Proc. Int. Symp. Foundations of
Quantum Mechanics, Tokyo, 1983, pp. 294–300

Delayed-Choice Experiments with the Neutron Interferometer*,**

Daniel M. GREENBERGER, Michael A. HORNE,
Clifford G. SHULL and Anton ZEILINGER

*Department of Physics, Massachusetts Institute of Technology,
Cambridge, Massachusetts 02139, USA*

In the neutron interferometer, the neutron wave packet is initially split into two sub-packets and then coherently recombined. The amplitudes and phases of the fringe pattern upon recombination can be changed by altering the experiments performed on the packets in midflight. Quantum mechanically, this is a straightforward procedure. However in a realistic theory this pattern is present by hidden variables. Several types of delayed-choice experiments are described, and they are related to a one-particle version of the Einstein-Podolsky-Rosen paradox, and to limits on the locality of realistic theories.

§1. The Neutron Interferometer

The neutron interferometer is a remarkable device which allows one to create a neutron wave packet of macroscopic size (about the size and shape of a small postage stamp), split the packet into two coherent parts separated by centimeters, and subsequently recombine them. It has made possible a number of experiments which were previously considered to be "gedanken experiments". A number of papers giving simple accounts of the interferometer are available[1-5] as well as more detailed reviews of developments in this field.[6-8] Since we will not need any detailed knowledge of the interferometer for the conceptual experiments we will discuss, we will briefly describe one type of interferometer and then replace it by a simple model for which results are easy to calculate.[5]

A typical interferometer consists of a single perfect crystal of silicon, cut so as to present three parallel slabs to the beam (Fig. 1(a)). The beam is split by Bragg scattering at the first slab, A, refocussed at the second, B and C, and coherently recombined at the third, D. The relative phase of the two beams is monitored by the counters K_1 and K_2. The simple model we shall use for the interferometer is to consider (as in Fig. 1(b)) a half-silvered mirror splitting the beam, A, two full mirrors reflecting it, B and C, and the combined beam being detected at a screen at D, where the diffraction pattern is produced.

§2. Delayed-Choice Experiments

Wheeler[9,10] has described a series of experiments in which a particle, from a classical point of view, at some point takes one of two (or more) alternative routes and each of these alternatives leads to a different distinct future course of events. However, from a quantum mechanical point of view, the wave functions representing each of these alternatives can be kept coherent with each other, so that one cannot say that the particle took one route *or* the other. In fact, by altering the experimental apparatus at some time after the initial choice has been made classically, the system can be made to appear to have chosen any combination of the possible alternatives. Thus it would appear, from a classical point of view, that by altering the apparatus at a later time, one seems to have affected what choice of routes the particle made at an earlier time. Wheeler has expressed this by his dictum: "No phenomenon is an actual phenomenon until it is a measured phenomenon".

One can realize this type of experiment easily in the neutron interferometer. For

* Presented by D. M. Greenberger.

** This work was supported in part by grants from the U.S. Department of Energy (No. DE-AC02-76ER03342.A010) and the U.S. National Science Foundation (No. DMR 802-1057A01).

† Permanent Address: Stonehill College, Department of Physics, Boston, Massachusetts, USA.

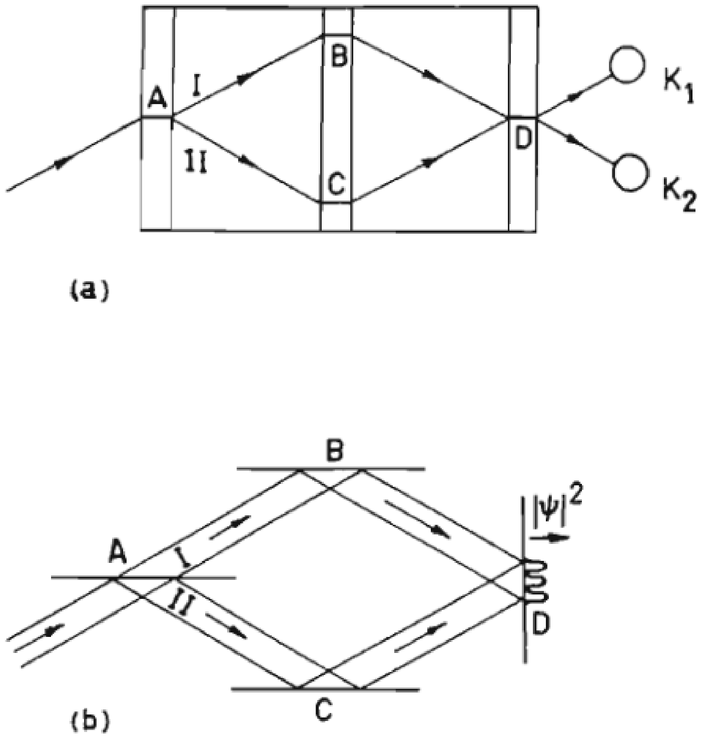

Fig. 1. The neutron interferometer. (a) Top view of an interferometer. (b) Simple mirror model for the interferometer. The interference pattern is produced at the screen D.

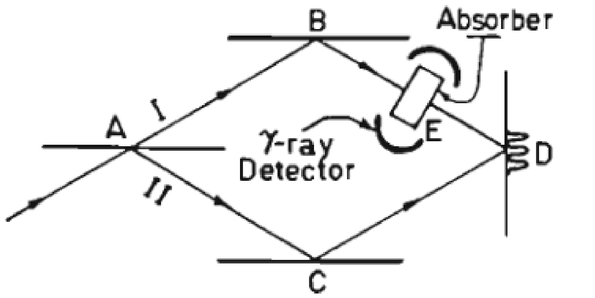

Fig. 2. Delayed choice experiment with the Interferometer.

example, one can place a total absorber in one of the beams (see Fig. 2). The situation can be so arranged (*e.g.*, with a cadmium absorber) that if the neutron is absorbed, a fast γ-ray is emitted, which can be detected with close to 100% efficiency. Thus, if a γ-ray is detected, one can say with near certainty, after the fact, that the neutron travelled along beam I.

On the other hand, one can choose to remove the absorber at the last instant, just before the neutron reaches it. This can easily be done, since the neutron wave packet is about 10^{-2} cm long, and is travelling at about 10^5 cm/sec, so that it takes 10^{-4} sec to traverse the interferometer, which is up to 10 cm long. In that case, there will be a diffraction pattern on the screen, and one can say that the neutron appears to be shared by both beams. But classically, of course, the neutron had to have chosen which beam to take back at point A, and so it appears to have been in a state of "suspended animation" between point A, and the point E at which it reached the absorber. (This state of suspended animation is similar to that experienced by Schrödinger's cat,[11] but in this case the experiment can be performed.)

Instead of the total absorber at E one can use a partial absorber so that, say, 50% of the beam is absorbed. Then one need not physically remove the absorber. Instead, there is a 50% chance that the neutron will be absorbed, so that it took beam I, and there is a 50% chance that it will be transmitted and create an interference pattern. In this case the random choice of whether to absorb the neutron or not is made not by the experimentalist, but by a cadmium atom in the absorber. (Of course, one can make classical hidden variable theories to explain these effects, but they are inherently non-local, and have other defects, which we shall discuss later.) It should be emphasized that these experiments are not speculative, but can be easily done today, as can the two-absorber experiments described below.

§3. The EPR Paradox and One-Particle Systems

In their original paper,[12] in which they suggested that quantum theory is incomplete, Einstein, Podolsky, and Rosen (EPR) gave one criterion for determining whether a physical quantity corresponded to an element of reality. This criterion was that if the physical quantity could be definitely determined without disturbing the system, then it corresponded to an element of reality. We shall call this criterion EPR-I. Though they did not state it explicitly, they also assumed a second postulate, which seems reasonable in light of the first. This was that if a physical quantity does correspond to an element of reality, and it has a definite value (known or not), then if one does not in any way disturb the system one cannot change this value, (beyond its normal time development). We shall call this postulate EPR-II.

Clearly if EPR-II were false, then one could not apply EPR-I to ascertain the reality of a quantity. In the Bohm variation of the EPR experiment[13] a two particle system, each of spin $\frac{1}{2}$, has total spin 0. In this system, if one measures S_z for one particle and finds $m_1 = +\frac{1}{2}$, then one knows that for the other particle $m_2 = -\frac{1}{2}$, even if the particles are far apart. The EPR-I criterion then yields S_z for each particle as an element of reality. But the assumption is also made that if m_2 is determined to be equal to $-\frac{1}{2}$ when you measure m_1, then it must have been equal to $-\frac{1}{2}$ before you measured m_1, *as you did not disturb this particle*. This reasoning uses EPR-II, as it takes for granted that as long as the particles are separated, and m_2 is not disturbed, it will remain definite. It follows from this that m_2 must have been determined when the particles separated.

In the neutron interferometer coherent neutrons pass through the system at a rate of about one per second. So they rarely interact and one is dealing with a one particle system. One can show that which beam the particle takes is an element of reality by virtue of EPR-I. Consider the interferometer with an absorber in beam I, which absorbs 100% of the beam, and emits an identifying γ-ray. Then if one detects a γ-ray, one knows with certainty that the particle took beam I. On the other hand, if there is no γ-ray then one knows that the particle is in beam II, without having disturbed beam II. It follows that being in the beam, by EPR-I, is an element of reality. It also follows, from EPR-II, that this was determined when the beam hit the mirror at point A and split. So one can certainly apply the EPR criteria to one-particle systems.

§4. The Interferometer with Two Absorbers

Consider an interferometer with two partial absorbers present (Fig. 3(a)), one at point E in beam I, in the last half of the beam (after point B), and the second at point F in beam II, in the first half of the beam (before point C). Assume the absorber at E transmits an amplitude a so that if the wave function approaching E is ψ_1, then after passing the absorber the wave-function will be

$$\psi_1' = a\psi_1. \tag{1}$$

Further assume that the absorber at F transmits an amplitude b, so that

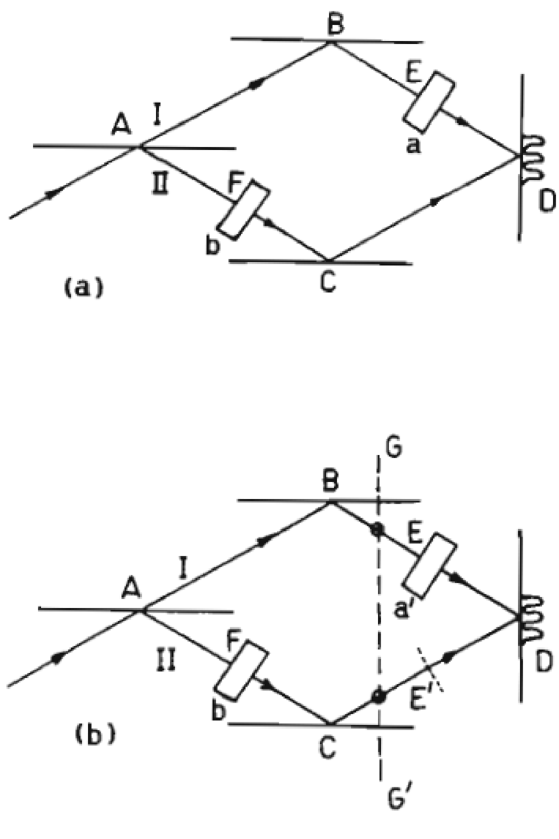

Fig. 3. The Interferometer with two absorbers.
(a) The absorbers are arranged so one (E) is in the second half of the beam, while the other (F) is in the first half.
(b) The absorber E is changed while the neutron is between them, along the plane GG′.

$$\psi'_2 = b\psi_2. \tag{2}$$

We will also assume for simplicity that the absorbers introduce no extra phase shift into the beams (so that a, b are real), and also that quantum theory gives the correct answers for any measurement we consider. The goal of any classical or hidden variable theory will be to reproduce the quantum results.

If the mirror at A equally divides the initial beam into two sub-beams I and II, and if there were no absorbers in the beams ($a=b=1$), then the wave function, and intensity at point D would be given by

$$\psi = \psi_{\mathrm{I}} + \psi_{\mathrm{II}} = e^{ik_z z}(e^{-ik_x x} + e^{ik_x x}) = 2\, e^{ik_z z}\cos k_x x,$$
$$I = |\psi|^2 = 4\cos^2 k_x x = 2(1+\cos 2k_x x)$$
$$\equiv 2(1+\cos\varphi(x)). \tag{3}$$

The contrast of the beam is given by

$$C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = 1. \tag{4}$$

(We will assume throughout for simplicity that the beams are as coherent as possible, and we do not need a density matrix to describe them.) Also we will suppress the z-dependence, which is irrelevant. With the absorbers present, we have ($k \equiv k_x$)

$$\psi' = \psi'_1 + \psi'_2 = a\, e^{-ikx} + b\, e^{ikx},$$
$$I' = |\psi'|^2 = a^2 + b^2 + 2ab\cos 2kx = a^2 + b^2 + 2ab\cos\varphi. \tag{5}$$

In this case (a^2+b^2) gives the average intensity, while the $\cos\varphi$ term gives the bunching of the beam to produce a diffraction pattern, with contrast

$$C_0 = \frac{(a+b)^2 - (a-b)^2}{(a+b)^2 + (a-b)^2} = \frac{2ab}{a^2+b^2}. \tag{6}$$

This is the quantum mechanical result. However if we prefer to explain the data by a "realistic" theory, then according to EPR-II, any change in the interference pattern at D must be caused by the absorbers placed in the beams. If one looks at the absorber in beam I at E, the intensity of the beam as it will impinge at D must be determined here, so that one can write

$$I_{\mathrm{E}} = A_{\mathrm{E}}(1 + \alpha_{\mathrm{E}}\cos\varphi). \tag{7}$$

(Perhaps this could better be written as $I_{\mathrm{D}}(\mathrm{E})$. It does not mean that one must necessarily observe fringes in the beam before it impinges on the screen at D. But after the neutron has passed the absorber at E any hidden variables present have been "set", so that the fringe pattern that will ultimately be seen at D has been determined by then, according to the EPR criteria.) Writing it in this form, one has taken into account that

$$I_{\mathrm{E}} \to 0 \quad \text{as} \quad A_{\mathrm{E}} \to 0, \tag{8}$$

where A_{E} represents the average intensity. It will also be true that

$$\alpha_{\mathrm{E}} \leqslant 1, \tag{9}$$

since one cannot have a negative number of particles striking the screen. We will call α the contrast parameter, as it is equal to the contrast produced by the beam.

Since both classically and quantum mechanically, the average intensity of the beam will be reduced by the absorber from $I=1$ to $I=a^2$, one can write

$$A = a^2, \quad I_{\mathrm{E}} = a^2(1 + \alpha(a,b)\cos\varphi). \tag{10}$$

Here we have taken into account the possibility that α will depend on the value of the absorber in beam II as well. Similarly, for the absorber at F, we can write

$$I_F = b^2(1+\beta(a, b)\cos\varphi). \tag{11}$$

Thus the total intensity at D will be given by

$$I_D = I_E + I_F = a^2 + b^2 + (a^2\alpha(a, b) + b^2\beta(a, b))\cos\varphi. \tag{12}$$

Both quantum theory, and experience, give the result that it does not matter where along the beam the point E is taken, and we will assume that this is true for the classical theory as well. Also the result does not depend on which beam is called beam I, so that if the absorbers a, b are interchanged, there will be no difference in the diffraction pattern. Thus

$$b^2\alpha(b, a) + a^2\beta(b, a) = 2ab. \tag{13}$$

The above symmetry also implies the result that

$$\beta(b, a) = \alpha(a, b), \tag{14}$$

so that eq. (12) becomes

$$a^2\alpha(a, b) + b^2\alpha(b, a) = 2ab. \tag{15}$$

One can find the most general solution to eq. (15). First, because of the symmetry between a and b quantum mechanically, one could make the (not essential) assumption

$$\alpha(a, b) = \alpha(b, a) \equiv \alpha_0. \tag{16}$$

This in turn would imply

$$\alpha_0 = \frac{2ab}{a^2+b^2} = C_0. \tag{17}$$

Then the most general solution to eq. (15) can be expressed as

$$\alpha(a, b) = \frac{2ab}{a^2+b^2} + \frac{b}{a}\delta(a, b). \tag{18}$$

If we substitute this into eq. (15), we find that δ is antisymmetrical,

$$\delta(a, b) = -\delta(b, a). \tag{19}$$

This is the most general solution for the behavior of the absorber. Two boundary conditions on the absorber can also be derived. One is given by the fact that if $a=b$, the contrast is $C=1$ (from eq. (6)), so that

$$\alpha(a, a) = 1. \tag{20}$$

The other is that if one of the beams is fully blocked, there will be no interference, so that

$$\alpha(a, 0) = \alpha(0, a) = 0. \tag{21}$$

These conditions (20) and (21) show that $\alpha(a, b)$ cannot be a constant, independent of b. Thus in general $\alpha(a, b)$ will depend on both a and b. This in turn implies that if one wants a "realistic" theory to describe the behavior of the interference pattern, it *must* be non-local, in the sense that the behavior of each absorber (or any hidden variable determined by passage through that absorber) also depends on the strength of the absorber which is in the other beam. In fact, the packet II at point F must know in advance that the packet I will strike an absorber when it reaches point E.

§5. Two-Absorber Delayed-Choice Experiments

In the previous section we have discussed the case of two absorbers placed in the beams, but left there throughout the experiment. However, one can wait until the neutron is in the middle of the system, having passed the point F, but not yet having reached the point E (say along the plane GG′ in Fig. 3(b)). At this time one can suddenly replace the absorber at point E by a different one, whose transmission amplitude is a'. The intensity at point D will then become

$$I'_D = a'^2 + b^2 + 2a'b\cos\varphi. \tag{22}$$

One then has the rather strange result, in a realistic theory, that not only must the beam passing through the absorber at E be changed from what it would have been, since $a \to a'$, but also the particles in beam II, now at point E′, must similarly change their intensity distribution, since their distribution depends upon both absorbers.

An even more striking conclusion can be reached in the case where the absorber a is removed entirely, so that $a'=1$. In this case, the particle in beam I also never hits any absorber, and so cannot change its distribution (according to EPR-II), and yet to get the correct intensity distribution at point D, *both* beams must change their intensity distribution beyond the plane GG′, even though neither of the beams is in any way disturbed.

One might think that possibly only those particles in beam I that would have been absorbed, had the absorber remained in place at E, will have to change their distribution. But this is not possible, as one can easily see. For the transmitted intensity would have been a^2. But since the absorber at E is removed, the extra particles not absorbed will have intensity $(1-a^2)$, and some contrast α', so that the new intensity will be (from eq. (5) with $a=1$)

$$I_{\text{new}} = I_{\text{old}} + I_{\text{extra}},$$
$$[1+b^2+2b\cos\varphi] = [(a^2+b^2)+2ab\cos\varphi] + [(1-a^2)(1+\alpha'\cos\varphi)]. \tag{23}$$

This implies that

$$2ab+(1-a^2)\alpha' = 2b,$$
$$\alpha' = 2b/(1+a), \tag{24}$$

which will yield a contradiction if $b>(1+a)/2$, since

$$b>(1+a)/2\alpha'>1. \tag{25}$$

So the hidden variables of particles in both beams must be adjusted to produce the quantum results, even though neither beam is disturbed!

Therefore, for a realistic theory to agree with the results of quantum theory, it must violate the postulate EPR-II. But a major justification for introducing realistic theories is that quantum theory violates the EPR postulates. However we have seen that a hidden variable theory that prescribes where at point D a particle will land must quickly alter the trajectory of the particles after the absorbers are suddenly switched, even though the particles are in no way disturbed. Thus such theories are open to the same criticism of being "incomplete" as is quantum theory. The inequality (25) is an analog of Bell's inequalities, applied to the one particle case, and shows that the EPR criteria are incompatible with a local theory. Many other such inequalities could be derived. This is so because eq. (12) says that for a realistic theory, the total intensity is the sum of the intensities in the sub-beams. It leads to a restriction on possible correlations, as factorizability does in the two particle case. This is a first step toward producing an actual experimental demarcation between quantum theory and certain classes of realistic theories.

In spite of their hidden variables, these theories do not pre-determine the trajectories of particles, even in the one-particle case, since they are subject to sudden non-local influences, according to the whim of a distant experimenter. In this sense the theories are even less local than quantum theory, and no longer meet the EPR criteria for being realistic. Similar conclusions are implied by the results of Aharonov and Kaufherr.[14] (We have not considered delayed choice experiments with communication between parts of the system limited by the speed of light. This would place even further restrictions on realistic theories.)

We would like to thank Dr. John Arthur and Profs. Y. Aharonov, A. Klein, A. Shimony, and J. Sheeler for helpful conversations.

References

1) S. A. Werner: Physics Today **33** (1980) 24.
2) D. W. Greenberger and A. W. Overhauser: Rev. Mod. Phys. **51** (1979) 43.
3) D. M. Greenberger and A. W. Overhauser: Sci. American **242** (1980) 66.
4) A. W. Overhauser and R. Colella: Amer. Scientist **68** (1980) 70.
5) D. M. Greenberger: to be published in Rev. Mod. Phys. (Oct. 1983).
6) H. Rauch and D. Petrascheck: *Topics in Current Physics* Vol. 6 (Springer, Berlin, 1979) Chap. 9.
7) U. Bonse and W. Graeff: *X-Ray Optics*, ed. H. J. Queisser (*Topics in Applied Physics* Vol. 22) (Springer, Berlin, 1977) p. 93.
8) *Neutron Interferometry*, eds. U. Bonse and H. Rauch (Oxford Univ. Press, Oxford, 1979).
9) J. A. Wheeler: *Mathematical Foundations of Quantum Theory*, ed. A. R. Marlow (Academic, New York, 1978) p. 9.
10) J. A. Wheeler: *Mind in Nature*, ed. R. Q. Elvee (Harper and Row, San Francisco, 1982) p. 1.
11) E. Schrödinger: Naturwissenschaften **23** (1935) 807.
12) A. Einstein, B. Podolsky and N. Rosen: Phys. Rev. **47** (1935) 777.
13) D. Bohm: *Quantum Theory* (Prentice-Hall, New York, 1951).
14) Y. Aharonov and T. Kaufherr: to be published in Phys. Rev. D.

M. Rauch: We are very grateful for the motivation, given by your contribution, for further experiments with the interferometer. I would like to announce the experiments we are preparing now. First we will observe the difference of the interference pattern when a 50% absorber or a 50% absorbing chopper is inserted into the beams and second we will insert two detectors with 50% efficiency into the beams and record their intensities and the interference pattern.

D. M. Greenberger: Good. Thank you. One reason absorption effects are good to consider is that, for example, if one beam has intensity 1, and the other beam has amplitude ε, its intensity will be ε^2, but the interference will be of order ε. So if $\varepsilon = 1\%$, from only 1% of the beam you can get 10% interference. This is a truly wave-like, quantum type effect.

A. J. Leggett: While I would agree fully that these experiments are very fundamental and worth doing, I've not considered that they are a substitute for the "Schrödinger's cat" type of macroscopic interference experiments (if they can be done). The important difference is that, assuming these experiments are done and give the quantum-mechanically predicted results, they can still be completely fitted into Bohr's original version of the Copenhagen interpretation, namely that one should not ever try to ascribe definite properties to a microscopic object in the absence of the specification of the macroscopic apparatus set up to measure it. The macroscopic interference experiments, on the other hand, cannot be so fitted in without a very dramatic extension of the interpretation.

D. M. Greenberger: I think there are two different aspects in Schrödinger's cat. One is the question of what a macroscopic object is. This is an important question, and you have been very concerned with it. But the second aspect is what I think bothered Schrödinger. This is that an object can really be in a state of "suspended animation", when considered classically. The neutron experiments exhibit this aspect very clearly. This is an intrinsic feature of delayed choice experiments, and this is what I meant.

C. N. Yang: I think I know what Leggett was saying. Namely, that no cat is a Schrödinger's cat unless it is a cat.

A. Shimony: Your claim that a local realistic treatment of a single neutron is incapable of accounting for known interference phenomena seems to me to overlook the power of the de Broglie and Bohm guiding wave theories. You may wish to argue that there is some non-locality implicit in these theories even in their treatment of single particles, but you haven't yet shown this explicitly.

D. M. Greenberger: This is what I mean in the paper by calling it a "first step" toward providing a resolution. If one says Bohm's quantum potential doesn't really disturb the system, then the argument given goes through. But one can take these theories seriously and say the quantum potential is as real as electromagnetic radiation. Then when you move an absorber, the result radiates out continuously to the rest of the universe, like any other force field. At the moment I would have to say, "okay, if it is real, it has some very strange properties, as shown by Aharonov in his talk." But actually, I think that ultimately I can say more than this.