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Great realisations

In a world of continuous trading, accurate estimation of realised volatility is vital. In the first of two empirically flavoured papers, Torben Andersen, Tim Bollerslev, Francis Diebold and Paul Labys show how high-frequency data can be used in an optimal way

Volatility is central to many applied issues in finance and financial engineering, from asset pricing and asset allocation to risk management. Hence financial economists have been intrigued by the very high precision with which volatility can be estimated under the diffusion assumption. Precise estimation of diffusion volatility does not require a long calendar span of data. Rather, volatility can be estimated well from an arbitrarily short span of data, provided that returns are sampled sufficiently frequently. This contrasts sharply with precise estimation of the drift, which generally requires a long calendar span of data, regardless of the frequency with which returns are sampled.

Consequently, the volatility literature has steadily progressed toward the use of higher-frequency data. This is true in the parametric autoregressive conditional heteroscedasticity (Arch) and stochastic volatility literatures (see Bollerslev, Engle & Nelson, 1994, for a review), as well as in the more traditional empirical finance literature. For example, Officer (1973) constructs annual volatilities from monthly returns on an equity index, whereas Merton (1980) and French, Schwert & Stambaugh (1987) use daily returns to estimate monthly volatilities. Even more recently, Schwert (1998) relies on 15-minute returns for construction of daily stock market volatilities, while Taylor & Xu (1997) and Andersen *et al* (1999a) exploit five-minute returns in the measurement of daily exchange rate volatilities.

Recent work has clarified the comparative desirability of alternative volatility estimators. This emerging theory emphasises the advantages of the so-called realised volatility estimator. Construction of realised volatility is trivial – one simply sums intra-period high-frequency squared returns (or cross products, for realised covariance), period by period. For example, for a 24-hour market, daily realised volatility based on five-minute underlying returns is defined as the sum of the 288 intra-day squared five-minute returns, taken day by day. Andersen & Bollerslev (1998) show that, under the usual diffusion assumptions, realised volatility calculated from high-frequency intra-day returns is effectively an error-free volatility measure.

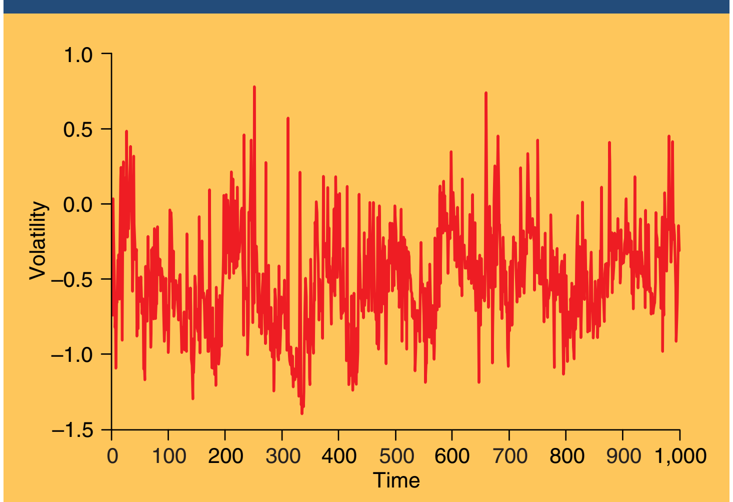
As realised volatility is, in principle, error-free, it is natural to treat volatility as observable. Observable volatility creates entirely new opportunities: we can analyse it, optimise it, use it and forecast it. This article exploits this insight. We describe our recent attempts at understanding both the unconditional and conditional distributions of realised asset return volatility. Second, we describe tools for optimising the construction of realised volatility measures. In the third section, we use realised volatilities to draw inferences about the conditional distributions of asset returns. We then discuss explicit modelling and forecasting of realised volatility.

Realised volatility and correlation

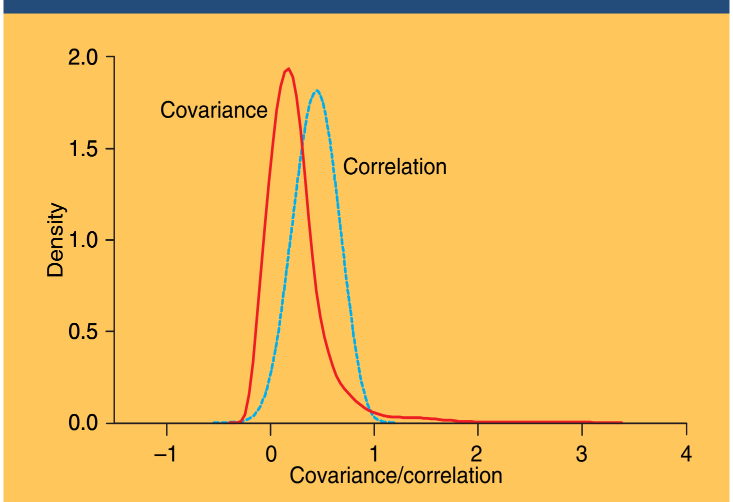
High-frequency data on Deutschmark and yen returns against the dollar are used to construct model-free estimates of daily exchange rate volatility and correlation, spanning an entire decade (Andersen *et al*, 1999a). Preliminary results indicate that the observed patterns apply more broadly to other types of assets, including the 30 individual stocks in the Dow Jones Industrial Average, as studied in Andersen *et al* (1999).

Figure 1 shows daily realised volatility for a representative asset return series of 1,000 days. (Unless otherwise noted, all of the graphs shown here are designed to be representative of daily returns.) It is clear that realised volatility changes from day to day, as one expects. Furthermore, its fluctuations display substantial persistence.

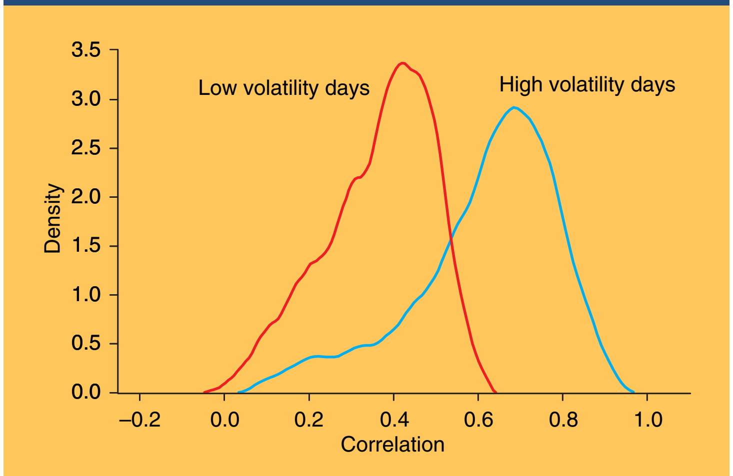
1. Representative realised volatility series



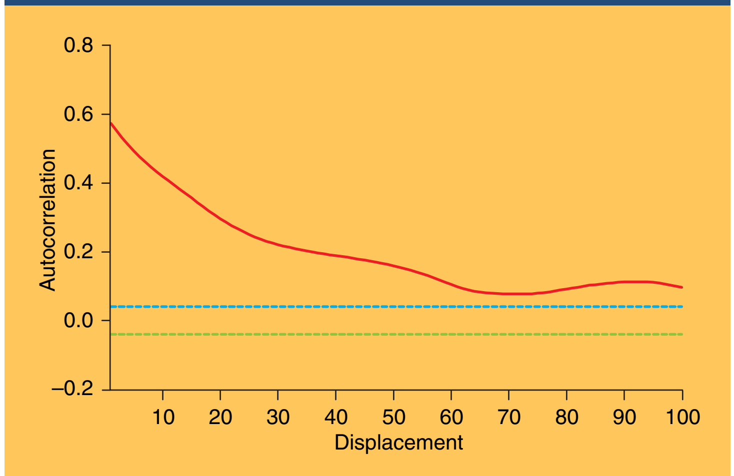
2. Representative distributions of realised covariance and correlation



3. Distributions of realised correlation in low volatility v. high volatility periods



4. Sample autocorrelations of realised volatility or correlation



Although not shown here, the distribution of the realised variance is skewed, but transforming to realised standard deviation moves it toward symmetry, and transforming to log standard deviations renders it approximately Gaussian.

Similarly, we find that realised covariance tends to be highly skewed, but that a simple transformation to correlation delivers approximate normality (see figure 2). Realised correlation is almost always positive, often strongly so, and it displays substantial variation. We also find that realised correlation is itself highly correlated with realised volatility, which we call the "volatility effect in correlation". In particular, return correlations tend to rise on high-volatility days, as we illustrate in figure 3.

We now move from unconditional to conditional aspects of the distributions of realised volatilities and correlations. Although correlograms of realised volatilities tend to exhibit a slow hyperbolic decay, as shown in figure 4, we routinely and soundly reject the unit-root hypothesis. However, such autocorrelation behaviour is also consistent with that of fractionally integrated long-memory processes. In fact, there is strong evidence to suggest that volatility is a long-memory process, an assertion we substantiate through various analyses. First, we estimate the long-memory parameter directly. The estimates tend to be about 0.4 for various realised volatility and correlation series, and the associated standard error is very small, about 0.02. Second, we verify that the degree of fractional integration is invariant to the horizon, which is a well-known property of long-memory processes, due to their self-similarity. Finally, we verify that our realised volatility and correlation series follow scaling laws, such that the logs of the variance of partial sums of the process are proportional to the logs of the horizon, which is also a well-known characteristic of long-memory processes.

Access to the high-frequency data necessary for constructing accurate realised volatilities is increasing rapidly, but it is far from universal, and we need simple and practical ways of characterising the measurement error remaining in realised volatilities constructed from insufficiently frequently sampled data. Moreover, even when high-frequency data is available, microstructure effects such as bid/ask bounce (occurring when transactions are priced between the bid and ask prices) and asynchronous trading may distort associated realised volatilities. We now turn to a tool for identifying and mitigating such effects.

Optimisation

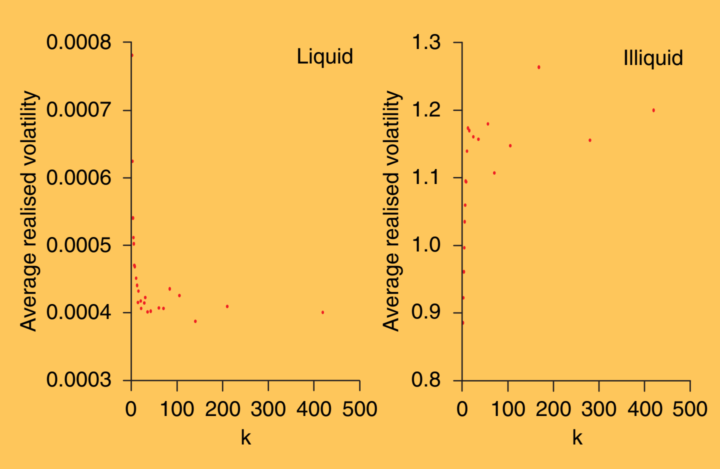
The appeal of realised volatility calculated from high-frequency data relies at least partially on the assumption that log asset prices evolve as diffusions. This assumption becomes progressively less tolerable as transaction time is approached and market microstructure effects emerge. Hence, a tension arises: the optimal sampling frequency will probably not be the highest available, but rather some intermediate value, ideally high enough to produce a volatility estimate with negligible sampling variation, yet low enough to avoid microstructure bias. The choice of underlying return frequency is therefore critical, but the literature currently offers little guidance for making that decision.

We developed a tool designed to provide some guidance (Andersen *et al*, 1999b). A key insight is that microstructure bias, if operative, will probably manifest itself as sampling frequency increases by distorting the average realised volatility. We construct a plot of average realised volatility against sampling frequency, which we call the "volatility signature plot". This helps to reveal the severity of microstructure bias as sampling frequency increases, and can be useful in guiding the selection of sampling frequency. We can also use the volatility signature plots to characterise different market microstructures. Interestingly, it turns out that the volatility signature has the same form as the variance-time function, which has been extensively studied in finance. However, while there is no information in the volatility signature that is not also present in high-frequency return autocorrelations, the two are complements, not substitutes, as the information relevant for construction and interpretation of realised volatilities is more directly and transparently revealed in the volatility signature plot.

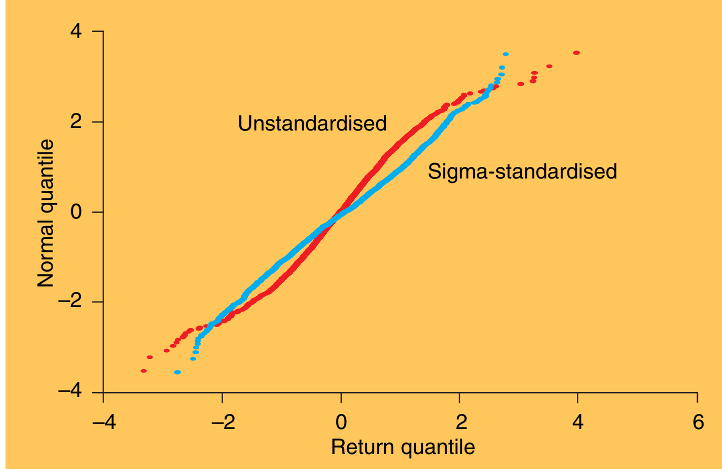
In figure 5, we show two representative volatility signature plots. The integer k represents multiples of the smallest sampling interval in the data. Thus, if we have a series for which the smallest available sampling interval is one minute, for $k = 1$ we construct average realised volatility using one-minute returns, for $k = 2$ we construct average realised volatility using two-minute returns, and so forth. The left panel of figure 5 represents a highly liquid asset for which the largest realised volatility estimates occur at the highest sampling rates, corresponding to the smallest values of k. This can be explained by negative serial correlation in the returns, most likely induced by bid-ask bounce. At the smallest sampling intervals, the volatility measures are very high, but as returns are aggregated across larger and larger sampling intervals the oscillating swings in the returns series tend to cancel, and overall volatility is lower. The volatility signature plot stabilises at roughly $k = 20$ (in this case corresponding to a 20-minute return sampling interval). Although high-frequency microstructural effects will also be small for sampling intervals larger than $k = 20$, realised volatility estimates constructed from larger return intervals will begin to suffer from a higher sampling error. Thus, for this particular example, we would recommend the use of a sampling interval of $k = 20$, which represents a reasonable trade-off between minimising microstructural bias and minimising sampling error.

The right panel of figure 5 represents a less liquid asset, whose volatility signature is quite different from that of the asset in the top panel. In this case, microstructural factors cause a positive serial correlation at high frequencies, resulting in a smaller estimate of realised volatility, which does not stabilise until the sampling interval reaches $k = 15$, or 15 minutes. In this case, the microstructure bias is probably induced by inactive trading.

5. Representative volatility signature plots (liquid and illiquid assets)



6. Representative QQ plots (standardised and unstandardised returns)



Again, much remains to be done. For example, the standard microstructural biases due to bid-ask bounce remain relevant in the multivariate case – and hence for estimation of correlation. Moreover, new complications arise due, for example, to asynchronous trading (see Epps, 1979). Nevertheless, we feel confident that high-quality realised volatilities and correlations can be constructed in liquid markets, and we are optimistic regarding the potential for using volatility and correlation signature plots to assist in the choice of underlying sampling frequency. We are also interested in assessing various volatility estimators' robustness to microstructural effects, particularly those based on the range, as in Alizadeh, Brandt & Diebold (1999).

Use

Andersen *et al* (1999c) characterise the distribution and temporal dependence of $\varepsilon_t = r_t/\sigma_t$, which we call the σ-standardised return. (σ denotes the realised standard deviation.)

There is a long tradition in the econometrics literature of needing and allowing for a fat-tailed conditional distribution of ε_t, as in Bollerslev (1987). But that literature typically works with returns standardised by volatilities obtained from discrete-time Arch or stochastic volatility models, in which day-t volatility depends only on information at day $t - 1$ and earlier. The situation is different with realised volatility. Day-t realised volatility is based on information within day t, and the theoretical predictions for distributions of returns standardised by realised volatility are unambiguous: under the diffusion assumption they should be Gaussian.

This is, in fact, what we tend to find. The red points on figure 6 displays a representative QQ plot for unstandardised returns. Because the points do not fall into a straight line, we conclude that the returns are not distributed normally. The blue points on figure 6 displays a representative QQ plot for σ-standardised returns. It is close to linear, indicating that the σ-standardised returns are approximately Gaussian.

Much work remains to be done. It is, for example, of practical importance to examine the distribution of returns standardised by forecasts of realised volatility, based on prior information only. On the theoretical side, it will be interesting to develop more formal tests for the presence of jumps from the distribution of the returns standardised by the realised volatility.

Forecasting

Our construction, optimisation and use of realised volatilities has helped us reach two general conclusions: realised variances tend to be lognormally distributed, and asset returns standardised by realised standard deviations tend to be normally distributed. In turn, this suggests that a lognormal-normal mixture may be a good model for asset returns, an idea that inspires a simple modelling and forecasting strategy. This section briefly outlines an operational procedure for the univariate case, but extensions to the multivariate setting, exploiting realised covariances as well as volatilities, are straightforward.

In essence, forecasting return volatility is equivalent to forecasting realised volatility (as long as high-quality intra-day return data are available). Because realised volatility is effectively observed, it is amenable to direct analysis via standard time-series methods. It is natural to assume that the log-volatility process falls within the usual Gaussian autoregressive moving average (Arma) class of models. However, we have already noted the long-memory characteristics of the realised volatility series. Consequently, it is desirable to allow for fractional integration in the specification, leading to a so-called autoregressive fractionally integrated moving average (Arfima) model.

First, one determines the degree of fractional integration, d, in the realised log-volatility series. As noted above, the typical estimates suggest a value of d around 0.4. Next, one obtains the fractionally differenced series, say, $y_t = (1 - L)^d \log \sigma_t$. This involves calculating a long (in theory infinite, in practice long, but truncated) distributed lag of the underlying log-volatility series. This transformation ideally removes the long-run dependence in the series. For illustration, we display such a fractionally differenced log-volatility series in figure 7 and the associated correlogram in figure 8. They provide remarkable contrasts to figures 1 and 4; any indication of long memory has been annihilated. The final step of the modelling procedure is to obtain a parsimonious Gaussian Arma representation for this fractionally differenced (residual) series.

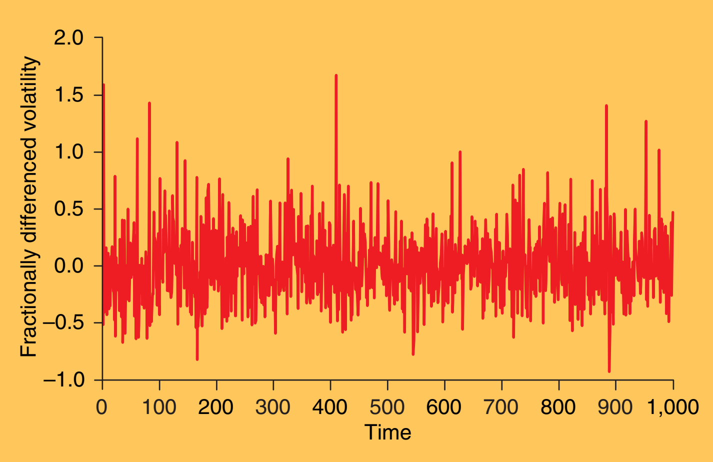
Standard Arfima procedures may now be applied to generate predictions of future realised log-volatility. The result is a sequence of volatility forecasts with associated prediction errors that are (approximately) lognormally distributed. Moreover, because returns are normally distributed conditional on realised volatility, one may readily calculate the fractiles of the conditional return distribution in closed form from the standard lognormal-normal mixture distribution.

The striking feature of this approach is that it builds directly on observed time series and utilises only standard linear Gaussian modelling and forecasting techniques. Hence, it is simple to assess in-sample performance and evaluate model forecasts through well-established out-of-sample procedures. It will be interesting in future work to investigate the actual performance of such an approach relative to popular frameworks such as Arch, stochastic volatility and RiskMetrics. Because our approach exploits an arguably superior volatility measure along with more sound distributional assumptions, it may outperform the standard procedures currently in use.

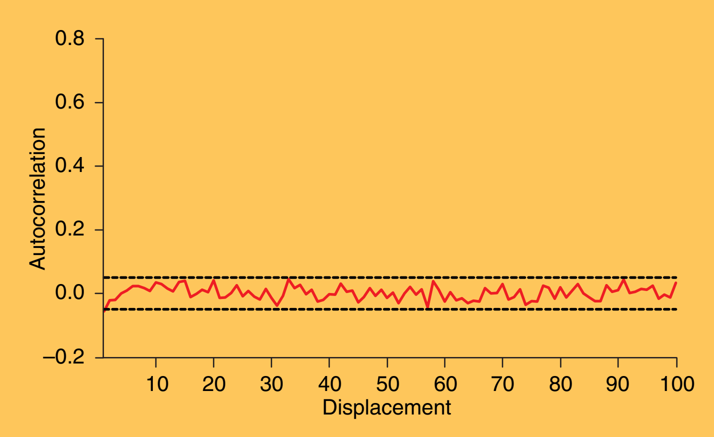
Conclusion

Our findings have potentially wide-ranging implications for applied finance. The results on the unconditional and conditional distributions of

7. Representative fractionally differenced realised volatility series



8. Sample autocorrelations of fractionally differenced realised volatility or correlation



asset return volatility are relevant for pricing derivatives. In fact, with the advent of volatility and covariance swaps, realised volatility itself is now the underlying. Such swaps are useful for, among others, holders of options who wish to vega-hedge their holdings. Proper pricing of derivatives on volatility depends critically on how volatility itself varies over time ("the volatility of volatility"). Our methodology allows for a direct approach to this issue through the construction and analysis of historical realised volatility series.

Improved volatility and correlation forecasts will also be useful for portfolio allocation and management. Concrete indications that more traditional volatility forecasts can be of value in guiding portfolio allocation decisions are provided by Fleming, Kirby & Ostdiek (1999). To the extent that our procedures are able to improve on the volatility forecast performance, the implied economic benefits could be high.

Finally, our forecasting procedures for realised volatility and correlation lead directly to a characterisation of the conditional return distribution (ignoring significant short-term variation in the conditional mean). The evaluation of fractiles of the conditional return distribution is, of course, a critical input into any active financial risk management programme. Hence, extensions of our methodology to a richer multi-asset setting should provide potentially valuable inputs for practical risk management. ■

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