Phase Retrieval by a Conditional Wavelet Flow: Applications to Near-field X-ray Holography

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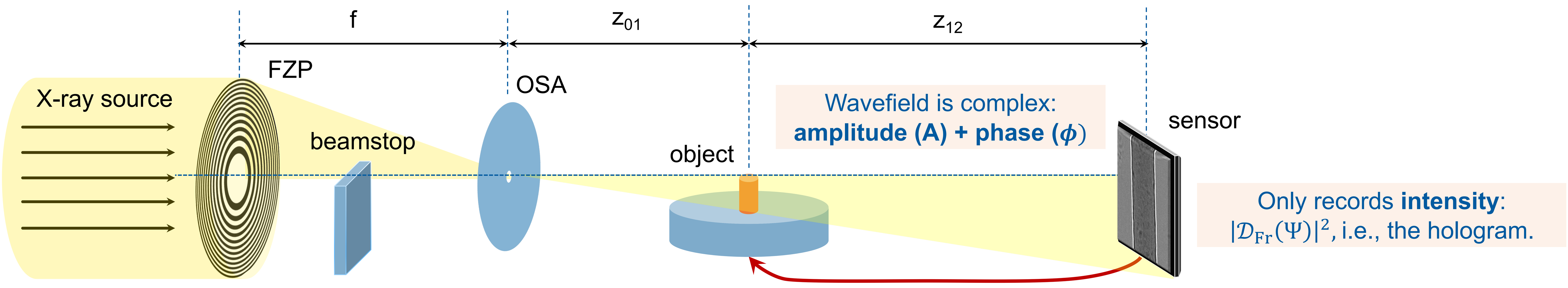
Near-Field X-ray Holography



FZP: Fresnel Zone Plate
OSA: Order Sorting Apertures

Phase information is **lost**: ill-posed problem

Conventional algorithms
- Assume certain object properties and optical propagation regimes
- Time-consuming process of tuning a wide range of free parameters
- Inference: minutes/projection

Machine learning-assisted phase retrieval
+ Fast inversion for large image datasets
+ Parallelized training of different image resolutions
+ Inference: seconds/projection

Fresnel free-space propagator:

$$\mathcal{D}_{\mathrm{Fr}}(\Psi) = \mathcal{F}^{-1}\{\exp((-i\pi)/(2\,\mathrm{Fr})(\varepsilon^2+\eta^2))\mathcal{F}[\Psi]\}$$

Ψ: complex wavefield ($\Psi = A\exp(i\phi)$)
ε, η: inverse space coordinates
Fr: Fresnel number ($\mathrm{Fr} = \Delta x^2/\lambda z$)
Δx: detector pixel size
λ: source wavelength
z: propagation distance

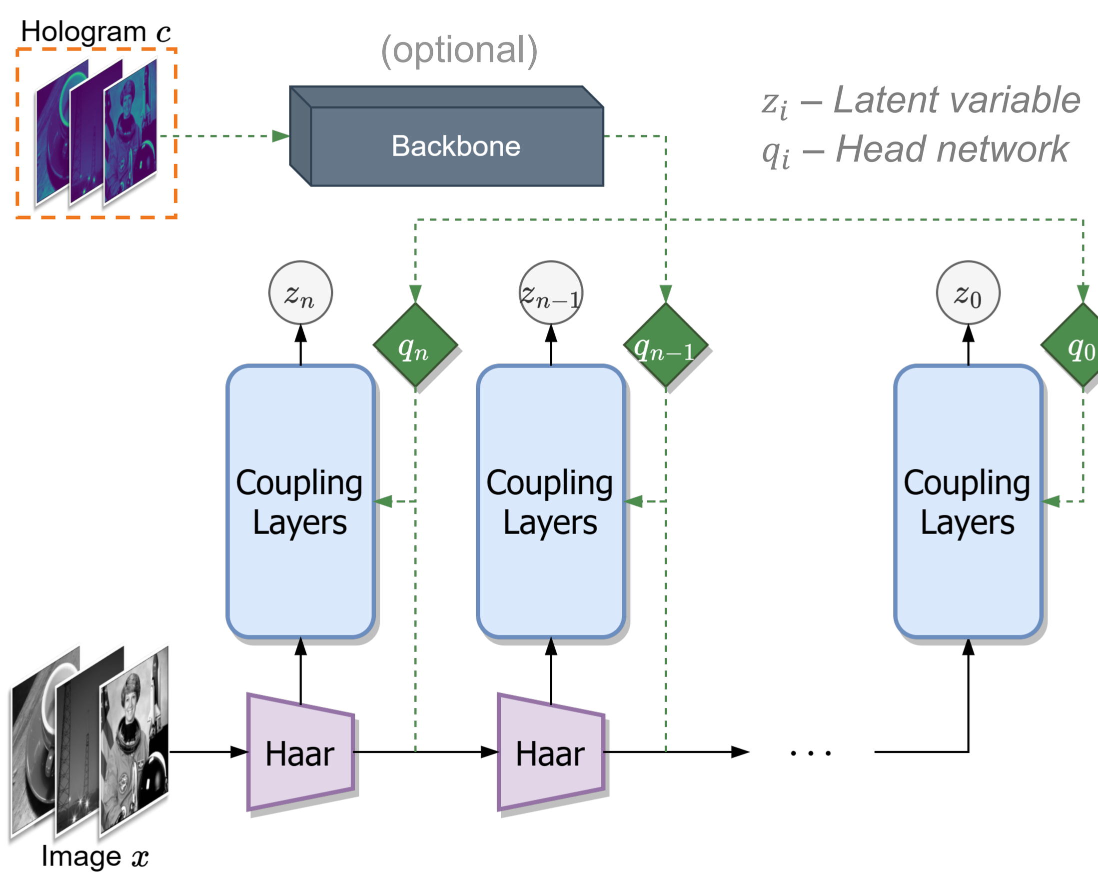
The dimensionless Fresnel number used as a single parameter shows the generality and transferability of the model.

Conditional Wavelet Flow

- A multi-scale normalizing flow architecture based on wavelets.
- Maps a complex distribution $p(x)$ to a distribution $p(z)$ which allows simple sampling by applying a series of **invertible transformations f_i** (coupling layers):

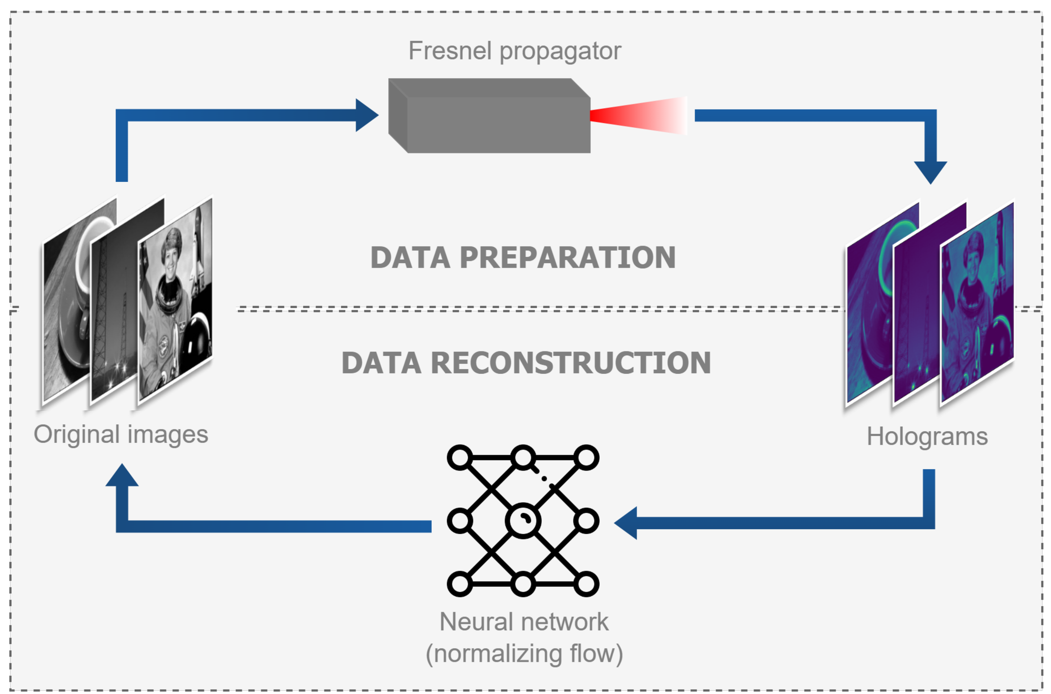
$$z = f(x) = f_1 \circ f_2 \circ \cdots \circ f_k(x)$$

Schematic diagram of a normalizing flow model.



Training path of Conditional Wavelet Flow.

Training
- **Input x** is decomposed by a **Haar wavelet transform**.
- **Coupling layers** learn the conditional distribution of the details. The remaining average is forwarded to the next Conditional Wavelet Flow level.
- **Hologram** is used as a **conditional input** from which features are extracted from.



Loss function is based on negative-log likelihood:

$$\mathrm{BPD} = \frac{-\log \mathcal{L}(\theta)}{\mathrm{H}\cdot\mathrm{W}\cdot\mathrm{C}\cdot\log 2}$$

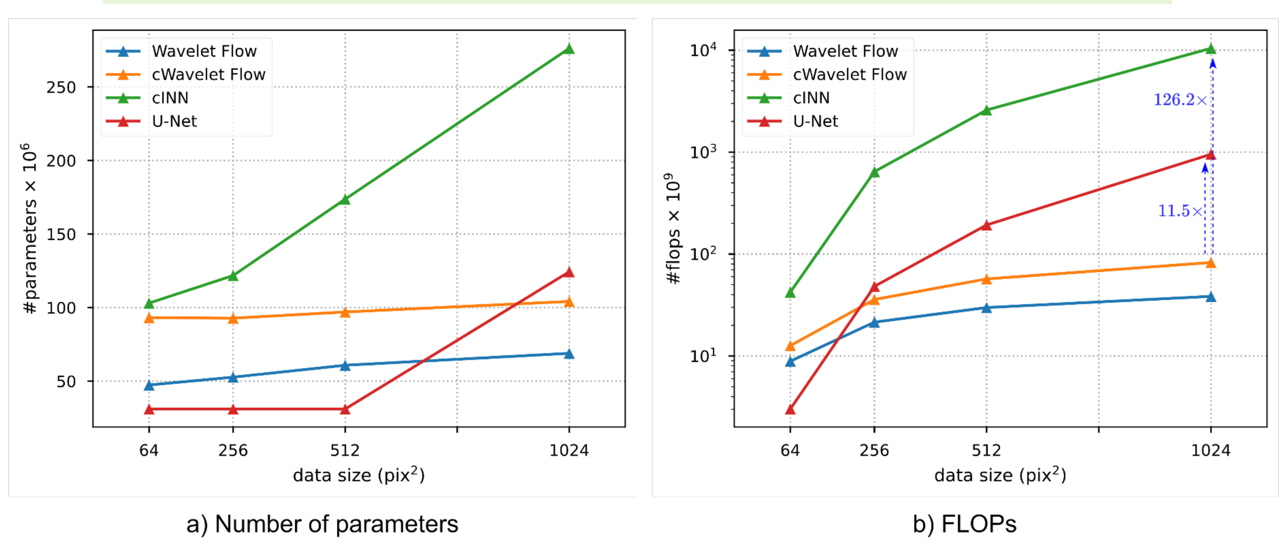
BPD: bits-per-dimension
$\mathcal{L}(\theta) = p_\theta(x)$
θ: model parameters
H, W, C: height, width, channel

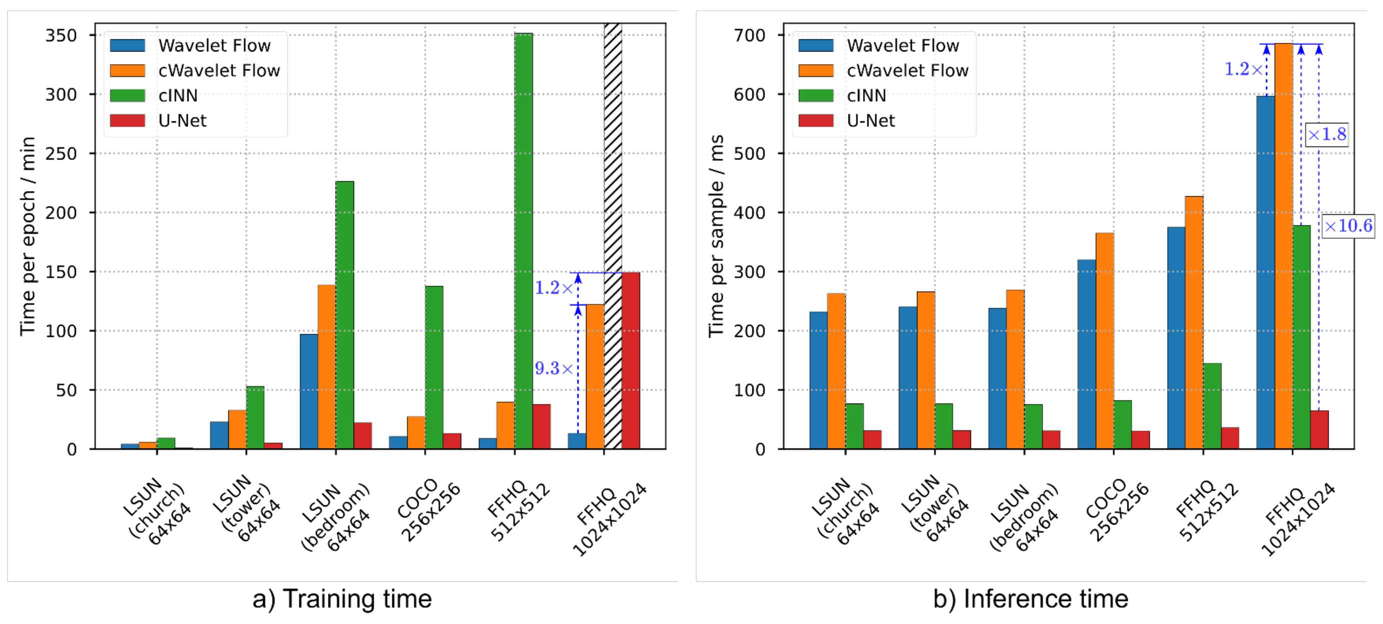
Reconstruction
- To reconstruct images with the trained model, **the "flow"** is simply reversed (from z to x).
- The **path of the hologram** remains the same.

Note: Poisson noise is added to the simulated holograms.

The number of trainable parameters and FLOPs stand for **model capacity and complexity**, respectively.

↑ model capacity and complexity, ↑ computational resources and time





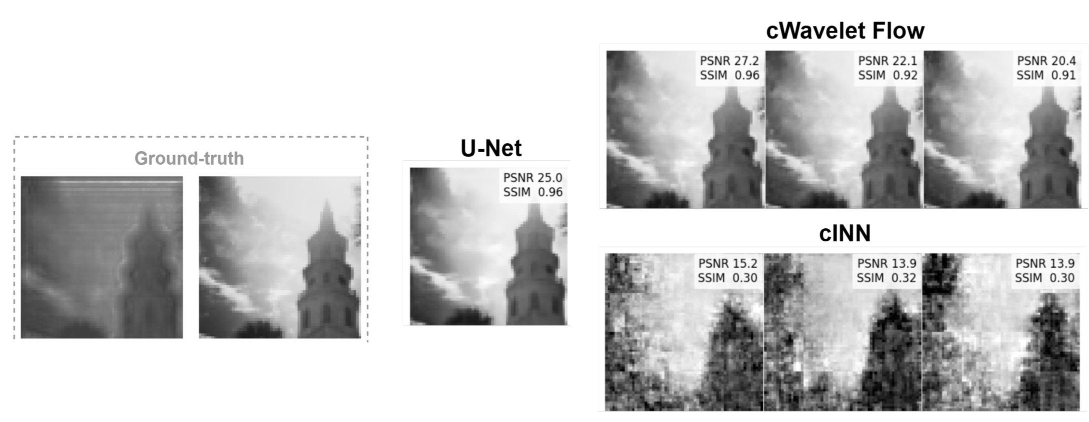
Conditional Wavelet Flow (cWavelet Flow) has **much more efficient training** compared to the other models.

Inference has a higher latency than cINN and U-Net.

cWavelet Flow and **U-Net** can both reconstruct **images of high visual fidelity**.

U-Net **cannot generate diverse reconstruction** given the same hologram.

cINN fails to generate images of meaningful contexts and only **outputs similar samples → mode collapse**.



References
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2. JJ Yu et al., "Wavelet Flow: Fast Training of High Resolution Normalizing Flows", NeurIPS 2020.

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