mathematics

Supplementary Material to

Potential-Growth Indicators Revisited: More Merits of Indication

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Matlab functions *diopha21_1* and *diopha21_2* find all the solutions to the Diophantine systems of equations and inequalities (17), (21) and (18), (22) respectively for 9 and 13 integer-valued parameters. Below is the Matlab code for the more cumbrous *diopha21_2* (Colony *II*), the reduction to Colony *I* being obvious.

```matlab
function [Pars, Lams] = diopha21_2
% finds all the solutions of the Diophantine system of equations&inequalities
% Colony 2 without excavation; hierarchy (6C) among parent Contributions
% reproductive-core 6x6 submatrix Lcor only (size L2 = 14x14)
% Output variables: Lams, vector of 2 λ1s extremal values over the set of solutions;
% Pars, 13x2 matrix of extremal parameters.
% @ Logofet 26.04.2021.
global ns % assign also in workspace
ns = 0;%number of solutions
Lcor=[0,    0,    0,   0,   0,   0;
    18/61,  0,    0,   0,   0,   0;
        0,  0,    0,   0,   0,   0;
    14/61,  0, 2/3,    0,   0,   0;
        0,  0,    0,   0,   0,   0;
```

```
25       16/61,  0,  0,  0,  1,  0]; % reproductive-core 6x6 submatrix, without birth rates
26    lam1min = 100; lam1max = 0;% unreal values to begin with
27    a_min=0;b_min=0;c_min=0;d_min=0;e_min=0;f_min=0;g_min=0;h_min=0;k_min=0;l_min=0;m_min=0;n_min=0;o_min=0;
28    a_max=0;b_max=0;c_max=0;d_max=0;e_max=0;f_max=0;g_max=0;h_max=0;k_max=0;l_max=0;m_max=0;n_max=0;o_max=0;
29    for a=122:178 a,% 122=207-(14+30+10+5+26),178=61*3-(1+1+1+1+1).
30        for b=1:14% 1 due to 207-(178+30+10+5+26)<0, 14 when all the rhizomes go to v1.
31            for c=1:29% 1 due to 207-(178+14+10+5+26)<0, 29=30-1.
32                for d=1:9% 1 due to 207-(187+14+30+...)<0, 9=10-1.
33                    for e=1:4% similarly, 4= 5-1.
34                        for f=1:24% similarly, 24= 26-(1+1).
35                            for g=1:50% 1 due to 54-(29+9+4+24)<0, 50=54-(1+1+1+1).
36                                for h=1:29% 1 due to 54-(50+9+4+24)<0, 29=30-1.
37                                    for k=1:9% 1 due to 54-(50+29+4+24)<0, 9=10-1.
38                                        for l=1:4% similarly, 4=5-1.
39                                            for m=1:24% similarly, 24=26-2.
40                                                for n=1:6% 6 = 7-1.
41                                                    for o=1:6% 6 = 7-1. Hierarchy (6C): a+g+n ≤ c+h ≤ b ≤ d + k ≤ m + o
42                                                        C1=a+g+n;C2=c+h;C3=f+m+o;C4=b;C5=d+k;C6=e+l;%for hierarchy (6C)
43                                                    if a+b+c+d+e+f)==207)&&((g+h+k+l+m)==54)&&((n+o)==7)&&(C1>=C2)&&(C2>=C3)&&(C3>=C4)&&(C4>=C5)&&(C5>=C6)
44                                                        ns = ns +1;% one more solution
45                                                        Lcor(1,1:8)=[a/61,b/8,0,c/6,d/6,0,e/1,f/6];
46                                                        Lcor(4,1:8)=[g/61,0,0,h/6,k/6,0,l/1,m/6];%'el'/1
47                                                        Lcor(7,1:8)=[n/61,0,0,0,0,0,0,o/6];% assigning the birth rates
48                                                        lam1 = max(eig(Lcor));
49                                                        if (lam1 < lam1min), lam1min = lam1;
50                                                            a_min = a;b_min = b;c_min = c;d_min = d;e_min = e;f_min = f;g_min = g;h_min = h;
51                                                            k_min = k;l_min = l;m_min = m;n_min = n;o_min = o; end
```

```matlab
52                                 if (lam1 > lam1max), lam1max = lam1;
53                                     a_max = a;b_max = b;c_max = c;d_max = d;e_max = e;f_max = f;g_max = g;h_max = h;
54                                     k_max = k;l_max = l;m_max = m;n_max = n;o_max = o;end
55                               end
56                          end
57                        end
58                      end
59                    end
60                  end
61                end
62              end
63            end
64          end
65        end
66      end
67    end
68 end
69 ns %the number of solutions
70 Lams = [lam1min, lam1max];
71 Pars = [a_min b_min c_min d_min e_min f_min g_min h_min k_min l_min m_min n_min o_min]';
72 Pars = [Pars, [a_max b_max c_max d_max e_max f_max g_max h_max k_max l_max m_max n_max o_max]'];
73
74       Below is a Matlab expression for RRT(Lcor2) as a function of 13 parameters a, b, …, n, o:
75 max([a/61 + (9*b)/244,
76 a/61 + (9*b)/244 + (7*d)/183 + h/6 + k/9 - (a*h)/366 - (3*b*h)/488 + (c*g)/366 + (d*g)/549 - (a*k)/549 - (7*d*h)/1098 - (b*k)/244 + (7*c*k)/1098,
77 a/61 + (9*b)/244 + (7*d)/183 + (8*f)/183 + h/6 + k/9 + o/6 - (a*h)/366 - (3*b*h)/488 + (c*g)/366 + (d*g)/549 - (a*k)/549 - (7*d*h)/1098 - (b*k)/244 + (7*c*k)/1098 -
78 (4*f*h)/549 - (a*o)/366 + (4*c*m)/549 - (3*b*o)/488 + (8*d*m)/1647 - (8*f*k)/1647 - (7*d*o)/1098 + (e*n)/61 + (8*e*o)/183 + (f*n)/366 - (h*o)/36 - (k*o)/54 + (a*h*o)/2196 +
```

79 (b*h*o)/976 - (c*g*o)/2196 - (d*g*o)/3294 + (a*k*o)/3294 + (7*d*h*o)/6588 - (e*h*n)/366 + (b*k*o)/1464 - (4*e*h*o)/549 - (f*h*n)/2196 - (c*k*o)/94 + (c*l*n)/366 +

80 (4*c*l*o)/549 + (c*m*n)/2196 + (d*l*n)/549 - (e*k*n)/549 + (8*d*l*o)/1647 + (d*m*n)/3294 - (8*e*k*o)/1647 - (f*k*n)/3294,

81 a/61 + (9*b)/244 + (7*d)/183 + h/6 + k/9 - (a*h)/366 - (3*b*h)/488 + (c*g)/366 + (d*g)/549 - (a*k)/549 - (7*d*h)/1098 - (b*k)/244 + (7*c*k)/1098 + (e*n)/61 - (e*h*n)/366 +

82 (c*l*n)/366 + (d*l*n)/549 - (e*k*n)/549, a/61 + (9*b)/244 + h/6 - (a*h)/366 - (3*b*h)/488 + (c*g)/366,

83 a/61])

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