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Local Deterministic Description of Einstein–Podolsky–Rosen Experiments

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We formulate a model of EPR experiments by including variables determining whether a photon will be detected or not. The resulting deterministic model satisfies Bell's original inequality even though it can agree exactly with the quantum mechanical predictions for the performed experiments. It violates variations of the inequality used in the interpretation of the experiments and deduced with the help of additional assumptions.

1. BOHM'S FORMULATION OF THE EPR PARADOX

A physical system M is given (atom, molecule, etc.) decaying into two spin-1/2 "particles" α and β. If $u_\alpha(+)$ and $u_\alpha(-)$ are the eigenvectors corresponding to the eigenvalues $+1$ and -1, respectively, of the Pauli matrix $\sigma_z(\alpha)$ representing the third component of the spin angular momentum for α; and if $u_\beta(+)$ and $u_\beta(-)$ are the corresponding eigenvectors of the Pauli matrix $\sigma_z(\beta)$ for β, there are some concrete physical situations in which the spin state vector for (α, β) must be the "singlet" state vector η_0 given by

$$\eta_0 = (1/\sqrt{2})[u_\alpha(+)\,u_\beta(-) - u_\alpha(-)\,u_\beta(+)] \tag{1}$$

Four properties of η_0 are important in Bohm's formulation of the EPR paradox[1]:

(*i*) *It is not a factorizable vector.*

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(*ii*) *It predicts the result zero for a measurement of the total squared spin of the particles* α *and* β.

(*iii*) *It is rotationally invariant.*

(*iv*) *It predicts opposite results for measurements of the components along* $\hat{n}$ *of the spins of particles* α *and* β, $\hat{n}$ *being an arbitrary unit vector.*

Another state vector important for the discussion of the EPR paradox is the "triplet" state, given by

$$\eta_1 = (1/\sqrt{2})[u_\alpha(+)\, u_\beta(-) + u_\alpha(-)\, u_\beta(+)] \tag{2}$$

One can show that η_1 shares with η_0 the properties (*i*) and (*iv*), but not the property (*iii*) since it is not rotationally invariant. Moreover, in place of (*ii*) η_1 has the following property: *Any measurement of the total squared spin of the two particles described by* η_1 *will give the result* $2\hbar^2$ *(spin* 1*)*.

By adding and subtracting (1) and (2) one gets:

$$\begin{aligned} u_\alpha(+)\, u_\beta(-) &= (1/\sqrt{2})[\eta_0 + \eta_1], \\ u_\alpha(-)\, u_\beta(+) &= (1/\sqrt{2})[\eta_0 - \eta_1]. \end{aligned} \tag{3}$$

Therefore, on invoking the quantum mechanical interpretation of superpositions, one concludes that

Measurements of the total squared spin on a set of (α, β) *pairs described as a mixture of the factorizable state vectors* (3) *will produce with equal probability the results* 0 *and* $2\hbar^2$.

Bohm's version of the EPR paradox is based on the Einstein, Podolsky and Rosen "reality criterion": "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity." In a slightly modified version, basically due to d'Espagnat,[2] the paradoxical reasoning goes as follows:

Consider a large set E of (α, β) pairs in the state (1). Measure $\sigma_z(\alpha)$ at time t_0 on all α's of a subset E_1 of E. If $+1(-1)$ is found, a future $(t > t_0)$ measurement of $\sigma_z(\beta)$ will give $-1(+1)$ *with certainty*. Using the EPR reality criterion, one can assign to all α's of E_1 an element of reality $\lambda_1(\lambda_2)$ fixing a priori the result $-1(+1)$ of the $\sigma_z(\beta)$ measurement.

But quantum mechanics treats an object β with predetermined value $-1(+1)$ of $\sigma_z(\beta)$ by assigning it the state vector $u_\beta(-)[u_\beta(+)]$: This is the completeness assumption, according to which every element of physical reality must have a counterpart in a complete physical theory. The strict correlation (iv), applied to the z axis, implies then, even for $t < t_0$, that E_1 had to be described in spin space with a 50–50 mixture of the vectors in Eq. (3) Excluding that $\lambda_1(\lambda_2)$ are created at a distance by the measurement of $\sigma_z(\alpha)$, it must be concluded that either λ_1 or λ_2 actually belong to all β's of E. Applying again completeness one concludes, like before, that a mixture of the vectors in Eq. (3) applies to all pairs of E.

But this contradicts at the empirical level the description provided by the singlet state in Eq. (1), as was previously shown. One reaches so an absurd conclusion (EPR paradox).

2. THE BOHM–AHARONOV CONCLUSION

Already the works of Schrödinger[3] and Furry[4] had evidenced the existence of a striking disagreement between the predictions of quantum theory and those to be expected on the assumption that a system free from dynamical interference can be regarded as possessing independently real properties. This conclusion is for some people very difficult to accept and can lead to the idea that something could be wrong with the existing quantum theory. The first organic examination of an eventual breakdown of quantum theory (with immediately negative conclusions) was made by Bohm and Aharonov.[5] They recalled that also Einstein had doubts about the validity of the quantum predictions in the EPR situation:

> "...Einstein has (in private communication) actually proposed such an idea; namely, that the current formulation of the many-body problem in quantum mechanics may break down when particles are far enough apart."

Bohm and Aharonov considered the electron-proton annihilation into two gamma rays and showed that the produced quantum state is

$$|0^-\rangle = \{|x_\alpha\rangle\, |y_\beta\rangle - |y_\alpha\rangle\, |x_\beta\rangle\}/\sqrt{2} \tag{4}$$

that is, the zero angular momentum negative parity state, where x and y denote orthogonal directions of linear polarizations of photon α and photon β. Also the latter state, like the singlet state of two spin-1/2 particles, is rotationally invariant. This means that each photon is always found in a linear polarization state orthogonal to that of the other photon, no matter what may be the choice of axes x and y. Bohm and Aharonov calculated the ratio $R(\theta) = \Gamma_1(\theta)/\Gamma_2(\theta)$, where $\Gamma_1(\theta)$ is the rate of double scattering of the two photons through a fixed angle θ, when the planes π_1

and π_2 formed by the lines of motion of the first and the second photon respectively (after scattering) with their common original direction of motion are perpendicular. $\Gamma_2(\theta)$ is the same rate when the planes π_1 and π_2 are parallel. The $|0^-\rangle$ state predicts

$$R(\theta)=\frac{(\gamma-2\sin^2\theta)^2+\gamma^2}{2\gamma(\gamma-2\sin^2\theta)} \tag{5}$$

where

$$\gamma=(k_0/k)+(k/k_0)$$

Here k_0 is the wave number of the incident photon, k that of the final photon, and k_0/k can easily be calculated from Compton scattering kinematics: For $\theta=82°$ one obtains $R(82°)=2.85$. This figure could not be compared directly with the result of the Wu–Shaknov experiment,[6] because photons had been detected with a considerable angular spread around the "ideal" value of 82°. To such a concrete situation applied instead the prediction

$$R=2.00$$

obtained with a suitable angular average of Eq. (5). This agreed very well with the experimental results, while Bohm and Aharonov could show that the hypothesis of a breakdown of the $|0^-\rangle$ state vector with increasing distance between the two photons and of its substitution with a mixture of factorizable vectors led necessarily to $R\leqslant 1.5$.

These results showed that the Wu–Shaknov experiment was well explained by the existing quantum theory, but not by any hypothesis implying a simple-minded breakdown of quantum theory that could avoid the paradox. It would however not be correct to conclude that this experimental evidence gives a proof against Einstein's locally realistic picture of atomic phenomena, since there are well known *local* models reproducing the quantum-mechanical predictions for experiments of the Wu–Shaknov type.[7]

The evidence found by Bohm and Aharonov is however conclusive against what has here been called "a simple-minded" breakdown of quantum theory, and it is surprising that some authors kept rediscovering this idea long after it had been discarded by its proponents. Examples are the papers by Jauch,[8] de Broglie,[9] and Piccioni and Mehlhop.[10]

3. QUANTUM PREDICTIONS FOR EPR EXPERIMENTS

The present section collects well known formulae predicted by quantum theory for all the physical quantities that can be measured in an EPR experiment performed with atomic photon pairs. At first we consider the well-known situation in which photon $\alpha(\beta)$ crosses a one-way polarizer with axis oriented along direction a or a' (b or b') before entering a detector with quantum efficiency $\eta_\alpha(\eta_\beta)$. If the polarizer is taken away from the path of the photon, the corresponding situation is denoted by ∞, as it is traditional to do.

Single photon probabilities for crossing the polarizer (when present) and for detection will be denoted with the letter $p(q)$ for photon α (β). Joint probabilities for transmission and detection of both photons will be denoted with the letter D: Thus $D(a, b)$ will be the probability of detecting both photons after photon $\alpha(\beta)$ has crossed a polarizer with axis $a(b)$; $D(a, \infty)$ will be the probability of detecting both photons after α has crossed a polarizer with axis a, if no polarizer is present on the trajectory of β; and so on.

An index "°" will indicate the quantum mechanical predictions: Thus $D_0(a, b)$ will be the quantum theoretical expression of $D(a, b)$, and so on. In experiments with atomic photon pairs the most widely used cascade is the $(J=0) \to (J=1) \to (J=0)$ cascade of calcium.[11] The quantum mechanical predictions following from the state $|0^-\rangle$ of the two emitted photons, which applies to this case, are

$$p_0(a) = p_0(a') = \tfrac{1}{2}\varepsilon_+^\alpha \eta_\alpha \tag{6}$$

$$p_0(\infty) = \eta_\alpha \tag{7}$$

$$q_0(b) = q_0(b') = \tfrac{1}{2}\varepsilon_+^\beta \eta_\beta \tag{8}$$

$$q_0(\infty) = \eta_\beta \tag{9}$$

for single photon probabilities, and

$$D_0(a, b) = \tfrac{1}{4}[\varepsilon_+^\alpha \varepsilon_+^\beta + \varepsilon_-^\alpha \varepsilon_-^\beta F \cos 2(a-b)]\, \eta_\alpha \eta_\beta \tag{10}$$

for the joint probability of transmission and detection of both photons. The expressions for $D_0(a, b')$, $D_0(a', b)$, $D_0(a', b')$ can easily be obtained from Eq. (10) with minor changes of the arguments a and b. Other joint probabilities that have been measured in EPR experiments are

$$D_0(a, \infty) = \tfrac{1}{2}\varepsilon_+^\alpha \eta_\alpha \eta_\beta \tag{11}$$

$$D_0(\infty, b) = \tfrac{1}{2}\varepsilon_+^\beta \eta_\alpha \eta_\beta \tag{12}$$

$$D_0(\infty, \infty) = \eta_\alpha \eta_\beta \tag{13}$$

In Eqs. (6), (8), (10), (11) and (12) one has:

$$\varepsilon_{\pm}^{\alpha} = \varepsilon_M^{\alpha} \pm \varepsilon_m^{\alpha}, \; \varepsilon_{\pm}^{\beta} = \varepsilon_M^{\beta} \pm \varepsilon_m^{\beta}$$

Here $\varepsilon_M^{\alpha}(\varepsilon_m^{\alpha})$ is the transmittance of the polarizer crossed by photon α for light polarized parallel (perpendicular) to the polarizer axis; and a similar notation has been used for the polarizer crossed by photon β. In Eq. (10), the factor F is a function of the angle subtended by the primary lenses and represents a depolarization due to noncollinearity of the two photons: In practice, F is often rather close to unity. Eq. (10) refers to the $(J=0) \to (J=1) \to (J=0)$ cascade. If the photon pair is obtained from a $(J=1) \to (J=1) \to (J=0)$ cascade with equal populations in the initial Zeeman sublevels and no coherence among them, but the used experimental arrangement is otherwise unchanged, then the quantum mechanical predictions of Eqs. (6)–(13) remain unchanged, the only exception being that F becomes $-F$ in Eq. (10).[12] Some experiments have actually been performed with cascades of the $(J=1) \to (J=1) \to (J=0)$ type.[13]

Garuccio and Rapisarda[14] studied an experiment in which a piece of calcite, monitored by two detectors put on the ordinary and on the extraordinary ray, was used as analyzer for each of the two photons. Of the same class are experiments with general two-way analyzers, where two orthogonal states of linear polarization are split and sent into two different directions. An experiment of this type was actually performed by Aspect and collaborators.[15] Every photon can be detected either in the transmitted beam or in the reflected beam, denoted by $\pm$ respectively. There are then four joint probabilities for detection of both photons, and the quantum mechanical predictions for the $(J=0) \to (J=1) \to (J=0)$ cascade, the only one used in practice, are:

$$D_0(a_+, b_+) = \tfrac{1}{4}[T_+^{\alpha} T_+^{\beta} + T_-^{\alpha} T_-^{\beta} F \cos 2(a-b)]\, \eta_\alpha \eta_\beta, \tag{14}$$

$$D_0(a_+, b_-) = \tfrac{1}{4}[T_+^{\alpha} R_+^{\beta} - T_-^{\alpha} R_-^{\beta} F \cos 2(a-b)]\, \eta_\alpha \eta_\beta \tag{15}$$

$$D_0(a_-, b_+) = \tfrac{1}{4}[R_+^{\alpha} T_+^{\beta} - R_-^{\alpha} T_-^{\beta} F \cos 2(a-b)]\, \eta_\alpha \eta_\beta \tag{16}$$

$$D_0(a_-, b_-) = \tfrac{1}{4}[R_+^{\alpha} R_+^{\beta} + R_-^{\alpha} R_-^{\beta} F \cos 2(a-b)]\, \eta_\alpha \eta_\beta \tag{17}$$

where

$$T_+^i = T_{\parallel}^i + T_{\perp}^i, \; T_-^i = T_{\parallel}^i - T_{\perp}^i$$

and

$$R_+^i = R_{\parallel}^i + R_{\perp}^i, \; R_-^i = R_{\parallel}^i - R_{\perp}^i$$

$(i = \alpha, \beta)$. The T and R parameters are transmittances defined in the following way. There are two prisms, denoted with $i = \alpha, \beta$. From each prism two beams are emitted, a reflected one and a transmitted one.

$T_{\|}(T_{\perp})$ denotes the prism transmittance along the transmitted path for incoming light polarized parallel (perpendicular) to the transmitted channel polarization plane;

$R_{\|}(R_{\perp})$ denotes the prism transmittance along the reflected path for incoming light polarized parallel (perpendicular) to the reflected channel polarization plane.

No other probabilities have been measured in experiments with two-way polarizers and we will correspondingly limit our considerations to Eqs. (14)–(17).

4. LOCAL DETERMINISTIC MODELS FOR EPR EXPERIMENTS

Following an idea that we first developed elsewhere[16] it will next be shown that local and deterministic models exist which are capable to reproduce exactly the quantum mechanical predictions for detector efficiencies η_α and η_β not too high, say below the 0.5 level (this condition is well satisfied in all the performed experiments[11,13]). It is rather obvious that if deterministic models of this type exist, also truly probabilistic local models should exist with the same basic physical property—indistinguishability from quantum theory for low enough detector efficiency. In fact the class of probabilistic local models is much wider than that of deterministic local models, and includes it: Deterministic is a model in which all probabilities are certainties (positive certainty → probability 1 and negative certainty → probability 0).

In our models, four different polarizer directions are considered; two for each photon; variables which determine whether a specific photon will trigger the detector or not are also included. Each individual photon is described by five dichotomic variables, two related to the transmission through a one-way polarizer, and three related to the detection in various conditions (the extension of the model to two-way polarizers will be made in a future section). Therefore every photon pair (α, β) is described by a set of ten dichotomic variables:

$$(s, s', \sigma, \sigma', \delta; t, t', \tau, \tau', \varepsilon) \tag{18}$$

with the first five variables pertaining to photon α and the second five variables to photon β. Each of the ten variables can only be zero or unity, and in this consists the deterministic nature of the model. In fact all the variables in Eq. (18) could be considered probabilities, and probabilities that can assume only the values zero or one are certainties.

These dichotomic variables determine the interactions that a given photon, α or β, will have with polarizers and detectors. More exactly:

$s=1(s'=1)$ determines that photon α will traverse its polarizer oriented along direction $a(a')$;

$\sigma=1(\sigma'=1)$ determines that photon α will be registered by its detector after having crossed a polarizer oriented along direction $a(a')$;

$\delta=1$ determines that photon α will be registered by its detector if no polarizer is set on its path;

$t=1(t'=1)$ determines that photon β will traverse its polarizer oriented along direction $b(b')$;

$\tau=1(\tau'=1)$ determines that photon β will be registered by its detector after having crossed a polarizer oriented along direction $b(b')$;

$\varepsilon=1$ determines that photon β will be registered by its detector if no polarizer is set on its path;

$s, s', \sigma, \sigma', \delta=0$ $(t, t', \tau, \tau', \varepsilon=0)$ determine that photon α (photon β) will not cross its polarizer or will not be registered by its detector.

Note that the future destiny of each individual photon, whether it will cross its polarizer or be absorbed, and whether it will be registered by its detector or not, is strictly determined by the set of five variables that the photon is assumed to carry locally with itself during its propagation. It is also assumed in our model that if photon $\alpha(\beta)$ does not encounter a polarizer in its flight from the source to the detector, the variable $\delta(\varepsilon)$ is active at the detector, but it is switched off—and the variable $\sigma(\tau)$ is switched on—upon interaction with the polarizer with axis $a(b)$, and so on.

The N_0 photon pairs emitted by the source may therefore be grouped into subsets, according to the specific set of variables [See Eq. (18)] they carry. If one defines by $n(s, s', \sigma, \sigma', \delta; t, t', \tau, \tau', \varepsilon)$ the population of that subset of photon pairs that carry the specific set $(s, s', \sigma, \sigma', \delta; t, t', \tau, \tau', \varepsilon)$ of variables, the normalization condition must hold

$$\sum n(s, s', \sigma, \sigma', \delta; t, t', \tau, \tau', \varepsilon)=N_0 \tag{19}$$

where the sum is extended over all the physically meaningful sets of values of the variables described in Eq. (18). Due to the ten different dichotomic variables of our model it might at first sight appear that one is dealing with $2^{10}=1024$ different subsets. Yet, it is easy to see that subsets for which, say,

$s=0$ and, simultaneously, $\sigma=1$ are not physically meaningful since they would imply detection with certainty of a photon that has been absorbed in the polarizer. The same considerations apply for the other three polarizer orientations, and this reduces the number of different subsets with nonzero population by a factor of $(3/4)^4$ to 324.

5. PREDICTED PROBABILITIES FOR ONE AND TWO PHOTONS

In order to define the experimental observables in terms of sums over populations of subsets it will be useful to adopt the abbreviated notation

$$n(...)=n(s, s', \sigma, \sigma', \delta; t, t', \tau, \tau', \varepsilon) \tag{20}$$

Measurable single photon probabilities can then be written

$$p(a)=N_0^{-1}\sum n(...)\, s\sigma \tag{21}$$

$$p(a')=N_0^{-1}\sum n(...)\, s'\sigma' \tag{22}$$

$$p(\infty)=N_0^{-1}\sum n(...)\, \delta \tag{23}$$

$$q(b)=N_0^{-1}\sum n(...)\, t\tau \tag{24}$$

$$q(b')=N_0^{-1}\sum n(...)\, t'\tau' \tag{25}$$

$$q(\infty)=N_0^{-1}\sum n(...)\, \varepsilon \tag{26}$$

In $p(a)$ for example, will survive multiplication by $s\sigma$ only the $n(...)$'s that lead to certain transmission in the polarizer $(s=1)$ and certain detection $(\sigma=1)$. Thus $p(a)$ is actually the frequency with which photons α will be detected after crossing a polarizer with axis a; the argument with the other probabilities is analogous.

The various joint probabilities for double detection of the two photons can similarly be written:

$$D(a, b)=N_0^{-1}\sum n(...)\, s\sigma t\tau \tag{27}$$

$$D(a, b')=N_0^{-1}\sum n(...)\, s\sigma t'\tau' \tag{28}$$

$$D(a', b)=N_0^{-1}\sum n(...)\, s'\sigma' t\tau \tag{29}$$

$$D(a', b')=N_0^{-1}\sum n(...)\, s'\sigma' t'\tau' \tag{30}$$

$$D(a', \infty) = N_0^{-1} \sum n(...)\, s'\sigma'\varepsilon \tag{31}$$

$$D(\infty, b) = N_0^{-1} \sum n(...)\, \delta t\tau \tag{32}$$

$$D(\infty, \infty) = N_0^{-1} \sum n(...)\, \delta\varepsilon \tag{33}$$

We avoided writing down $D(a, \infty)$ and $D(\infty, b')$ that are not used in Bell's type inequalities discussed next.

We define a linear combination of joint probabilities as follows:

$$\begin{aligned} \Gamma &\equiv D(a, b) - D(a, b') + D(a', b) + D(a' + b') \\ &= N_0^{-1} \sum n(...)[s\sigma(t\tau - t'\tau') + s'\sigma'(t\tau + t'\tau')] \end{aligned} \tag{34}$$

The latter quantity will be useful in the deduction of inequalities for EPR experiments.

6. STRONG AND WEAK INEQUALITIES OF THE BELL TYPE

Next it will be shown that our model can satisfy two different types of inequalities: (*i*) The *weak inequalities* essentially identical to Bell's original inequalities, that are a necessary consequence of our local realistic model; (*ii*) The *strong inequalities*, that have been used in the analysis of the performed experiments, and that can only be deduced by means of some additional assumption. The strong inequalities provide, in general limits, for Γ that are numerically much more stringent than those provided by the weak inequalities. In the limit of high efficiency detectors weak and strong inequalities coincide, and the additional assumptions become certainly true as a consequence of the very definition of "perfect detector."

In order to demonstrate the inequalities one can start from the Clauser–Horne theorem[17]

$$-XY \leqslant xy - xy' + x'y + x'y' - x'Y - Xy \leqslant 0 \tag{35}$$

always valid if $0 \leqslant x, x' \leqslant X$ and $0 \leqslant y, y' \leqslant Y$. Taking now

$$x = s\sigma;\ x' = s'\sigma';\ y = t\tau;\ y' = t'\tau' \tag{36}$$

and thus $X, Y = 1$ one obtains

$$-1 \leqslant s\sigma(t\tau - t'\tau') + s'\sigma'(t\tau + t'\tau') - s'\sigma' - t\tau \leqslant 0 \tag{37}$$

an inequality that must be valid for each individual photon pair. Multiplication with the population density $n(...)/N_0$ and summation over all the 324 sets of values of the dichotomic variables yields

$$-1 \leqslant \Gamma - p(a') - q(b) \leqslant 0 \tag{38}$$

where the definition in Eq. (34) of Γ was used. These are exactly the weak inequalities. Therefore, as expected, our local model is in agreement with Bell's inequality and thus necessarily in disagreement with quantum mechanics. However, it will later be shown that the disagreement can concern only experiments made with high efficiency detectors, meaning in our model that the parameters σ, σ', τ, τ', δ, ε should almost always equal $+1$, and only rarely equal 0.

Inequalities stronger than Eq. (38) can only be obtained by using some additional assumption such as the "no-enhancement assumption" of Clauser and Horne[17]:

> *For every photon the probability of a detection with a polarizer in place on its trajectory is less than or equal to the detection probability with the polarizer removed.*

With our dichotomic variables the Clauser–Horne assumption can be written:

$$\begin{aligned} &0 \leqslant s\sigma, s'\sigma' \leqslant \delta \\ &0 \leqslant t\tau, t'\tau' \leqslant \varepsilon \end{aligned} \tag{39}$$

These relations imply that a photon which is registered by the detector with the polarizer in place would always be detected if the polarizer were removed, but not necessarily *vice versa.* Using again Eq. (35), this time with $X = \delta$ and $Y = \varepsilon$, one obtains

$$-\delta\varepsilon \leqslant s\sigma(t\tau - t'\tau') + s'\sigma'(t\tau + t'\tau') - s'\sigma'\varepsilon - \delta t\tau \leqslant 0 \tag{40}$$

valid for every photon pair if the Clauser–Horne assumption is true. Multiplication with the population density $n(...)/N_0$ and summation over all 324 populations now yields

$$-D(\infty, \infty) \leqslant \Gamma - D(a', \infty) - D(\infty, b) \leqslant 0 \tag{41}$$

Here we have so obtained the strong inequality, consequence of our local realistic model only if the "no-enhancement" assumption is made. A numerical comparison of the weak and the strong inequalities for three EPR experiments is shown in Table I.

The numerical comparison of Table I has been made by assuming correct the quantum mechanical predictions for the nonparadoxical

Table I. Comparison of Weak and Strong Inequalities.

Experiment	ε_+^α	ε_+^β	η_α	η_β	weak inequality	strong inequality
Freedman–Clauser[11]	1.01	1.00	0.13	0.28	$-0.794 \leqslant \Gamma \leqslant 0.206$	$0.000 \leqslant \Gamma \leqslant 0.037$
Holt–Pipkin[13]	0.91	0.88	0.08	0.27	$-0.845 \leqslant \Gamma \leqslant 0.155$	$-0.002 \leqslant \Gamma \leqslant 0.019$
Aspect *et al.*[11]	1.00	1.00	0.25	0.06	$-0.845 \leqslant \Gamma \leqslant 0.155$	$0.000 \leqslant \Gamma \leqslant 0.015$

probabilities $p(a')$, $q(b)$, $D(\infty, \infty)$, $D(a', \infty)$, $D(\infty, b)$. It should be stressed that the strong inequalities restrict the acceptable values of Γ to an interval that is 27 times smaller for the Freedman–Clauser experiment, 48 times smaller for the Holt–Pipkin experiment, and 67 times smaller for the Orsay experiment. The quantum mechanical predictions for Γ violate the strong inequalities, but are fully compatible with the weak ones in the experiments of Table I, as in all known EPR experiments.

7. DEDUCTION OF QUANTUM PROBABILITIES WITHIN OUR MODEL

The "no-enhancement" assumption Eq. (39) has no justification within a general deterministic model, and cannot even be checked experimentally as a matter of principle, since it refers to different detections of *individual photons*. One can give a simple example where our (local) model violates the strong inequalities: Assume that for all photon pairs

$$s\sigma = s'\sigma' = t\tau = t'\tau' = 1$$

Therefore $\Gamma = 2$ and the right hand side of the inequality in Eq. (41) becomes

$$2 \leqslant N_0^{-1} \sum n(...)(\delta + \varepsilon)$$

Clearly, this inequality is violated, if δ or ε vanish even for a single photon only! Admittedly this example is not a very physical one both because of the assumptions made and because it exhibits average enhancement, i.e., the total counting rate with polarizers present is larger than that without polarizers. Yet, our model having a large number of adjustable parameters (populations $n(...)$ of the subensembles) is certainly rich enough to yield physically more reasonable cases which still violate the strong inequality. That this is true will be shown next.

In order to simplify notation, note that in all the expressions in Eqs. (21)–(34), the variables s, s', t, t', σ, σ', τ, τ' appear always in pairs. It is therefore useful to define the new variables

$$S = s\sigma,\ S' = s'\sigma',\ T = t\tau\ T' = t'\tau' \tag{42}$$

This means for example, that $S = 1$ determines that photon α traverses its polarizer oriented at direction a and that it is registered by its detector, while $S = 0$ determines that a similar photon is either absorbed in the polarizer or transmitted but not detected.

The populations can now be written as functions of six variables only:

$$N(S, S', \delta; T, T', \varepsilon) \tag{43}$$

with some of these $N(...)$ populations being sums over some of the populations $n(...)$ previously discussed. There are only two restrictions that these populations must satisfy:

(i) *They must be non negative*;

(ii) *Their sum must equal N_0, the total number of pairs.*

Whatever the choice of the $N(...)$ satisfying (*i*) and (*ii*) one could easily conceive a source producing the right ensemble of pairs. We are thus free to assume that

$$N(S, S', \delta; T, T', \varepsilon) = N_0 F(S, S'; T, T')\, G(\delta; \varepsilon) \tag{44}$$

where $F(...)$ and $G(...)$ are nonnegative functions of their dichotomic arguments satisfying

$$\sum_{S, S', T, T'} F(S, S'; T, T') = 1 \tag{45}$$

$$\sum_{\delta, \varepsilon} G(\delta; \varepsilon) = 1 \tag{46}$$

In order to reproduce the quantum mechanical predictions of Eqs. (21)–(34) it is enough to assume that only the following 9 quantities F are nonzero:

$$\begin{aligned}
F_1 &= F(1, 0; 1, 0) = D_0(a, b)\\
F_2 &= F(1, 0; 0, 1) = D_0(a, b')\\
F_3 &= F(0, 1; 1, 0) = D_0(a', b)\\
F_4 &= F(0, 1; 0, 1) = D_0(a', b')\\
F_5 &= F(1, 0; 0, 0) = p_0(a) - [D_0(a, b) + D_0(a, b')]
\end{aligned} \tag{47}$$

$$
\begin{aligned}
F_6 &= F(0, 1; 0, 0) = p_0(a') - [D_0(a', b) + D_0(a', b')] \\
F_7 &= F(0, 0; 1, 0) = q_0(b) - [D_0(a, b) + D_0(a', b)] \\
F_8 &= F(0, 0; 0, 1) = q_0(b') - [D_0(a, b') + D_0(a', b')] \\
F_9 &= F(0, 0; 0, 0) = 1 - (F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8) \\
&= 1 - p_0(a) - p_0(a') - q_0(b) - q_0(b') + D_0(a, b) \\
&\quad + D_0(a, b') + D_0(a', b) + D_0(a', b')
\end{aligned}
\tag{47}
$$

For the $G(\ldots)$ quantities we assume

$$
\begin{aligned}
G_1 &= G(1; 1) = D_0(\infty, \infty) \\
G_2 &= G(1; 0) = p_0(\infty) - D_0(\infty, \infty) \\
G_3 &= G(0; 1) = q_0(\infty) - D_0(\infty, \infty) \\
G_4 &= G(0; 0) = 1 - (G_1 + G_2 + G_3) = 1 - p_0(\infty) - q_0(\infty) + D_0(\infty, \infty)
\end{aligned}
\tag{48}
$$

Now every interesting probability can be calculated by using Eqs. (44), (47) and (48):

$$p, q, D \Rightarrow B(f; g) \equiv \left[\sum F(S, S'; T, T') \cdot f \right] \left[\sum G(\delta; \varepsilon) \cdot g \right] \tag{49}$$

where f and g are suitable products of dichotomic variables and $B(f; g)$ is the bilinear form of f and g defined by the right-hand side of Eq. (49). It is now a simple matter to obtain all the interesting probabilities by using Eqs. (45) or (46) whenever possible:

$$D(a, b) = B(ST; 1) = F_1 = D_0(a, b) \tag{50}$$

$$D(a, b') = B(ST'; 1) = F_2 = D_0(a, b') \tag{51}$$

$$D(a', b) = B(S'T; 1) = F_3 = D_0(a', b) \tag{52}$$

$$D(a', b') = B(S'T'; 1) = F_4 = D_0(a', b') \tag{53}$$

$$D(a', \infty) = B(S'; \varepsilon) = (F_3 + F_4 + F_6)(G_1 + G_3) = p_0(a')\, q_0(\infty) \tag{54}$$

$$D(a, \infty) = B(S; \varepsilon) = (F_1 + F_2 + F_5)(G_1 + G_3) = p_0(a)\, q_0(\infty) \tag{55}$$

$$D(\infty, b) = B(T; \delta) = (F_1 + F_3 + F_7)(G_1 + G_2) = p_0(\infty)\, q_0(b) \tag{56}$$

$$D(\infty, b') = B(T'; \delta) = (F_2 + F_4 + F_8)(G_1 + G_2) = p_0(\infty)\, q_0(b') \tag{57}$$

$$D(\infty, \infty) = B(1; \delta\varepsilon) = G_1 = D_0(\infty, \infty) \tag{58}$$

$$p(a) = B(S; 1) = F_1 + F_2 + F_5 = p_0(a) \tag{59}$$

$$p(a') = B(S'; 1) = F_3 + F_4 + F_6 = p_0(a') \tag{60}$$

$$p(\infty) = B(1; \delta) = G_1 + G_2 = p_0(\infty) \tag{61}$$

$$q(b) = B(T; 1) = F_1 + F_3 + F_7 = q_0(b) \tag{62}$$

$$q(b') = B(T'; 1) = F_2 + F_4 + F_8 = q_0(b') \tag{63}$$

$$q(\infty) = B(1; \varepsilon) = G_1 + G_3 = q_0(\infty) \tag{64}$$

From Eqs. (6)–(13) one sees that $p_0(a)\, q_0(\infty) = D_0(a, \infty)$, etc. Therefore the calculated probabilities of Eqs. (50)–(64) coincide exactly with their quantum mechanical counterparts. *We have so obtained a complete reconstruction of the quantum formulae with a local deterministic model.* As stated before, the quantities $F(...)$ and $G(...)$ can in no case be negative. We must next find out in which cases this is actually so.

The sum of the nine $F(...)$ has been set equal to unity, by Eq. (47). Therefore, only the positivity of the $F(...)$ needs worry us. Positivity is certainly satisfied for F_1, F_2, F_3, F_4 because of their definition in Eq. (47). Considering F_5, one sees that positivity is certainly satisfied if it is when the two cosines entering in $D_0(a, b)$ and $D_0(a, b')$ are set equal to unity. This leads to the condition

$$1 \geqslant \left[\varepsilon_+^\beta + \frac{\varepsilon_-^\alpha}{\varepsilon_+^\alpha}\, \varepsilon_-^\beta F \right] \eta_\beta$$

which is in turn certainly satisfied, in practical experiments, if $\eta_\beta \leqslant 1/2$. The same condition guarantees the nonnegativity of F_6. Similarly, $\eta_\alpha \leqslant 1/2$ guarantees nonnegativity of F_7 and F_8. Coming to the last term we see that F_9 is certainly nonnegative if it is when all D_0's are set equal to zero. The obtained condition is satisfied again if $\eta_\alpha \leqslant 1/2$ and $\eta_\beta \leqslant 1/2$.

Finally, G_1 is never negative; G_2 and G_3 cannot be negative if η_α and η_β do not exceed unity; The nonnegativity of G_4 is certainly true if neither η_α nor η_β exceed 1/2.

We conclude that in our model all probabilities are nonnegative and correctly normalized if $\eta_\alpha \leqslant 1/2$ and $\eta_\beta \leqslant 1/2$. That these conditions are well satisfied in real life can be seen from Table I. For detectors having an efficiency above the 50% level our model blows up and no reproduction of the quantum mechanical predictions in local deterministic terms is possible, consistently with Bell's theorem.

8. THE CASE OF TWO-WAY POLARIZERS

In the case of two-way polarizers we introduce, like before, variables S, S' (for photon α) and T, T' (for photon β) determining the future behavior of each photon. This time our variables shall however be three-valued: For example, $S=+1$, -1, 0 will determine that photon α will be transmitted by the polarizer oriented along a and that it will be detected, that it will instead be reflected and detected (in the extraordinary ray), that it will not be detected in any channel, respectively. We will not deal in the present case with terms such as $D(a+, \infty)$, that have never been measured, but the complications needed to do so consist again of the introduction of G functions similar to those of the previous section.

Therefore our $F(\ldots)$ functions are of the type

$$F(S, S'; T, T')$$

and there are *a priori* 3^4 of them. They can be reduced to 25 with nonzero population if it is assumed that only the configurations

$$(S, S'), (T, T') = (1, 0), (0, 1), (-1, 0), (0, -1), (0, 0)$$

present themselves. We furthermore assume:

$$F(1, 0; 1, 0) = D_0(a+, b+), F(-1, 0; -1, 0) = D_0(a-, b-) \tag{65}$$

$$F(1, 0; 0, 1) = D_0(a+, b'+), F(-1, 0; 0, -1) = D_0(a-, b'-) \tag{66}$$

$$F(0, 1; 1, 0) = D_0(a'+, b+), F(0, -1; -1, 0) = D_0(a'-, b-) \tag{67}$$

$$F(0, 1; 0, 1) = D_0(a'+, b'+), F(0, -1; 0, -1) = D_0(a'-, b'-) \tag{68}$$

$$F(1, 0; -1, 0) = D_0(a+, b-), F(-1, 0; 1, 0) = D_0(a-, b+) \tag{69}$$

$$F(1, 0; 0, -1) = D_0(a+, b'-), F(-1, 0; 0, 1) = D_0(a-, b'+) \tag{70}$$

$$F(0, 1; -1, 0) = D_0(a'+, b-), F(0, -1; 1, 0) = D_0(a'-, b+) \tag{71}$$

$$F(0, 1; 0, -1) = D_0(a'+, b'-), F(0, -1; 0, 1) = D_0(a'-, b'+) \tag{72}$$

$$\begin{aligned} F(1, 0; 0, 0) = p_0(a+) - D_0(a+, b+) - D_0(a+, b-) \\ - D_0(a+, b'+) - D_0(a+, b'-) \end{aligned} \tag{73}$$

$$\begin{aligned} F(-1, 0; 0, 0) = p_0(a-) - D_0(a-, b+) - D_0(a-, b-) \\ - D_0(a-, b'+) - D_0(a-, b'-) \end{aligned} \tag{74}$$

$$\begin{aligned} F(0, 1; 0, 0) = p_0(a'+) - D_0(a'+, b+) - D_0(a'+, b-) \\ - D_0(a'+, b'+) - D_0(a'+, b'-) \end{aligned} \tag{75}$$

$$F(0,\,-1;0,0)=p_0(a'-)-D_0(a'-,b+)-D_0(a'-,b-)$$
$$-D_0(a'-,b'+)-D_0(a'-,b'-) \tag{76}$$
$$F(0,0;1,0)=q_0(b+)-D_0(a+,b+)-D_0(a-,b+)$$
$$-D_0(a'+,b+)-D_0(a'-,b+) \tag{77}$$
$$F(0,0;\,-1,0)=q_0(b-)-D_0(a+,b-)-D_0(a-,b-)$$
$$-D_0(a'+,b-)-D_0(a'-,b-) \tag{78}$$
$$F(0,0;0,1)=q_0(b'+)-D_0(a+,b'+)-D_0(a-,b'+)$$
$$-D_0(a'+,b'+)-D_0(a'-,b'+) \tag{79}$$
$$F(0,0;0,\,-1)=q_0(b'-)-D_0(a+,b'-)-D_0(a-,b'-)$$
$$-D_0(a'+,b'-)-D_0(a'-,b'-) \tag{80}$$

The remaining $F(...)$ function, that is $F(0,0;0,0)$, is assumed to be unity minus the sum of all other 24 $F(...)$ functions given by Eqs. (65)–(80). In the present case one has, for example:

$$D(a+,b+)=\sum F(S,S';T,T')[S(1+S)/2][T(1+T)/2]$$
$$=F(1,0;1,0)=D_0(a+,b+) \tag{81}$$

the sum being extended over all values of S, S', T, T'. The factor $S(1+S)/2$ is such that only terms with $S=1$ can contribute, and similarly for $T(1+T)/2$.

One sees from Eq. (81) that the quantum mechanical predictions are obtained once more. This result holds for all possible single particle probabilities and joint probabilities for two particles, as one can easily check from Eqs. (65)–(80), and we will not insist on this. Again, our model blows up for detector efficiencies above the 50% level.

9. CONCLUSIONS

A broad class of local deterministic models indistinguishable from quantum mechanics for low detector efficiencies has been shown to exist. It should also be stressed that within our model populations are also possible in principle which would violate the strong inequalities more than the violations observed hitherto in experiments. It is remarkable that Nature is such that the strong inequality is violated just the way quantum mechanics predicts and neither in a stronger nor in a weaker way as would both be possible in a realistic model of the kind proposed here.

Finally we stress again that our model violates the strong inequality only because it violates the no-enhancement hypothesis. Only if experimental efficiencies can be significantly improved over the present status of EPR experiments, this hypothesis can be ruled out definitively. The difference in the expectations of the present authors whether this will happen or not is indicative of the diversity of opinion among physicists at large.

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