A study of the mechanics of an oscillating mechanism

Christopher G. Provatidis

***Abstract*—**This paper discusses aspects of the mechanics involved in the Dean-like Space Drives, which are mechanisms with two eccentric lumped masses that rotate in opposite direction about parallel linear axes which are rigidly connected and driven by electric motors. The typical mechanism is based on the induced centrifugal forces and it achieves to convert rotary motion to unidirectional motion. In case of a constant angular velocity, we derived closed form analytical expressions for all mechanical quantities related to the motion of an object to which the inertial mechanism is attached. Based on these formulas, we found that although this device is not capable of achieving long-term propulsion, it can operate like a catapult thus reaching a maximum altitude. We also discuss the effect of predefined time varying angular velocity imposed by the motors, the oblique motion, the possible absence of power supply, the conservation of the linear momentum and the efficiency in the conversion process from rotary to unidirectional motion.

***Keywords*—** Centrifugal force, Dean space drive, Mechanical antigravity toys.

I. Introduction

NON-ROCKET space-launch is the idea of reaching outer space specifically from the Earth's surface predominately without the use of conventional chemical rockets, which today is the only method in use. The main disadvantages of rocket systems is the increased cost of space flights, the serious damage to the atmosphere and the difficulties in full control of the spacecraft after its launch. Therefore, many scientists and technicians have proposed several alternative propulsive systems, for which reviews are available [1,2]. The interested reader may also consult a textbook concerning ion and Hall thrusters [3].

Apart from the abovementioned electric propulsion, several inventors have proposed some purely mechanical means and particularly inertial propulsion mechanisms based either on centrifugal forces [4,5] or on Coriolis force [6]; in all these cases rotating members have been utilized. The general concept they claimed is that when those mechanisms (sometimes called *inertial propulsion devices*) are attached to a rigid object, the inertial forces could achieve long-term propulsion. As they claimed [4,5], the high efficiency is due to disappearing reaction forces (so-called 'reactionless' propulsion). In contrast to these claims, a recent report on '*mechanical antigravity*' has concluded that since the total impulse per cycle becomes zero, the gravity forces lead the object back to the earth [7,8]. Unfortunately, the aforementioned report is quite qualitative in the sense that it presents neither detailed mechanics nor any graphs.

Within the context of the abovementioned mechanical antigravity devices, one of the earliest concepts was to utilize two synchronized eccentric masses that rotate in opposite direction about parallel linear axes which are rigidly connected and are driven by electric motors. It is remarkable that Dean [4] demonstrated that his experimental prototype was capable of converting rotary motion to unidirectional motion and it travelled a certain distance along vertical guides (a photo is found at the link http://en.wikipedia.org/wiki/Dean-drive). Nevertheless, this topic has not been covered in a scientific manner so far. A careful search reveals an old paper in the Russian language [9], two remarks in a textbook [10], as well as the abovementioned references [7,8], which are very general reports dealing with many possible mechanical antigravity concepts (qualitatively including Dean's drive). A preliminary study has been recently published to investigate the variation of the angular velocity in the rotating parts [11], while another preliminary report is concerned with the modification of the circular path to an eight-shaped one [12].

Therefore, many questions have to be definitely answered. It should be also noted that the significance of the eccentric masses and their synchronization has become a matter of detailed analysis in the excitation of ground machines and structures [13,14] but not in the particular case under investigation in which the object is left to fly or fall.

The purpose of this paper is fourfold. First, to obtain closed-form equations of motion in the Dean space drive for the first time. Second, to provide a definite conclusion whether contra-rotating mass particles are capable of producing long-term propulsion. Third, to investigate the effect of constant or predefined variable revolutions per minute in the motors. Fourth, to investigate the differences involved in the absence of power supply. Moreover, oblique motion and the conservation of linear momentum are discussed. Topics of ongoing research are mentioned.

Manuscript received April 27 2011: Revised version received August 5, 2011.

C. G. Provatidis is with the National Technical University of Athens, GR-15780 Athens, GREECE (corresponding author to provide phone: +30-210-7721520; fax: +30-210-7722347; e-mail: cprovat@central.ntua.gr).

II. Basic Formulation

A. Problem definition

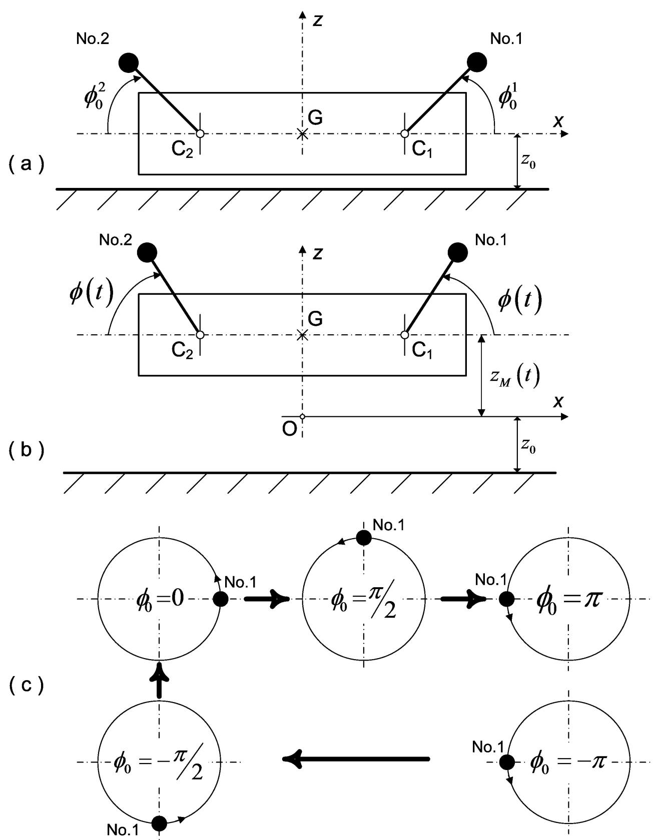

Fig. 1 Setup of the mechanism (a) when it is left to fall, (b) at the arbitrary time t, while (c) are characteristic positions of the initial polar angle ϕ_0 increasing as the arrow shows.

Let us consider an object (body B) of mass M on which two rigid rods (No.1 and No.2), of the same radius r, are articulated as shown in Fig. 1; the rods bring at their ends lumped masses, of equal size m. The two aforementioned eccentric masses are driven by electric motors and are synchronized to rotate at *constant* angular velocities of equal and opposite magnitude, i.e. $\omega_1(t) = -\omega_2(t) \equiv \omega$. Obviously, the mass of the electric motors is included into the body B. The initial position of the rods are denoted by ϕ_1^0 and ϕ_2^0, respectively, for which it is assumed that $\phi_1^0 = -\phi_2^0 = \phi_0$; therefore, for a later time instance, it holds that $\phi_1(t) = -\phi_2(t) = \phi(t)$. Without loss of generality, we assume that the articulation of the rigid rods is chosen at the level of the centroid G of the mass M.

For the sake of briefness, we simplify the problem as follows:

1. The shape of the object B appears no variation along the x- and y-axis (the latter is not shown).
2. All participating bodies are considered to be rigid.
3. The two lumped masses of magnitude m have always the same z-coordinate.
4. The motors are fixed to the object at points that define a segment C_1C_2 parallel to the horizontal x-axis (Fig. 1) and their shafts are parallel to the y-axis.
5. At the initial time instance, $t = 0$, the mechanism is left to fall from a height z_0.
6. The effect of the air is neglected.

Due to the abovementioned assumptions, the components of the centrifugal forces parallel to the x-axis are perfectly cancelled, and any possible motion of the object will be in the z-direction only. In other words, no rotation of the object B will occur, thus its position can be written in the simple form:

$$z(t) = z(\omega, \phi_0) \tag{1}$$

B. Equations of motion

Let $\mathbf{f}_0 = f_0\hat{\mathbf{e}}_z$ stand for the reaction force, which the ground imposes to the object, with $\hat{\mathbf{e}}_z$ denoting the unit vector of the z-axis. According to the Newton's Second Law, it holds:

$$(2m+M)\ddot{\mathbf{r}}_c = \mathbf{f}_0 + (2m+M)\mathbf{g} \tag{2}$$

where $(2m+M)$ is the total mass, $\mathbf{g} = -g\hat{\mathbf{e}}_z$ is the acceleration of gravity vector, and $\ddot{\mathbf{r}}_c$ is the acceleration of the center of mass (CM) of the system of which the ordinate is given as

$$z_c = (2mz_m + Mz_M)/(2m+M) \tag{3}$$

In (3), z_M, z_m and z_c are the ordinates of the centroid of mass M, the masses m and the overall centroid, respectively. The relationship between z_M and z_m (mass No.1) is:

$$z_m = z_M + r\sin\phi \tag{4}$$

Eliminating the ordinate z_m between (3) and (4), one receives:

$$z_c = z_M + \frac{2m}{(2m+M)} r\sin\phi \tag{5}$$

Taking the second temporal derivative, the vertical acceleration is given by

$$(2m+M)\ddot{z}_c = (2m+M)\ddot{z}_M + 2mr(\sin\phi)^{\cdot\cdot} \tag{6}$$

Since the term $(2m+M)\ddot{z}_c$ represents the sum of the external forces, which consist of only the gravitational ones, the combination of (2) with $\mathbf{f}_0 = 0$, and (6) leads to:

$$(2m+M)\ddot{z}_M + 2mr(\sin\phi)^{\cdot\cdot} + (2m+M)g = 0 \tag{7}$$

Furthermore, the second temporal derivative of $(\sin\phi_1)$ becomes

$$(\sin\phi)^{\cdot\cdot} = -(\dot{\phi})^2\sin\phi + \ddot{\phi}\cos\phi \tag{8}$$

Considering the instantaneous angular velocity:

$$\omega = \frac{d\phi(t)}{dt} = \dot{\phi}(t) \ , \tag{9}$$

and substituting (8) and (9) into (7), the final differential equation of motion of the free object B becomes:

$$\ddot{z}_M = \mu\left[\omega^2 \sin\phi - \dot{\omega}\cos\phi\right] - g \triangleq f(t) \ , \tag{10}$$

where ω^2 and $\dot{\omega}$ are related to the centrifugal and the tangential components, respectively, and

$$\mu = 2mr/(2m+M) \tag{11}$$

C. Solution of equation of motion

C1. General solution

According to classical dynamics [15], the vertical position of the mass *M* is described by the formula

$$z_M(t) = z_0 + v_0 t + \int_0^t \int_0^\xi f(\tau)\, d\tau\, d\xi \,, \tag{12}$$

while the velocity will be

$$\dot{z}_M(t) = v_0 + \int_0^t f(\tau)\, d\tau \,, \tag{13}$$

where z_0 and v_0 denote the initial height and initial velocity of the object B, respectively [$z_M(0) = z_0$, $\dot{z}_M(0) = v_0$]. Considering that at the initial time *t* = 0 the angular velocity and the angular position are ω_0 and ϕ_0, respectively, the exact solution is obtained as follows:

$$z_M(t) = \left(z_0 + v_0 t - 1/2\, gt^2\right) + \left(\mu\omega_0 \cos\phi_0\right)t \\ -\mu\left(\sin\phi - \sin\phi_0\right) \tag{14}$$

and

$$\dot{z}_M(t) = \left(v_0 - gt\right) - \mu\left(\omega\cos\phi - \omega_0\cos\phi_0\right) \tag{15}$$

As previously mentioned, the horizontal motion of the object vanishes, i.e.

$$x_M(t) \equiv 0 \tag{16}$$

It is remarkable that the altitude $z_M(t)$ in (14) includes the gravitational term $\left(z_0 + v_0 t - 1/2\, gt^2\right)$ which is responsible for the free fall, a second linear term $\left(\mu\omega_0\cos\phi_0\right)t$, and also a small compound term $-\mu\left[\sin\phi(t) - \sin\phi_0\right]$ (its absolute value is smaller than 2μ). Whatever the value of the coefficient $\left(\mu\omega_0\cos\phi_0\right)$ is, there is always a minimum value of time *t* for which and further the term $\left(-1/2\, gt^2\right)$ dominates thus leading to the fall of the object.

In the sequence, two types of temporal variation of the angular velocity in the motors will be mainly investigated. A third variation (section *IV, E*) concerns the absence of power supply.

C2. Constant angular velocity

In case of a constant angular velocity, (14) and (15) are valid with

$$\omega = \omega(t) = \omega_0 \quad \text{and} \quad \phi = \phi(t) = \phi_0 + \omega_0 t \tag{17}$$

C3. Exponential angular velocity

Alternatively, we choose that:

$$\phi(t) = \phi_0 + \left(e^{\lambda t} - 1\right)\omega_0 / \lambda \tag{18}$$

whence

$$\omega(t) = \dot{\phi}(t) = \omega_0 e^{\lambda t} = \omega_0 + \lambda\left(\phi - \phi_0\right) \tag{19}$$

where $\lambda = \Delta\omega / \Delta\phi$ is a constant, while ω_0 and ϕ_0 are the initial angular velocity and the initial angular (polar) position of the rods, respectively. Obviously, the relationship between time and polar angle is

$$t = \frac{1}{\lambda}\ln\left[\left(\phi - \phi_0\right)\lambda/\omega_0 + 1\right] \tag{20}$$

Also, the temporal derivative of the angular velocity does not generally vanish and it becomes:

$$\dot{\omega} = \lambda\omega_0 e^{\lambda t} = \lambda\omega \tag{21}$$

III. Breakdown of Energy

A. General

By virtue of (4), at every time instance the velocity of the *i*-th eccentric mass will be:

$$\dot{z}_{m,i} = \dot{z}_M + r\left(\sin\phi_i\right)^{\cdot} = \dot{z}_M + r\omega_i\cos\phi_i, \ i = 1,2 \tag{22}$$

$$\dot{x}_{m,i} = \dot{x}_M + r\left(\cos\phi_i\right)^{\cdot} = \dot{x}_M - r\omega_i\sin\phi_i, i = 1,2\,; x_M \equiv 0 \tag{23}$$

Therefore, due to the symmetrical arrangement of the two rotating masses so as with respect to the inertial reference frame (fixed to the ground) both velocities have the same measure given by $\left|\mathbf{v}_m\right| = \left(\dot{x}_m^2 + \dot{z}_m^2\right)^{1/2}$, the kinetic energy of the system '*object + eccentric masses*' will be:

$$E_{kin} = \frac{1}{2}\left[M\dot{z}_M^2 + 2m\left|\mathbf{v}_m\right|^2\right] = \frac{1}{2}\left[M\dot{z}_M^2 + 2m\left(\dot{x}_m^2 + \dot{z}_m^2\right)\right] \\ = \frac{1}{2}\left\{M\dot{z}_M^2 + 2m\left[\left(r\omega\sin\phi\right)^2 + \left(\dot{z}_M + r\omega\cos\phi\right)^2\right]\right\} \tag{24}$$

while the potential energy of the system will be:

$$E_{pot} = \left[Mz_M + 2m\left(z_M + r\sin\phi\right)\right]g \\ = \left(2m + M\right)gz_M + 2mgr\sin\phi \tag{25}$$

B. Operation when the object has fixed support to the ground

When the object is fixed to the ground ($z_M = 0, \dot{z}_M = 0$), in virtue of (22)-(23), (24) and (25) lead to the relationships:

$$E_{kin}^{fixed} = mr^2\omega^2$$
$$E_{pot}^{fixed} = 2mgr\sin\phi = 2mgr\sin(\phi_0 + \omega t) \qquad (26)$$

In other words, the potential energy continuously changes with respect to the varying polar angle ϕ (the latter, of course, varies with time). Obviously, the aforementioned variation of energy is due to the variation of energy consumption by the electric motor. In more details, when the rods move from the lower to the upper vertical position the motor has to produce energy to achieve this lift ($\Delta z = +2r$), while when they move from the upper to the lower vertical position ($\Delta z = -2r$) the aforementioned increase of potential energy is given back to the motor but it is transformed mainly into heat. In other words, although the angular velocity has been assumed to be constant, the entire mechanical energy is *not preserved* but is harmonically fluctuates around the constant value of the kinetic energy (mean average), E_{kin}^{fixed} given in (26).

C. Object free to move vertically with energy supply

Assuming that at *t* = 0 the rigid rods are in the angular position ϕ_0 and the object starts from a vertical rest ($z_M = 0, \dot{z}_M = 0$), the total initial energy becomes

$$E_{tot,0} = m\omega^2 r^2 + 2mgr\sin\phi_0 \qquad (27)$$

At the arbitrary time instance, *t*, after the object has been left to fall, it will be at the height $z_M(t)$ and the rigid rods will be at the angular position $\phi = \phi_0 + \omega t$. Under these conditions, after some manipulations, the total mechanical energy is found equal to

$$E_{tot} = m\omega^2 r^2 \left[1 + \frac{2m}{(2m+M)}\left(\cos^2\phi_0 - \cos^2\phi\right)\right] + 2mgr\sin\phi_0 \qquad (28)$$

Therefore, when the object is left to leave the ground, in addition to the permanent term ($E_{tot,0}$), (28) includes also an harmonic term [$E_h = \dfrac{2m^2r^2\omega^2}{(2m+M)}\left(\cos^2\phi_0 - \cos^2\phi\right)$]. Obviously, the latter term is not only due to the periodical rising of the rigid rods, but also due to the lift $z_M(t)$. As previously, the variation of the entire mechanical energy is due to the variation of the electric energy consumed by the motor. It is remarkable that for a *given inclination* of the rods (given polar angle ϕ), the *same* mechanical energy is obtained. Therefore, when the object obtains its highest position (zero linear velocity), the linear kinetic energy becomes zero (only rotational kinetic energy exists). Comparing the latter situation with the same on the ground level (vertical position of the rods in both cases), it becomes evident that the abovementioned term E_h approximately represents the variation of the potential energy. The aforementioned 'approximation' refers to the fact that it is not a-priori clear which is the polar angle ϕ at the position of the highest height, $z_{\max}$.

C1. Approximate analytical solution

Equating the above-mentioned harmonic term E_h by $(2m+M)gz_{\max}$ we can approximately write that

$$z_{\max} \cong (2/g)\left[m\omega r/(2m+M)\right]^2 \sin^2\phi_0 \qquad (29)$$

Also, the time required to reach the abovementioned upper point is obtained by setting the object velocity equal to zero.

Thus,

$$\dot{z}_M = -\mu\omega(\cos\phi - \cos\phi_0) - gt_{\max} = 0 \qquad (30)$$

Ignoring the term $-\mu\omega\cos\phi$ compared to the rest ones, we can approximately write that

$$t_{\max} \cong \mu\omega\cos\phi_0/g \qquad (31)$$

C2. Accurate numerical approach

We can estimate the accurate time instance at which the maximum possible height is achieved by requiring that the velocity of the object becomes zero ($\dot{z}_M = 0$). However, the solution of $\dot{z}_M = -\mu\omega(\cos\phi - \cos\phi_0) - gt = 0$ implies *all* those time instances, $t_{l\max}$, where the aforementioned local maxima or minima appear. These values are given by (30) from which we obtain:

$$t_{local,opt} = (\mu\omega/g)\left[\cos(\phi_0 + \omega t_{local,opt}) - \cos\phi_0\right] \qquad (32)$$

Equation (32) is a transcendental equation and gives a range of values of which only one corresponds to the upper height. In general, it is sufficient to apply (32) for every round of the rigid rods, in the angle interval $[\phi_0 + 2n\pi,\ \phi_0 + 2(n+1)\pi]$, $n = 0,1,\ldots,n_{rounds}$, where n_{rounds} is an arbitrary chosen high number of rounds. Numerical solution can be performed using the standard Newton-Raphson method, in which the initial choice is in the middle of the aforementioned angle intervals. For each of the aforementioned solutions, the time is calculated using (17) and, then, the height is calculated using (14). If both aforementioned values are stored in two vectors, each of dimensions $n_{rounds} \times 1$, the estimation of the global maximum is trivial.

Alternatively, it is suggested to start with an initial value given by (31) and then continue with some local maxima on the interval in which the initial value belongs, as well as the left and right neighbouring intervals.

D. Efficiency of the mechanism

As was shown in section *III, B*, the initial kinetic energy $E_{kin}^{fixed} = mr^2\omega^2$ (cf. (26)) at the ground level is transformed into potential and kinetic energy, which at the maximum possible altitude (considering that the instantaneous object velocity vanishes) becomes:

$$E_{tot} \cong (2m+M)\,gz_{\max} \cong \frac{2(m\omega r)^2}{(2m+M)}\cos^2\phi_0 \qquad (33)$$

Therefore, the efficiency of the propulsive mechanism could be expressed by:

$$\eta = E_{tot} \big/ E_{kin}^{fixed} \cong 2m\big/(2m+M)\cos^2\phi_0 \qquad (34)$$

E. Energy conservation without power supply

So far we have assumed a continuous power supply through a motor. In the absence of a motor, if the object is suddenly left free to move by dragging the floor, the total energy is preserved. Taking into consideration that the total energy produced by adding in parts (24) and (25) is maintained constant, obviously equal to its initial value at time $t = 0$, after manipulations we obtained that the angular velocity varies according to the formula

$$\omega(\phi) = \dot{\phi}(t) = \omega_0 \left[\frac{2m\sin^2\phi_0 + M}{2m\sin^2\phi + M}\right]^{1/2} \qquad (35)$$

As we could not find an explicit solution in the time t, (35) was numerically solved using the forward Euler method in conjunction with a very small time step.

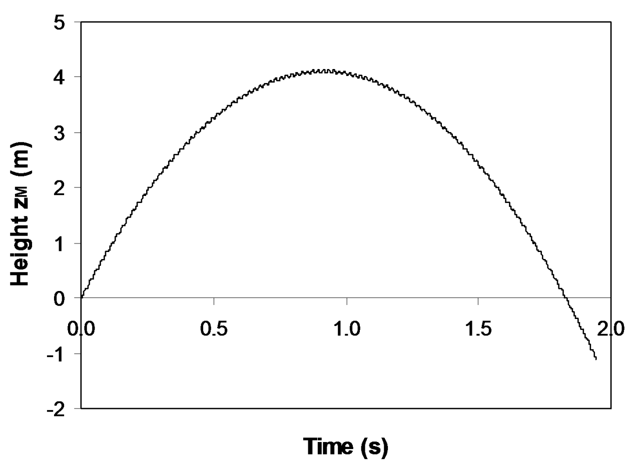

Fig. 2 Calculated height of the object versus the elapsed time (ω = 3000 rpm, $\phi_0 = 0$).

IV. NUMERICAL SIMULATION

A. General data

In the beginning, the following set up is selected:

- Rotating mass at the end of every rigid rod: m = 1·0 kg
- Mass of the object B: M = 5·0 kg
- Radius of rigid rod: r = 0·10 m
- Angular velocity: ω = 314·16 s^{-1} (3000 rpm)
- Initial polar angle of the rods: $\phi_0 = 0$ deg
- Acceleration of gravity: g = 9·81m/s^2

In the sequence, the analysis includes constant and time-varying angular velocity of the electric motors.

B. Constant angular velocity

B1. Basic results

A typical graph of the time varying height of the object is shown in Fig. 2. It can be noticed that the object obtains its maximum height at the time $t_{\max} \cong \mu\omega/g = 0\cdot 915$ s, and then it returns back to the ground level at $t_{back} \cong 2\mu\omega/g = 1\cdot 83$ s.

Concerning the linear velocity of the object B, a typical graph is shown in Fig. 3. It can be noticed that:

- The object's velocity is subjected to high variations (it fluctuates) but its lower value continuously coincides with that of a hypothetical free fall and also the amplitude is preserved.
- Close to the abovementioned time t_{back}, the maximum value of the fluctuating velocity becomes zero, thus being equal to the zero linear velocity at t = 0.

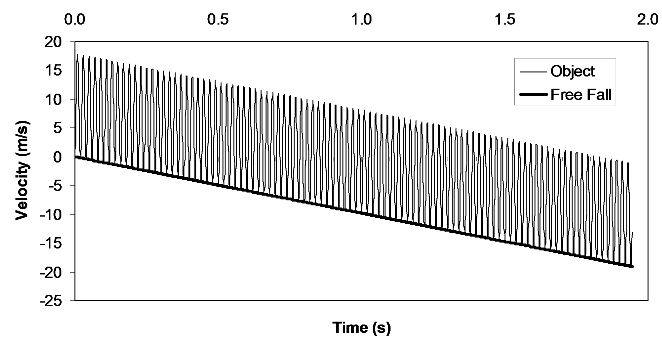

Fig. 3 Calculated velocity of the object versus the elapsed time (ω = 3000 rpm, $\phi_0 = 0$).

For the entire system, i.e. object and the attached rotating rigid rods, the graphs of the kinetic and potential energy are shown in Fig. 4, where one can notice a progressive decrease of the kinetic energy and an increase of the potential energy up to the time instance $t_{\max}$ = 0·915s, which corresponds to the highest possible height.

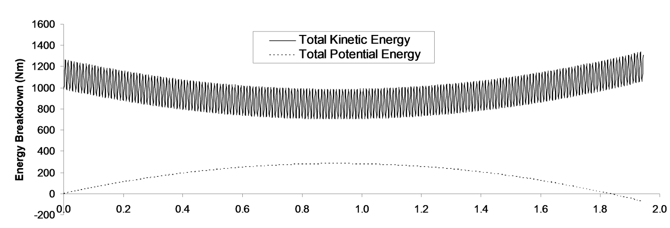

Fig. 4 Breakdown of total energy of the system object-rods versus time (ω = 3000 rpm, $\phi_0 = 0$).

A better understanding is obtained in Fig. 5, which focuses on the early period of the lift; it should be noted that the waveform remains unaltered and preserves the same minimum and maximum values in all subsequent instances. It can be noticed that at the initial time $t = 0$ ($\phi = \phi_0 = 0$ degrees), the total energy of the system coincides with the kinetic energy that possesses its minimum value of 987 Nm. Within the first round of the rods, it obtains twice its maximum value of 1269 Nm, once at the angular position of 90 degrees and again at 270 degrees; the same minimum value of 987 Nm is obtained at 180 degrees. All the latter values are repeated for all subsequent rounds.

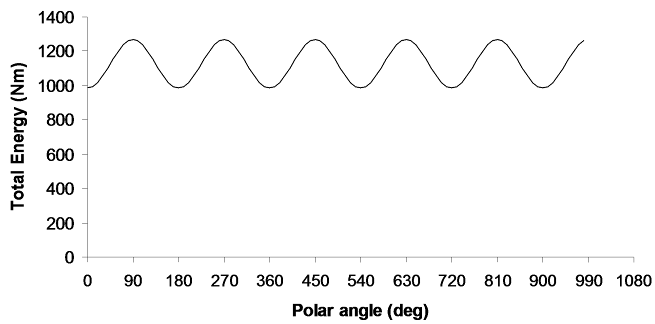

Fig. 5 Total mechanical energy (kinetic and potential) of the entire system (object plus rods) versus time ($\omega = 3000$ rpm, $\phi_0 = 0$).

In the sequence, we investigate the influence of the rod masses, *m* on the propulsion performance. When every mass *m* equals to *M*, the corresponding (reference) maximum achieved height is denoted by $Z_{\max}(M)$ while for any other lower value of *m* it is denoted by $Z_{\max}(m)$. In Fig. 6, the normalized height $Z_{\max}(m)/Z_{\max}(M)$ is illustrated in terms of the mass ratio (m/M), and their relation is not very far from the linear interpolation.

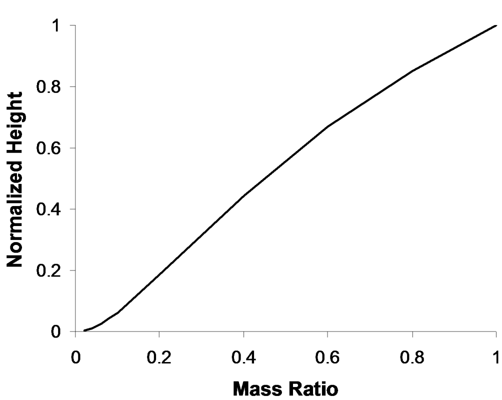

Fig. 6 Normalized maximum height versus the mass ratio (*m*/*M*).

B.2 Remarks

In this study the angular velocity was considered to be preserved constant, a fact that could give the wrong impression that the kinetic energy of the rigid rods had to be preserved, too. In contrast, the variation of the latter is shown in Fig. 7, where one can notice that it continuously decreases until the upper height is reached. This happens because the analytical expression for the kinetic energy, i.e. $E_{kin,rods} = m\left[\left(r\omega\right)^2 + \dot{z}_M^2 + 2\dot{z}_M r\omega\cos\phi\right]$, is coupled with the motion of the object, as it includes the terms $\dot{z}_M^2$ (non-negative) and $2\dot{z}_M r\omega\cos\phi$ (of variable sign).

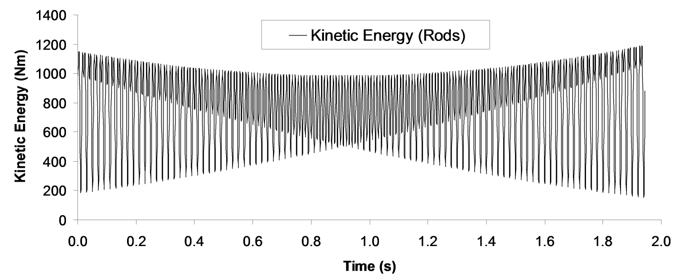

Fig. 7 Variation of the kinetic energy of the rotating rigid rods versus time ($\omega = 3000$ rpm, $\phi_0 = 0$).

The exchange of the kinetic energy between the rods and the object is clearly shown in Fig. 8. It can be noticed that during approximately the first 70 degrees the kinetic energy of the rods increases and then it decreases obtaining its minimum value at the position of 180 degrees. In contrast, the kinetic energy of the object increases monotonically up to the 180 degrees and then it decreases. The latter position of 180 degrees is the point where the total kinetic energy obtains its local minimum value, which is -0·62% smaller than the initial value (980·83 Nm instead of 986·96 Nm). It is noted that the aforementioned difference of 6l3 Nm equals t o the increase of total potential energy) but this happens only for the particular case of 180 degrees at which the total mechanical energy obtains its initial value.

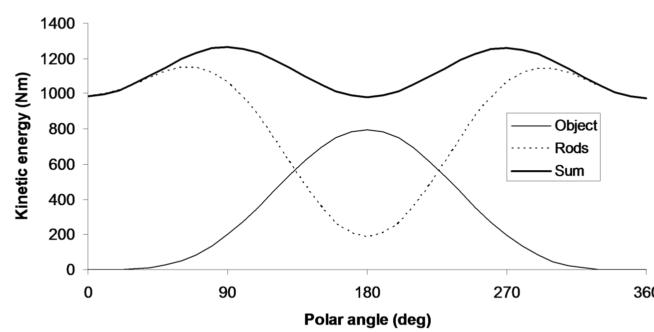

Fig. 8 Exchange of kinetic energy between rotating rods and lifted object during the first round of 360 degrees ($\omega = 3000$ rpm, $\phi_0 = 0$).

In more details, concerning the results shown in Fig. 5, the fact that the total mechanical energy of the system is not preserved (except of the positions of 180, 360, 540 degrees, and so on), suggests that the fluctuations are due to the electric motor that consumes additional energy within the intervals [0,180]deg, [180,360]deg, [360,540]deg and so on; thus it is capable of elevating the object while maintaining a constant angular velocity.

It is also noted that the initial available kinetic energy required to achieve the lift of the object at a certain height is many times larger than that required to achieve the same maximum height by fully transforming the rotational kinetic energy into (a zero linear kinetic energy plus) potential energy. For example, the initial kinetic energy of 98696 Nm would be sufficient to lift the object up to 14·4m but the calculations of this study suggest a maximum lift of only 4·1m. In this particular case where the mass ratio is $m/M = 0\cdot 2$, these calculations determine the efficiency of the mechanism at about $\eta = 0{\cdot}286$ (28·6%). Similar calculations, using (34), for $m/M = 0{\cdot}3$, 0·5, and 1·0 lead to $\eta =$ 0·375, 0·444, and 0·667, respectively. Obviously, when the rotating masses become infinitely large compared to the object mass ($m >> M$), the efficiency tends to the unity ($\eta \to 1$).

Moreover, let us now assume that close to the vertical orbit of the object there is a vertical wall of infinite height or a tether. When the object reaches the maximum altitude ($z_{\max}$), we assume that either an external mechanical system or an internal grip is used to temporarily immobilize it. We also assume that the object remains there for a while, particularly until the rigid bars obtain again the initial horizontal position ($\phi_0 = 0$) and at this specific time it is suddenly released to fall down. Obviously, the analysis of Section II and Section III dictates that the object will again rise for another $z_{\max}$ from the latest datum ($z_{datum} = z_{\max}$), thus leading to a total altitude $2z_{\max}$, measured from the ground level. Repeating the latter procedure, the object can climb as high as we wish provided a neighbouring vertical wall exists. Having said this, it should become clear that the capability of climbing at a multiple of $z_{\max}$ is ought to the external or internal mechanical system that continuously offers the required reaction to support the object for its next climb.

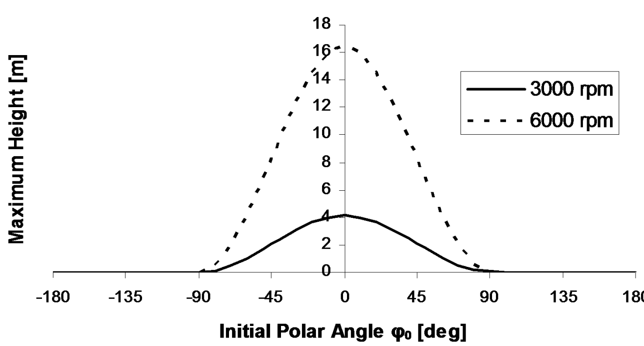

Fig. 9 Maximum achieved height versus the initial polar angle ϕ_0, for two typical angular velocities.

So far, the initial angular position was set at $\phi_0 = 0$ and the angular velocity at 3000 rpm (314·16 s^{-1}). Furthermore, the dependence of the maximum possible height that can be reached by the object, in terms of the initial polar angle and the angular velocity, is shown in Fig. 9. It can be noticed that the best result corresponds to the horizontal position of the rigid rods ($\phi_0 = 0$), while for greater than 90 degrees the maximum height practically vanishes. Theoretically, the maximum height becomes absolutely zero only when the position angle equals to 180 degrees. Also, the higher the angular velocity is, the higher altitude is obtained.

C. Linearly increasing angular velocity

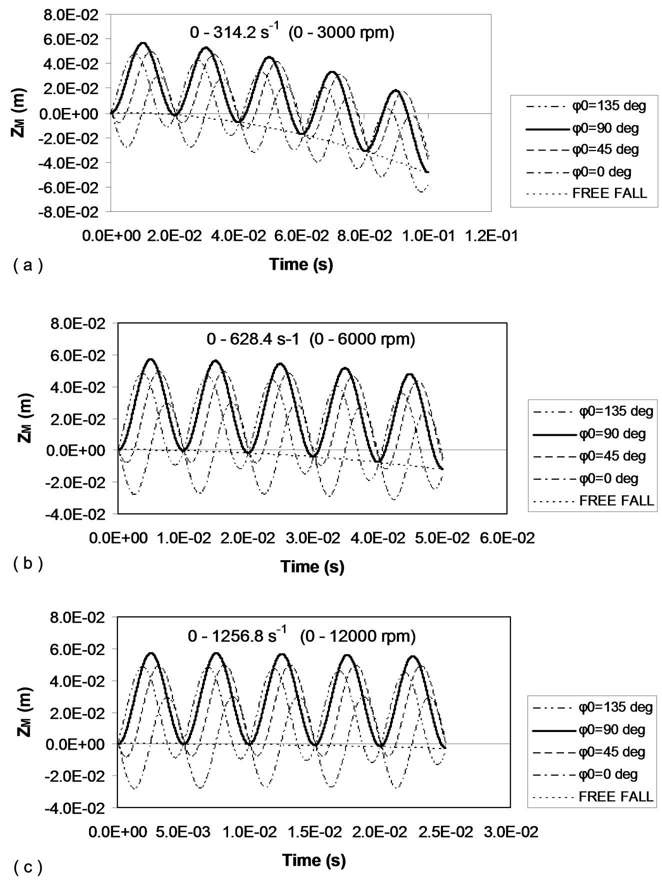

Fig. 10 The vibration of the object when it is first left to fall and then the angular velocity increases linearly in time up to the value of (a) 3000, (b) 6000 and (c) 12000 rpm.

While all previous results and above remarks refer to an object that is left to fall when the attached rigid rods have already obtained a *considerable* angular velocity, $\omega = \omega_0 > 0$, here we investigate what happens when the object is left free to fall at a *zero* angular velocity ($\omega_0 = 0$). Then, we assume that it obtains a final angular velocity of magnitude ω_{fin} = 1×314·16, 2×314·16 and 3×314·16 s^{-1} (3000, 6000 and 12000 rpm), respectively, within a time period of $\Delta t = 1.0$E-04 seconds. In all these three cases, the transition from zero to the final value of the angular velocity varies linearly in time and then it is preserved constant at 3000, 6000, and 12000 rpm, respectively, as follows:

$$\omega(t)=\begin{cases}(\omega_{fin}t)/\Delta t, & 0\le t\le \Delta t\\ \omega_{fin}, & t>\Delta t\end{cases}$$

Starting from four different initial polar positions of the rigid rods ($\phi_0=0$, 45, 90, and 135 degrees), the results are shown in Fig. 10. It can be noticed that, in contrast to the previous findings, the object *cannot* now rise up but it *continuously falls down*. The only essential difference between these four angular velocities is the time scale, in the sense that the higher the angular velocity is the shortest the time interval is (for the first five rounds shown). Another interesting fact is that only when $\phi_0=90$ degrees, the lower height of the object is very similar to that of the free fall case; otherwise it is lower.

D. Exponentially varying angular velocity

In this subsection we assume that the angular velocity is governed by (19), i.e. as $\omega(t)=\omega_0 e^{\lambda t}$. Typical graphs of the object's velocity $\dot{z}_M(t)$ and the corresponding angular velocity $\omega(t)$ are illustrated in Fig. 11. Finally, it is noted that in the graphs of the vertical displacement of the object, coming from the differences in the selection of the λ-variable, can be only hardly noticed.

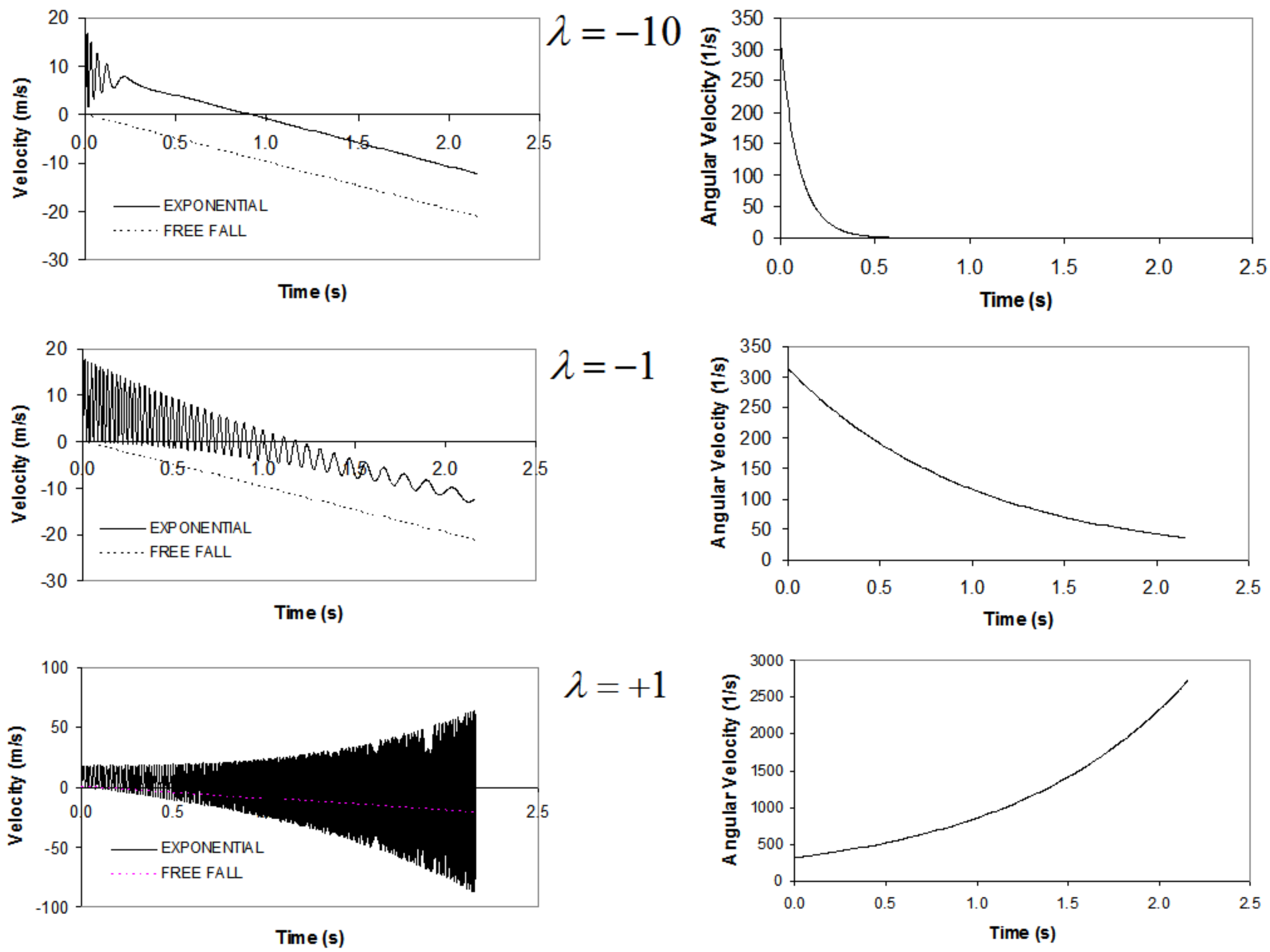

Fig. 11 Object velocity, $\dot{z}_M(t)$, and angular velocity versus time, $\omega(t)$, for exponentially varying angular velocity [exponents: λ = -1, -10, and +1 are involved in (18)-(21)].

E. Motion without energy supply

Similar to the above conclusion, when the motors stop supplying energy, the numerical solution of (35) shows that neither the maximum height nor the time required for the object to reach the ground is highly influenced. The differences with the previous cases (power supply) are not visible in the graphs. The only quantity that seriously changes is the angular velocity that varies periodically between approximately 266 and 314 s^{-1}, i.e. the rigid rods preserve the initial value as an upper limit but they also obtain a minimum value, which is about 15 percent less than the initial angular velocity.

I. FURTHER DISCUSSION

A. Oblique motion

Let us consider the straight line segment that joins the two centers (C_1 and C_2) of the circular paths on which the rotated masses move. As previously mentioned, the two rigid rods are articulated at these centers as shown in Fig. 1. Let us also assume that the segment C_1C_2 is inclined with respect to the horizontal plane. In this case, the object will follow an inclined orbit, similar to the motion of a projectile thrown obliquely into the air [16]. In more detail, the resultant propulsive force is no further vertical but perpendicular to the line segment C_1C_2, while a small moment is produced by the weights of the rotating masses thus causing rotation of the object B. In this framework, a number of applications such as toys or locomotion devices could be developed by extending the mechanics of the elementary mechanism studied in this work.

B. Conservation of linear momentum

B1. General

For all simulated results obtained in this work, either the motors supply energy or not, it has been numerically validated that the horizontal component of the resultant vector of system's linear momentum are *preserved*. In contrast, the vertical component changes due to the gravitational (external) force, so as it always equals to the quantity $-(2m+M)gt$, where t denotes the elapsed time.

B2. Absence of gravity and the first revolution of the lumped masses

In the absence of gravity, in conjunction with the ideal condition of no frictional losses, the motion of the object continues for ever while the momentum conservation holds.

Let us for simplicity consider the case where the two rods rotate at a constant angular velocity ω. As an initial condition, we consider that the mass M is at rest (velocity $v_M = 0$), while the two bars are horizontal moving towards their upper point (Fig. 12). At the aforementioned time instance t = 0, the object is released from its support and it is left to fall down.

At every time t, the absolute velocity of any lumped mass equals to the sum of the relative velocity plus the velocity of the object, i.e.:

$$\mathbf{v}_m = \omega r \hat{\mathbf{e}}_\theta + \mathbf{v}_M \tag{36}$$

Obviously, the vertical component of the initial linear momentum of the mechanical system is

$$(\mathbf{p}_i)_y = 2m\omega r \tag{37}$$

After rotation by 90 degrees ($\phi = \pi/2$), the two bars become vertical, so as the vertical component of the tangential unit vector $\hat{\mathbf{e}}_\theta$ vanishes. Therefore, each of the two lumped masses obtains identical vertical velocity v_M to the object; thus the momentum of the lumped masses becomes $2mv_M$, while that of the object becomes Mv_M.

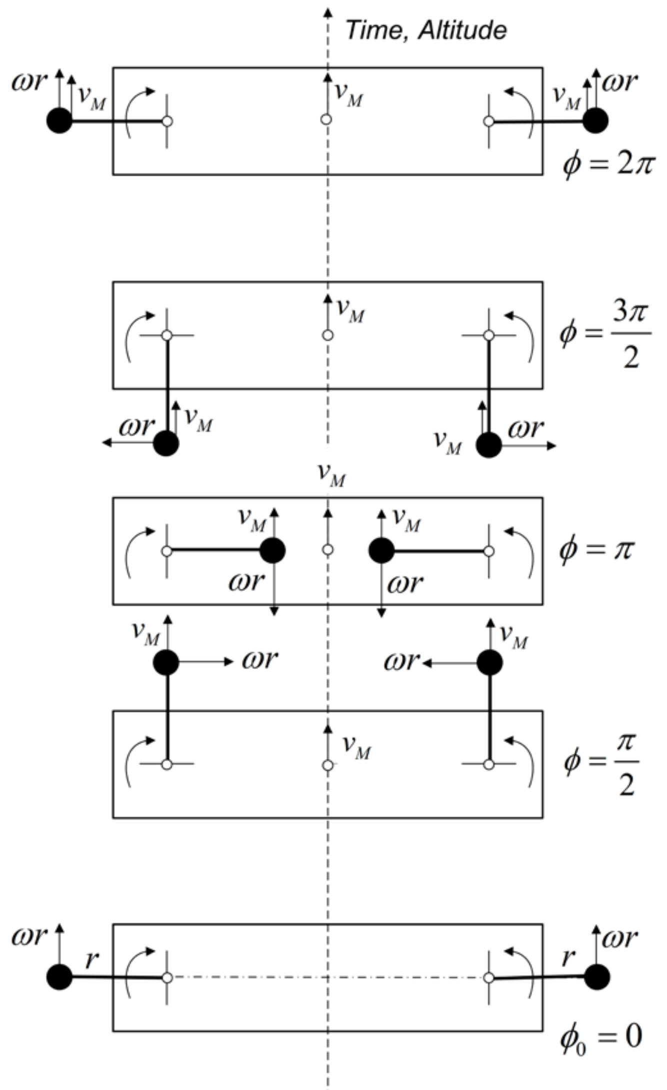

Fig. 12 Four successive positions of the object, starting from the bottom (where: $t_0 = 0,\ \phi = \phi_0 = 0$) to the top (where: $t = 2\pi/\omega,\ \phi = 2\pi$).

Similarly, when $\phi = \pi$ the lumped masses undertake absolute vertical velocity component equal to $v_M - \omega r$, while when $\phi = 3\pi/2$ the vertical velocity component becomes again equal to v_M. Finally, when $\phi = 2\pi$ the two bars become again horizontal having again vertical velocity components equal to $\omega r + v_M$, which implies that $v_M = 0$ as it initially was at t = 0.

Based on the conservation of the linear momentum in the vertical direction we obtain Table 1. Obviously, for all subsequent rotations, the same pattern in the velocity of the object M will be repeated.

Table 1: Calculated object velocity and object height at four characteristic positions of the rods, in the absence of gravity.

Polar Angle, ϕ [deg]	Object Velocity v_M (m/s)	Object Height z_M (m)
0	0	0
90	$\mu\omega$	$\mu\left(\pi/2-1\right)$
180	$2\mu\omega$	$\mu\pi$
270	$\mu\omega$	$\mu\left(3\pi/2+1\right)$
360	0	$2\mu\pi$
$\mu = 2mr/\left(2m+M\right)$		

C. Limitations of this study

In the particular case studied in this paper (i.e., negligible moments of inertia for all three, the rod and the two eccentric masses), we will show below that Lagrange equations *coincide* with the '*Centre of Mass Theorem*' we applied in this work. In fact, if we start from (24) and (25), in which however we enter the initially supposed 'unknown' horizontal motion as a non-vanishing variable, i.e. $x_M\left(t\right)\neq 0$, we can write the Lagrangian *L* of the dynamical system as follows:

$$\begin{aligned} L &= T - V \equiv E_{kin} - E_{pot} \\ &= \frac{1}{2}\Big[M\left(\dot{x}_M^2+\dot{z}_M^2\right)+m\left(\dot{x}_M - r\omega\sin\phi\right)^2 \\ &\quad +m\left(\dot{x}_M + r\omega\sin\phi\right)^2+2m\left(\dot{z}_M + r\omega\cos\phi\right)^2\Big] \\ &\quad -\Big[\left(2m+M\right)gz_M + 2mgr\sin\phi\Big] \end{aligned} \tag{38}$$

The two generalized coordinates of the dynamical system may be chosen as follows:

$$q_1 = x_M\left(t\right),\quad q_2 = z_M\left(t\right). \tag{39}$$

By virtue of (39) and considering $\phi=\phi_0+\omega t$ while all four quantities (m, g, r, ω) are constants, (38) becomes:

$$\begin{aligned} L\left(q_1,q_2,\dot{q}_1,\dot{q}_2\right) &= \frac{1}{2}\Big[M\left(\dot{q}_1^2+\dot{q}_2^2\right)+m\left(\dot{q}_1 - r\omega\sin\left(\phi_0+\omega t\right)\right)^2 \\ &\quad +m\left(\dot{q}_1 + r\omega\sin\left(\phi_0+\omega t\right)\right)^2+2m\left(\dot{q}_2 + r\omega\cos\left(\phi_0+\omega t\right)\right)^2\Big] \\ &\quad -\Big[\left(2m+M\right)gq_2 + 2mgr\sin\left(\phi_0+\omega t\right)\Big] \end{aligned} \tag{40}$$

Following [15], the equation of motion is given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0 \qquad \text{for} \qquad i=1,2 \tag{41}$$

For i = 1, (41) implies that:

$$\ddot{q}_1 = 0 \qquad , \tag{42}$$

which means that the object does not accelerate and therefore preserves its initial (zero) horizontal velocity, i.e. $x_M\left(t\right)\equiv 0$, as anticipated due to the symmetry.

For i = 2, after manipulation (41) implies (7), as was anticipated.

Restrictions of this study are as follows:

- As previously mentioned, this study reduces to mechanical analysis in which we have ignored the rotational motion of the rigid bodies. In general, we have to consider the moment of inertia I_r of the rod and the corresponding moment of inertia I_m for each of the two eccentric masses. These two mechanical quantities modify the kinetic energy in the Lagrangian (38), which in this general case becomes still more convenient and safe than applying the aforementioned '*Centre of Mass Theorem*'.
- The angular velocities ω of both electric motors (and their associated eccentric masses) have been assumed to be identically equal, while in engineering practice a slight difference may occur ($\omega_1 \cong \omega_2$). In this case, in addition to the vertical motion $z_M\left(t\right)$, the object (body B) will obtain a horizontal motion $x_M\left(t\right)\neq 0$ and a rotation; as a result, the complexity in relevant mechanics increases.
- In the most part of this study the angular velocity ω has been assumed to be constant. In general, when ω changes with time, tangential inertial forces are applied on the eccentric masses; therefore both elastic rods should be designed thick so as to be capable of undertaking high bending deformation [18], which has been ignored in this work.
- Since every electric motor is identified by a characteristic curve (shaft torque M_d versus angular velocity ω), each time the couple (M_d, ω) must be a point in the diagram that follows (moves along) the aforementioned curve [13]. It should be further clarified that:
    - The case of constant angular velocity ($\omega=\omega_0$) refers to a motor having an ideal characteristic curve that is perpendicular to the horizontal axis ω, at least in the neighborhood of ω_0.
    - The angular velocity may also deliberately vary on a large extent by properly controlling the electric motor (guidance), for example, according to an exponential law [11]. Again, each time the couple (shaft torque, angular velocity) must be a point (in the diagram) moving along the characteristic curve of the electric motor; the aforementioned condition has not been considered in this study.

- In this study we have assumed that the mechanism is left to fall from a height z_0. Therefore, the case of an object (body B), which initially lies on a floor with dry friction [19], is not included; preliminary numerical results depict that –in the beginning– the problem is nonlinear due to bouncing effects.
- The wave propagation phenomena along the elastic rod (or beam), which occur either in the transient (initial) phase when the motor is accelerated or particularly when the angular velocity later varies with time, have been ignored.
- Within the framework of the abovementioned wave propagation analysis, the strong dependence of the natural frequency of the elastic rods versus the angular velocity has to be taken into consideration. In more details, the induced centrifugal force per unit volume will alter the stress equilibrium thus resulting in a modified eigenfrequency, as happens, for example, in rapidly rotating turbomachinery [20] to which the finite element method is usually applied [21].
- The common angular velocity of the two eccentric masses ($\omega_1 = \omega_2 \equiv \omega$) must be significantly different than the natural frequency of the elastic rods, so as to avoid resonance [20].
- Ongoing research deals with the abovementioned elastic deformation of the rotating rods (bending due to tangential force, and tension due to centripetal force) as well as the associated dynamic and gyroscopic phenomena that appear either when the path of the masses changes from circular to a more complex smooth curve or the entire mechanism is rotated about its vertical axis of revolution [12,17]. Although slowly moving mechanical components are usually properly analyzed using general purpose finite element codes such as ANSYS, COSMOS/M [22] etc, the mechanism of this paper requires the use of a dedicated software such as ADAMS-ANSYS or the Motion Analysis module in SolidWorks™, among others. At the moment, an in-house customized finite element computer code is under construction and is anticipated to be a useful tool for further engineering analysis.

II. CONCLUSION

We showed that the combination of two contra-rotating masses moving along two circular paths can conditionally produce motion on their plane and particularly in the direction perpendicular to the line of centers. The induced velocity of the object becomes oscillatory while the displacement increases progressively with time. However, due to the gravitational acceleration *g*, an additional free fall motion must be superposed to the previous state, in a vector sense. In more details, and focusing in the case of a vertical setup of the circular paths, when the motors start operating at the initial time at which the object is left to fall, they are not capable of lifting the object. In contrast, when a high value of initial angular velocity has been obtained before the object is left free to move a short-term lift is possible; the more horizontal the initial position of the rods is the higher altitude the object achieves. For given initial angular velocity and initial position of the rods, we found that the maximum height does not practically depend on the way the angular velocity varies with time, whereas the profile of the oscillating object's velocity is drastically influenced. We also found that if at the initial time instance the synchronized motors stop supplying energy, the maximum altitude is practically the same as that obtained when they were working; the basic difference is that now the angular velocity varies with time and the total energy is preserved. Ongoing research deals with the elasto-dynamic analysis of the same mechanical system.

REFERENCES

[1] M. Tajmar, *Advanced Space Propulsion Systems*. New York-Wien, Springer, 2002.

[2] M. G. Millis and E. W. Davis, *Frontiers of Propulsion Science*. American Institute of Aeronautics and Astronautics, Inc., Reston, VA, 2009.

[3] D. M. Goebel and I. Katz, *Fundamentals of Electric Propulsion: Ion and Hall Thrusters*. New Jersey, Wiley, 2008.

[4] N. L. Dean, "System for converting rotary motion into unidirectional motion." *US Patent 2,886,976* (Filed Jul. 13, 1954, granted May 19, 1959).

[5] R. L. Cook, "Device for Conversion of Centrifugal Force to Linear Force and Motion." *US Patent 4,238,968*, granted December 16, 1980.

[6] T. B. Chung, "Internal propulsion apparatus of closed system using a Coriolis force." *International Patent WO 03/087574 A1*, 23 (Filing date: 08 April 2003; Publication date: 23 October 2003)

[7] M. G. Millis and N. E. Thomas, *Responding to Mechanical Antigravity*. NASA/TM-2006-214390, AIAA-2006-4913, December 2006. Available at: http://gltrs.grc.nasa.gov/reports/2006/TM-2006-214390.pdf

[8] M. G. Millis, "Assessing Potential Propulsion Breakthroughs." In *New Trends in Astrodynamics and Applications*, Edward Belbruno, Ed. New York: Annals of the New York Academy of Sciences 1065:441–461, 2005 (doi: 10.1196/annals.1370.023).

[9] G. Yu. Stepanov, "Why is it impossible to have 'Dean's Apparatus'?," Jour. Priroda, vol. 7, 1963, pp. 85-91 (in Russian).

[10] I. I. Blekhman, *Vibrational Mechanics: Nonlinear dynamic effects, General approach, Applications*. Singapore, World Scientific, 2000 (p.15 and p.19).

[11] C. G. Provatidis, "Some issues on inertia propulsion mechanisms using two contra-rotating masses," *Theory of Mechanisms and Machines*, vol. 8, no. 1, pp. 34-41, Apr. 2010.

[12] C. G. Provatidis, "A device that can produce net impulse using rotating masses," Engineering, vol. 2, no. 8, pp. 648-657, Aug. 2010.

[13] V. O. Kononenko, *Oscillatory Systems with Limited Excitation*. Moscow, Nauka, 1964; also: V. O. Kononenko, Vibrating Systems with a Limited Power Supply. London, Ilife Books, 1969 (in *English*).

[14] I. I. Blekhman II, *Synchronization in science and technology*. ASME Press, NY, 1988 (in English, translated from Russian 1981).

[15] H. Goldstein, *Classical Mechanics*, 2nd edn. Reading, Addison-Wesley, 1980.

[16] D. Halliday and R. Resnick, *Physics*. NY, Wiley International Edition, pp.59-61, 1966.

[17] C. G. Provatidis, "*Progress in an Antigravity Mechanism using Inertial Masses*," Proceedings "Space, Propulsion & Energy Sciences International Forum (SPESIF-2011)" [CR-ROM], presented on March 15-17, 2011, University of Maryland, College Park, MD, USA.

[18] A. S. Sayyad, Y. M. Ghugal, "Flexure of Thick Beams using New Hyperbolic Shear Deformation Theory," *International Journal of Mechanics*, vol. 5, no. 3, 2011.

[19] D. Guiad, F. Nilvetti, and C. M. Pappalardo, "Dry Friction Influence on Cart Pendulum Dynamics," *International Journal of Mechanics*, vol. 3, no. 2, 2009.

[20] M. Behzad and A. R. Bastami, "Effect of centrifugal force on natural frequency of lateral vibration of rotating shafts," *Journal of Sound and Vibration*, vol**.** 274, pp.985–995, 2004.

[21] Z. I. Mahri, M. S. Rouabah, "Calculation of Dynamic Stresses using Finite Element Method and Prediction of Fatigue Failure for Wind Turbine Rotor," *WSEAS Transactions on Applied and Theoretical Mechanics*, vol. 3, no. 1, January 2008.

[22] C. B. Pinca, G. O. Tirian, A. Josan, G. Chete, "Quantitative and Qualitative Study on the State of Stresses and Strains of the Strength Structure of a Crane," *WSEAS Transactions on Applied and Theoretical Mechanics*, volume 5, no. 4, October 2010

Christopher G. Provatidis was born in Athens, Greece, in November 3, 1956. He has a BS in Mechanical-Electrical Engineering from the National Technical University of Athens, Greece (NTUA), 1977, a MSc in Mechanical Engineering from the NTUA, 1979 and a PhD also in Mechanical Engineering from the NTUA, 1987. The author's major field of study is applied mechanics.

He has experience in the design of steel structures (as IAESTE student, in the chemical industry SUPRA AB, Sweden, June-August, 1978). He served the Greek Army (Technical Branch 1980-1981) and then worked in projects concerning mechanical equipment of buildings and hospitals (1981-1982). In 1982, he moved to the National Technical University of Athens (NTUA) to work at the 'Machine Elements & Dynamics Laboratory'. He was appointed as Lecturer (1989-1993), Assistant Professor (1994-2002), Associate Professor (2002-2009) and he is currently Full Professor in the School of Mechanical Engineering at NTUA, Greece. In the same school, he served as a Vice-Chairman (2005-2007). He has participated in 35 European and national research projects. He has supervised over 120 diploma-works or MSc theses and eight PhD theses. He has written two books and three monographs (all in Greek). Over the past 30 years, he has worked across a wide discipline to include components of several sectors in mechanical simulation (elastostatics, crack and fatigue analysis, elastodynamics, acoustics, structural optimization, light-weight structures, textile micromechanics, thermal analysis, biomechanics: orthodontics, dental implants, orthopedics, inverse problems, system identification, gearless differentials, dynamics, CAD/CAE integration, etc) in conjunction with the finite element method, the boundary element method and other computational methods. As a result, he has more than 300 publications in refereed journals and conference proceedings. Current research interests include collocation methods, biomechanics and inertial propulsion, as well as rapid prototyping and other design tools.

Prof. Provatidis is a member of ASME, AIAA, European Society of Biomechanics (ESB), Greek Society of Biomechanics (Vice-president, 2008-2010), and Greek Association of Computational Mechanics (General Secretary, 2007-2009). In March 2011 he was elected as an active member (Class VI: Technical and Environmental Sciences) of the European Academy of Sciences and Arts (Salzburg, Austria).