Computational Learning and Complexity Theory

Supervisor(s):

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Project description:

The project aims to introduce the students to research in current themes across the mathematical foundations of computational learning and its interface with complexity theory. The overarching objective is to show explicit computational limitations on the ability to efficiently learn certain function families, and to provide new applications for these limitations in deep questions about our ability to establish central open problems in computational complexity. This objective is multi-faceted, and the student is expected to focus on one concrete model and application.
Project Aims:
The aim is to show that certain Boolean or algebraic circuit classes cannot be efficiently learned in different learning models (PAC-learning or uniform learning with membership queries). Moreover, to apply these results to obtain limitations on the ability to establish non-provability results for certain central statements in complexity theory.
Project synopsis:
1. Write a background report detailing rigorously *two* of the following (based on relevance to the problem tackled):
    a. PAC-learning (confidence, accuracy, learning circuits model) and learning with equality membership, uniform models of learning.
    b. Algebraic circuits, learning of algebraic circuits (reconstruction algorithms).
    c. Rudich-Razborov Natural Proofs notion.
    d. Algebraic natural proofs.
2. Develop a way to translate between algebraic natural proofs to learning of algebraic circuits ("reconstruction"), mimicking the Boolean translation of Carmosino et al. 2016. This should be considered in different algebraic circuit models (e.g., constant-depth circuits, or other restricted variants).
3. Variants and expansions of this project are possible and expected.

Timeline (tentative):

Month 1-2: Agree on a precise plan.
Month 3-8: Research.
Month 9-11: Thesis writing-up.

Minimum viable thesis:

Write a report detailing rigorously for the following concepts:
a. PAC-learning (confidence, accuracy, learning circuits model) and learning with equality membership, uniform models of learning.
b. Algebraic circuits, learning of algebraic circuits (reconstruction algorithms).
c. Provability concepts in weak theories of arithmetic (Vi, PV, S^1_2).
d. Rudich-Razborov Natural Proofs notion.
e. Algebraic natural proofs.

Required background & skills:

Mathematical maturity mostly. Background in undergraduate level computational complexity theory (and depending on the direction taken, possibly also first-order logic) is required.

Representative References:

[1] Marco Carmosino, Valentine Kabanets, Antonina Kolokolova, Igor C. Oliveira. LEARN-Uniform Circuit Lower Bounds and Provability in Bounded Arithmetic. STOC '21.

[2] M. J. Kearns and U. V. Vazirani. An Introduction to Computational Learning Theory. MIT Press, Cambridge, MA, USA, 1994.
[3] Stephen Cook and Phuong Nguyen. Logical Foundations of Proof Complexity. Cambridge University Press, 2010. doi: 10.1017/CBO9780511676277.
[4] Iddo Tzameret and Stephen A. Cook. Uniform, Integral and Feasible Proofs for the Determinant Identities. Journal of the ACM (JACM), Vol. 68, No. 2 (2021).
[5] Carmosino M., Impagliazzo R., Kabanets V., Kolokolova A.; Learning algorithms from natural proofs, Conference on Computational Complexity (CCC), 10:1-24, 2016.