Inflation Risk Premia in the Euro Area and the United States*

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We use a joint model of macroeconomic and term structure dynamics to estimate inflation risk premia and inflation expectations in the United States and the euro area. To sharpen our estimation, we include in the information set macro data and survey data on inflation and interest rate expectations at various future horizons, as well as term structure data from both nominal and index-linked bonds.

Our results indicate that, over the post-2004 period when index-linked bond markets were sufficiently developed in both monetary areas, inflation risk premia across various maturities had strikingly similar properties in the United States and in the euro area: their dynamics and their levels, especially over the years until mid-2011, have remained quite close to each other, even if premia appear to be subject to somewhat greater high-frequency volatility in the United States.

After correcting for liquidity and inflation risk premia, long-term inflation expectations extracted from bond prices have remained remarkably stable at the peak of the financial crisis and throughout the Great Recession. For the United States, we also document a downward shift in the perceived

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inflation target, from approximately 3 percent until 2011 to levels closer to 2 percent following the FOMC announcement of a numerical long-term inflation goal.

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1. Introduction

As markets for inflation-linked securities have grown in recent years, the prices of such securities have increasingly become an important source of information about the state of the economy for market participants as well as central banks and other public institutions. Index-linked bonds, for example, provide a means of measuring ex ante real yields at different maturities. In combination with nominal yields, observable from markets for nominal bonds, real rates also allow us to calculate a "break-even inflation rate," i.e., the rate of inflation for which the payoff from the two types of bonds would be equal. Because of their timeliness and simplicity, break-even inflation rates are seen as useful indicators of the markets' expectations of future inflation. Moreover, implied forward break-even inflation rates for distant horizons are often viewed as providing information about the credibility of the central bank's commitment to maintaining price stability.

Break-even rates on default-free bonds do not, however, only reflect inflation expectations. They also include risk premia, notably to compensate investors for inflation risk and for differential liquidity risk in the nominal and index-linked bond markets. At the peak of the financial crisis in the last quarter of 2008, for example, ten-year break-even rates fell markedly in both the euro area and the United States, to a minimum of 1.2 and 0.4 percentage points, respectively. Should such levels be interpreted as correct measures of long-term inflation expectations, or do they rather reflect changes in liquidity and/or inflation risk premia?

The effects of liquidity premia on the price of index-linked bonds, and specifically their impact on break-even rates in the last quarter of 2008, have already been studied in the literature—see, e.g., Gürkaynak, Sack, and Wright (2010) and Pflueger and Viceira (2013). In this paper we complement those studies with an analysis of investors' inflation expectations and inflation risk premia. In so doing, we adopt a consistent modeling framework across the euro area and the United States. We can therefore provide a comparative analysis of the main features of inflation risk premia for the two largest world monetary areas.

Our methodology has three distinctive features. First, we model jointly output, inflation, monetary policy, and term structure dynamics in order to analyze the relationship between inflation risk premia and the macroeconomy. More specifically, building on Ang and Piazzesi (2003), we adopt the framework developed in Hördahl, Tristani, and Vestin (2006), in which bonds are priced based on the dynamics of the short rate obtained from the solution of a forward-looking macro model and using an essentially affine stochastic discount factor (see Duffie and Kan 1996; Dai and Singleton 2000; Duffee 2002).

Second, we estimate the model using a broad information set, which includes macro variables, nominal and index-linked yields, and survey information on expectations of future inflation and future policy interest rates. Macro variables allow us to identify some stylized facts on the cyclical features of inflation risk premia. Survey expectations help us to disentangle expectations dynamics from movements in risk premia—see Kim and Orphanides (2012).

Finally, to reduce the risk that liquidity factors might distort our estimates, we pre-filter index-linked data based on commonly used measures of liquidity risk. Our results can therefore provide indications on inflation expectations embodied in bond prices, after correcting for both inflation and liquidity risk premia.

We document that ten-year inflation expectations incorporated in bond prices in the United States fluctuate around levels slightly below 3 percent up until 2011 and then move close to 2 percent over the course of 2012. This timing is consistent with a realignment of long-term inflation expectations following the Federal Open Market Committee (FOMC) press release on January 25, 2012, which specified a 2 percent long-run inflation goal. In the euro area, long-term inflation expectations remain very close to levels slightly below 2 percent over the whole sample period. These results provide further support to the view that the availability of an official numerical definition of price stability plays an important role in anchoring long-term inflation expectations.

The estimated model-implied expected long-run inflation rates have been characterized by somewhat greater high-frequency volatility in the United States than in the euro area throughout much of our sample, consistent with the findings of Beechey, Johannsen, and Levin (2011). We find that despite the severe turbulence in bond markets during the financial crisis, long-run inflation expectations remained quite stable at the peak of the crisis and throughout the Great Recession in both the United States and the euro area. This stability in inflation expectations during the crisis is in line with survey-based evidence reported by Galati, Poelhekke, and Zhou (2011) for the crisis period. Financial markets therefore appear to be similarly confident about the ability of the Federal Reserve and of the European Central Bank (ECB) to deliver a stable inflation rate over medium and long-run horizons.

Concerning inflation risk premia, the broad conclusion of our investigation is that, over the post-2004 period when index-linked bond markets were sufficiently developed in both monetary areas, such premia display remarkably similar properties in the United States and in the euro area across various maturities: their dynamics and their levels, especially over the years of the financial crisis until mid-2011, remain quite close to each other, even if U.S. premia appear to be subject to greater high-frequency volatility. Over 2012 and the initial months of 2013, however, there is a decoupling of inflation risk premia, especially at three- to five-year maturities: such premia increase temporarily to higher levels in the United States, while they fall to slightly negative territory in the euro area. In our model these developments are mainly related to differences in growth prospects.

The dynamics of inflation risk premia are positively correlated with actual inflation rates in both monetary areas: premia become higher when inflation increases. Over the past five years, they are also somewhat procyclical and, in particular, they fall during the Great Recession.

The rest of this paper is organized as follows. The next section describes our model, its implications for the inflation risk premium, and the econometric methodology, while section 3 discusses the data and our methodology to correct index-linked yields for liquidity premia. The empirical results are presented in section 4, where we show our parameter estimates and their implications for term premia and inflation risk premia. In this section, we also relate premia to their macroeconomic determinants and calculate premium-adjusted break-even inflation rates. Section 5 concludes the paper.

2. The Model

We rely on a simple economic model in the New Keynesian tradition and specified directly at the aggregate level. The model includes a forward-looking Phillips curve (e.g., Galí and Gertler 1999) and a consumption-Euler equation (e.g., Fuhrer 2000). Compared with the alternative of using a microfounded model, the advantage of this approach is that it imposes milder theoretical constraints. This flexibility allows us to provide descriptive evidence on the dynamics of risk premia, conditional on a widely used law of motion for macroeconomic variables and on the assumption of rational expectations. The evidence, in turn, can be interpreted as a stylized fact that successful microfounded models should be able to match. The flexibility, however, comes at a price: in the absence of a microfounded stochastic discount factor, we are unable to explain why certain risks appear to be priced more than others from an empirical viewpoint.

The specification of the model is similar to that in Hördahl, Tristani, and Vestin (2006), and we therefore describe it only very briefly here. The model includes two equations which describe the evolution of inflation, π_t in deviation from its mean $\bar{\pi}$, and the output gap, x_t:

$$\pi_t = \frac{\mu_\pi}{12}\sum_{i=1}^{12} E_t\left[\pi_{t+i}\right] + (1-\mu_\pi)\sum_{i=1}^{3}\delta_{\pi,i}\pi_{t-i} + \delta_x x_t + \varepsilon_t^\pi, \quad (1)$$

$$x_t = \frac{\mu_x}{12}\sum_{i=1}^{12} E_t\left[x_{t+i}\right] + (1-\mu_x)\sum_{i=1}^{3}\zeta_{x,i}x_{t-i} - \zeta_r(r_t - E_t\left[\pi_{t+1}\right]) + \varepsilon_t^x, \quad (2)$$

where r_t is the one-month nominal interest rate (in deviation from its mean $\bar{r}$), and inflation is defined as the year-on-year change in the log-price level. The lead and lag structure reflects the fact that we will be using monthly data for the estimations.

In this setup, inflation can be due to demand shocks ε_t^x, which increase output above potential and create excess demand, and to cost-push shocks ε_t^π, which have a direct impact on prices. In addition, monetary policy can affect inflation by changing the real interest rate $r_t - E_t[\pi_{t+1}]$ or influencing expectations. Specifically, we assume that the central bank sets the nominal short rate according to a forward-looking rule of a type proposed by Taylor (1993):

$$r_t = (1-\rho)\left\{\beta\left(E_t\left[\pi_{t+11}\right] - \pi_t^*\right) + \gamma x_t\right\} + \rho r_{t-1} + \eta_t, \qquad (3)$$

where π_t^* is the perceived inflation target and η_t is a monetary policy shock. The inflation target is allowed to be time varying to allow for some evolution in the behavior of monetary policy over time, or at least in the way monetary policy was perceived by market participants. The coefficient ρ captures interest rate smoothing, reflecting the central bank's desire to avoid making nominal interest rates excessively volatile.

Finally, we assume that the monetary policy shock is serially uncorrelated, while the other structural shocks may be correlated.[1] All shocks are assumed to be normally distributed with constant variance. The unobservable inflation target is assumed to follow an AR(1) process

$$\pi_t^* = (1-\phi_{\pi^*})\,\bar{\pi} + \phi_{\pi^*}\pi_{t-1}^* + u_{\pi^*,t}, \qquad (4)$$

where $u_{\pi^*,t}$ is a normal disturbance with constant variance uncorrelated with the other structural shocks.[2]

The linear Taylor rule in equation (3) is known to provide a good characterization of monetary policy decisions under normal circumstances—see, e.g., Hördahl and Tristani (2012), where the Taylor rule is used to describe euro-area policy rates over the January 1999 to June 2007 period. A linear policy rule is, however, less suitable when interest rates are near the zero lower bound, as it does not rule out the possibility of negative interest rates. Ignoring the

[1]In principle, the monetary policy shock could also be allowed to be serially correlated. We assume instead that the interest rate smoothing component, in combination with the other persistent elements in the policy rule, will be sufficient to capture all of the persistence in the short rate.

[2]To ensure stationarity of the inflation target process, we impose an upper limit of 0.99 on the ϕ_{π^*} parameter during the estimation process. This restriction is binding.

zero lower bound constraint is particularly problematic in the United States, where policy rates have been very close to zero since December 2008. It could produce incorrect inference concerning expected future policy rates and, in turn, for the term structure of nominal risk premia.

These drawbacks are mitigated by our use of survey expectations on future short-term interest rates, which help discipline model-implied interest rate forecasts. In any case, we focus on inflation risk premia at medium to long-term horizons, which should be less affected by the non-linearity induced by the zero bound constraint.

In order to obtain the rational expectations solution to the model, we write it in state-space form and proceed to solve it numerically (see the appendix). The result is two matrices $\mathbf{M}$ and $\mathbf{C}$ which give the law of motion of the predetermined variables $\mathbf{X}_{1,t} = [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_t^*, \eta_t, \varepsilon_t^\pi, \varepsilon_t^x, r_{t-1}]'$ and which describe how the non-predetermined variables $\mathbf{X}_{2,t} = [E_t x_{t+11}, \ldots, E_t x_{t+1}, x_t, E_t \pi_{t+11}, \ldots, E_t \pi_{t+1}, \pi_t]'$ depend on $\mathbf{X}_{1,t}$. Specifically, we obtain

$$\mathbf{X}_{1,t} = \mathbf{M}\mathbf{X}_{1,t-1} + \Sigma \xi_{1,t}, \tag{5}$$

$$\mathbf{X}_{2,t} = \mathbf{C}\mathbf{X}_{1,t}, \tag{6}$$

where ξ_1 is a vector of independent, normally distributed shocks, and we also get an expression for the equilibrium short-term interest rate in terms of the state variables: $r_t = \mathbf{\Delta}'\mathbf{X}_{1,t}$.

Because the state variables follow a first-order Gaussian VAR and the short-term interest rate is expressed as a linear function of the state vector, we can derive bond prices once we impose the assumption of absence of arbitrage opportunities and specify a process for the pricing kernel. The appendix describes the pricing kernel we choose. One important aspect here is the choice of the market prices of risk. Following Duffee (2002), we assume that these risk prices are affine functions of the states, or, more precisely, affine functions of a particular transformation of the original states. Specifically, we define the transformed state vector $\mathbf{Z}_t \equiv [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_t^*, r_t, \pi_t, x_t, r_{t-1}]'$, which is obtained as $\mathbf{Z}_t = \hat{\mathbf{D}}\mathbf{X}_{1,t}$ for a suitably defined matrix $\hat{\mathbf{D}}$. Working with $\mathbf{Z}_t$ rather than $\mathbf{X}_{1,t}$ facilitates the interpretation of the results later on. Our specification for the market prices of risk therefore has the following form:

$$\lambda_t = \lambda_0 + \lambda_1 \mathbf{Z}_t, \tag{7}$$

so that the risk premium associated with each of the four shocks in $\xi_{1,t}$ can depend on the level of all the state variables. To keep the number of parameters manageable, we allow only the 4×4 submatrix in λ_1 corresponding to the non-lagged states $[\pi_t^*, r_t, \pi_t, x_t]$ in $\mathbf{Z}_t$ to be non-zero.

Given the setup described above, we can write the continuously compounded yield y_t^n on a zero-coupon nominal bond with maturity n as

$$y_t^n = A_n + B_n' \mathbf{Z}_t, \tag{8}$$

where the A_n and B_n' matrices can be derived using recursive relations (see the appendix). Stacking all yields in a vector $\mathbf{Y}_t$, we write the above equations jointly as $\mathbf{Y}_t = \mathbf{A} + \mathbf{B}\mathbf{Z}_t$ or, equivalently, $\mathbf{Y}_t = \mathbf{A} + \tilde{\mathbf{B}}\mathbf{X}_{1,t}$, where $\tilde{\mathbf{B}} \equiv \mathbf{B}\hat{\mathbf{D}}$. Similarly, for real bonds y_t^{*n} we obtain

$$y_t^{*n} = A_n^* + B_n'^* \mathbf{Z}_t, \tag{9}$$

and $\mathbf{Y}_t^* = \mathbf{A}^* + \tilde{\mathbf{B}}^* \mathbf{X}_{1,t}$, with $\tilde{\mathbf{B}}^* \equiv \mathbf{B}^* \hat{\mathbf{D}}$.

Given the solutions for real and nominal bonds, we can derive the inflation risk premium as the difference between historical and risk-adjusted expectations of future inflation rates. In so doing, we follow closely Hördahl and Tristani (2012)—see also the appendix.

2.1 Estimation

Given the large number of parameters involved in the estimation, we do not directly maximize the likelihood, but we introduce priors and proceed by relying on Bayesian estimation methods. Specifically, we maximize the posterior density function obtained by combining the log-likelihood function with the prior density for the model parameters. An advantage of such an approach is that it allows us to exploit prior information on structural economic relationships available from previous studies. Moreover, the inclusion of prior distributions brings an added advantage in that it tends to make the optimization of the highly non-linear estimation problem more stable.

We evaluate the model likelihood using the Kalman filter. We first define a vector $\mathbf{W}_t$ containing the observable contemporaneous variables,

$$\mathbf{W}_t \equiv \begin{bmatrix} \mathbf{Y}_t \\ \mathbf{Y}_t^* \\ \mathbf{X}_{2,t}^o \\ \mathbf{U}_t \end{bmatrix},$$

where $\mathbf{Y}_t$ and $\mathbf{Y}_t^*$ denote vectors of nominal and real zero-coupon yields, $\mathbf{X}_{2,t}^o = [x_t, \pi_t]'$ contains the macro variables, and $\mathbf{U}_t$ denotes survey expectations that are included in the estimation (see below). The dimension of $\mathbf{W}_t$ is denoted n_y. Next, we partition the vector of predetermined variables into observable ($\mathbf{X}_{1,t}^o$) and unobservable variables ($\mathbf{X}_{1,t}^u$) according to

$$\mathbf{X}_{1,t}^o = [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, r_{t-1}]',$$
$$\mathbf{X}_{1,t}^u = [\pi_t^*, \eta_t, \varepsilon_t^\pi, \varepsilon_t^x]'.$$

We treat survey expectations of future interest rates and inflation as noisy observations of the model-implied forecasts. Survey data can then be written as suitable linear functions of the states plus an observation error:

$$\mathbf{U}_t = \mathbf{G}\mathbf{X}_{1,t} + u_t.$$

For example, for an inflation forecast j periods ahead, we know, based on the model solution, that $E_t[\pi_{t+j}] = \mathbf{C}_\pi E_t[\mathbf{X}_{1,t+j}] = \mathbf{C}_\pi \mathbf{M}^j \mathbf{X}_{1,t}$, where $\mathbf{C}_\pi$ picks the row in $\mathbf{C}$ corresponding to the inflation equation. Thus, if $\mathbf{U}_t$ contained survey data reflecting this expectation, the corresponding row in $\mathbf{G}$ would be defined as $\mathbf{C}_\pi \mathbf{M}^j$. Similar expressions would be derived for the interest rate survey forecasts included in the estimations (the specific survey data used is discussed in the next section).

When defining $\mathbf{G}$, we take into account that some of the survey forecasts refer to average expectations over the future—e.g., over a quarter or over several years. For the exact form of the Kalman filter matrices, see the appendix. We start the filter from the unconditional mean and the unconditional mean squared error (MSE) matrix (see Hamilton 1994). The Kalman filter produces forecasts of the states and the associated MSE, which feed into the likelihood function.

Our assumptions regarding the prior distribution of the macro parameters, which are broadly in line with those implied by typical calibrations of the New Keynesian model, are listed in tables 3 and 4. For the market prices of risk (the elements in λ_0 and λ_1), we assume normal priors centered at zero with very large standard errors, reflecting our lack of prior information regarding these parameters. We proceed to find the mode of the posterior density using the simulated annealing algorithm (see Goffe, Ferrier, and Rogers 1994) and simulate the posterior by drawing from a distribution centered at the mode using the Metropolis-Hastings sampling algorithm (see An and Schorfheide 2007).[3]

3. Data

We estimate the model using monthly data on nominal and real zero-coupon Treasury yields, inflation, a measure of the output gap, and survey expectations of the short-term interest rate and inflation. The model is applied to U.S. and euro-area data. To avoid obvious structural break issues associated with the introduction of the single currency, we limit our euro-area sample to the period January 1999–April 2013. For the United States, we include more historical data and start our sample in January 1990.

We treat the yields on index-linked bonds as reflecting risk-free real yields, i.e., we assume that the inflation risk borne by investors because of the indexation lag (the fact that there exists a lag between the publication of the inflation index and the indexation of the bond) is negligible. Evans (1998) estimates the indexation-lag premium for UK index-linked bonds and finds that it is likely to be quite small, around 1.5 basis points. Since the indexation lag in the United Kingdom is 8 months, while the lag in the United States and the euro area is only 2.5–3 months, it seems likely that any indexation-lag premium for these two markets would be even smaller than Evans's estimate.

[3]The estimations were performed using modified versions of Frank Schorfheide's Gauss code for Bayesian estimation of DSGE models. His original code is available at http://www.econ.upenn.edu/~schorf/.

In addition to the aforementioned premium, the indexation lag can induce deviations in index-linked yields away from the true underlying real yield due to inflation seasonality and to “carry” effects. Inflation seasonality matters because index-linked bonds are linked to the seasonally unadjusted price level, which means that bond prices will be affected due to the indexation lag, unless the seasonal effect at a given date is identical to that corresponding to the indexation lag (which is, in general, not the case; see, e.g., Ejsing, Grothe, and Grothe 2012). The carry effect refers to the fact that index-linked yields often contain some amount of realized inflation, due to predictable changes in inflation during the indexation-lag period (see D’Amico, Kim, and Wei 2008 for a discussion of the carry effect). While these lag effects can be sizable for short-dated bonds, they tend to be quite small for longer-term bonds. In this paper, we abstract from such effects, as they are likely to be of second-order importance for our purposes. By excluding short-term real yields (below three years) in the estimations, we reduce the risk that index-lag effects might influence our results to any significant extent.

3.1 U.S. Data

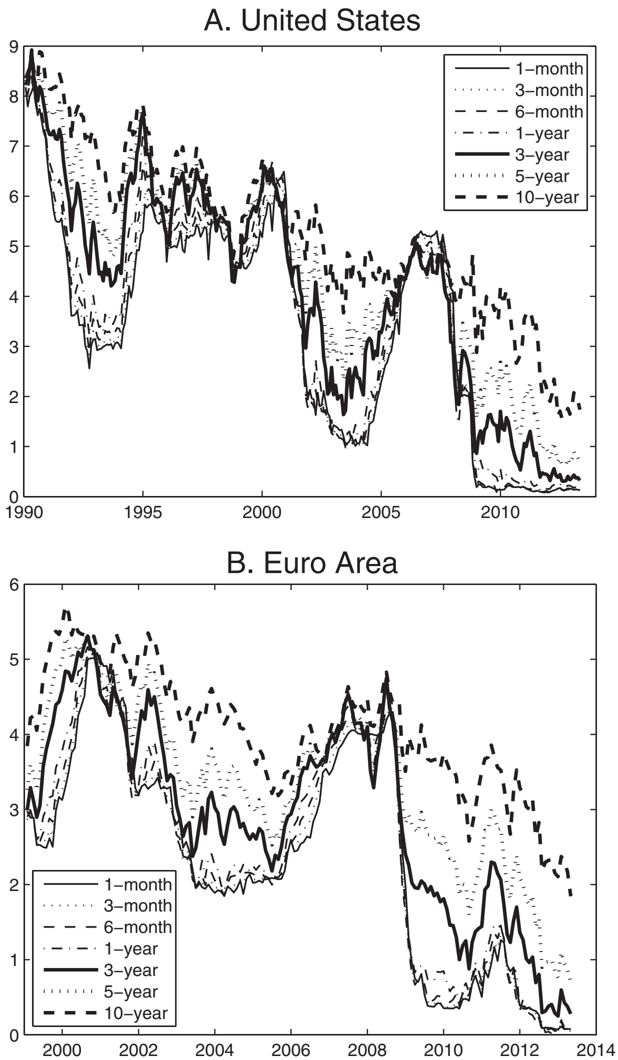
The U.S. real and nominal term structure data consists of zero-coupon yields based on the Nelson-Siegel-Svensson (NSS) method, which are available from the Federal Reserve Board.[4] For the nominal bonds, seven maturities ranging from one month to ten years are used in the estimation, while for the real bonds we include four maturities from three to ten years (figures 1A and 2A).

While nominal yield data is available from the beginning of the sample, real zero-coupon yields can be obtained only from 1999. We therefore treat them as unobservable variables prior to 1999 and include them in the measurement equation only thereafter. We also correct real yield for liquidity premia—see section 3.3.

Our inflation data is year-on-year (YoY) CPI (seasonally adjusted) log-differences, observed at a monthly frequency and scaled by 12 to get an approximate monthly measure, while the output gap is computed as the log-difference of real GDP and the

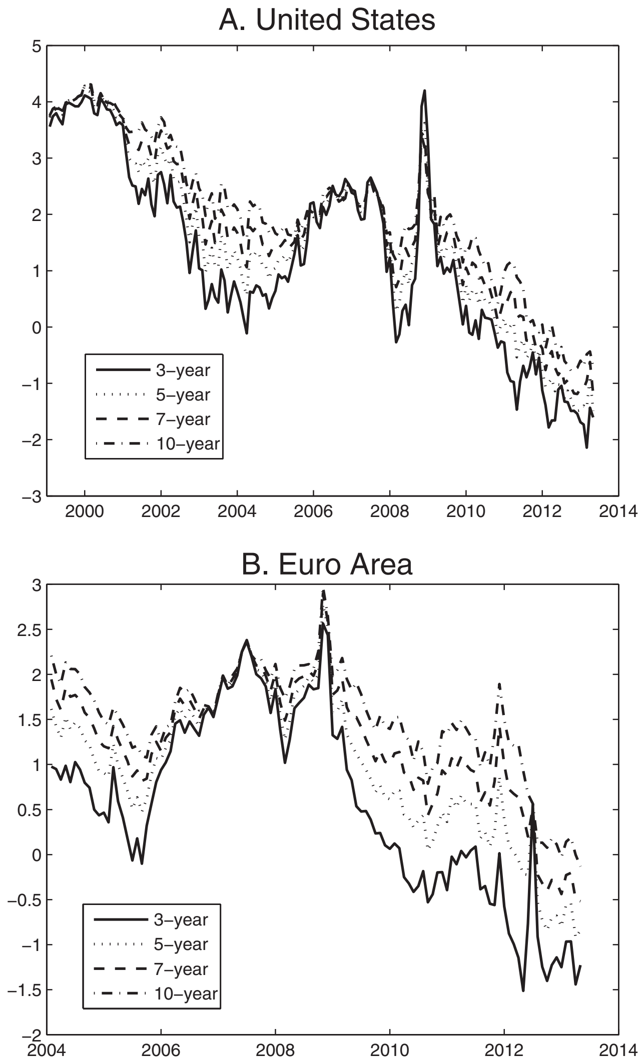
[4]This data is described in detail in Gürkaynak, Sack, and Wright (2007, 2010).

Figure 1. Nominal Zero-Coupon Yields



Congressional Budget Office's estimate of potential real GDP, which is a quarterly series. Since we estimate the model using a monthly frequency, and since the output gap is a state variable, we need a monthly series for the gap. This is obtained by fitting an ARMA(1,1) model to the quarterly gap series, forecasting the gap one quarter

Figure 2. Real Zero-Coupon Yields



ahead, and computing one- and two-month-ahead values by means of linear interpolation. This exercise is conducted in "real time," in the sense that the model is reestimated at each quarter using data only up to that quarter.

Following Kim and Orphanides (2012), we also use data on survey forecasts for inflation and the three-month interest rate obtained from the Federal Reserve Bank of Philadelphia's quarterly Survey of Professional Forecasters (SPF).[5] As argued by Kim and Orphanides (2012), survey data is likely to contain useful information for pinning down the dynamics of the state variables that determine the bond yields, which, due to the high persistence of interest rates, is a challenging task. For the United States, we include six survey series: the expected three-month interest rate two quarters ahead, four quarters ahead, and during the coming ten years, and the expected CPI inflation for the same horizons. These survey forecasts are available at a quarterly frequency, with the exception of the ten-year forecast of the three-month interest rate, which is reported only once per year. The surveys therefore enter the measurement equation only in those months when they are released.

3.2 Euro-Area Data

The data setup for the euro area is similar to that for the United States. We use nominal and real zero-coupon yields for the same maturities as in the U.S. case (figures 1B and 2B). The nominal yields are zero-coupon yields on French government bonds, reported by Bloomberg. An alternative could have been to instead use German nominal bond yields, which often are seen as a natural benchmark for the euro area. However, since our sample includes the most recent period when German yields have come under pressure due to flight-to-safety flows, we view the French government bond yields as more likely to reflect underlying macroeconomic fundamentals in the euro area during this period (and before the crisis, French and German yields were virtually indistinguishable).

For the real yields, we estimate zero-coupon rates using NSS, based on prices of government bonds linked to the euro-area HICP issued by France (obtained from Bloomberg). We exclude HICP-linked bonds issued by other countries—e.g., Italy and Greece—to avoid mixing bonds with different credit ratings, a problem that

[5]D'Amico, Kim, and Wei (2008) discuss at length various survey forecasts available for the United States. They conclude that inflation surveys based on the forecasts of business economists, such as the SPF, are preferable to consumer surveys.

became particularly acute during the euro-area sovereign debt crisis. Moreover, the French segment of the market is the largest in the euro area, which suggests that liquidity conditions in this market are likely to be relatively good.

The first HICP-linked government bond was issued by the French Treasury in November 2001, and the issuance of additional bonds was very gradual. For this reason, we were able to estimate a euro-area real zero-coupon curve only as of January 2004, which is the date as of which we include real yields in the estimation of our model.[6] The fact that we do not include the first few years of real yield data in the estimation is likely to reduce potential effects on our estimates arising from initial illiquid conditions in the index-linked market, similar to the U.S. case. Moreover, prior to the introduction of HICP-linked bonds, a market for French bonds linked to the French CPI had been growing since 1998, which may have had a positive impact on the overall liquidity situation for the euro-area index-linked bond market.

As in the U.S. case, our measure of inflation is monthly YoY HICP log-differences. For the output gap, we rely on the estimates by the European Commission—see D'Auria et al. (2010).[7]

The euro-area survey data we include in the estimation consists of forecasts for inflation obtained from the ECB's quarterly Survey of Professional Forecasters and three-month interest rate forecasts available on a monthly basis from Consensus Economics. The inflation forecasts refer to expectations of HICP inflation one, two, and five years ahead. The survey data for the short-term interest rate correspond to forecasts three and twelve months ahead.

3.3 Correction for Liquidity Premia

Well-known liquidity problems affect both the U.S. Treasury Inflation-Protected Securities (TIPS) and the euro-area index-linked

[6]Due to data limitations at the beginning of the sample, we included in the calculation of real zeros one A+ rated bond issued by the Italian Treasury during the first ten months of our sample. During this period, the yield difference between French and Italian government bonds was minimal.

[7]In a previous version of the paper, we followed Clarida, Galí, and Gertler (1998) and measured the output gap as deviations of real GDP from a quadratic trend. The results are not insensitive to this change of variable.

bonds, both during the years when these markets were initially created and at the peak of the financial crisis in 2008. D'Amico, Kim, and Wei (2008) provide an extensive discussion on the illiquidity of the TIPS market in the early years and argue that it resulted in severe distortions in TIPS yields. Gürkaynak, Sack, and Wright (2010) and Pflueger and Viceira (2013) provide estimates of liquidity premia during the financial crisis.

For these reasons, we correct real yields for liquidity premia prior to using them in estimation. Specifically, we regress break-even inflation rates on proxies for market liquidity and interpret the (negative of the) fitted values from the regressions as a measure of the time variation in liquidity premia on index-linked bonds.

For the United States, we follow Gürkaynak, Sack, and Wright (2010) and use two proxies for liquidity: the trading volume among primary dealers in TIPS, expressed as a share of total Treasury trading volume, and the spread between Resolution Funding Corporation (Refcorp) strips and Treasury strips. Refcorp-issued bonds are less liquid than Treasury bonds, but they are guaranteed by the U.S. Treasury. The spreads between these yields can therefore be interpreted as capturing a liquidity premium, which we use as a proxy for the liquidity premium on TIPS.

For the euro area, we do not have information on the volume of index-linked bonds. We therefore rely solely on the spread between Kreditanstalt für Wiederaufbau (KfW) bonds and German bunds as a proxy for the liquidity premium. KfW is a development bank involved in supporting public policies, and its bonds have an explicit guarantee from the Federal Republic of Germany. Ejsing, Grothe, and Grothe (2012) argue that this spread is a reliable and timely indicator of liquidity premia in the euro area.

The regression coefficients are shown in tables 1 and 2. All regressors have the expected sign and are significant for the United States as well as for the euro area. The U.S. spread regressors are, however, larger in absolute value. An increase in the Refcorp or KfW spread lowers the break-even inflation rate, but the elasticity at the ten-year maturity is almost -0.8 in the United States, compared with -0.35 in the euro area. In the United States, a higher TIPS volume increases the break-even inflation rate, as it reduces the TIPS liquidity premium and thereby lowers the real yield.

Table 1. Regression Results to Estimate Liquidity Premia in the United States (Sample Period: January 1999–April 2013)

Regressor	Three Year	Five Year	Seven Year	Ten Year
TIPS Relative Volume	0.365	0.299	0.249	0.246
	(0.134)	(0.083)	(0.077)	(0.053)
Refcorp Spread	−2.627	−1.836	−1.531	−0.770
	(0.709)	(0.516)	(0.504)	(0.296)
R^2	0.47	0.45	0.40	0.32

Notes: This table reports the results from regressions of three-, five-, seven- and ten-year break-even inflation rates (in percentage points) onto the TIPS volume as a share of total Treasury Primary Dealer trading volume (in percentage points), and the spread of ten-year Refcorp strips over their Treasury counterparts (in percentage points). Newey-West standard errors with a lag truncation parameter of 20 are shown in parentheses. The number of observations is 172.

Table 2. Regression Results to Estimate Liquidity Premia in the Euro Area (Sample Period: January 2004–April 2013)

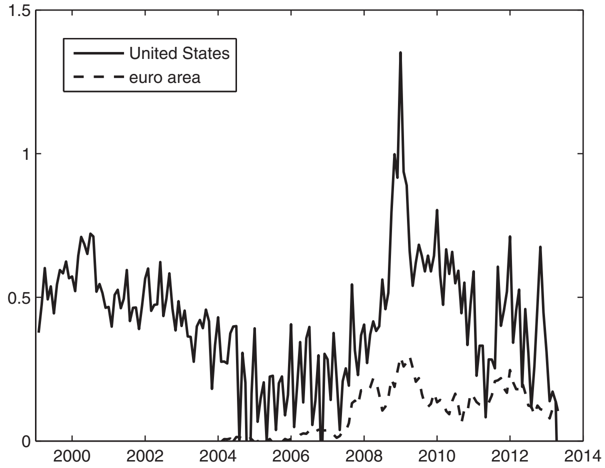
Regressor	Three Year	Five Year	Seven Year	Ten Year
KfW Spread	−0.814	−0.599	−0.544	−0.358
	(0.325)	(0.183)	(0.165)	(0.131)
R^2	0.15	0.14	0.18	0.13

Notes: This table reports the results from regressions of three-, five-, seven- and ten-year euro-area break-even inflation rates (in percentage points) onto the spread of ten-year KfW bonds over ten-year bunds (in percentage points). Newey-West standard errors with a lag truncation parameter of 20 are shown in parentheses. The number of observations is 115.

The fitted values from these regressions do not identify the level of liquidity premia, but only their time variation. As in Gürkaynak, Sack, and Wright (2010), we therefore normalize the U.S. premium to be zero in April 2005. For consistency, we do the same for the euro area.

Our estimated liquidity premia are highly correlated across the two monetary areas that we study (see figure 3 for the ten-year maturity). Their absolute levels, however, are markedly different.

Figure 3. Estimated Liquidity Premia on Ten-Year Index-Linked Bonds



Notes: See section 3.3 for the methodology. Sample period: January 1999 (United States) and 2004 (euro area) to February 2013 (percent per year).

Over the January 2004 to April 2013 sample, liquidity premia on ten-year bonds are on average equal to 10 basis points in the euro area and 37 basis points in the United States. At the peak of the crisis, these premia increase to 30 basis points in the euro area and soar to 1.3 percentage points in the United States. These large differences in liquidity premia are consistent with anecdotal evidence that the flight to safety/liquidity during the financial crisis was mostly a flight towards U.S. nominal bonds. A temporary spike in TIPS yields was apparently also due to leveraged investors that had built up huge positions in U.S. TIPS that needed to be quickly unwound after the Lehman bankruptcy (see Bank for International Settlements 2009).

In the rest of our analysis we take these liquidity corrections at face value and proceed to estimate our model using liquidity-adjusted index-linked yield data.

4. Empirical Results

Tables 3 and 4 report parameter estimates and associated posterior distributions for the United States as well as the euro area,

Table 3. U.S. Parameter Estimates (Sample Period: January 1990–April 2013)

	Prior Distribution			Posterior Mode		Posterior Distr.		
Parameter	**Type**	**Mean**	**St. Error**[a]	**Mode**	**St. Error**	**5%**	**Median**	**95%**
ρ	Beta	0.9	0.05	0.786	0.004	0.783	0.792	0.799
β	Normal	1.5	0.05	1.745	0.008	1.698	1.735	1.778
γ	Normal	0.25	0.05	0.309	0.009	0.278	0.306	0.329
μ_π	Beta	0.5	0.05	0.365	0.005	0.334	0.343	0.351
δ_x	Gamma	0.13	0.02	0.018	0.001	0.016	0.017	0.018
μ_x	Beta	0.5	0.05	0.102	0.007	0.092	0.103	0.114
ζ_r	Gamma	0.09	0.01	0.070	0.005	0.076	0.081	0.086
ϕ_π	Normal	0.5	0.1	0.806	0.002	0.800	0.803	0.806
ϕ_x	Normal	0.9	0.1	0.978	0.001	0.973	0.976	0.977
$\bar{\pi} \times 1200$	Beta	2.5	0.4	2.755	0.093	2.594	2.679	2.750
$\bar{r} \times 1200$	Beta	4	0.4	4.357	0.187	4.267	4.397	4.522
$\sigma_{\pi^*} \times 10^3$	Inv. Gamma	0.05	4	0.076	0.005	0.062	0.069	0.078
$\sigma_\eta \times 10^3$	Inv. Gamma	0.35	4	0.168	0.003	0.157	0.166	0.173
$\sigma_\pi \times 10^3$	Inv. Gamma	0.3	4	0.105	0.000	0.105	0.105	0.105
$\sigma_x \times 10^3$	Inv. Gamma	0.075	4	0.025	0.000	0.028	0.029	0.030
$\lambda_0(\pi^*)$	Normal	0	100	0.000	0.005	−0.083	−0.052	−0.025
$\lambda_0(\eta)$	Normal	0	100	−0.330	0.008	−0.451	−0.413	−0.376
$\lambda_0(\pi)$	Normal	0	100	0.010	0.001	−0.026	−0.022	−0.019
$\lambda_0(x)$	Normal	0	100	−0.090	0.005	−0.060	−0.032	−0.003

[a]For the inverted gamma distribution, the degrees of freedom are indicated.
Note: Based on 500,000 simulations.

(continued)

Table 3. (Continued)

$\lambda_1 \times 10^{-2}$: Posterior Distribution[b]				
	π^*	η	π	x
π^*	3.601 (2.991, 4.476)	−0.033 (−0.135, 0.086)	−0.436 (−0.530, −0.317)	0.412 (0.332, 0.515)
η	0.058 (−1.264, 1.447)	−4.960 (−5.535, −4.557)	1.511 (1.023, 1.978)	3.297 (2.998, 3.701)
π	5.670 (5.396, 5.928)	1.364 (1.298, 1.457)	−4.208 (−4.394, −4.004)	0.618 (0.535, 0.739)
x	−4.811 (−5.369, −4.156)	−0.288 (−0.389, −0.158)	1.178 (1.052, 1.285)	−0.793 (−0.897, −0.690)

[b]For all lambda parameters, the prior distribution is normal with mean zero and standard error 100.
Note: Median values; 5% and 95% percentiles in parentheses.

Table 4. Euro-Area Parameter Estimates (Sample Period: January 1999–April 2013)

	Prior Distribution			Posterior Mode		Posterior Distr.		
Parameter	**Type**	**Mean**	**St. Error**[a]	**Mode**	**St. Error**	**5%**	**Median**	**95%**
ρ	Beta	0.9	0.05	0.976	0.001	0.972	0.974	0.976
β	Normal	1.5	0.05	1.497	0.008	1.448	1.500	1.600
γ	Normal	0.3	0.05	0.216	0.001	0.200	0.218	0.226
μ_π	Beta	0.75	0.05	0.374	0.001	0.371	0.374	0.377
δ_x	Gamma	0.13	0.02	0.022	0.000	0.018	0.019	0.023
μ_x	Beta	0.5	0.05	0.213	0.002	0.180	0.188	0.209
ζ_r	Gamma	0.1	0.02	0.057	0.000	0.062	0.063	0.066
ϕ_π	Normal	0	0.1	0.689	0.003	0.671	0.691	0.706
ϕ_x	Normal	0.9	0.1	0.952	0.001	0.956	0.958	0.960
$\bar{\pi} \times 1200$	Beta	2	0.3	1.778	0.031	1.781	1.818	1.866
$\bar{r} \times 1200$	Beta	4	0.4	3.788	0.073	3.599	3.728	3.868
$\sigma_{\pi^*} \times 10^3$	Inv. Gamma	0.01	4	0.020	0.001	0.019	0.020	0.023
$\sigma_\eta \times 10^3$	Inv. Gamma	0.1	4	0.164	0.004	0.151	0.157	0.163
$\sigma_\pi \times 10^3$	Inv. Gamma	0.1	4	0.091	0.001	0.089	0.098	0.112
$\sigma_x \times 10^3$	Inv. Gamma	0.05	4	0.048	0.001	0.037	0.039	0.043
$\lambda_0(\pi^*)$	Normal	0	100	0.500	0.018	0.486	0.514	0.539
$\lambda_0(\eta)$	Normal	0	100	0.710	0.001	0.827	0.832	0.836
$\lambda_0(\pi)$	Normal	0	100	−0.110	0.005	−0.141	−0.124	−0.110
$\lambda_0(x)$	Normal	0	100	0.450	0.008	0.492	0.504	0.518

[a]For the inverted gamma distribution, the degrees of freedom are indicated.

Note: Based on 500,000 simulations.

(continued)

Table 4. (Continued)

$\lambda_1 \times 10^{-2}$: Posterior Distribution[b]				
	π^*	η	π	x
π^*	27.558 (27.135, 28.041)	−0.016 (−0.025, −0.013)	−0.648 (−0.747, −0.565)	0.420 (0.258, 0.535)
η	82.144 (81.698, 82.494)	−2.264 (−2.329, −2.187)	−1.581 (−1.643, −1.470)	4.467 (4.436, 4.500)
π	−2.952 (−3.108, −2.815)	−0.641 (−0.709, −0.542)	−8.762 (−9.520, −8.274)	1.642 (1.458, 2.076)
x	52.619 (52.436, 52.778)	−1.270 (−1.310, −1.227)	−0.400 (−0.440, −0.337)	1.823 (1.795, 1.868)

[b]For all lambda parameters, the prior distribution is normal with mean zero and standard error 100.

Note: Median values; 5% and 95% percentiles in parentheses.

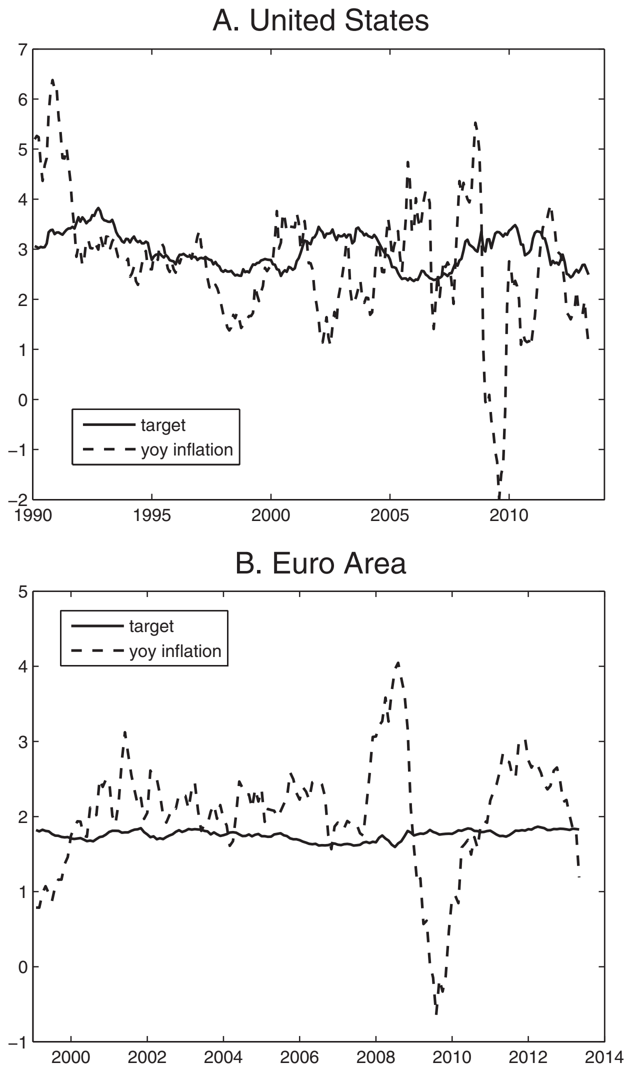
respectively. The results show that our model seems empirically plausible, with estimated macro parameters that are broadly within the range of estimates which can be found in the literature (see, e.g., Rudebusch 2002).

In both sets of estimates, the policy rule is characterized by a high degree of interest rate smoothing—however, more so for the euro area ($\rho = 0.97$) than in the case of the United States ($\rho = 0.79$). This might reflect the shorter sample used in the estimation of the euro-area model, during which the short rate remained relatively stable. The responses to inflation deviations from the objective and the output gap (β and γ, respectively) are estimated to be similar in the two economies, and also in line with typical values reported in the literature. The degree of forward-lookingness of the output-gap equation (μ_x) is somewhat higher for the euro area than for the United States. As for the estimated standard deviations of fundamental shocks, these tend to be higher for the United States than for the euro area. The only exception is demand shocks, whose standard deviation is smaller, but the persistence higher, in the United States. These differences are likely to be due to the relatively low macroeconomic volatility during the euro sample, compared with the longer U.S. sample. Concerning the λ_1 parameters, the prices of inflation target, monetary policy, and aggregate demand risk appear to be much more sensitive to changes in the perceived inflation target in the euro area (see the first column of the lower table in tables 3 and 4). The other elements of the λ_1 matrix appear to be more similar across countries in absolute value.

As already mentioned, our assumed perceived policy rule allows for a time-varying inflation target. This is an unobservable variable that needs to be filtered out from available observable data. Panels A and B of figure 4 display the estimates obtained for the United States and the euro area, respectively. From an intuitive viewpoint, these estimates appear reasonable: in both cases the filtered targets move slowly and with little variability compared with realized inflation. In the euro area, the estimated target fluctuates narrowly around levels below but close to 2 percent over the whole sample period. This is consistent with the numerical definition of price stability provided by the ECB Governing Council in 1998.

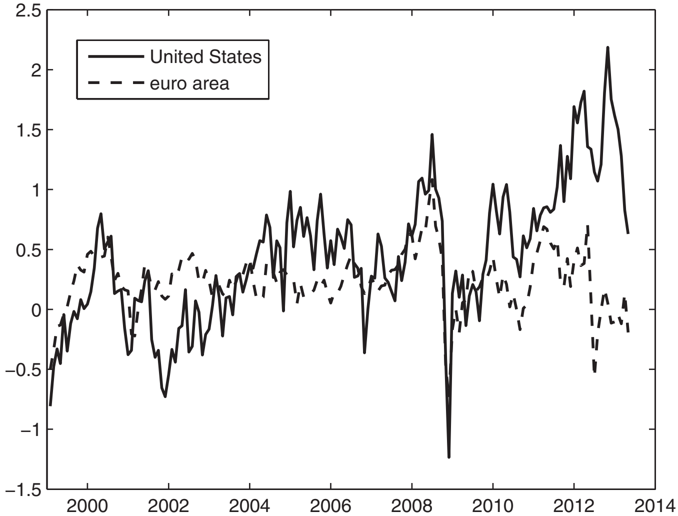
For the United States, we document a slight reduction in the estimated inflation target from levels around 3 percent before

Figure 4. Inflation and Estimated Perceived Inflation Target



2011 to levels closer to 2 percent at the end of 2012. This shift is consistent with the implications of the FOMC announcement of January 25, 2012, which stated as follows: "The Committee judges that inflation at the rate of 2 percent, as measured by

Figure 5. Estimated Three-Year Inflation Risk Premia



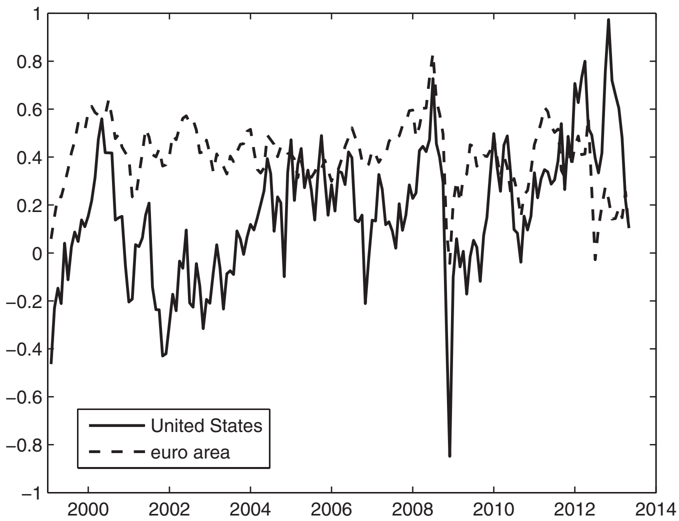
the annual change in the price index for personal consumption expenditures, is most consistent over the longer run with the Federal Reserve's statutory mandate." Our results suggest that the announcement produced the intended effects. Together with the euro-area evidence, they provide further support to the view that the availability of an official numerical definition of price stability plays an important role in anchoring long-term inflation expectations.

4.1 Inflation Risk Premia

Given a set of parameters and a specific realization of the state-variable vector, we can compute the model-implied inflation risk premium.[8] The dynamics of our estimated inflation premia are displayed in figures 5 and 6, with a focus on the three-year and ten-year maturities, respectively, over the sample for which data are

[8]Our model can also be used to compute nominal and real term premia for any maturity. We do not report these variables, because our focus is on inflation risk premia. Note that here we disregard the component due to Jensen's inequality, which is in the order of only a few basis points.

Figure 6. Estimated Ten-Year Inflation Risk Premia



available for both the United States and the euro area. The overall magnitude of our estimated inflation premia are comparable, for the United States, with recent empirical evidence reported by Christensen, Lopez, and Rudebusch (2010) and Chernov and Mueller (2012) and, for the euro area, with the results in Hördahl and Tristani (2012).[9]

In spite of the different evolution of inflation, of the output gap, and of inflation expectations, inflation risk premia appear to be remarkably close to each other in the United States and in the euro area. Both at the three-year and the ten-year horizons, their levels are broadly comparable and their fluctuations at business-cycle frequencies are relatively closely aligned. Only at higher frequencies are U.S. premia subject to higher volatility. Financial market participants appear to attach very similar prices to inflation risk in the United States and in the euro area. This suggests that they are equally confident about the ability of the Federal Reserve and the

[9]Amongst other papers that estimate inflation risk premia on U.S. data, see Durham (2006) and D'Amico, Kim, and Wei (2008); García and Werner (2008) study inflation risk premia in the euro area.

European Central Bank to deliver a stable inflation rate over the medium-to-long run.

Nevertheless, temporary divergences between inflation risk premia across the two sides of the Atlantic can occasionally be observed over our sample period. One occurs over 2006 and 2007, but it is confined to ten-year premia, which fall to near-zero levels in the United States, while they remain relatively stable in the euro area. This deviation is linked to a small reduction in the U.S. inflation rate in a period of positive output gap—see section 4.3 for a more detailed discussion of the cyclical features of inflation risk premia.

The second period in which U.S. and euro-area inflation risk premia diverge starts around 2012 and continues until the end of our sample. Over this period we observe a decoupling of premia across the two areas: they fall towards zero or slightly negative levels in the euro area, while they tend to increase temporarily in the United States. This divergence is visible both at the three-year and, to a lesser degree, at the ten-year maturity—as well as the five-year maturity, which is not displayed in the figure.

The model interprets these recent developments in light of the diverging cyclical conditions. Specifically, over the past two years we observe an improvement in our output-gap measure and a mild increase in the inflation rate in the United States. In contrast, the output gap falls further in the euro area, and so does the inflation rate.

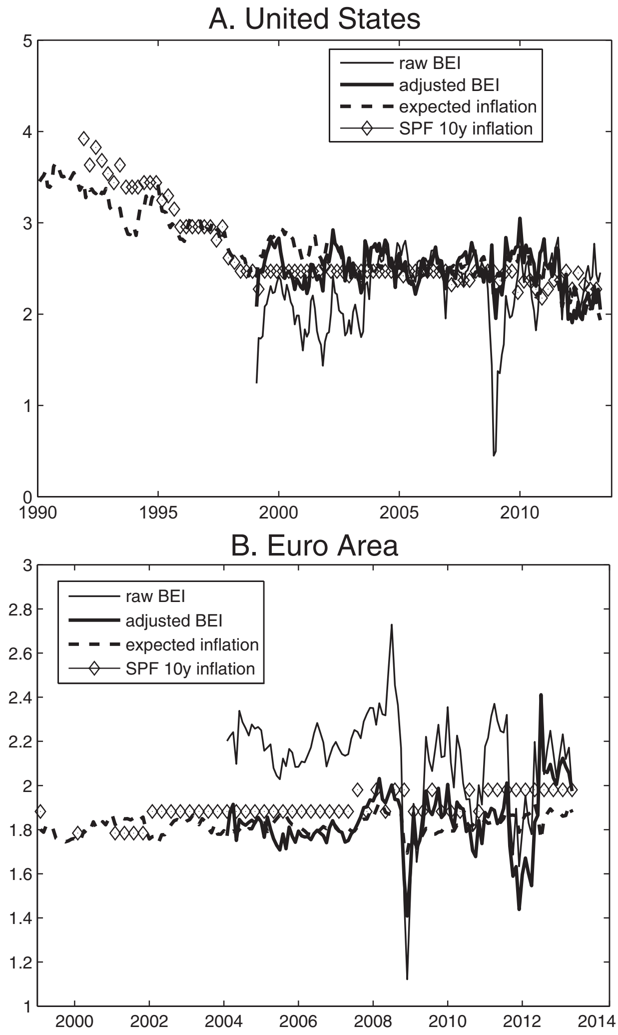
4.2 Premium-Adjusted Break-Even Inflation Rates

Given our estimates of the inflation risk premium, we can strip out this component from our measures of the break-even inflation rates which have already been corrected for liquidity risk. The results are premium-adjusted break-even inflation rates, which provide a more accurate estimate of inflation expectations over the life of the bonds.

Figure 7 reports raw and premium-adjusted ten-year break-even inflation rates in the United States and the euro area for the period over which we have reliable estimates of zero-coupon real rates (see the data section above).

In the euro area, liquidity risk premia are relatively small over most of the sample period and the inflation premium is somewhat larger. The adjusted break-even rate is consequently also lower relative to the raw rate. While the raw euro-area break-even rate has

Figure 7. Ten-Year Break-Even Inflation Rates and Survey Inflation Forecasts



Notes: The solid thin line is the unadjusted observed ten-year break-even rate; the solid thick line is the break-even rate adjusted for liquidity and inflation risk premia; the dashed line is the model-implied average expected inflation over the next ten years. The diamonds are SPF survey expectations of inflation during the next ten years (United States) and five years ahead (euro area). All values are expressed in percent per year.

been mostly fluctuating above a level of 2 percent since 2004, the premium-adjusted measure has generally been below but close to 2 percent, suggesting long-term euro-area inflation expectations more in line with the ECB's price stability objective than what would be concluded based on the unadjusted break-even rate.

In the United States, liquidity risk premia are very large—larger than inflation risk premia—from 2008 onwards. Over much of this period, adjusted break-even rates are therefore higher than the raw rates, since the positive liquidity premium on TIPS pushes break-even rates downward. While the raw U.S. break-even rate reached a minimum of 0.4 percentage points at the end of 2008, the premium-adjusted measure has hovered around 2.5 to 3 percentage points until mid-2011 and has then fallen slightly towards a level just above 2 percent. For the period 1999–2008, our estimates are quite similar to those reported by Gürkaynak, Sack, and Wright (2010) for that period.

Our results suggest that, both in the euro area and in the United States, long-term inflation expectations have remained remarkably stable at the peak of the financial crisis and throughout the Great Recession. Specifically, premium-adjusted break-even rates have remained stable even over 2009, when our measures record the largest negative output gap and actual inflation rates temporarily reached negative territory in both monetary areas.

Figure 7 also displays the estimated model-implied average expected inflation over the next ten years for each point in time during the sample periods. This is the expected inflation produced by the macro dynamics of the model, which would fully coincide with the premium-adjusted break-even rate discussed above if all yield measurement errors were always zero. While this is not the case, the difference is always very small, in the order of a few basis points, indicating that our model does well in terms of capturing the dynamics of both nominal and real yields. It also shows that the most recent uptick in the euro-area adjusted break-even rate above 2 percent is due to increased yield measurement errors rather than higher inflation expectations in the model.

Finally, figure 7 reports measures of long-horizon inflation expectations from available survey forecasts: ten-year U.S. inflation expectations from the Federal Reserve's Survey of Professional Forecasters (SPF) and five-year euro-area inflation expectations from the ECB's

SPF. Clearly, inclusion of inflation survey data in the estimation has been useful in getting the model to capture the broad movements of investors' inflation expectations, as reported by these survey measures. In both monetary areas, the adjusted break-even rate is much closer to the survey forecasts than the unadjusted rate.

4.3 The Inflation Risk Premium and the Macroeconomy

In this section, we look closer at the relationship between inflation risk premia and macroeconomic developments. In essentially affine models, movements in risk premia are related to changes in the state variables. Since the state variables in our model are inflation, the output gap, the monetary policy shock, and the perceived inflation target, we can relate the dynamics of our estimated inflation risk premia directly to developments in these variables.

From a descriptive perspective, inflation risk premia are procyclical over the past five years. In particular, they fall during the Great Recession. Inflation risk premia are also positively correlated with actual inflation rates in both monetary areas: premia become higher when inflation increases. These patterns are apparent from figures 8–11, which plot movements in ten-year inflation risk premia together with rescaled measures of the output gap and of inflation.

Both in the United States and in the euro area, ten-year inflation risk premia fall sharply when output contracts at the end of 2008. They then tend to recover, and in the euro area edge down again, alongside the measured output gaps. Over the longer sample available for the United States, however, inflation risk premia are on average countercyclical. This result is in line with the widely documented countercyclicality of term premia (see, e.g., Stambaugh 1988; Hördahl, Tristani, and Vestin 2006).

Figures 8–11 also show that the positive correlation between month-to-month changes in inflation and the inflation premium takes place mostly at a higher frequency. This pattern is again common to both the United States and the euro area.

Through the lens of our model, we can also look at how long-term inflation expectations and inflation risk premia react to macroeconomic shocks. Figures 12 and 13 compare the impulse responses of long-term inflation expectations and inflation risk premia, respectively, to the four structural shocks in our model. The figures show

Figure 8. U.S. Ten-Year Inflation Risk Premium and Output Gap

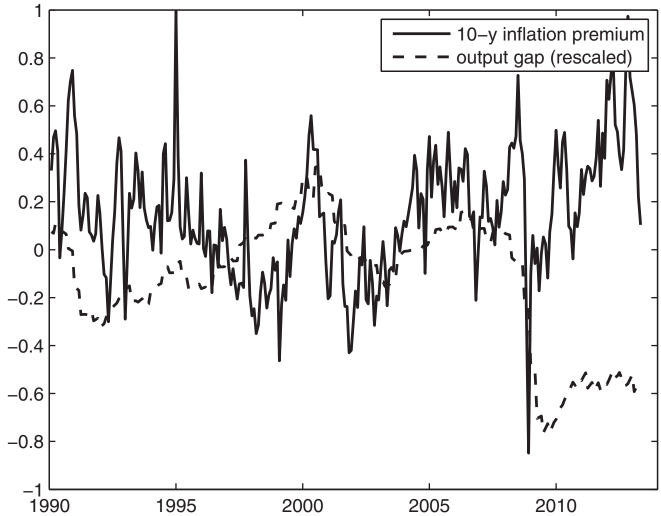


Figure 9. U.S. Ten-Year Inflation Risk Premium and Year-on-Year Inflation

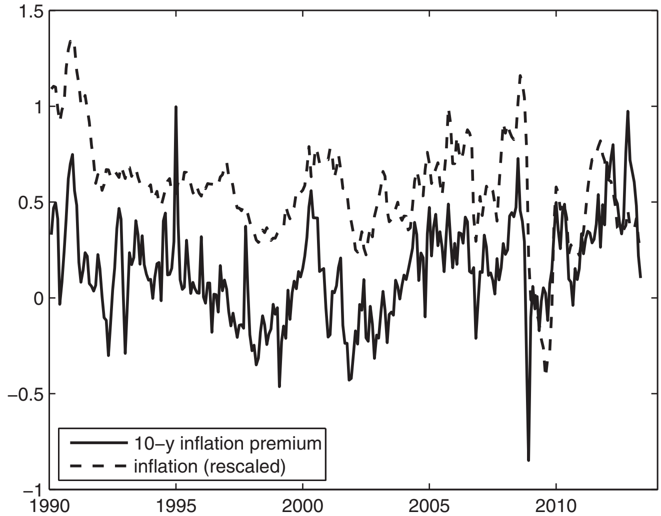


Figure 10. Euro-Area Ten-Year Inflation Risk Premium and Output Gap

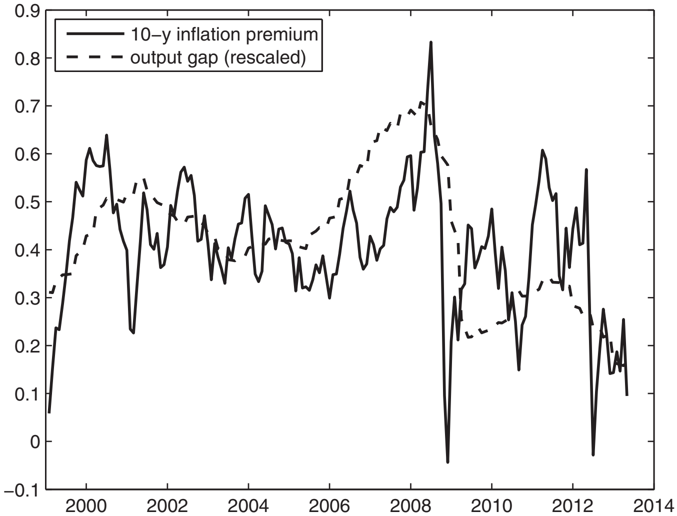


Figure 11. Euro-Area Ten-Year Inflation Risk Premium and Year-on-Year Inflation

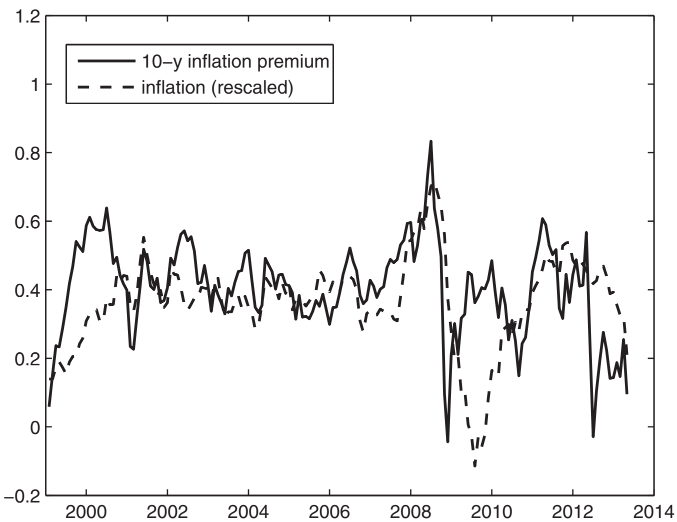
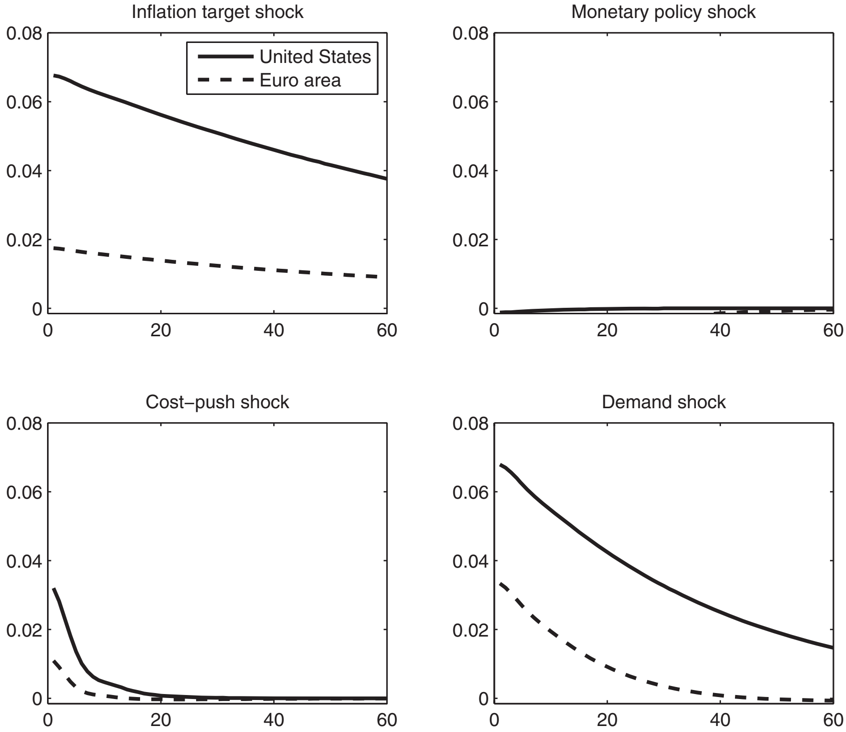


Figure 12. Impulse Responses of Ten-Year Inflation Expectations

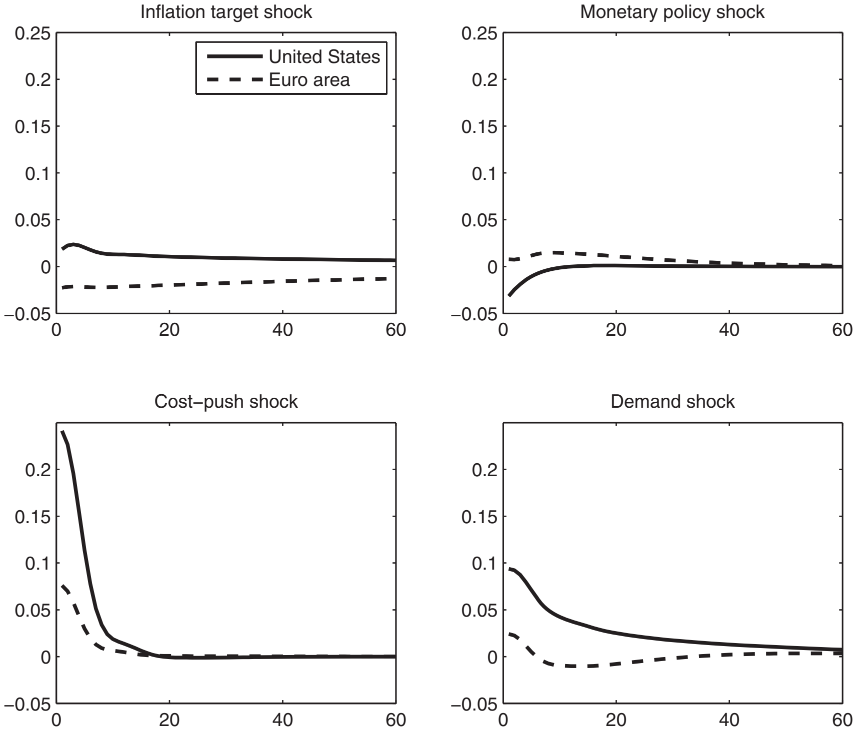


Note: The impulses are equal to one standard deviation for all shocks.

that demand shocks are the most important drivers of long-term inflation expectations (together with target shocks in the United States case); inflation risk premia are mostly driven by cost-push shocks.

Specifically, an aggregate demand shock leads to an increase in ten-year average inflation expectations in both monetary areas, but the increase is approximately twice as big in the United States compared with the euro area—almost 7 basis points and approximately 3 basis points, respectively. This difference is especially striking taking into account that the standard deviation of the shocks is higher in the euro area.

Figure 13. Impulse Responses of Ten-Year Inflation Risk Premia



Note: The impulses are equal to one standard deviation for all shocks.

The cross-country differences in the response of inflation risk premia to aggregate demand shocks are even larger. At the ten-year horizon, inflation premia increase by almost 10 basis points in the United States and by 2 basis points in the euro area. All in all, demand shocks have comparable effects on long-term inflation expectations and long-term inflation risk premia, but these effects are larger in the United States.

For a given size of the inflation target shock, we observe similar effects on long-term inflation expectations in the United States and in the euro area. A hypothetical 1-percentage-point upward shock to the target would result in a surge in ten-year average inflation expectations by approximately 0.8 percentage points in both cases.

The estimated average size of inflation target shocks, however, is four times larger in the United States (0.08 percent) compared with the euro area (0.02 percent). This result helps explain the higher volatility of inflation expectations conditional on target shocks in the United States.

The size of monetary policy and cost-push shocks is very similar across the two monetary areas. A 0.2-percentage-point monetary policy shock has negligible effects on ten-year average inflation expectations and a small impact on ten-year inflation risk premia.

For a given shock size, however, cost-push shocks have a much larger impact on long-term inflation expectations and inflation risk premia in the United States. A positive shock of this type of 0.1 percentage point produces an increase in average ten-year inflation expectations by about 3 basis points in the United States, while the same expectations only edge up by 1 basis point in the euro area. At the same time, the shock triggers a temporary 25-basis-point increase in the ten-year inflation risk premium in the United States, while the premium only increases by 8 basis points in the euro area.

All in all, our results suggest that ten-year inflation expectations and risk premia are somewhat more volatile in the United States compared with the euro area. A distinguishing feature of positive aggregate demand shocks is to increase long-term inflation expectations and inflation risk premia by comparable amounts. In contrast, cost-push shocks have a much larger impact on inflation risk premia, especially in the United States.

5. Conclusions

Break-even inflation rates are often used as timely measures of market expectations of future inflation, and are therefore viewed as useful indicators for central banks, among others. However, some care should be exercised when interpreting break-even inflation rates in terms of inflation expectations, because they include risk premia, most notably to compensate investors for liquidity and inflation risks. In this paper we model and estimate the inflation risk premium in order to obtain a "cleaner" measure of investors' inflation expectations embedded in bond prices. In addition, we investigate the macroeconomic determinants of inflation risk premia, in order to better understand their dynamics.

We estimate our model on U.S. and euro-area data. This provides us with an opportunity to examine the main features of inflation risk premia for the two largest economies, including similarities and differences in the determinants of such premia.

Our results suggest that ten-year inflation expectations have remained quite stable over time in both monetary areas. In the United States, since the late 1990s, they have fluctuated around levels slightly below 3 percent up until 2011 when, further to the FOMC announcement of a 2 percent inflation goal, they adjusted downwards towards this level over the course of 2012. In the euro area, long-term inflation expectations have generally moved very little around levels slightly below 2 percent.

Inflation risk premia at various maturities have remarkably similar properties in the United States and in the euro area, especially over the years of the financial crisis until mid-2011, even if U.S. premia appear to be subject to greater high-frequency volatility. The higher volatility in U.S. premia appears to be due to their greater sensitivity to aggregate demand shocks and especially to cost-push shocks.

These results should provide a useful benchmark for comparison to further analyses of inflation risk premia based on fully microfounded models.

Appendix

Solving the Model

In order to solve the model, we write it in the general form

$$\begin{bmatrix} \mathbf{X}_{1,t+1} \\ E_t\mathbf{X}_{2,t+1} \end{bmatrix} = \mathbf{H} \begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{2,t} \end{bmatrix} + \mathbf{K} r_t + \begin{bmatrix} \Sigma\xi_{1,t+1} \\ \mathbf{0} \end{bmatrix}, \qquad (10)$$

where $\mathbf{X}_{1,t} = [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_t^*, \eta_t, \varepsilon_t^\pi, \varepsilon_t^x, r_{t-1}]'$ is the vector of predetermined variables, $\mathbf{X}_{2,t} = [E_t x_{t+11}, \ldots, E_t x_{t+1}, x_t, E_t\pi_{t+11}, \ldots, E_t\pi_{t+1}, \pi_t]'$ includes the variables which are not predetermined, r_t is the policy instrument, and ξ_1 is a vector of independent, normally distributed shocks. The short-term rate can be written in the feedback form

$$r_t = -\mathbf{F} \begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{2,t} \end{bmatrix}. \qquad (11)$$

The solution of the model can be obtained numerically following standard methods. We choose the methodology described in Söderlind (1999), which is based on the Schur decomposition. The result is two matrices $\mathbf{M}$ and $\mathbf{C}$ such that $\mathbf{X}_{1,t} = \mathbf{M}\mathbf{X}_{1,t-1} + \Sigma\xi_{1,t}$ and $\mathbf{X}_{2,t} = \mathbf{C}\mathbf{X}_{1,t}$.[10] Consequently, the equilibrium short-term interest rate will be equal to $r_t = \mathbf{\Delta}'\mathbf{X}_{1,t}$, where $\mathbf{\Delta}' \equiv -(\mathbf{F}_1 + \mathbf{F}_2\mathbf{C})$ and $\mathbf{F}_1$ and $\mathbf{F}_2$ are partitions of $\mathbf{F}$ conformable with $\mathbf{X}_{1,t}$ and $\mathbf{X}_{2,t}$.

Pricing Real and Nominal Bonds

To build the term structure of interest rates, we first note that the solution of the macro model is the same as that in standard affine term structure models. Specifically, the short-term interest rate is expressed as a linear function of the state vector ($\mathbf{X}_1$), which in turn follows a first-order Gaussian VAR.[11] To derive the term structure, we therefore only need to impose the assumption of absence of arbitrage opportunities, which guarantees the existence of a risk-neutral measure, and to specify a process for the stochastic discount factor, or pricing kernel.

The (nominal) pricing kernel m_{t+1} is defined as $m_{t+1} = \exp(-r_t)\psi_{t+1}/\psi_t$, where ψ_{t+1} is the Radon-Nikodym derivative assumed to follow the log-normal process $\psi_{t+1} = \psi_t \exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\xi_{1,t+1}\right)$, and where λ_t denotes the market prices of risk. As described in section 2, we assume that these risk prices are affine functions of a transformed state vector $\mathbf{Z}_t \equiv [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_t^*, r_t, \pi_t, x_t, r_{t-1}]'$, defined as $\mathbf{Z}_t = \hat{\mathbf{D}}\mathbf{X}_{1,t}$ for a suitably defined matrix $\hat{\mathbf{D}}$. Given this transformation, the solution equation for the short-term interest rate can be rewritten as a function of $\mathbf{Z}_t$,

$$r_t = \overline{\mathbf{\Delta}}'\mathbf{Z}_t. \tag{12}$$

[10]The presence of non-predetermined variables in the model implies that there may be multiple solutions for some parameter values. We constrain the system to be determinate in the iterative process of maximizing the likelihood function.

[11]Note, however, that in our case both the short-rate equation and the law of motion of vector $\mathbf{X}_1$ are obtained endogenously, as functions of the parameters of the macroeconomic model. This contrasts with the standard affine setup based on unobservable variables, where both the short-rate equation and the law of motion of the state variables are postulated exogenously.

From the macro model solution, we also know that

$$\begin{aligned}\pi_{t+1} &= \mathbf{C}_{\pi}\mathbf{M}\mathbf{X}_{1,t} + \mathbf{C}_{\pi}\Sigma\xi_{1,t+1} \\ &= \mathbf{C}_{\pi}\mathbf{M}\hat{\mathbf{D}}^{-1}\mathbf{Z}_t + \mathbf{C}_{\pi}\Sigma\xi_{1,t+1},\end{aligned}$$

where $\mathbf{C}_{\pi}$ is the relevant row of $\mathbf{C}$.

Now assume that the real pricing kernel is m^*_{t+1}, so that the following fundamental asset pricing relation holds:

$$E_t\left[m^*_{t+1}\left(1 + R^*_{t+1}\right)\right] = 1,$$

where R^*_{t+1} denotes the real return on some asset.

If we now want to price an n-period nominal bond, p^n_t, we get

$$\frac{p^n_t}{q_t} = E_t\left[m^*_{t+1}\frac{p^{n-1}_{t+1}}{q_{t+1}}\right],$$

where q_t is the price level in the economy. In terms of inflation rates, $\pi_{t+1} \equiv \ln q_{t+1} - \ln q_t$, we obtain

$$p^n_t = E_t\left[m^*_{t+1}\frac{p^{n-1}_{t+1}}{\exp\left(\pi_{t+1}\right)}\right].$$

Notice that this is equivalent to postulating a nominal pricing kernel $m_{t+1} \equiv m^*_{t+1}/\exp(\pi_{t+1})$, such that

$$p^n_t = E_t\left[m_{t+1}p^{n-1}_{t+1}\right].$$

Given our assumption on the nominal pricing kernel and on the market prices of risk, we can postulate that nominal bond prices will be exponential-affine functions of the state variables, to obtain

$$p^n_t = \exp\left(\bar{A}_n + \bar{B}'_n\mathbf{Z}_t\right), \tag{13}$$

where $\bar{A}_n$ and $\bar{B}'_n$ are recursive parameters that depend on the maturity n in the following way:

$$\bar{A}_{n+1} = \bar{A}_n - \bar{B}'_n\hat{\mathbf{D}}\Sigma\lambda_0 + \frac{1}{2}\bar{B}'_n\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_n, \tag{14}$$

$$\bar{B}'_{n+1} = \bar{B}'_n\hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) - \overline{\mathbf{\Delta}}'. \tag{15}$$

Nominal bond yields are then given by

$$\begin{aligned} y_t^n &= -\frac{\ln\left(p_t^n\right)}{n} \\ &= -\frac{\bar{A}_n}{n} - \frac{\bar{B}_n'}{n}\mathbf{Z}_t \\ &\equiv A_n + B_n'\mathbf{Z}_t. \end{aligned} \tag{16}$$

The definition of the pricing kernel implies

$$r_t = -\ln m_{t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\xi_{1,t+1},$$

which translates into a real pricing kernel

$$m_{t+1}^* = \exp\left(-r_t + \pi_{t+1}\right)\frac{\psi_{t+1}}{\psi_t},$$

or

$$m_{t+1}^* = \exp\left(-\overline{\mathbf{\Delta}}'\mathbf{Z}_t + \mathbf{C}_\pi\mathbf{M}\mathbf{X}_{1,t} + \mathbf{C}_\pi\Sigma\xi_{1,t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\xi_{1,t+1}\right).$$

We postulate again that real bond prices will be exponential-affine functions of the state variables,

$$p_t^{n*} = \exp\left(\bar{A}_n^* + \bar{B}_n^{*\prime}\mathbf{Z}_t\right),$$

where $\bar{A}_n^*$ and $\bar{B}_n^*$ are parameters that depend on the maturity n, and which can be identified using

$$\begin{aligned} p_t^{n+1*} &= E_t\left[m_{t+1}^* p_{t+1}^{n*}\right] \\ &= \exp\left(\bar{A}_n^* - \overline{\mathbf{\Delta}}'\mathbf{Z}_t + \mathbf{C}_\pi\mathbf{M}\mathbf{X}_{1,t} + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\mathbf{M}\mathbf{X}_{1,t} - \frac{1}{2}\lambda_t'\lambda_t\right) \\ &\quad \times E_t\left[\exp\left(\left(\mathbf{C}_\pi\Sigma - \lambda_t' + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\Sigma\right)\xi_{1,t+1}\right)\right], \end{aligned}$$

where we used

$$\mathbf{Z}_{t+1} = \hat{\mathbf{D}}\mathbf{M}\mathbf{X}_{1,t} + \hat{\mathbf{D}}\Sigma\xi_{1,t+1}.$$

Noting that

$$E_t\left[\exp\left(\left(\mathbf{C}_\pi\Sigma+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\Sigma-\lambda_t'\right)\xi_{1,t+1}\right)\right]$$
$$=\exp\left(\frac{1}{2}\left(\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma-\lambda_t'\right)\left(\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma-\lambda_t'\right)'\right),$$

and rearranging terms, we obtain

$$p_t^{n+1*}=\exp\left(\bar{A}_n^*+\frac{1}{2}\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma\Sigma'\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)'\right.$$
$$-\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma\lambda_0$$
$$\left.+\left(\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\left(\mathbf{M}\hat{\mathbf{D}}^{-1}-\Sigma\lambda_1\right)-\overline{\boldsymbol{\Delta}}'\right)\mathbf{Z}_t\right).$$

We can therefore identify $\bar{A}_n^*$ and $\bar{B}_n^*$ recursively as

$$\bar{A}_{n+1}^*=\bar{A}_n^*+\frac{1}{2}\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma\Sigma'\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)'-\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma\lambda_0,$$
$$\bar{B}_{n+1}^{*\prime}=\left(\mathbf{C}_\pi+\bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\left(\mathbf{M}\hat{\mathbf{D}}^{-1}-\Sigma\lambda_1\right)-\overline{\boldsymbol{\Delta}}'.$$

For a one-month real bond, in particular, we obtain

$$p_t^{1*}=E_t\left[m_{t+1}\right]$$
$$=\exp\left(\left(-\overline{\boldsymbol{\Delta}}'+\mathbf{C}_\pi\left(\mathbf{M}\hat{\mathbf{D}}^{-1}-\Sigma\lambda_1\right)\right)\mathbf{Z}_t-\mathbf{C}_\pi\Sigma\left(\lambda_0-\frac{1}{2}\Sigma'\mathbf{C}_\pi'\right)\right),$$

which can be used to start the recursion. Note that the short-term real rate is

$$r_t^*=\mathbf{C}_\pi\Sigma\left(\lambda_0-\frac{1}{2}\Sigma'\mathbf{C}_\pi'\right)+\left(\overline{\boldsymbol{\Delta}}'-\mathbf{C}_\pi\left(\mathbf{M}\hat{\mathbf{D}}^{-1}-\Sigma\lambda_1\right)\right)\mathbf{Z}_t.$$

Derivation of Inflation Risk Premium and Break-Even Inflation Rates

For all maturities, recall that the continuously compounded yield is, for nominal and real bonds, respectively,

$$y_{t,n}=-\frac{\bar{A}_n}{n}-\frac{\bar{B}_n'}{n}\mathbf{Z}_t$$

$$y^*_{t,n} = -\frac{\bar{A}^*_n}{n} - \frac{\bar{B}^{*\prime}_n}{n}\mathbf{Z}_t.$$

The yield spread is therefore simply

$$y_{t,n} - y^*_{t,n} = -\frac{1}{n}\left(\bar{A}_n - \bar{A}^*_n\right) - \frac{1}{n}\left(\bar{B}'_n - \bar{B}^{*\prime}_n\right)\mathbf{Z}_t,$$

where

$$\begin{aligned}
\bar{A}_{n+1} - \bar{A}^*_{n+1} &= \bar{A}_n - \bar{A}^*_n - \left(\bar{B}'_n - \bar{B}^{*\prime}_n\right)\hat{\mathbf{D}}\Sigma\lambda_0 + \mathbf{C}_\pi\Sigma\lambda_0 \\
&\quad - \frac{1}{2}\mathbf{C}_\pi\Sigma\Sigma'\mathbf{C}'_\pi - \mathbf{C}_\pi\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}^*_n \\
&\quad + \frac{1}{2}\left(\bar{B}'_n\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_n - \bar{B}^{*\prime}_n\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}^*_n\right) \\
\bar{B}'_{n+1} - \bar{B}^{*\prime}_{n+1} &= \left(\bar{B}'_n - \bar{B}^{*\prime}_n\right)\hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) \\
&\quad - \mathbf{C}_\pi\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right).
\end{aligned}$$

Note that the nominal bond equation can be solved explicitly as

$$\bar{A}_n = \bar{A}_1 + \sum_{i=1}^{n-1}\left(\frac{1}{2}\overline{\mathbf{B}}'_i\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\overline{\mathbf{B}}_i - \overline{\mathbf{B}}'_i\hat{\mathbf{D}}\Sigma\lambda_0\right),$$

$$\overline{B}'_n = -\overline{\mathbf{\Delta}}'\sum_{i=0}^{n-1}\left[\hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)\right]^i.$$

Similarly, for the real bond $\bar{A}^*_n$ we obtain

$$\begin{aligned}
\bar{A}^*_n &= n\mathbf{C}_\pi\Sigma\left(\frac{1}{2}\Sigma'\mathbf{C}'_\pi - \lambda_0\right) \\
&\quad + \sum_{i=1}^{n-1}\left(\bar{B}^{*\prime}_i\hat{\mathbf{D}}\Sigma\Sigma'\mathbf{C}'_\pi + \frac{1}{2}\bar{B}^{*\prime}_i\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}^*_i - \bar{B}^{*\prime}_i\hat{\mathbf{D}}\Sigma\lambda_0\right)
\end{aligned}$$

$$\bar{B}^{*\prime}_n = \left(\mathbf{C}_\pi\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) - \overline{\mathbf{\Delta}}'\right)\sum_{i=0}^{n-1}\left[\hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)\right]^i.$$

Note that the law of motion of the transformed state vector can be written as $\mathbf{Z}_{t+1} = \hat{\mathbf{D}}\mathbf{M}\hat{\mathbf{D}}^{-1}\mathbf{Z}_t + \hat{\mathbf{D}}\Sigma\xi_{1,t+1}$, so that the term $\hat{\mathbf{D}}(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1)$ represents the expected change in $\mathbf{Z}_t$ under Q. We can then define a new matrix $\widehat{\mathbf{M}} = \hat{\mathbf{D}}(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1)$. Note also that the sum $\sum_{i=0}^{n-1}\widehat{\mathbf{M}}^i$ can be solved out as $\sum_{i=0}^{n-1}\widehat{\mathbf{M}}^i = (\mathbf{I} - \widehat{\mathbf{M}})^{-1}(\mathbf{I} - \widehat{\mathbf{M}}^n)$ for bonds of finite maturity.[12] Note that we could equivalently write $\sum_{i=0}^{n-1}\widehat{\mathbf{M}}^i = (\mathbf{I} - \widehat{\mathbf{M}}^n)(\mathbf{I} - \widehat{\mathbf{M}})^{-1}$.

For the state-dependent component of bond prices, it follows that

$$\overline{B}_n' = -\overline{\mathbf{\Delta}}'\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right)$$
$$\bar{B}_n^{*\prime} = \left(\mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}} - \overline{\mathbf{\Delta}}'\right)\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right),$$

and

$$\bar{B}_n^{*\prime} - \overline{B}_n' = \mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right).$$

Note also that

$$E_t\left[\pi_{t+n}\right] = \mathbf{C}_\pi\mathbf{M}^n\hat{\mathbf{D}}^{-1}\mathbf{Z}_t,$$

and that expected average inflation up to $t+n$, $\overline{\pi}_{t+n}$ is

$$E_t\overline{\pi}_{t+n} = \frac{1}{n}\sum_{i=1}^{n}E_t\pi_{t+i}$$
$$= \mathbf{C}_\pi\frac{\sum_{i=1}^{n}\mathbf{M}^i}{n}\hat{\mathbf{D}}^{-1}\mathbf{Z}_t,$$

or, writing this out explicitly,

$$E_t\overline{\pi}_{t+n} = \frac{1}{n}\mathbf{C}_\pi\left(\mathbf{I} - \mathbf{M}^n\right)\left(\mathbf{I} - \mathbf{M}\right)^{-1}\mathbf{M}\hat{\mathbf{D}}^{-1}\mathbf{Z}_t.$$

[12]For bonds of infinite maturity, the sum will only be defined if all eigenvalues of $\widehat{\mathbf{M}}$ are inside the unit circle. This is not necessarily true, even if the eigenvalues of $\mathbf{M}$ are within the unit circle by construction.

We are now ready to define the break-even inflation rate as

$$y_{t,n} - y^*_{t,n} = \frac{1}{n}\left(\bar{A}^*_n - \bar{A}_n\right) + \frac{1}{n}\left(\bar{B}^{*\prime}_n - \bar{B}^\prime_n\right)\mathbf{Z}_t$$
$$= \frac{1}{n}\left(\bar{A}^*_n - \bar{A}_n\right) + \frac{1}{n}\mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right)\mathbf{Z}_t,$$

where $\bar{A}^*_n$ and $\bar{A}_n$ are defined above.

The inflation risk premium can then be defined as

$$y_{t,n} - y^*_{t,n} - E_t\overline{\pi}_{t+n} = \frac{1}{n}\left(\bar{A}^*_n - \bar{A}_n\right) + \frac{1}{n}\left(\bar{B}^{*\prime}_n - \bar{B}^\prime_n\right)\mathbf{Z}_t - E_t\overline{\pi}_{t+n},$$

whose state-dependent component can be written explicitly as

$$\frac{1}{n}\left(\bar{B}^{*\prime}_n - \bar{B}^\prime_n\right)\mathbf{Z}_t - E_t\overline{\pi}_{t+n} = \frac{1}{n}\mathbf{C}_\pi\Big[\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right)$$
$$- \mathbf{M}\left(\mathbf{I} - \mathbf{M}^n\right)\left(\mathbf{I} - \mathbf{M}\right)^{-1}\hat{\mathbf{D}}^{-1}\Big]\mathbf{Z}_t.$$

Note that the time-varying component of the inflation risk premium is zero *at all maturities* when the λ_1 prices of risk are zero. To see this, note that for $\lambda_1 = 0$ we obtain $\widehat{\mathbf{M}} = \hat{\mathbf{D}}\mathbf{M}\hat{\mathbf{D}}^{-1}$, so that $(\hat{\mathbf{D}}\mathbf{M}\hat{\mathbf{D}}^{-1})^n = \hat{\mathbf{D}}\mathbf{M}^n\hat{\mathbf{D}}^{-1}$, and

$$\frac{1}{n}\left(\bar{B}^{*\prime}_n - \bar{B}^\prime_n\right)\mathbf{Z}_t - E_t\overline{\pi}_{t+n}$$
$$= \frac{1}{n}\mathbf{C}_\pi\mathbf{M}\Big[\left(\mathbf{I} - \mathbf{M}\right)^{-1}\hat{\mathbf{D}}^{-1}\left(\mathbf{I} - \hat{\mathbf{D}}\mathbf{M}^n\hat{\mathbf{D}}^{-1}\right)$$
$$- \left(\mathbf{I} - \mathbf{M}^n\right)\left(\mathbf{I} - \mathbf{M}\right)^{-1}\hat{\mathbf{D}}^{-1}\Big]\mathbf{Z}_t$$
$$= \frac{1}{n}\mathbf{C}_\pi\mathbf{M}\left[\left(\mathbf{I} - \mathbf{M}\right)^{-1}\left(\mathbf{I} - \mathbf{M}^n\right) - \left(\mathbf{I} - \mathbf{M}^n\right)\left(\mathbf{I} - \mathbf{M}\right)^{-1}\right]\hat{\mathbf{D}}^{-1}\mathbf{Z}_t$$
$$= \frac{1}{n}\mathbf{C}_\pi\mathbf{M}\left[\sum_{i=0}^{n-1}\mathbf{M}^i - \sum_{i=0}^{n-1}\mathbf{M}^i\right]\hat{\mathbf{D}}^{-1}\mathbf{Z}_t = 0.$$

Kalman Filter Matrices

Given the model solution, the nominal and real bond pricing equations, and the expressions for the survey forecasts, we can proceed to define the observation equation as

$$\mathbf{W}_t = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^* \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{B}} \\ \tilde{\mathbf{B}}^* \\ \mathbf{C} \\ \mathbf{G} \end{bmatrix} \mathbf{X}_{1,t}$$

$$= \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^* \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{B}}^o \\ \tilde{\mathbf{B}}^{o*} \\ \mathbf{C}^o \\ \mathbf{G}^o \end{bmatrix} \mathbf{X}^o_{1,t} + \begin{bmatrix} \tilde{\mathbf{B}}^u \\ \tilde{\mathbf{B}}^{u*} \\ \mathbf{C}^u \\ \mathbf{G}^u \end{bmatrix} \mathbf{X}^u_{1,t}$$

$$\equiv \mathbf{K} + \mathbf{L}'\mathbf{X}^o_{1,t} + \mathbf{H}'\mathbf{X}^u_{1,t},$$

and the measurement equation as

$$\mathbf{X}^u_{1,t} = \mathbf{F}\mathbf{X}^u_{1,t-1} + \mathbf{v}^u_{1,t},$$

where $\mathbf{F}$ selects the sub-matrix of $\mathbf{M}$ corresponding to $\mathbf{X}^u_1$.

Next, the unobservable variables are estimated using the Kalman filter. In doing so, we first introduce a vector $\mathbf{w}_t$ of serially uncorrelated measurement errors corresponding to the observable variables $\mathbf{W}_t$. Letting $\mathbf{R}$ denote the variance-covariance matrix of the measurement errors and $\mathbf{Q}$ the variances of the unobservable state variables $\mathbf{X}^u_{1,t}$, we have

$$\mathbf{W}_t = \mathbf{K} + \mathbf{L}'\mathbf{X}^o_{1,t} + \mathbf{H}'\mathbf{X}^u_{1,t} + \mathbf{w}_t, \quad \mathrm{E}\left[\mathbf{w}_t\mathbf{w}_t'\right] = \mathbf{R}$$
$$\mathbf{X}^u_{1,t} = \mathbf{F}\mathbf{X}^u_{1,t-1} + \mathbf{v}^u_{1,t}, \quad \mathrm{E}\left[\mathbf{v}^u_{1,t}\mathbf{v}^{u\prime}_{1,t}\right] = \mathbf{Q}.$$

While we assume that all observable variables are subject to measurement error, we limit the number of parameters to estimate by assuming that all yield measurement errors have identical variance and that all errors are mutually uncorrelated:

$$\mathbf{R} = \begin{bmatrix} \sigma^2_{m,y} & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{m,y} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{m,x} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{m,\pi} & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots \end{bmatrix}.$$

Note also that

$$\mathbf{Q} = \begin{bmatrix} \sigma^2_{\pi^*} & 0 & 0 & 0 \\ 0 & \sigma^2_{\eta} & 0 & 0 \\ 0 & 0 & \sigma^2_{\pi} & 0 \\ 0 & 0 & 0 & \sigma^2_{x} \end{bmatrix}.$$

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