The second term, however, arises from the fact that investors are biased in their perception of the growth rate and does not constitute a reward for bearing risk. In particular, this term depends on the idiosyncratic volatility and the investors' perception of the growth rate which are both firm-specific quantities.

Unconditional tests performed on data generated from the model, do not capture the distributional properties of stock returns as perceived by investors. Instead, such tests measure a combination of perceived returns that are solely due to exposure to aggregate risk, and estimation biases that are due to incomplete information. In other words, basic regressions provide coefficient estimates that are drawn from the true underlying distribution and therefore depend on the unconditional expected value $E_o[\iota_{it}\hat{\eta}_{it}|\mathcal{F}_0]$ of the second term under the objective probability measure.

If investors are biased at the aggregate level then this expected value is non zero and will mechanically influence the regression results. But whether or not this can explain the cross-sectional evidence on the relation between idiosyncratic volatility and stock returns depends on the covariance between the idiosyncratic volatility of stock returns ι_{it} and the estimation bias $\hat{\theta}_{it} - m_{it}$. Indeed, we have

$$\begin{aligned} E_o[\iota_{it}\hat{\eta}_{it}|\mathcal{F}_0] &= E_o[\iota_{it}|\mathcal{F}_0]E_o[\hat{\eta}_{it}|\mathcal{F}_0] + \mathrm{Cov}_o[\iota_{it}; \hat{\eta}_{it}|\mathcal{F}_0] \\ &= E_o[\iota_{it}|\mathcal{F}_0]E_o[\hat{\eta}_{it}|\mathcal{F}_0] + \sigma_{ii}^{-1}\mathrm{Cov}_o[\iota_{it}; \hat{\theta}_{it} - m_{it}|\mathcal{F}_0]. \end{aligned}$$

If the covariance on the right-hand side is zero then the investors' estimation biases will impact the regression results since $E_o[\iota_{it}|\mathcal{F}_0]E_o[\hat{\eta}_{it}|\mathcal{F}_0] \neq 0$. But this cannot be invoked as an explanation for the empirical relation between returns and idiosyncratic volatility because in this case sorting stocks according to their idiosyncratic volatilities does not carry any information regarding the size of the corresponding estimation biases.

To illustrate this point, let the risk free rate and risk premium be constant and assume that $m_{it} = m_i$ for some m_i. In this case, the results of Appendix A imply that the idiosyncratic volatility of the firm's stock returns is $\iota_{it} = \sigma_{ii}$ and it follows that the second term in equation (7) is given by

$$\iota_{it}\hat{\eta}_{it} = \sigma_{ii}\hat{\eta}_{it} = \hat{\theta}_{it} - m_i$$

which is completely unrelated to idiosyncratic volatility.

To obtain a relation between idiosyncratic volatility and stock returns it is necessary that the idiosyncratic volatility ι_{it} covaries with the estimation bias $\hat{\theta}_{it} - m_{it}$. In the preceding example this covariance was zero due to the fact that the investors' perception of the growth rate, and hence also the idiosyncratic volatility of returns, was constant. But there is no reason to believe that it should be so otherwise. For example, if the investors' perception evolves according to a diffusion process of the form

$$dm_{it} = \mu_i(t, m_{it})dt + \psi_i(t, m_{it})dB_{it}$$

then it follows from the results of Appendix A that the idiosyncratic volatility of the stock is given by

$$\iota_{it} = \sigma_{ii} + \psi_i(t, m_{it})\frac{\partial \log q_i(t, m_{it})}{\partial m} \tag{8}$$

for some nonnegative function q_i that represents the firm's price/dividend ratio. Using this expression together with the definition of the estimation bias then shows that the model generates an idiosyncratic volatility effect provided that

$$\text{Cov}_o \left[\psi_i(t, m_{it})\frac{\partial \log q_i(t, m_{it})}{\partial m}; \hat{\theta}_{it} - m_{it} \middle| \mathcal{F}_0 \right] \neq 0. \tag{9}$$

While we cannot assert that this condition holds as soon as the volatility of the investors' perception is non zero it is quite easy to construct examples of reasonable models in which this covariance is non zero. To obtain such an example consider an infinite horizon economy in which the risk free rate and risk premium are constant and assume that the true underlying distribution and the investors' perception are such that

$$m_{it} = b\hat{\theta}_{it}$$

for some nonnegative constant $b \leq 1$ so that investors are in aggregated terms more pessimistic than a Bayesian investor in periods where $\hat{\theta}_{it} \geq 0$, and more optimistic otherwise. To derive a closed-form solution for the stock price we further assume that the investor's perception evolves according to

$$dm_{it} = \lambda_i(\overline{m}_i - m_{it})dt + \sigma_{ii}^{-1}(m_{it} - m_{il})(m_{ih} - m_{it})dB_{it} \tag{10}$$

with initial condition

$$m_{i0} = \overline{m}_i = m_{il} + \frac{\mu_i}{\lambda_i}(m_{ih} - m_{il})$$

for some constants $\lambda_i > \mu_i > 0$ and $m_{ih} > m_{il}$ where the process B_{it} is the investors perceived idiosyncratic shock as defined in equation (2).[1] In this case the results of Appendix A show that under appropriate parametric restrictions the price/dividend ratio of the firm is explicitly given by

$$q_i(m) = q_{i1} + q_{i2}m_{it}$$

for some strictly positive constants $q_{i1} > q_{i2}$. Using this expression in conjunction with equation (8) then shows that the idiosyncratic volatility is

$$\iota_{it} = \sigma_{ii} + \frac{q_{i1}(m_{it} - m_{il})(m_{ih} - m_{it})}{\sigma_{ii}(q_{i0} + q_{i1}m_{it})} = \sigma_{ii} + \frac{q_{i1}}{\sigma_{ii}} g_i(m_{it}).$$

and it now follows from equation (9) that the model under consideration produces an idiosyncratic volatility effect if

$$\mathrm{Cov}_o\left[\iota_{it}; \hat{\theta}_{it} - m_{it} \middle| \mathcal{F}_0\right] = \frac{q_{i1}}{\sigma_{ii}}\left(\frac{1}{b} - 1\right)\mathrm{Cov}_o\left[m_{it}; g_i(m_{it})\right| \mathcal{F}_0] \neq 0,$$

or equivalenty

$$\mathrm{Corr}_o\left[m_{it}; g_i(m_{it})\right| \mathcal{F}_0] \neq 0.$$

This correlation cannot be computed in closed form and, since the function g_i is non-monotonic, it cannot be proved a priori that the covariance is non zero. To circumvent this difficulty we fix the values of all the parameters but m_{il} as in Table 1 and use Monte Carlo simulations to estimate the above correlation coefficient at a one year horizon for different values of the parameter m_{il}.

As shown by Figure 1 the resulting correlation estimates are clearly different from zero. For example, in the symmetric case where $m_{il} = -m_{ih} = -0.15$, the estimate of the correlation at a one year horizon is

$$\mathrm{Corr}_o\left[m_{i1}; g_i(m_{i1})\right| \mathcal{F}_0] = -0.15076 \ (\pm 0.00795)$$

where the numbers in parenthesis give a 99% confidence interval. This shows that the model indeed produces an idiosyncratic volatility effect but it remains to be seen whether this firm-level effect can explain the cross-sectional relation between idiosyncratic volatility and stock returns. To do so in the context of the current example we proceed as

[1] As is well-known (see e.g. Pastor and Veronesi (2003)) this process gives the perception of an agent who believes that the unobserved underlying growth rate process evolves according to a continuous time Markov chain with states (m_{il}, m_{ih}) and transition intensities $(\mu_i, \lambda_i - \mu_i)$.

follows. First, we simulate the cash flows and perceived growth rates of a panel of 1,000 ex-ante identical firms over 60 years at a daily frequency and drop the first 10 years of data so as to eliminate the impact of the homogenous initial conditions. Following the approach of Ang et al. (2006) we then approximate the idiosyncratic volatility of each firm in each month by the standard deviation of the errors from the market model

$$R^e_{it+\Delta} - R^e_{it} = \alpha_i + \beta_i(R^e_{Mt+\Delta} - R^e_{Mt}) + \varepsilon_{it+\Delta}$$

run at a daily frequency ($\Delta = 1/365$) over non overlapping periods of one month. Using these estimates we form ten value weighted portfolios of firms sorted on idiosyncratic volatility and estimate the α of each of these portfolios by running the market model

$$R^e_{pt+\Delta} - R^e_{pt} = \alpha_P + \beta_p(R^e_{Mt+\Delta} - R^e_{Mt}) + \varepsilon_{pt+\Delta}$$

at a daily frequency over the whole sample period. As shown by the first two columns of Table 2 the data generated from the model exhibit a strong negative idiosyncratic volatility effect. Indeed, the alphas of the portfolio decrease almost monotonically from 10.20 for the low idiosyncratic volatility portfolio to 2.28 for the high volatility portfolio and most are significative at 5% level.

To verify whether these alphas are accounted for by the idiosyncratic volatility effect predicted by our model we use the simulated data to compute the quantity

$$\iota_{it}\hat{\eta}_{it} = \frac{1}{\sigma_{ii}}\left(\frac{1}{b} - 1\right) m_{it} q_i(m_{it})$$

for each firm at a daily frequency, then take a weighted average to obtain the corresponding quantity for each of the deciles portfolios at a daily frequency and finally take the time series average to obtain an estimate of the expected idiosyncratic volatility effect for each decile portfolio. The results of this procedure are reported in the third column of Table 2 and clearly show that the model implied idiosyncratic volatility effects are decreasing in idiosyncratic volatility and closely match the alpha for most of the ten portfolio.

This example clearly shows that the existence of a covariance between idiosyncratic volatility of returns and estimation biases at the firm-level allows to explain the cross-sectional relation between idiosyncratic volatility and returns within the context of a simulated model. In order to verify whether this effect can explain the empirical evidence we construct in Section 4 a proxy for the contribution of idiosyncratic volatility.

It is important to note that the deviation from the CAPM generated by our incomplete information model does not represent an arbitrage opportunity in practice. To see this assume, as we did in the above example, that an investor identifies a bias by running regressions on volatility sorted portfolios and then constructs a long-short portfolio to take advantage of the significant alphas. The performance of such a strategy depends on the future levels of the covariance between idiosyncratic volatility of returns and, since the latter is time-varying, the investor cannot be sure ex-ante that over the holding period the sign of the realized covariance will be the same as that estimated from the data. As a result, the long-short strategy still includes a significant risk component, and thus does not constitute an arbitrage.[2]

3.2 Testable implications

If estimation biases indeed covary with the idiosyncratic volatility of stock returns then our theoretical results can be used to derive two novel implications related to earning forecasts, idiosyncratic volatility and stock returns. We discuss them in this section and perform formal tests in the next section.

Equation (6) shows that the loading of a firm's stock returns on the investors' aggregated estimation bias, $\theta_{it} - m_{it}$, is proportional to the idiosyncratic volatility of returns, ι_{it}. This suggests that stocks with larger idiosyncratic volatility should be more responsive to estimation errors. In particular, when the realized growth rate of a firm's cash flows is higher than anticipated, a situation we refer to as *good news*, firms with larger idiosyncratic volatility should experience higher risk-adjusted returns than firms with lower idiosyncratic volatility. On the contrary, these same firms should exhibit lower risk-adjusted returns following bad news (i.e. when $\theta_{it} < m_{it}$). This directly leads to the following:

Implication 1: *Following good news, firms with larger idiosyncratic volatility produce relatively larger risk-adjusted returns, and following bad news they produce relatively lower risk-adjusted returns.*

Since there is neither debt nor investment in our model, the cash flow X_{it} paid by the firm to its shareholders can be interpreted as the firm's earnings. As a result, the above implication is related to the vast literature documenting the predictability of stock returns following earning announcements, see Ball and Brown (1968), Watts (1978), Foster et al. (1984) and Bernard and Thomas (1990) among others. The fact that good (bad) news are followed by positive (negative) returns is referred to as the post-earning announcement drift.[3] Most of the theories proposed to explain this anomaly are behavioral. In particular,

[2] A perfectly informed investor who observes the true growth rates of all firms would be able to identify the aggregated estimation biases exactly, as opposed to on average, and would thus be able to arbitrage the market. However, the presence of such investors is ruled out by assumption.

[3] Since information is revealed continuously through time there is no formal earnings announcement in our model. However, Implication 1 deals with instantaneous returns and thus describes the contemporaneous relation between news and stock returns.

Bernard and Thomas (1990) suggest that investors under-react to news while Barberis, Shleifer, and Vishny (1998) rely on the representative heuristic and conservatism bias.

In contrast, we propose a rational explanation based on incomplete information and which, in addition, predicts that the effect of good/bad news should be stronger among high idiosyncratic volatility firms. Importantly, this implication of our model holds irrespective of whether the investors' perception are biased or not in aggregated terms. Indeed, conditional on good (bad) news the contribution of the forecast errors to the expected stock returns is always positive (negative) even if the forecast m_{it} is an unbiased estimator of the true underlying growth rate.

According to Proposition 1 the expected excess return on a stock is the sum of two components. The first one is generated by exposure to aggregate risk and can be measured by the sensitivity of a firm's stock returns to standard aggregate risk factors. The second $\iota_{it}\hat{\eta}_{it}$, which we will refer to as the idiosyncratic volatility effect (IDEF), is an idiosyncratic component that is the product of the firm's idiosyncratic volatility and a normalized measure of the investors' estimation bias. This suggests that if one could control for IDEF then idiosyncratic volatility should not play any role in explaining the cross-section of stock returns and naturally leads to the following:

Implication 2: *Risk-adjusted returns on idiosyncratic volatility sorted portfolios do not significantly differ after controlling for IDEF.*

In a series of papers Ang et al. (2006; 2008), Jiang et al. (2007) and Brockman and Yan (2008) show that risk-adjusted returns on idiosyncratic volatility sorted portfolios decrease with the level of idiosyncratic volatility. On the contrary, Malkiel and Xu (2001), Spiegel and Wang (2005) and Fu (2005) find positive relations using different samples and alternative testing procedures. Our model provides a potential explanation for these findings by showing that, under incomplete information, returns and idiosyncratic volatility are linked through the investors' estimation biases. Note that, while our model predicts the existence of a relation between stock returns and idiosyncratic volatility, it is silent about the sign of this relation and thus could be consistent with both a positive and a negative relation.

4 Tests and results

In order to test the implications of our model the first step consists in constructing a proxy for the idiosyncratic volatility effect. We start by detailing the methodology and data used in this process before we present the results of empirical tests.

4.1 Data and proxy construction

Assume for simplicity that there is a single source of aggregate risk so that the standard CAPM holds from the point of view of investors. Under this assumption our model predicts that the excess return on the stock of firm i over a time period of length $\Delta = 1$ month started at t is given by

$$\begin{aligned} R^e_{it+\Delta} - R^e_{it} &= \int_t^{t+\Delta} a_{i\tau}(dB_{a\tau} + \kappa_\tau d\tau) + \int_t^{t+\Delta} \iota_{i\tau}(dW_{i\tau} + \eta_{i\tau} d\tau) \\ &= \int_t^{t+\Delta} \beta_{i\tau} dR^e_{M\tau} + \int_t^{t+\Delta} \iota_{i\tau}(dW_{i\tau} + \eta_{i\tau} d\tau) \end{aligned} \tag{11}$$

where R^e_M denotes the excess return on the market portfolio. To test this relation we need to approximate both integrals by observable quantities. Following the standard approach we assume that the beta of the firms are constant so that

$$\int_t^{t+\Delta} \beta_{i\tau} dR^e_{M\tau} = \beta_i (R^e_{Mt+\Delta} - R^e_{Mt}).$$

To approximate the second integral we proceed in two steps. First, we assume that idiosyncratic volatility is constant within each month and, following Ang et al. (2006), we approximate the constant value in the month starting at t by the standard deviation $\sigma^\varepsilon_{it+\Delta}$ of the residuals from the 3 factor model:

$$\begin{aligned} R^e_{it+k\delta} - R^e_{it+(k-1)\delta} = \alpha_{it} &+ \beta_{1it}(R^e_{it+k\delta} - R^e_{it+(k-1)\delta}) \\ &+ \beta_{2it}\mathrm{HML}_{t+k\delta} + \beta_{3it}\mathrm{SMB}_{t+k\delta} + \varepsilon_{it+k\delta} \end{aligned} \tag{12}$$

run at a daily frequency ($\delta = 1$ day) between t and $t + \Delta$ where $\mathrm{HML}_{t+k\delta}$ and $\mathrm{SMB}_{t+k\delta}$ are the book-to-market and size factors as defined by Fama and French (1993). Given this estimate we approximate the second integral in equation (11) as

$$\int_t^{t+\Delta} \iota_{i\tau}(dW_{i\tau} + \eta_{i\tau} d\tau) \approx \sigma^\varepsilon_{it+\Delta} \int_t^{t+\Delta} (dW_{i\tau} + \eta_{i\tau} d\tau)$$

and it now remains to approximate the integral on the right hand side. To this end, assume that the investors' perception of the growth rate evolves according to

$$dm_{it} = \mu_{it} dt + \psi_{it} dB_{it} = \mu_{it} dt + \psi_{it}(dW_{it} + \eta_{it} dt)$$

for some μ_{it}, ψ_{it} where the second equality follows from the definition of the perceived innovation process in equation (2). This implies

$$dW_{it} + \eta_{it}dt = \psi_{it}^{-1}(dm_{it} - \mu_{it}dt)$$

and assuming that the second term in negligible[4] we conclude that the second integral on the right hand side of equation (11) can be approximated as

$$\int_t^{t+\Delta} \iota_{i\tau}(dW_{i\tau} + \eta_{i\tau}d\tau) \approx \sigma_{it+\Delta}^{\varepsilon} \left(\frac{m_{it+\Delta} - m_{it}}{\psi_{it}} \right). \tag{13}$$

To construct a proxy for the term inside the brackets we rely on earnings forecast revisions obtained from the I/B/E/S detail files.[5] In our model cash flows are equivalent to earnings and are perceived to grow at rate m_{it}. Therefore, we can use variations in the expected growth rate of earnings normalized by their standard deviation in order to approximate this term. Analysts provide forecasts for end-of-year earnings that we aggregate each month for every firm to obtain an average forecast which we denote by FY_{it}. The average forecast revision for firm i in the month starting at t is computed as

$$\Delta FY_{it+\Delta} \equiv FY_{it+\Delta} - FY_{it}.$$

Dividing throughout by the previous year's realized earnings EPS_i then allows us to approximate the change in the perceived growth rate of cash flows as:[6]

$$m_{it+\Delta} - m_{it} \approx FE_{it+\Delta} \equiv \frac{\Delta FY_{it+\Delta}}{\text{EPS}_i}.$$

dividing this measure by its standard deviation $S_{FE,i}$ and inserting the result into the approximation of equation (13) gives us a proxy for the idiosyncratic volatility effect applicable to firm i in the month starting at t:

$$\int_t^{t+\Delta} \iota_{i\tau}(dW_{i\tau} + \eta_{i\tau}d\tau) \approx \sigma_{it+\Delta}^{\varepsilon} \times \frac{FE_{it+\Delta}}{S_{FE,i}} \equiv \text{IDEF}_{it+\Delta}. \tag{14}$$

[4]To justify this assumption we may for example assume that investors filter the growth rate of the firm's cash flows under the assumption that it is a constant in which case $\mu_{it} \equiv 0$. Alternatively we may assume that investors filter the growth rate of the firm's cash flows under the assumption that it is a strongly mean reverting process in which case $\mu_{it} \approx 0$.

[5]The data used in our empirical analysis was downloaded after April 2009 and, therefore, uses the new time-stamped version of the I/B/E/S database.

[6]Since some firms occasionally report negative earning per share one might be concerned that dividing by EPS_i instead of $|\text{EPS}_i|$ introduces some kind of bias. This is not the case. In particular, we have verified that our empirical results remain qualitatively unchanged when we use the absolute value of the realized earnings rather than the raw earnings. Details are available upon request.

Putting everything back together finally shows that the approximate relation implied by the model is given by

$$R^e_{it+\Delta} - R^e_{it} = \beta_i(R^e_{Mt+\Delta} - R^e_{Mt}) + \mathrm{IDEF}_{it+\Delta}. \tag{15}$$

In the empirical investigation to follow we measure the standard deviation $S_{FE,i}$ by using the whole sample period to obtain a precise estimate. This makes the variable IDEF forward looking but has a negligible impact as we use this information to measure the realized idiosyncratic volatility effect but not to construct portfolios. We verify in Section 4.4 that our results remain qualitatively unchanged when using only backward looking information in the construction of the proxy. We also perform a number of robustness checks that show that our results still hold under alternative proxy constructions.

The daily stock return data used in our tests is from CRSP and the sample period for all our tests is January 1982 to December 2007. Summary statistics are presented in Table 3. On average, the sample contains 2848 firms per month. Each firm has on average 4 analyst forecast revisions per month. Large firms tend to be followed by more analysts and display lower idiosyncratic volatility on average. These characteristics are in line with previous studies (see Diether et al. (2002) and references therein).

4.2 The effect of good and bad news

The relation between idiosyncratic volatility and stock returns predicted by our model is contemporaneous. Indeed, equation (15) and the definition of IDEF show that expected returns over the month starting at t are affected by the idiosyncratic volatility for that month. In order to test the validity of this relation we start by performing a series of tests in which firms are sorted in month t according to their idiosyncratic volatility in that same month.

According to Implication 1, firms with high idiosyncratic volatility should earn higher returns during good news episodes even on a risk-adjusted basis and the model predicts an opposite relation during bad news events. To test this implication we first define good and bad news as a positive, respectively negative, average forecast revision. Each month we allocate firms to either the good or the bad news group depending on the direction of the average forecast revision, and then sort firms in deciles based on the level of idiosyncratic volatility computed for that month from the 3 factor model of equation (12). As a result of this procedure, there are each month on average 80 firms per decile for the good news group, and 73 firms per decile for the bad news group. We compute the value weighted return for each decile portfolio and estimate the alpha by running the

3 factor model at a monthly frequency. We compare the alpha to the time series average of the value weighted idiosyncratic volatility effect computed according to our proxy.

Figure 2 and Table 4 summarize the results of this procedure and show that the predictions of our model are remarkably well verified in the data. Indeed, for the good news group, the alpha increases with idiosyncratic volatility and ranges from 0.51 to 3.86 in monthly percentage points while the idiosyncratic volatility effect, IDEF, is also increasing and ranges from 0.27 to 3.29. For the bad news group, the alpha decreases with idiosyncratic volatility and ranges from -0.18 to -5.58 while IDEF decreases from -0.21 to -5.15. For the ten decile portfolios our incomplete information based explanation produces an effect which closely matches the fraction of returns that is not explained by the 3 factor model. In particular, the difference between alpha and the average IDEF is not statistically significant for eighteen out of the twenty deciles portfolios.

According to implication 2, there should not be a statistically significant difference between the returns on high and low idiosyncratic volatility portfolios after controlling for the idiosyncratic volatility effect predicted by our model. To test this implication, we first verify whether it holds when firms are sorted into good and bad news groups by estimating the regressions

$$\begin{aligned} R^e_{pt+\Delta} - R^e_{pt} = \alpha_p + \beta_{1p}(R^e_{Mt+\Delta} - R^e_{Mt}) \\ + \beta_{2p}\mathrm{HML}_{t+\Delta} + \beta_{3p}\mathrm{SMB}_{t+\Delta} + \beta_{4p}(\Theta \cdot \mathrm{IDEF}_{pt+\Delta}) + \nu_{pt+\Delta} \end{aligned} \tag{16}$$

at a monthly frequency for each decile portfolio where Θ is a dummy variables that takes the value one when a control for IDEF is included. Table 5 summarizes the results of this estimation by comparing the alphas obtained with and without controlling IDEF. As shown by the table, the idiosyncratic volatility effect has little effect on the initial estimation results and we therefore cannot conclude from this test that Implication 2 holds in the split sample. As this negative result may be due to the presence of outliers in the sample[7] we repeat the procedure after applying a monthly filter that eliminates from both groups the 1% of most extreme IDEF observations as well as those firms that are not followed by at least five analysts.

The results of this estimation are presented in Table 6. We first note that the values of the alpha are almost identical to those of the full sample. This is reassuring as it implies that the filters we applied have little effect on the initial estimation results. In the bad news group, the alphas are all highly significant in the absence of control for IDEF except for the low volatility portfolio which has a $t-$statistic of -0.99.[8] When

[7] In particular, firms with very low earnings can significantly affect the results of the testing procedure, given that IDEF is standardized using the previous year EPS.

[8] All the reported $t-$statistics are robust Newey-West $t-$statistics.

controlling for IDEF the alphas of six out of the ten decile portfolios are still significant but all $t-$statistics are much lower. In particular, the alpha of the high idiosyncratic volatility portfolio is no longer significant and has a much smaller magnitude than that of the ninth decile portfolio, -1.27 against -2.16. In the good news group, the control has a much weaker impact. The alpha remain significant for all portfolios but the $t-$statistics are reduced.

To gain further insight into the above estimation results we consider a portfolio that is long in the high volatility stocks and short in the low volatility stocks. Here also, the data is filtered by removing 1% of most extreme IDEF observations and taking into consideration a firm's return only if it is followed by at least 5 analysts during that month. As shown in Table 7, the alpha of the long-short portfolio is significant and positive in the good news group, even after controlling for IDEF. In the bad news group, controlling for the idiosyncratic volatility effect eliminates the impact of idiosyncratic volatility on the return of the long/short portfolio. Indeed, controlling for IDEF drives the alpha down from -4.73 to -1.23 percent per month while its $t-$statistics decreases from -4.76 to -1.22. The coefficient on IDEF for the bad news group is highly significant ($t-$statistics of 4.77) and equal to 1.59. As predicted by the model, the value of the coefficient is not statistically different from one. Indeed the $t-$statistic for the difference is 1.78.

Summarizing the results of this section, we find that Implication 1 is strongly supported by the data and that Implication 2 holds in the bad news group. In the good news group, our results indicate that IDEF is not sufficient to fully explain the cross-section of risk-adjusted returns as there might be other forces at play. In particular, the positive relation between idiosyncratic volatility and stock returns in the good news group seems to be in line with the under-diversification explanation proposed by Merton (1987) and documented empirically by Malkiel and Xu (2001), Spiegel and Wang (2005) and Fu (2005) among others.

4.3 Idiosyncratic volatility and stock returns

Our model predicts opposite effects during good and bad news episodes but it is silent as to the direction of the effect when all firms are taken into consideration. If earning forecasts are unbiased, and unaffected by the level of idiosyncratic volatility, our model is unlikely to produce a significant effect for the cross section of all firms as explained in Section 2. On the other hand, if earning forecasts are biased, and there is ample evidence of such biases in the literature (see, for example, O'Brien (1988), Mendenhall (1991)), then we can verify empirically if these biases and the idiosyncratic volatility effect can explain the cross sectional relation between idiosyncratic volatility and stock returns.

The relation between idiosyncratic volatility and stock returns predicted by our model is contemporaneous. However, to facilitate comparison with previous results in the literature we follow Ang et al. (2006) in sorting firms according to the previous month's idiosyncratic volatility. Using this procedure we form 10 value weighted portfolios that each contain 285 firms on average and measure their returns and alphas relative to the 3 factor model. Table 8 compares the results of this estimation to the value weighted IDEF measured according to equation (14). The first and fifth column show that, even though we use a different sample period, we obtain results similar to those of Ang et al. (2006; 2008). In particular, the portfolio that is long in high idiosyncratic volatility stocks and short in low idiosyncratic volatility stock has a negative and significant alpha of −0.78 and produces an average return of −0.70 percent per month. Furthermore, firms in the high idiosyncratic volatility decile perform significantly worse than firms in the low volatility decile. Indeed, the corresponding alphas range from 0.25 for low volatility firms to −0.53 for high volatility firms. For the 3 deciles with the highest idiosyncratic volatility, the alpha ranges from −0.27 to −0.53 while IDEF ranges from −0.12 to −0.39.

As shown by Table 8, high idiosyncratic volatility stocks produce negative IDEFs whose magnitude closely matches that of the corresponding alphas. Consistent with the findings of Lim (2001), this suggests that earning forecasts are positively biased for high idiosyncratic volatility stocks, i.e. that analysts are overly optimistic about risky stocks. Furthermore, and as can be seen by comparing the third and fifth column of the table, this bias can explain a large part of the negative performance of high idiosyncratic volatility stocks documented in the literature. We verify in Table 9 that this finding remains valid when the observations are filtered as in the previous section. This procedure significantly reduces the sample size — each decile portfolio now contains on average 120 firms as opposed to 285 in the original sample — but produces qualitatively similar results. In particular, the idiosyncratic volatility effect predicted by the model still explains a sizeable fraction of the negative risk-adjusted return on the long/short portfolio.

To gain further insights into the above results we estimate the regressions of equation (16) using the returns on the long/short portfolio as the dependent variable and report the results in Table 10. Considering the entire sample and not controlling for IDEF, the alpha of the long/short portfolio is negative (−0.78) and statistically significant with a $t-$statistics of −2.11. In accordance with the predictions of the model, controlling for IDEF makes the alpha of the long/short portfolio insignificant ($t-$statistic of −1.62), however the coefficient on IDEF is not significant. As this may be due to the influence of outliers, we present in Panel B of Table 10 the results of the same regression analysis for the filtered sample. As before, the alpha is negative and significant (−0.66 with a $t-$statistic of −1.94) in the absence of control and is no longer significant (−0.35 with a

$t-$statistic of -0.98) when a control for IDEF is included in the set of regressors. In line with model, the regression coefficient on IDEF is now significant ($t-$statistic of -2.09) and equal to 1.01 which is exactly the value predicted by Proposition 1.

Since the relation between idiosyncratic volatility and stock returns predicted by the model is contemporaneous, we expect the results of Tables 8 and 9 to be amplified when stocks are sorted on contemporaneous rather than lagged idiosyncratic volatility. Table 11 summarizes the results of this procedure and confirms this intuition. Here also, firms in the highest volatility decile underperform firms in the lowest volatility deciles. The alpha ranges from 0.22, for the lowest volatility firms, to -0.44 for the highest volatility firms but, as predicted by the model, the idiosyncratic volatility effect is now of much larger magnitude ranging from 0.02 to -1.00. This clearly suggests that if we were able to construct IDEF at a higher frequency the model should be able to explain the whole cross-sectional relation between idiosyncratic volatility and stock returns.

4.4 Robustness checks

In this section, we discuss the robustness of our results to alternative constructions of the proxy for the idiosyncratic volatility effect and alternative measures of forecast errors. We also verify that our results still hold when controlling for alternative explanations based on return reversals and the dispersion of analysts' forecasts.

4.4.1 Alternative proxy construction

As a first robustness check, we repeat the cross-sectional tests of Table 8 with the following alternative definition of the forecast error

$$m_{it+\Delta} - m_{it} \approx FE2_{it+\Delta} \equiv \frac{\Delta FY_{it+\Delta}}{FY_{it}}.$$

To obtain comparable results across firms we standardize this measure by dividing it by its standard deviation which we denote by $S_{FE2,i}$. Multiplying the resulting standardized measure of forecast errors by the firm's idiosyncratic volatility then gives us the following alternative proxy for the idiosyncratic volatility effect applicable to firm i in the month starting at t:

$$\text{IDEF2}_{it+\Delta} \equiv \sigma^{\varepsilon}_{it+\Delta} \times \frac{FE2_{it+\Delta}}{S_{FE2,i}} \approx \int_t^{t+\Delta} \iota_{i\tau}(dW_{i\tau} + \eta_{i\tau} d\tau) \tag{17}$$

Compared to equation (14) we replace the previous year realized earning per share, EPS_i, with the previous month average forecast FY_{it}. As shown by Table 12 the cross sectional test results remain qualitatively unchanged when using this alternative proxy. In particular, the new proxy ranges from 0.02 for the low volatility portfolio to −0.18 for the high volatility portfolio and explains about a fourth of the negative risk-adjusted performance of the long/short portfolio.

As a second robustness check, we repeat the cross-sectional tests of Table 8 with the following definition of the forecast error

$$FE3_{it+\Delta} \equiv \Delta FY_{it+\Delta}.$$

Standardizing this variable by its standard deviation and multiplying the result by the firm's idiosyncratic volatility then gives us the following proxy for the idiosyncratic volatility effect applicable to firm i in the month starting at t:

$$\text{IDEF3}_{it+\Delta} \equiv \sigma^{\varepsilon}_{it+\Delta} \times \frac{FE3_{it+\Delta}}{S_{FE3,i}}. \tag{18}$$

The cross sectional test results for this specification of the model are displayed in Table 13. In this case the idiosyncratic volatility effect ranges from 0.01 for the low volatility portfolio to −0.43 for the high volatility portfolio and explains about half of the negative risk-adjusted performance of the long/short portfolio. These results are close to those displayed in Table 8 for the benchmark proxy.

To check whether our empirical results are affected by the fact that we rely on the entire sample to compute the standard deviation of changes in forecast errors, $S_{FE,i}$, we now repeat the cross-sectional tests of Table 8 with the following unstandardized proxy for the idiosyncratic volatility effect

$$\text{IDEF4}_{it+\Delta} \equiv \sigma^{\varepsilon}_{it+\Delta} \times \Delta FY_{it+\Delta}. \tag{19}$$

As shown by the results of Table 14, this new proxy for the idiosyncratic volatility effect also decreases as idiosyncratic volatility increases but is shifted upward compared to Table 8. It ranges from 0.63 for the low volatility portfolio to −0.10 for the high volatility portfolio and matches the magnitude of the alpha on the long/short portfolio.

In summary, these alternative proxy constructions show that we obtain qualitatively similar results even when not relying on forward looking data to standardize the variations in the forecasted earning growth rates.

4.4.2 Alternative measure of estimation errors

We now check the robustness of our results to the sampling frequency and to the assumption that the forecast errors which are relevant to the idiosyncratic volatility effect are those that pertain to the growth rate of a firm cash-flows.

To do so we repeat the cross-sectional tests of Table 8 using a quarterly frequency and the following alternative proxy for the investors' estimation errors

$$FE5_{it+Q} = \frac{EPSQ_{i,t+Q} - FQ_{it}}{EPSQ_{i,t}}$$

where $Q = 1$ quarter, $EPSQ_{it}$ is the realized earning for firm i in the quarter ending at t and FQ_{it} is the average of all the end-of-quarter earnings forecasts available for firm i in that same quarter. The idiosyncratic effect applicable to firm i in the quarter starting at t is obtained by multiplying the above measure by the firm's idiosyncratic volatility:

$$\text{IDEF5}_{it+Q} = \sigma^{\varepsilon}_{it+Q} \times FE5_{it+Q}$$

where $\sigma^{\varepsilon}_{it+Q}$ is the idiosyncratic volatility of firm i estimated by running equation (12) at a daily frequency over the quarter starting at t. Since many firms in our sample do not report quarterly earnings the amount of data available to run our cross-sectional tests based on the above specification proxy is much smaller than in our previous tests. To facilitate the comparison with our previous results, we therefore need to limit the number of volatility sorted portfolios so as to keep roughly the same number of firms in each of them. With this in mind, we construct quintile rather than decile portfolios. The resulting quintile portfolios each include on average 264 firms which is comparable to the figure we obtained in the previous section.

The results for this specification of the model are presented in Table 15. The idiosyncratic volatility effect measured in quarterly percentage points ranges from -0.06 for the low idiosyncratic volatility portfolio to -0.58 for the high idiosyncratic volatility portfolio. The corresponding alphas range from -0.08 to -1.68, but only the alpha of the high volatility portfolio is significant at the 10% level.

4.4.3 Return reversal and the idiosyncratic volatility effect

Huang et al. (2010) have shown that the relatively low returns of high idiosyncratic volatility portfolios can potentially be explained by return reversal. In order to show that the idiosyncratic volatility effect predicted by our model is distinct from this return reversal explanation we construct a panel of portfolios double sorted on past idiosyncratic volatility and returns, and check whether the idiosyncratic volatility effect predicted by our model is still present in this panel.

Table 16 reports the number of firms in each double sorted portfolio and shows that, even though we use a different sample, the high idiosyncratic volatility portfolio contains a large number of past winners and past losers and few intermediate stocks as in Huang et al. (2010). However, and as can be seen from Table 17, the idiosyncratic volatility effect measured as in equation (14) decreases with the level of idiosyncratic volatility independently of past returns and is even stronger for past losers. This does not contradicts the results in Huang et al. (2010) but clearly indicates that the idiosyncratic volatility effect implied by our model is different from the return reversal.

4.4.4 Dispersion of analysts' forecasts and the idiosyncratic volatility effect

Diether et al. (2002) find that when sorting firms on the basis of analysts forecasts dispersion, the portfolio of low dispersion firms performs significantly better than the portfolio of high dispersion firms. In order to show that the idiosyncratic volatility effect predicted by our model is distinct from this dispersion effect we construct a panel of portfolios double sorted on idiosyncratic volatility and analysts forecasts' dispersion and check whether the idiosyncratic volatility effect is still present.

Following Diether et al. (2002), we define dispersion of analysts forecasts as the standard deviation of earnings forecasts scaled by the absolute value of the mean earnings forecast. Each month, firms are sorted using forecast dispersion and then within each group using lagged idiosyncratic volatility. We use fewer groups than in the main empirical section (quintiles instead of deciles) to maintain a sufficiently large number of firms within each group. The results displayed in Table 18 show that the idiosyncratic volatility effect that we identify is present within each forecast dispersion group. It is however interesting to notice that the effect is much stronger within the group with large dispersion of analyst forecasts. The two effects appear to be correlated although imperfectly.

The idiosyncratic volatility effect is present whether the double sort is performed using lagged values as reported in Table 18 or contemporaneous values as reported in Table 19. Finally, Table 20 displays the average number of firms present in each group. If there were no differences between the sorting procedure based on dispersion and the sorting procedure based on idiosyncratic volatility then most firms would be on the diagonal, i.e. low dispersion would imply low volatility and high dispersion would imply high volatility. We can see from Table 20 that it is not the case as no clear pattern emerges from the distribution of firms across groups.

5 Conclusion

Explaining the relation between idiosyncratic volatility and stock return does not necessarily require the use of behavioral models or the introduction of anomalies. In particular, we show in this paper that incomplete information allows to capture a significant part of this relation. The key to our explanation is that when there is incomplete information about idiosyncratic shocks, any firm-specific forecast error appears in the return equation scaled by the idiosyncratic volatility. The model we develop to illustrate this mechanism is standard in all aspects as agents behave rationally conditional on their beliefs.

Taking the model to the data requires the construction of a proxy for the idiosyncratic volatility effect implied by the model. We do so by relying on earning forecasts and measure idiosyncratic volatility from the residuals of a standard asset pricing model. We document a strong link between the unexplained part of the risk-adjusted return (the alpha) and the proxy of the idiosyncratic volatility effect. The effect is particularly strong when the sample is split between good and bad news events.

After performing a number of robustness tests, we conclude that incomplete information explains a significant part of the relation between idiosyncratic volatility and stock returns and we propose a new variable, IDEF, that should be included as a control to attenuate this relation.

Appendix A: Dynamics of the firm value process

In this appendix we show how to obtain the dynamics of the firm value process and provide a couple of examples that illustrate the procedure.

According to equation (4) of the main text we have that the market value of firm i at time t satisfies the asset pricing relation

$$\xi_t V_{it} + \int_0^t \xi_\tau X_{i\tau} d\tau = M_{it} \equiv E_s \left[\int_0^{T_i} \xi_\tau X_{i\tau} d\tau \middle| \mathcal{F}_t \right]$$

where the process ξ_t is the state price density, or pricing kernel, of the economy as defined in equation (3). Since the right hand side is a martingale with respect to the investors' information set under their subjective measure it follows from the martingale representation theorem (see Duffie (2001, Appendix D)) that

$$M_{it} = E_s \left[\int_0^{T_i} \xi_\tau X_{i\tau} d\tau \right] + \int_0^t \varphi_{i\tau}^\top dB_{a\tau} + \int_0^t \phi_{i\tau} dB_{i\tau}$$

for some $\mathbb{F}-$adapted processes φ_i and ϕ_i such that the above stochastic integrals are well-defined. Using this expression together with the dynamics of the state price density and Itô's lemma we get that the dynamics of the firm value are given by

$$dV_{it} = (r_t V_{it} - X_{it})dt + (\kappa_t V_{it} + \varphi_{it}/\xi_t)^\top (dB_{at} + \kappa_t dt) + (\phi_{it}/\xi_t)dB_{it}$$

and setting

$$\begin{aligned} a_{it} &\equiv \kappa_t V_{it} + (\varphi_{it}/\xi_t), \\ \iota_{it} &\equiv (\phi_{it}/\xi_t) \end{aligned}$$

delivers the stock return dynamics of equation (5). The processes φ_i and ϕ_i can be identified once we specify a model for the perceived growth rate, the risk free rate and the market risk premium.

Consider for example a world where the interest rate and the risk premium are constant and assume that the perceived growth rate evolves according to

$$dm_{it} = \mu_i(t, m_{it}, X_{it})dt + \psi_i(t, m_{it}, X_{it})dB_{it}$$

for some deterministic functions μ_i, ψ_i. In this case, standard results on stochastic differential equations guarantee that

$$V_{it} = V_i(t, m_{it}, X_{it})$$

for deterministic function and an application of Itô's lemma shows that the corresponding volatility coefficients are given by

$$a_{it} = \frac{\partial V_i(t, m_{it}, X_{it})}{\partial x} X_{it} \sigma_{ia},$$

and

$$\iota_{it} = \frac{\partial \log V_i(t, m_{it}, X_{it})}{\partial x} X_{it} \sigma_{ii} + \frac{\partial \log V_i(t, m_{it}, X_{it})}{\partial m} \psi_i(t, m_{it}, X_{it}).$$

In particular, if the perceived growth rates are autonomous in the sense that the functions μ_i, ψ_i do not depend on the firm's cash flow then

$$V_{it} = q_i(t, m_{it}) X_{it} \tag{20}$$

where the function q_i represents the firm's price/dividend ratio, and the expressions for the volatility coefficients simplify to $a_{it} = \sigma_{ia}$ and

$$\iota_{it} = \sigma_{ii} + \frac{\partial \log q_i(t, m_{it})}{\partial m} \psi_i(t, m_{it}). \tag{21}$$

In the special case of the example developed in the second part of Section 3.1 we have that the drift and volatility of the perceived growth rate are given by

$$\mu_i(m) = \lambda_i(m_{il} - m) + \mu_i(m_{ih} - m_{il}),$$

and

$$\psi_i(m) = \sigma_{ii}^{-1}(m - m_{il})(m_{ih} - m)$$

and, as shown by the following lemma, it is possible to obtain closed form expression for the value of the firm and the two volatility coefficients.

Lemma 1: *Consider an infinite horizon economy with constant coefficients and assume that the perceived growth rates evolve according to equation (10) for some constants parameters such that*

$$q_{i1} = \frac{r + \sigma_{ia}^{\top}\kappa + \lambda_i - m_{il} - m_{ih}}{(r + \sigma_{ia}^{\top}\kappa)(r + \sigma_{ia}^{\top}\kappa + \lambda_i - m_{il} + m_{ih}) - \lambda_i \overline{m}_i + m_{il} m_{ih}} > 0,$$

and

$$q_{i2} = \frac{q_{i1}}{r + \sigma_{ia}^{\top}\kappa + \lambda_i - m_{il} - m_{ih}} > 0.$$

Then the value of the firm and its volatility coefficients are explicitly given by equation (20), $a_{it} = \sigma_{ia}$ and equation (21) with $q_i(m) = q_{i1} + q_{i2} m$.

Proof. Let the constants $q_{i1} > 0$ and $q_{i2} > 0$ be as in the statement and consider the nonnegative $\mathbb{F}-$adapted process defined by

$$M_{it} = \xi_t X_{it}(q_{i1} + q_{i2} m_{it}) + \int_0^t \xi_\tau X_{i\tau} d\tau.$$

Using the dynamics of the pair (m_i, X_i) in conjunction with the definition of the constants q_{ij} and applying Itô's lemma we deduce that M_i is a local martingale under P_s. On the other hand, using the boundedness of m_i together with the assumptions of the statement and well–know results on geometric Brownian motion it can be shown that

$$E_s\left(\sup_{t\in[0,T]}|M_t|^2\right)<\infty$$

for any finite T. This implies that the local martingale M is a true martingale up to any finite time and it follows that

$$\xi_t X_{it}(q_{i1}+q_{i2}m_{it}) = E_s\left[\xi_T X_{iT}(q_{i1}+q_{i2}m_{iT}) + \int_t^T \xi_\tau X_{i\tau}d\tau \,\middle|\, \mathcal{F}_t\right].$$

Taking the limit as $T\to\infty$ on both sides of the previous expression and using the dominated convergence theorem gives

$$\xi_t X_{it}(q_{i1}+q_{i2}m_{it}) = \xi_t V_{it} + \lim_{T\to\infty} E_s\left[\xi_T X_{iT}(q_{i1}+q_{i2}m_{iT})\right|\mathcal{F}_t]$$

and the proof will be complete once we show that the second term on the right is equal to zero. This follows from the boundedness of the perceived growth rate and the assumptions of the statement, we omit the details. □

Appendix B: Proof of Proposition 1

By standard results (see e.g. Liptser and Shiryaev (2001)) we have that the process

$$\hat{W}_{it} = W_{it} + \int_0^t \sigma_{ii}^{-1}(\theta_{i\tau}-\hat{\theta}_{i\tau})d\tau$$

is an observed Brownian motion under the objective probability measure. Combining this definition with equation (6) then gives

$$\begin{aligned}
dR^e_{it} &= a_{it}^\top(dB_{at}+\kappa_t dt) + \iota_{it}(dW_{it}+\eta_{it}dt)\\
&= a_{it}^\top(dB_{at}+\kappa_t dt) + \iota_{it}(d\hat{W}_{it} - \sigma_{ii}^{-1}(\theta_{it}-\hat{\theta}_{it})dt + \sigma_{ii}^{-1}(\theta_{it}-m_{it})dt)\\
&= a_{it}^\top(dB_{at}+\kappa_t dt) + \iota_{it}(d\hat{W}_{it} + \sigma_{ii}^{-1}(\hat{\theta}_{it}-m_{it})dt)\\
&= a_{it}^\top(dB_{at}+\kappa_t dt) + \iota_{it}(d\hat{W}_{it} + \hat{\eta}_{it}dt)
\end{aligned}$$

and the desired result now follows by taking expectation conditional of $\mathcal{F}_t$ under the objective probability measure on both sides.

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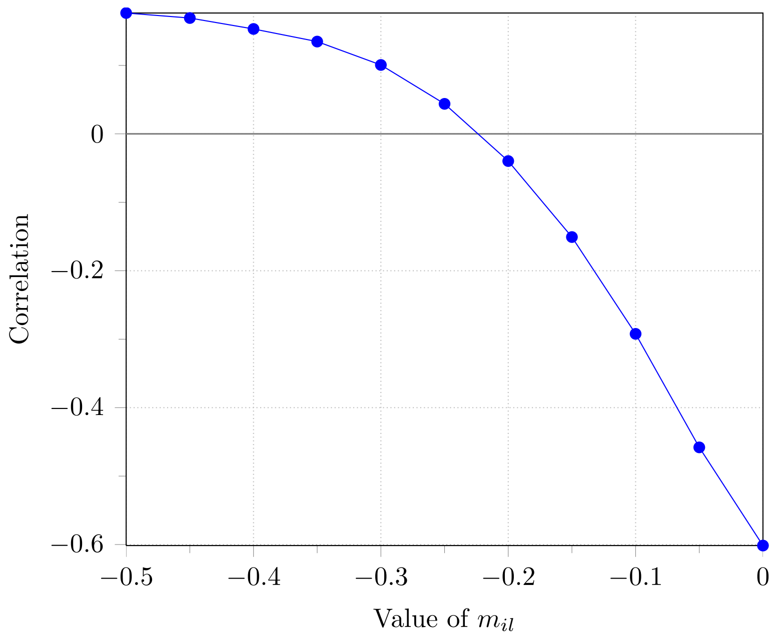

Figure 1: Correlation $\text{Corr}_o[\iota_{i1}, \hat{\eta}_{i1}|\mathcal{F}_0]$ estimated from 100,000 simulated paths of the perceived growth rate m_{it} for various values of the parameter m_{il}. The values of the other parameters of the model are given by Table 1.

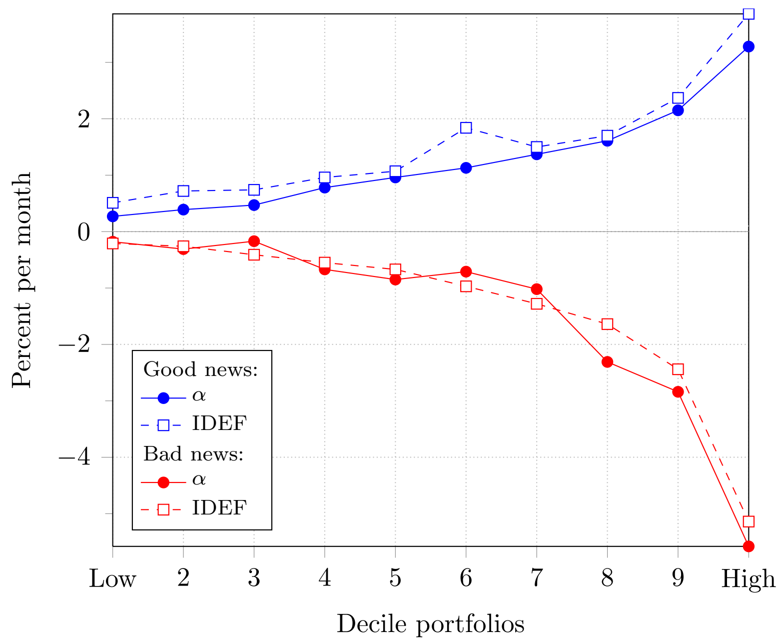

Figure 2: Squares represent the average idiosyncratic volatility effect measured as in (14) while circles represent the alpha from a 3 factor model. Both statistics are computed for ten value weighted portfolios sorted monthly on idiosyncratic volatility over the sample period January 1982 to December 2007.

Symbol	b	r	σ_{ia}	σ_{ii}	κ	λ_i	μ_i	m_{ih}
Value	0.5	0.045	0.13	0.152	0.3	1	0.5	0.15

Table 1: Parameter values for the model of equation (10)

	alpha	$t-$stat	IDEF
Low Vol.	10.20	7.97	10.61
2	9.59	7.55	9.48
3	5.83	4.41	8.65
4	9.69	6.91	7.71
5	6.06	4.32	6.34
6	4.92	3.45	5.49
7	2.21	1.44	4.32
8	3.50	2.24	3.26
9	2.88	1.83	2.35
High Vol.	2.28	1.32	0.89

Table 2: Average idiosyncratic volatility effect (IDEF) and CAPM alpha for 10 value weighted portfolios sorted on idiosyncratic volatility. The data used in the regressions and the computation of the average idiosyncratic volatility effect were obtained by simulating the cash flows and perceived growth rates of equations (1) and (10) for a panel of 1,000 firms at a daily frequency for 60 years and dropping the first 10 years of data. The parameters used in this procedure are given by $m_{il} = -m_{ih} = -0.15$ and the values of Table 1.

Panel A: Descriptive statistics			
	Mean	Median	Std.
Number of firms per month	2848	2972	828
Analysts per firm per month	4.03	2.00	5.7
Idiosyncratic volatility (Percent/Month)	2.89	2.18	3.25
Log market cap ($Mio.)	5.21	5.08	1.95
Panel B: Correlations			
	IVOL	NUM	log Mcap
Idiosyncratic volatility: IVOL	1		
Analysts per firm per month: NUM	−0.14	1	
Logarithm of mkt. capitalization: log Mcap	−0.34	0.60	1

Table 3: Summary statistics for the sample period January 1982 to December 2007. Panel A reports the monthly mean, median and standard deviation of the number of firms, the number of analyst providing a forecast during that month (NUM), the level of idiosyncratic volatility (IVOL) measured by the standard deviation of the errors from the 3 factor model of equation (12) run at a daily frequency, and the log market capitalization in million dollars (log Mcap). Panel B reports the correlations between IVOL, NUM and log Mcap.

	Good news		Bad news	
	IDEF	alpha	IDEF	alpha
Low Vol.	0.27	0.51	-0.21	-0.18
2	0.40	0.73	-0.26	-0.31
3	0.48	0.74	-0.42	-0.17
4	0.78	0.96	-0.55	-0.67
5	0.97	1.06	-0.68	-0.85
6	1.13	1.84	-0.97	-0.71
7	1.38	1.50	-1.28	-1.02
8	1.62	1.70	-1.65	-2.31
9	2.16	2.37	-2.44	-2.84
High Vol.	3.29	3.86	-5.15	-5.58

Table 4: Idiosyncratic volatility effect (IDEF) measured as in equation (14) and 3-factor model alpha for 10 value weighted portfolios sorted on the contemporaneous month's idiosyncratic volatility for the sample period January 1982 to December 2007. The sample is split in good and bad news months obtained using average analyst forecast revisions. All values are monthly percentage points.

	Good news				Bad news			
	Not controlling for IDEF		Controlling for IDEF		Not controlling for IDEF		Controlling for IDEF	
	alpha	$t-$stat.	alpha	$t-$stat.	alpha	$t-$stat.	alpha	$t-$stat.
Low Vol.	0.51	3.50	0.46	3.02	-0.18	-1.26	-0.24	-1.59
2	0.73	5.77	0.68	5.05	-0.31	-2.62	-0.24	-1.56
3	0.74	5.13	0.81	5.18	-0.17	-1.07	-0.12	-0.61
4	0.96	6.83	0.73	4.82	-0.67	-4.01	-0.59	-2.85
5	1.06	6.93	0.94	5.31	-0.85	-4.29	-1.11	-4.92
6	1.84	6.92	1.62	6.77	-0.71	-2.94	-0.90	-2.34
7	1.50	6.57	1.76	6.86	-1.02	-3.44	-0.87	-2.76
8	1.70	6.59	1.70	5.44	-2.31	-5.27	-1.71	-2.39
9	2.37	6.91	2.86	6.89	-2.84	-5.14	-2.85	-3.80
High Vol.	3.86	6.06	4.55	5.31	-5.58	-5.45	-4.23	-3.54

Table 5: 3-factor alpha and 3-factor alpha controlling for the idiosyncratic effect for 10 value weighted portfolios sorted on the contemporaneous month's idiosyncratic volatility over the sample period January 1982 to December 2007. The sample is split in good and bad news months obtained using average analyst forecast revisions. All values are monthly percentage points and the reported test statistics are robust Newey-West $t-$statistics.

	Good news				Bad news			
	Not controlling for IDEF		Controlling for IDEF		Not controlling for IDEF		Controlling for IDEF	
	alpha	$t-$stat.	alpha	$t-$stat.	alpha	$t-$stat.	alpha	$t-$stat.
Low Vol.	0.50	3.64	0.59	3.45	-0.13	-0.85	-0.18	-0.91
2	0.60	4.74	0.63	3.48	-0.31	-2.41	-0.01	-0.07
3	0.91	5.57	1.12	5.17	-0.34	-2.33	-0.43	-2.09
4	0.87	6.34	0.90	6.02	-0.33	-1.91	0.02	0.08
5	0.88	5.45	1.09	5.25	-0.86	-4.95	-1.26	-5.10
6	1.30	7.05	1.24	4.88	-0.82	-3.50	-0.78	-2.22
7	1.89	9.22	1.88	5.28	-0.79	-2.74	-0.96	-1.97
8	1.75	7.11	1.63	4.56	-1.43	-4.03	-1.26	-2.01
9	1.85	6.33	2.21	5.32	-2.60	-4.38	-2.16	-3.34
High Vol.	2.96	5.31	3.45	5.06	-4.82	-5.18	-1.27	-1.30

Table 6: 3-factor alpha and 3-factor alpha controlling for the idiosyncratic effect for 10 value weighted portfolios sorted on the contemporaneous month's idiosyncratic volatility over the sample period January 1982 to December 2007. The sample is split in good and bad news months obtained using average analyst forecast revisions. Outliers are removed using a 99 % confidence interval and we only consider firms month return where 5 or more analysts provide a forecast. All values are monthly percentage points and the reported test statistics are robust Newey-West $t-$statistics..

Panel A: Good news						
		Constant	MKT	SMB	HML	IDEF
No control	Coefficient	2.41	0.87	1.44	-1.03	–
	t−stat.	*3.81*	*5.14*	*7.13*	*-3.29*	–
Control	Coefficient	2.77	0.88	1.41	-1.03	-0.38
	t−stat.	*3.66*	*5.21*	*7.27*	*-3.17*	*-1.00*
Panel B: Bad news						
		Constant	MKT	SMB	HML	IDEF
No control	Coefficient	-4.73	0.80	1.24	-0.82	–
	t−stat.	*-4.76*	*3.52*	*3.73*	*-2.17*	–
Control	Coefficient	-1.23	0.73	1.22	-0.85	1.59
	t−stat.	*-1.22*	*3.43*	*3.68*	*-2.43*	*4.77*

Table 7: Regression analysis for the long short portfolios. The dependent variable for all estimation is the value weighted monthly return of a strategy that is long in the high volatility portfolio and short in the low volatility portfolio. The constant is expressed in monthly percentage points and IDEF is the idiosyncratic volatility effect measured according to equation (14). The sample is split in good and bad news events. Outliers are removed using a 99 % confidence interval and we only consider firms month return where 5 or more analysts provide a forecast. The sample period is January 1982 to December 2007, all values are monthly percentage points and the reported test statistics are robust Newey-West $t-$statistics.

	Avg. R	Std.	IDEF	Mkt. Cap.	alpha	$t-$stat.	beta
Low Vol.	1.30	3.73	0.01	26.23	0.25	2.68	0.83
2	1.41	4.10	0.01	22.31	0.31	2.75	0.94
3	1.17	4.63	-0.02	16.07	-0.02	-0.26	1.07
4	1.24	4.89	-0.02	11.21	0.11	1.11	1.07
5	1.17	5.53	-0.05	7.74	-0.03	-0.26	1.16
6	1.32	6.42	-0.09	5.71	0.14	1.04	1.22
7	1.18	7.04	-0.03	4.22	0.09	0.63	1.21
8	0.84	7.94	-0.12	3.07	-0.27	-1.37	1.28
9	0.82	8.74	-0.23	2.19	-0.31	-1.47	1.33
High Vol.	0.60	9.70	-0.39	1.26	-0.53	-1.69	1.33
10–1	-0.70		-0.40		-0.78	-2.18	0.50

Table 8: Portfolios sorted on the previous month's idiosyncratic volatility. Avg. R is the monthly average return, Std. is the standard deviation of the portfolio return, IDEF is the idiosyncratic volatility effect measured according to equation (14), Mkt. Cap. is the market capitalization of the portfolio in percent, alpha is the 3 factor alpha, $t-$stat. is the robust Newey-West $t-$statistics for alpha and beta is the market beta from the 3 factor model. The line labelled $10 - 1$ provides the same statistics for the long/short portfolio. The sample period is January 1982 to December 2007 and all values are monthly percentage points.

	Avg. R	Std.	IDEF	Mkt. Cap.	alpha	$t-$stat.	beta
Low Vol.	1.19	3.74	0.01	22.52	0.21	2.35	0.81
2	1.41	4.16	0.01	19.58	0.31	2.34	0.93
3	1.26	4.49	-0.01	15.49	0.13	1.40	1.01
4	1.03	4.96	-0.01	11.75	-0.13	-1.13	1.08
5	1.23	5.08	-0.05	9.31	0.12	1.04	1.07
6	1.34	5.68	-0.06	6.81	0.23	1.65	1.12
7	1.23	6.70	-0.08	5.32	0.07	0.47	1.22
8	1.10	7.28	-0.07	4.10	0.04	0.24	1.23
9	1.05	8.49	-0.19	3.10	0.02	0.11	1.28
High Vol.	0.77	9.62	-0.37	2.02	-0.45	-1.49	1.44
10–1	-0.51		-0.38		-0.66	-1.94	0.62

Table 9: Portfolios sorted on the previous month's idiosyncratic volatility. Avg. R is the monthly average return, Std. is the standard deviation, IDEF is the idiosyncratic volatility effect measured according to equation (14), Mkt. Cap. is the market capitalization of the portfolio in percent, alpha is the 3 factor alpha, $t-$stat. is the robust Newey-West $t-$statistics for alpha and beta is the market beta from the 3 factor model. Outliers are removed using a 99 % confidence interval and we only consider firms month return where 5 or more analysts provide a forecast. The line labelled $10 - 1$ provides the same statistics for the long/short portfolio. The sample period is January 1982 to December 2007 and all values are monthly percentage points.

Panel A: Entire sample						
		Constant	MKT	SMB	HML	IDEF
No control	Coefficient	-0.78	0.5	1.28	-0.8	–
	t−stat.	*-2.11*	*3.52*	*9.19*	*-3.33*	
Control	Coefficient	-0.66	0.49	1.28	-0.81	0.29
	t−stat.	*-1.62*	*3.50*	*9.13*	*-3.40*	*0.77*
Panel B: 99 % trim and at least 5 analysts						
		Constant	MKT	SMB	HML	IDEF
No control	Coefficient	-0.66	0.62	1.08	-0.74	–
	t−stat.	*-1.94*	*3.80*	*8.44*	*-3.65*	
Control	Coefficient	-0.35	0.61	1.09	-0.75	1.01
	t−stat.	*-0.98*	*3.90*	*8.75*	*-3.89*	*2.09*

Table 10: Regression analysis for the long/short portfolio. The dependent variable for all regressions is the value weighted monthly return on a strategy that is long in the high volatility portfolio and short in the low volatility portfolio. The constant is expressed in monthly percentage points, IDEF is the idiosyncratic volatility effect measured according to equation (14) and the reported test statistics are robust Newey-West t−statistics.

	Avg. R	Std.	IDEF	Mkt. Cap.	alpha	t−stat.	beta
Low Vol.	1.24	3.67	0.02	26.04	0.22	1.85	0.80
2	1.25	3.98	0.02	22.39	0.17	1.64	0.91
3	1.42	4.55	-0.00	16.03	0.25	2.68	1.05
4	1.37	4.95	0.01	11.21	0.20	2.02	1.09
5	1.49	5.55	-0.01	7.82	0.23	1.95	1.19
6	1.40	6.30	-0.08	5.71	0.28	2.38	1.17
7	1.43	7.61	-0.06	4.24	0.22	1.26	1.31
8	0.96	8.76	-0.17	3.10	-0.23	-0.86	1.41
9	0.81	10.53	-0.28	2.23	-0.32	-0.87	1.41
High Vol.	1.03	15.04	-1.00	1.26	-0.44	-0.51	1.80

Table 11: Portfolios sorted on the contemporaneous month's idiosyncratic volatility. Avg. R is the monthly average return, Std. is the standard deviation of the portfolio return, IDEF is the idiosyncratic volatility effect measured according to equation (14), Mkt. Cap. is the market capitalization of the portfolio in percent, alpha the 3 factor alpha, t−stat. is the t−statistic of the alpha, beta is the market beta from the 3 factor model. The sample period is January 1982 to December 2007 and all values are monthly percentage points.

	Avg. R	Std.	IDEF2	Mkt. Cap.	alpha	$t-$stat.	beta
Low Vol.	1.31	3.75	0.02	26.00	0.26	2.77	0.83
2	1.38	4.08	0.02	22.29	0.28	2.53	0.94
3	1.20	4.62	0.02	16.17	0.02	0.21	1.06
4	1.25	4.91	0.03	11.23	0.12	1.17	1.07
5	1.19	5.51	0.01	7.81	-0.02	-0.20	1.16
6	1.31	6.42	-0.01	5.71	0.14	1.06	1.22
7	1.23	7.02	0.01	4.25	0.14	0.98	1.21
8	0.81	7.94	0.01	3.08	-0.32	-1.58	1.28
9	0.81	8.75	-0.03	2.21	-0.31	-1.51	1.33
High Vol.	0.62	9.69	-0.18	1.25	-0.51	-1.66	1.32
10–1	-0.70		-0.20		-0.78	-2.18	0.49

Table 12: Portfolios sorted on the previous month's idiosyncratic volatility. Avg. R is the monthly average return, Std. is the standard deviation of the portfolio return, IDEF2 is the idiosyncratic volatility effect measured according to equation (17), Mkt. Cap. is the market capitalization of the portfolio in percent, alpha is the 3 factor alpha, $t-$stat. is the robust Newey-West $t-$statistics for alpha and beta is the market beta from the 3 factor model. The line labelled $10-1$ provides the same statistics for the long/short portfolio. The sample period is January 1982 to December 2007 and all values are monthly percentage points.

	Avg. R	Std.	IDEF3	Mkt. Cap.	alpha	$t-$stat.	beta
Low Vol.	1.31	3.74	0.01	26.02	0.27	2.78	0.83
2	1.38	4.09	0.01	22.33	0.28	2.65	0.94
3	1.20	4.62	-0.03	16.17	0.01	0.10	1.06
4	1.23	4.90	-0.05	11.21	0.09	0.92	1.07
5	1.21	5.50	-0.09	7.80	0.01	0.09	1.16
6	1.31	6.36	-0.07	5.70	0.13	1.00	1.22
7	1.21	7.01	-0.07	4.24	0.11	0.78	1.21
8	0.87	7.93	-0.12	3.07	-0.25	-1.29	1.28
9	0.79	8.71	-0.20	2.21	-0.34	-1.62	1.32
High Vol.	0.65	9.69	-0.43	1.26	-0.48	-1.53	1.32
10–1	-0.66		-0.44		-0.74	-2.08	0.49

Table 13: Portfolios sorted on the previous month's idiosyncratic volatility. Avg. R is the monthly average return, Std. is the standard deviation of the portfolio return, IDEF3 is the idiosyncratic volatility effect measured according to equation (18), Mkt. Cap. is the market capitalization of the portfolio in percent, alpha is the 3 factor alpha, $t-$stat. is the robust Newey-West $t-$statistics for alpha and beta is the market beta from the 3 factor model. The line labelled $10-1$ provides the same statistics for the long/short portfolio. The sample period is January 1982 to December 2007 and all values are monthly percentage points.

	Avg. R	Std.	IDEF4	Mkt. Cap.	alpha	$t-$stat.	beta
Low Vol.	1.31	3.75	0.63	26.00	0.26	2.77	0.83
2	1.38	4.08	0.13	22.29	0.28	2.53	0.94
3	1.20	4.62	-0.18	16.17	0.02	0.21	1.06
4	1.25	4.91	0.43	11.23	0.12	1.17	1.07
5	1.19	5.51	0.12	7.81	-0.02	-0.20	1.16
6	1.31	6.42	-0.03	5.71	0.14	1.06	1.22
7	1.23	7.02	-0.03	4.25	0.14	0.98	1.21
8	0.81	7.94	-0.04	3.08	-0.32	-1.58	1.28
9	0.81	8.75	-0.04	2.21	-0.31	-1.51	1.33
High Vol.	0.62	9.69	-0.10	1.25	-0.51	-1.66	1.32
10–1	-0.70		-0.73		-0.78	-2.18	0.49

Table 14: Portfolios sorted on the previous month's idiosyncratic volatility. Avg. R is the monthly average return, Std. is the standard deviation of the portfolio return, IDEF4 is the idiosyncratic volatility effect measured according to equation (19), Mkt. Cap. is the market capitalization of the portfolio in percent, alpha is the 3 factor alpha, $t-$stat. is the robust Newey-West $t-$statistics for alpha and beta is the market beta from the 3 factor model. The line labelled $10-1$ provides the same statistics for the long/short portfolio. The sample period is January 1982 to December 2007 and all values are monthly percentage points.

	Avg. R	Std.	IDEF5	Mkt. Cap.	alpha	$t-$stat.	beta
Low Vol.	2.79	6.40	-0.06	55.66	-0.08	-0.28	0.84
2	3.13	7.38	0.01	24.05	-0.00	-0.01	0.93
3	3.62	10.34	-0.14	11.83	0.64	1.09	1.04
4	2.56	13.89	-0.35	5.84	-0.25	-0.25	1.18
High Vol.	1.08	17.29	-0.58	2.61	-1.68	-1.76	1.27
5–1	-1.82		-0.52		-1.55	-1.55	0.42

Table 15: Portfolios sorted on the previous month's idiosyncratic volatility. Avg. R is the monthly average return, Std. is the standard deviation of the portfolio return, IDEF5 is the idiosyncratic volatility effect measured according to equation (17), Mkt. Cap. is the market capitalization of the portfolio in percent, alpha is the 3 factor alpha, $t-$stat. is the robust Newey-West $t-$statistics for alpha and beta is the market beta from the 3 factor model. The line labelled $5-1$ provides the same statistics for the long/short portfolio. The sample period is January 1982 to December 2007 with quarterly sampling frequency and all values are quarterly percentage points.

	Losers	2	3	4	5	6	7	Winners
Low Vol.	4	25	50	65	62	48	24	6
2	9	32	46	52	53	48	34	10
3	13	36	44	45	47	45	38	15
4	20	38	41	40	41	43	41	20
5	26	40	38	35	36	39	42	28
6	34	40	35	32	31	34	42	35
7	43	41	31	27	28	31	40	43
8	53	39	28	24	24	27	37	53
9	65	36	24	20	20	23	33	63
High Vol.	88	27	18	15	14	16	25	86

Table 16: Average number of firms in portfolios double sorted on the previous month's idiosyncratic volatility and the previous month return. The sample period is January 1982 to December 2007. All values are rounded to the nearest integer.

	Losers	2	3	4	5	6	7	Winners
Low Vol.	-0.29	0.51	-0.19	0.26	-0.11	0.28	0.27	0.13
2	-0.02	-0.32	-0.35	0.37	-0.26	1.11	0.70	0.24
3	-0.41	-1.29	0.34	-0.43	0.01	-0.14	-0.19	-0.24
4	-0.91	-0.93	-0.99	-0.20	0.42	0.32	0.38	0.17
5	-2.04	-1.95	-0.47	-0.89	-2.06	-0.00	-0.48	2.48
6	-2.99	-1.92	-2.59	-1.33	-0.66	0.83	-0.31	-0.52
7	-4.01	-2.57	-0.09	-1.51	0.02	0.81	0.66	3.57
8	-5.98	-3.97	-0.97	-1.12	-0.77	-0.52	1.09	0.26
9	-16.64	-3.05	-1.83	0.13	-1.03	-0.46	-0.69	1.05
High Vol.	-25.67	-4.63	-0.69	-1.38	-0.61	-0.94	0.09	-5.46

Table 17: Average idiosyncratic volatility effect measured according to equation (14) in portfolios double sorted on the previous month's idiosyncratic volatility and the previous month's return. The sample period is January 1982 to December 2007 and all values are monthly basis points.

	Low DISP	2	3	4	High DISP
Low Vol.	-0.19	0.08	1.52	0.65	-0.97
2	-0.80	0.65	0.81	-1.10	-1.64
3	-0.47	0.71	-0.48	-2.30	-5.14
4	-0.07	1.72	-0.90	-2.64	-4.79
High Vol.	-0.81	-0.58	-6.81	-9.82	-11.96
alpha (High - Low)	0.13	-8.50	-24.25	-25.07	-4.41
t-stat	0.04	-1.04	-2.35	-2.49	-0.37
IDEF (High - Low)	-0.62	-0.67	-8.33	-10.47	-10.99

Table 18: Average idiosyncratic volatility effect in portfolios double sorted on the lagged month's idiosyncratic volatility and the previous month's dispersion of analysts' forecasts. Alpha (High - Low) is the alpha obtained by regressing the return of a portfolio long in high volatility and short in low volatility within each dispersion group on the 3 Fama- French factors. The sample period is January 1982 to December 2007 and all values are monthly basis points.

	Low DISP	2	3	4	High DISP
Low Vol.	-0.13	0.60	1.61	0.51	-0.87
2	-0.26	0.19	1.49	-0.16	-1.14
3	-0.51	0.97	-0.22	-1.63	-2.81
4	-0.18	0.89	0.46	-4.80	-7.64
High Vol.	-1.54	-3.91	-16.63	-15.32	-18.79
alpha (High - Low)	18.69	-18.98	-45.70	-35.08	27.47
t-stat	4.05	-1.13	-2.00	-1.89	1.81
IDEF (High - Low)	-1.41	-4.50	-18.24	-15.83	-17.92

Table 19: Average idiosyncratic volatility effect in portfolios double sorted on the contemporaneous month's idiosyncratic volatility and the previous month's dispersion of analysts' forecasts. Alpha (High - Low) is the alpha obtained by regressing the return of a portfolio long in high volatility and short in low volatility within each dispersion group on the 3 Fama- French factors. The sample period is January 1982 to December 2007 and all values are monthly basis points.

	Low DISP	2	3	4	High DISP
Low Vol.	92	151	156	107	63
2	90	123	140	125	92
3	101	107	116	125	119
4	121	97	91	118	142
High Vol.	166	91	66	94	154

Table 20: Average number of firms in portfolios double sorted on the lagged month's idiosyncratic volatility and the previous month's dispersion of analysts' forecasts. The sample period is January 1982 to December 2007 and all values are monthly basis points.