

Neural Network Control Based on Adaptive Observer for Quadrotor Helicopter

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ABSTRACT

A neural network control scheme with an adaptive observer is proposed in this paper to Quadrotor helicopter stabilization. The unknown part in Quadrotor dynamical model was estimated on line by a Single Hidden Layer network. To solve the non measurable states problem a new adaptive observer was proposed. The main purpose here is to reduce the measurement noise amplification caused by conventional high gain observer by introducing some changes in observer's original structure that can minimize the variance and the amplitude of the noisy signal without increasing tracking error. The stability analysis of the overall closed-loop system/ observer is performed using the Lyapunov direct method. Simulation results are given to highlight the performances of the proposed scheme.

KEYWORDS

Adaptive observer; high gain observer; Neural Network control; Quadrotor; Lyapunov stability.

1. INTRODUCTION

As their application potential both in the military and industrial sector strongly increases, miniature unmanned aerial vehicles (UAV) constantly gain in interest among the research community [1]. Quadrotor Helicopter is considered as one of the most popular UAV platform. The main reasons for all this attention have stemmed from its simple construction and its large payload as compared with the conventional helicopter [2].

The Quadrotor is an under actuated system from where it has six degrees of freedom controlled only by four control inputs. To solve the Quadrotor UAV tracking control problem many techniques have been proposed [2-7] where the control objective is to control three desired Cartesian positions and a desired yaw angle.

In [1-3] Backstepping and sliding mode techniques have been used to control the Quadrotor. In [4] the H_∞ robust control law was proposed and a PID and LQR controls have been applied in [5]. Unfortunately these techniques are limited if the dynamic model is completely or partially unknown and/or if the states variables are unavailable.

The unknown nonlinear functions problem in the control law can be solved by using adaptive control technique based on universal approximators as artificial neural network [6]. Several Neural Networks adaptive schemes have been proposed for Quadrotor control [6-10].

To avoid the state variables measurement assumptions, state observers can be introduced in the control scheme such as in [9] where a neuro-sliding mode observer was proposed in order to reduce the noise measurement amplification. In [10] other observer was used to estimate the velocity, the corrective term here is the superposition of a observation errors proportional term and a feed-forward neural networks output used to estimate unknown functions.

High-gain observers have evolved over the past two decades as an important tool for the design of output feedback control of nonlinear systems [11]. However this observer has two drawbacks, the first one is the peak phenomena and the second one is its high sensitivity to noise measurements. Several methods have been proposed in the literature to reduce the last phenomena as in [11] were two observation gains values have been used. The biggest value was used in the transient response and the smallest observation gain was used in permanent regime. In [12] and [13] an adaptation laws is utilized to compute the optimal observation gain. However those methods present some limitations in particular when the observation gain has to be too large as in the case of quadrotor dynamic.

In the present paper a new robust adaptive scheme that does not require any prior information on model dynamics and state measurement is proposed. The SHL NN in the control loop is used to estimate the unknown part in Quadrotor's dynamic model. States variables reproduction is made by a proposed adaptive observer. The main purpose of the new observer is to reduce noise measurements amplification by the minimization of the variance and the amplitude of the noisy signal without introducing an increase in the tracking error. The learning law is derived based on the Lyapunov stability theory.

This paper is organized as follows. In Section 2, the dynamical model of the Quadrotor will be presented. Problem formulation will be posed in section 3. In Sections 4 and 5, neural adaptive controller based on high gain observer and the proposed observer will be developed respectively. Simulation results are given in section 6 to show the effectiveness and feasibility of the proposed observer at noise measurement presence. The conclusion is given in section 7.

2. QUADROTOR DYNAMICS

Such its name indicates Quadrotor helicopter is composed of four propellers in cross configuration where each rotors pair situated on the same branch; turn in opposite direction of the other pair to avoid the device rotation on her (Figure. 1).

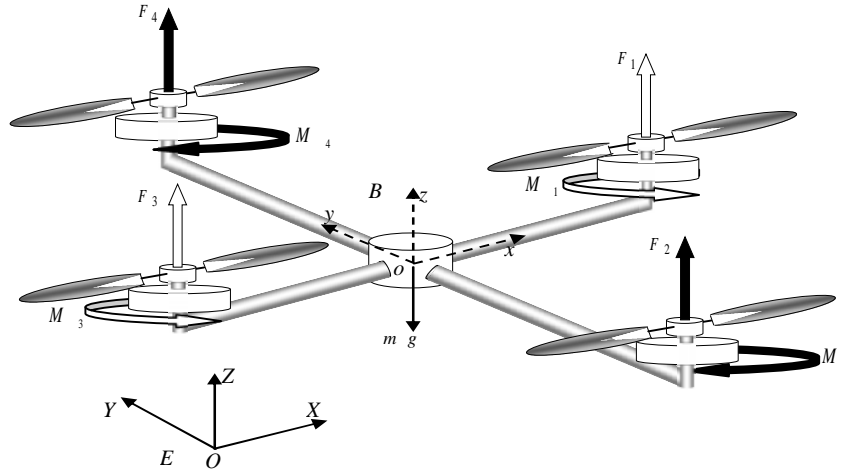


Figure 1. Model of conventional Quadrotor.

Flights mode in quadrotor are defined according to the direction and the velocity of each rotor. Vertical ascending (descending) flight is created by increasing (decreasing) thrust forces. Applying speed difference between front and rear rotors we will have pitching motion which is defined as a rotation motion around Y axis coupled with a translation motion along X axis. The same analogy is applied to obtain rolling motion, but by changing the side motors speed this time and as result we will have a rotation motion around X axis coupled with a translation motion along Y axis. The last flight mode is yaw motion; this one is obtained while increasing (decreasing) speed of motors (1.3) compared to (2.4) motors speed. Unlike pitch and roll motions, yaw rotation is the result of reactive torques produced by rotors rotation.

By using the formalism of Newton-Euler [2, 3, 6, 9] the dynamic model of a Quadrotor can be expressed as:

$$\left\{ \begin{array}{l} \ddot{\varphi} = \frac{U_{\varphi}(t)}{I_x} + \frac{(I_y - I_z)}{I_x} \dot{\psi} \dot{\theta} - \frac{J_r \Omega_r(t)}{I_x} \dot{\theta} - \frac{k_{frx}}{I_x} \dot{\varphi}^2 \\ \ddot{\theta} = \frac{U_{\theta}(t)}{I_y} + \frac{(I_z - I_x)}{I_y} \dot{\psi} \dot{\varphi} + \frac{J_r \Omega_r(t)}{I_y} \dot{\varphi} - \frac{k_{fry}}{I_y} \dot{\theta}^2 \\ \ddot{\psi} = \frac{U_{\psi}(t)}{I_z} + \frac{(I_x - I_y)}{I_z} \dot{\varphi} \dot{\theta} - \frac{k_{frz}}{I_z} \dot{\psi}^2 \\ \ddot{x} = \frac{1}{m} ((C_{\psi} S_{\theta} C_{\varphi} + S_{\varphi} S_{\psi}) U_1(t) - k_{fzx} \dot{x}) \\ \ddot{y} = \frac{1}{m} ((S_{\psi} S_{\theta} C_{\varphi} - S_{\varphi} C_{\psi}) U_1(t) - k_{fzy} \dot{y}) \\ \ddot{z} = \frac{1}{m} ((C_{\theta} C_{\varphi}) U_1(t) - k_{fz} \dot{z}) - g \end{array} \right. \quad (1)$$

Where : m : Quadrotor mass, k_p : Thrust factor, k_d : drag factor, ω_i : angular rotor speed, $J = \text{diag}(I_x, I_y, I_z)$: inertia matrix, $K_f = \text{diag}(k_{fzx}, k_{fzy}, k_{fz})$, drag translation matrix, $K_{fr} = \text{diag}(k_{frx}, k_{fry}, k_{frz})$: friction aerodynamic coefficients, $\xi = [x \ y \ z]^T$: position vector, $\eta = [\varphi \ \theta \ \psi]^T$ represents the angles of roll, pitch and yaw and $\Omega_r = \sum_{i=1}^4 (-1)^{i+1} \omega_i$.

The control inputs according to the angular velocities of the four rotors are given by:

$$\begin{bmatrix} U_1(t) \\ U_\phi(t) \\ U_\theta(t) \\ U_\psi(t) \end{bmatrix} = \begin{bmatrix} k_p & k_p & k_p & k_p \\ 0 & -lk_p & 0 & lk_p \\ -lk_p & 0 & lk_p & 0 \\ k_d & -k_d & k_d & -k_d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (2)$$

2.2. Virtual control

In this section, three virtual control inputs will be defined to ensure that the Quadrotor follows a specified trajectory. Those virtual controls are [9]:

$$\begin{cases} U_x(t) = (C_\psi S_\theta C_\phi + S_\phi S_\psi)U_1(t) \\ U_y(t) = (S_\psi S_\theta C_\phi - S_\phi C_\psi)U_1(t) \\ U_z(t) = (C_\theta C_\phi)U_1(t) \end{cases} \quad (3)$$

The physical interpretation of these virtual control, means that the control of translation motion depends on three common inputs are: $U_x(t)$, $U_y(t)$ and $U_z(t)$. This requires that the rolling and pitching motions must take a desired trajectory to guarantee the control task of translation motion. Using equation (3) the desired trajectories in rolling and pitching are defined as follow [9]:

$$\begin{cases} \phi_d = \arcsin\left(\frac{U_x(t)S_{\psi d} - U_y(t)C_{\psi d}}{\sqrt{U_x^2(t) + U_y^2(t) + U_z^2(t)}}\right) \\ \theta_d = \arctan\left(\frac{C_{\psi d}U_x(t) + S_{\psi d}U_y(t)}{U_z(t)}\right) \end{cases} \quad (4)$$

with $U_x(t)$, $U_y(t)$ and $U_z(t)$ are the desired trajectories in roll, pitch and yaw respectively.

3. PROBLEM FORMULATION

After the use of the virtual control defined in eq. (3), the Quadrotor helicopter dynamical model can be described by the following differential equation system:

$$\begin{cases} \dot{y}_1^{(r_1)} = f_1(X) + g_1(X)u_1(t) \\ \vdots \\ \dot{y}_p^{(r_p)} = f_p(X) + g_p(X)u_p(t) \end{cases} \quad (5)$$

Where the state space form is represented by:

$$\Sigma_i = \begin{cases} \dot{x}_{i1} = x_{i2} \\ \vdots \\ \dot{x}_{i r_i} = f_i(X) + g_i(X)u_i(t) \\ y_i = x_{i,1} + b_i \end{cases} \quad (6)$$

with: $X = [y_1 \ \dot{y}_1 \ \dots \ y_1^{(r_1-1)}, \dots, y_p \ \dot{y}_p \ \dots \ y_p^{(r_p-1)}]^T$: state space vector, $U = [u_1 \ \dots \ u_p]^T$: the input vector, $Y = [y_1 \ \dots \ y_p]^T$: output vector which is assumed available for measurement, $f_i(X)$, $g_i(X)$ are smooth nonlinear unknown functions and b_i is the measurement noise.

Assumption.1: The desired output trajectory $y_{di}, i = \overline{1:p}$ and its first r_i derivatives are smooth and bounded.

Assumption.2: The gain $g_i(X)$ is bounded, positive definite and slowly time varying.

Tracking error e_i and filtered errors s_i are defined by following equation:

$$\begin{cases} e_i = y_{di}(t) - y_i(t) \\ s_i = \left(\frac{d}{dt} + \lambda_i \right)^{r_i-1} e_i = \Phi_i E_i, \quad \lambda_i > 0 \end{cases} \quad (7)$$

From filtered error expression we can conclude that the convergence of tracking error and its derivatives to zero is guaranteed when s_i converge to zero [14]. For that reason, our control objective will be based on the synthesis of control law that allows the convergence to zero of the filtered error.

The filtered errors time derivative of can be expressed by:

$$\dot{s}_i = v_i - f_i(X) - g_i(X)u_i(t) = e_i^{(r_i)} + \Phi_{0i} E_i, \quad i = \overline{1:p} \quad (8)$$

With:

$$\begin{cases} \Phi_i = [v_{i1} \ \dots \ v_{i,r_i-1} \ 1], \\ \Phi_{0i} = [0 \ v_{i1} \ \dots \ v_{i,r_i-1}], \\ E_i = [e_i \ \dot{e}_i \ \dots \ e_i^{(r_i-1)}]^T, \quad i = \overline{1:p}, j = \overline{1:r_i-1} \\ v_i = y_d^{(r_i)} + \sum_{j=1}^{r_i-1} v_{ij} e_i^{(j)} \\ v_{ij} = \frac{(r_i-1)!}{(r_i-i)!(j-1)!} \lambda_i^{r_i-j} \end{cases} \quad (9)$$

The following equation expresses the ideal control that can guarantee the closed loop system performances:

$$u_i^*(t) = u_i^{eq^*}(t) + u_i^{pd^*}(t), \quad i = \overline{1:p} \quad (10)$$

With:

$$\begin{cases} u_i^{eq^*}(t) = g_i^{-1}(X)(v_i - f_i(X)) \\ u_i^{pd^*}(t) = k_i s_i = k_i \Phi_i E_i, \quad k_i > 0 \end{cases} \quad (11)$$

By substituting (10) in (8), we will have:

$$\dot{s}_i = -g_i k_i s_i \Rightarrow s_i \dot{s}_i \leq 0 \quad i = \overline{1:p} \quad (12)$$

Which implying that $s_i \rightarrow 0$ when $t \rightarrow \infty$, and therefore $e_i^{(j)} \rightarrow 0 \quad j = \overline{1:r_i-1}, i = \overline{1:p}$ when $t \rightarrow \infty$ [14].

Unfortunately the application of this control law is impossible if the dynamic model and the state variables are unknown as this case. From this fact, six SHL NN will be implemented in control loop in order to approximate online the equivalent controller defined in (11). The non measured states problem will be solved by using a proposed observer.

4. SYNTHESIS OF NEURAL ADAPTIVE CONTROL WITH A HIGH GAIN OBSERVER

The proposed adaptive control is given by the following expression:

$$\begin{cases} u_i(t) = u_i^{eq}(t) + u_i^r(t) + u_i^{pd}(t) \\ u_i^{pd}(t) = k_i \hat{s}_i = k_i \Phi_i \hat{E}_i \quad i = \overline{1:p} \\ u_i^r(t) = \hat{w}_i \text{sign}(\hat{s}_i) \end{cases} \quad (13)$$

Where:

- ↪ u_i^{pd} : is the PD term, the ideal PD control is given by (11), However the state vector X is not available to measure so u_i^{pd*} is replaced by u_i^{pd} ,
- ↪ u_i^r : is the robust term used for errors approximation compensation,
- ↪ u_i^{eq} : is the neural adaptive control term which is an approximate of ideal equivalent control defined in (11).

The vector \hat{E}_i is the estimated of E_i defined in eq. (9) and $\hat{w}_i \in \mathfrak{R}$ is the estimated of \bar{w}_i , which will be defined later.

4. 1. Adaptive control Design

According to approximation theorem [15] a SHL network presented in Figure.2 can approximate the unknown implicit ideal equivalent controller defined in (11) as follows:

$$u_i^{eq*}(t) = W_i^T \sigma(V_i^T \chi_i) + \varepsilon_i \quad i = \overline{1:p} \quad (14)$$

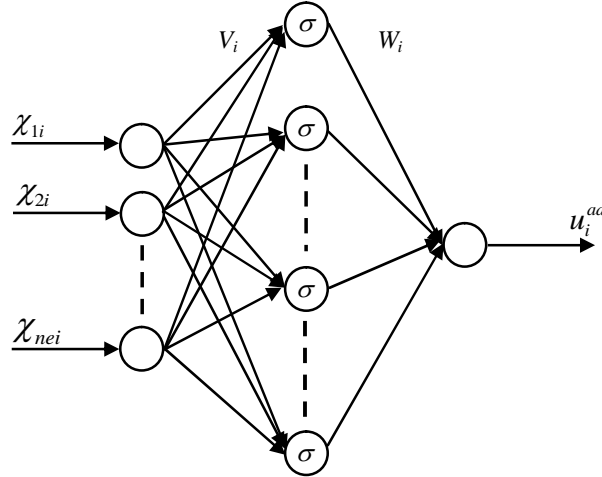


Figure 2: Network used in the control schemes.

where : ne_i input neurons number, nc_i hidden neurons number, $ns_i=1$ output neurons number, $\chi_i = [\dot{y}_{di} \quad \dot{y}_{di} \quad y_{di} \quad e_i \quad \dot{e}_i]^T$ input vector and σ is sigmoid activation function, $W_i \in \mathfrak{R}^{nc_i \times ns_i}$ and $V_i \in \mathfrak{R}^{ne_i \times nc_i}$ are the ideal weight and ε_i is the reconstruction error.

Since the optimal weights and the input vector are unknown, it is necessary to estimate them by an adaptation mechanism so that the output feedback control law can be realized. \hat{W}_i and \hat{V}_i are the estimate of W_i and V_i . Thus, the adaptive control approximating the ideal SHLNN output defined in (14) is given by :

$$u_i^{eq} = \hat{W}_i^T \sigma(\hat{V}_i^T \hat{\chi}_i) \quad i = \overline{1:p} \quad (15)$$

with : $\hat{\chi}_i = [\hat{y}_{di} \quad \dot{\hat{y}}_{di} \quad y_{di} \quad \hat{e}_i \quad \dot{\hat{e}}_i]^T$ is χ_i estimated.

Assumption.3: $\|W_i\|_F \leq W_{i,m}$, $\|V_i\|_F \leq V_{i,m}$. with : $W_{i,m}$ et $V_{i,m}$ are unknown positive constants.

The identification error of the equivalent control is :

$$\begin{aligned} \tilde{u}_i^{eq}(t) &= u_i^{eq*}(t) - u_i^{eq}(t) \\ &= \tilde{W}_i^T \sigma(\hat{V}_i^T \hat{\chi}_i) + \hat{W}_i^T \sigma'(\hat{V}_i^T \hat{\chi}_i) \tilde{V}_i^T \hat{\chi}_i + w_i \end{aligned} \quad (16)$$

Where, w_i presents the estimation errors:

$$w_i = \left(\begin{aligned} &W_i^T \{ \sigma(V_i^T \chi) - \sigma(\hat{V}_i^T \hat{\chi}_i) \} + W_i^T O(\tilde{V}_i^T \hat{\chi}_i)^2 \\ &+ \tilde{W}_i^T \sigma'(\hat{V}_i^T \hat{\chi}_i) \tilde{V}_i^T \hat{\chi}_i + \varepsilon_i \end{aligned} \right) \quad (17)$$

Where : $\tilde{W}_i = W_i - \hat{W}_i$ and $\tilde{V}_i = V_i - \hat{V}_i$ are parameter estimation errors.

Assumption.4 : $\|w_i\| \leq \bar{w}_i$. with : \bar{w}_i is an unknown positive constant.

To achieve the goal of controlling the weights adaptation laws are defined by:

$$\begin{cases} \dot{\hat{W}}_i = F_{W_i} \left(\sigma(\hat{V}_i^T \hat{\chi}_i) \hat{s}_i - \kappa_i \hat{W}_i \right) \\ \dot{\hat{V}}_i = F_{V_i} \left(\hat{\chi}_i \hat{s}_i \hat{W}_i^T \sigma'(\hat{V}_i^T \hat{\chi}_i) - \kappa_i \hat{V}_i \right) \end{cases} \quad (18)$$

with : $F_{W_i} > 0$, $F_{V_i} > 0$ are the adaptive gains and κ_i is a positive constant. The estimated of \bar{w}_i is given by:

$$\dot{\hat{w}}_i = \gamma_i |\hat{s}_i|, \gamma_i > 0 \quad (19)$$

4. 2. High gain Observer Design

The first time derivative of E_i is giving by :

$$\begin{cases} \dot{E}_i = (A_i - B_i \Phi_{0i}) E_i + B_i \dot{s}_i \\ e_i = C_i E_i \end{cases} \quad (20)$$

With:

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C_i^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

For this equation system; the conventional high gain observer used has the following form:

$$\begin{cases} \dot{\hat{E}}_i = (A_i - B_i \Phi_{0i}) \hat{E}_i + \delta_i K_i \tilde{e}_i \\ \hat{e}_i = C_i \hat{E}_i \end{cases} \quad (21)$$

With:

$$K_i = [\mathcal{g}_{i,1} \quad \mathcal{g}_{i,2} \delta_i \quad \dots \quad \mathcal{g}_{i,r_i-1} \delta_i^{r_i-1}]^T \quad (22)$$

And : $\delta_i \gg 1$ and $\mathcal{g}_{i,1}, \mathcal{g}_{i,2}, \dots, \mathcal{g}_{i,r_i-1} > 0$ are design parameter. The notation \hat{E}_i is the estimate of E_i , and $\tilde{e}_i = e_i - \hat{e}_i$.

As we already noted in first section this observer is characterized by its sensibility to noise measurement. For that raison a new adaptive observer is proposed in this paper.

5. PROPOSED OBSERVER DESIGN

The proposed adaptive observer is given by the following equation:

$$\begin{cases} \dot{\hat{E}}_i = (A_i - B_i \Phi_{0i}) \hat{E}_i + \mu_i(t) K_i \tilde{e}_i - \beta_i \mu_i(t) K_i e_i \\ \hat{e}_i = C_i \hat{E}_i \end{cases} \quad (23)$$

Where:

$$\dot{\mu}_i = \varsigma_{i,1} (\tilde{e}_i | - \bar{e}_i) \mu_i + \varsigma_{i,2} (\mu_{i,f} - \mu_i) \quad (24)$$

With: $\varsigma_{1,i}, \varsigma_{2,i}, \bar{e}_i > 0$, $\mu_i(0) = \delta_i$, $1 < \mu_{i,f} < \delta_i$, and, $\beta_i \in [0;1[$.

In this case the filtered error dynamic can express by the following equation:

$$\dot{s}_i = -g_i (k_i s_i - k_i \tilde{s}_i - w_{i,1}) \quad (25)$$

Where :

$$w_{i,1} = \begin{pmatrix} \tilde{W}_i^T \sigma(\hat{V}_i^T \hat{\chi}_i) + \hat{W}_i^T \sigma'(\hat{V}_i^T \hat{\chi}_i) \tilde{V}_i^T \hat{\chi}_i \\ + w_i - u_i^r \end{pmatrix} \quad (26)$$

Assumption.5 : $\|w_{i,1}\| \leq c_{i,1} \|\tilde{W}_i\| + c_{i,2} \|\tilde{V}_i\| + c_{i,3}$. With $c_{i,1}, c_{i,2}, c_{i,3}$ are unknown positive constants.

Assumption.6 : $|e_i| \leq \gamma_{i,1} |s_i| + \gamma_{i,2}$, with $\gamma_{i,1}, \gamma_{i,2} > 0$.

The observer error dynamic is given by :

$$\dot{\tilde{E}}_i = \mu_i(t) \bar{A}_i \tilde{E}_i - B_i \Phi_{0i} \tilde{E}_i + B_i \dot{s}_i + \mu_i(t) K_i (\beta_i - 1) e_i \quad (27)$$

Where :

$$\tilde{E} = E - \hat{E}, \quad \bar{A}_i = \begin{bmatrix} -g_{i,1} & 1 & 0 & \dots & 0 \\ -g_{i,2} \mu_i(t) & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -g_{i,r_i-2} \mu_i^{r_i-3}(t) & 0 & 0 & \dots & 1 \\ -g_{i,r_i-1} \mu_i^{r_i-2}(t) & 0 & 0 & \dots & 0 \end{bmatrix}$$

The matrix \bar{A}_i is stable, then it exists $P_i = P_i^T > 0$ and $Q_i = Q_i^T > 0$, as : [14]

$$P_i A_i + A_i^T P_i = -Q_i \quad i = \overline{1:p} \quad (28)$$

5. 1. Proof

Consider the following Lyapunov function candidate:

$$\begin{cases} L = L_1 + L_2 \\ L_1 = \frac{1}{2} \sum_{i=1}^p (\tilde{E}_i^T P_i \tilde{E}_i) \\ L_2 = \frac{1}{2} \sum_{i=1}^p \left(g_i^{-1} s_i^2 + \text{tr}(\tilde{W}_i^T F_{\tilde{W}_i}^{-1} \tilde{W}_i) + \text{tr}(\tilde{V}_i^T F_{\tilde{V}_i}^{-1} \tilde{V}_i) + \frac{1}{\gamma_i} \tilde{w}_i^2 \right) \end{cases} \quad (29)$$

After introducing the following equations (13), (18), (19), (25) and (26) and by using the 5th and the 6th assumption the first time derivative of Lyapunov function can express by:

$$\dot{L} \leq \sum_{i=1}^p \left(- \left(\frac{\mu_i(t)}{2} \lambda_{\min}(Q_i) - c_{i,9} \right) \|\tilde{E}_i\|^2 - k_i (1 - c_{i,5}) |s_i|^2 - \left(\frac{\kappa_i}{2} - c_{i,6} \right) \|\tilde{W}_i\|^2 - \left(\frac{\kappa_i}{2} - c_{i,7} \right) \|\tilde{V}_i\|^2 + c_{i,10} \right) \quad (30)$$

Where :

$$\begin{aligned} c_{i,4} &= \|P_i B_i \Phi_{0i}\| + k_i \|P_i B_i g_i \Phi_i\|, \quad c_{i,5} = \frac{1}{2} \left(\|\Phi_i\| + \frac{k_i \|P_i B_i g_i\| + \beta_i \gamma_{i,1} \|P_i \mu_i(t) K_i\|}{k_i} \right), \\ c_{i,6} &= \frac{c_{i,1} (\|\Phi_i\| + \|P_i B_i g_i\|)}{2}, \quad c_{i,7} = \frac{c_{i,2} (\|\Phi_i\| + \|P_i B_i g_i\|)}{2}, \quad c_{i,8} = \frac{(c_{i,3} (\|\Phi_i\| + \|P_i B_i g_i\|) + \beta_i \gamma_{i,2} \|P_i \mu_i(t) K_i\|)}{2}, \\ c_{i,9} &= c_{i,4} + k_i c_{i,5} + c_{i,6} + c_{i,7} + 1, \quad c_{i,10} = c_{i,8}^2 + \frac{\kappa_i}{2} (\|\hat{s}_i\| (\|\tilde{W}_i\|^2 + \|\tilde{V}_i\|^2)). \end{aligned}$$

Let's suppose that: $c_{i,5} < 1$, $\mu_i(t) > c_{i,9} / \min(Q_i)$ and $i > 2 \text{Max}(c_{i,6}, c_{i,7})$. So the Lyapunov function is decreasing if s_i , \tilde{E}_i , \tilde{W}_i and \tilde{V}_i are outsiders of the compact sets Ω_{s_i} , $\Omega_{\tilde{E}_i}$, $\Omega_{\tilde{W}_i}$ and $\Omega_{\tilde{V}_i}$ respectively, where:

$$\begin{cases} \Omega_{s_i} = \left\{ |s_i| \leq \sqrt{\frac{c_{i,10}}{k_i (1 - c_{i,5})}} \right\}, \\ \Omega_{\tilde{E}_i} = \left\{ \|\tilde{E}_i\| \leq \sqrt{\frac{2c_{i,10}}{\mu_i(t) \lambda_{\min}(Q_i) - 2c_{i,9}}} \right\}, \\ \Omega_{\tilde{W}_i} = \left\{ \|\tilde{W}_i\| \leq \sqrt{\frac{2c_{i,10}}{\kappa_i - 2c_{i,6}}} \right\}, \\ \Omega_{\tilde{V}_i} = \left\{ \|\tilde{V}_i\| \leq \sqrt{\frac{2c_{i,10}}{\kappa_i - 2c_{i,7}}} \right\} \end{cases} \quad (31)$$

6. SIMULATION RESULTS

The simulation experiments are performed to compare the ordinary high gain observe and the proposed observer.

The simulation parameters are given in Tables I and II. The 3rd Figure show simulation results obtained by using the proposed and the conventional approach at the presence of noise

measurement and with 50% parametric variation from time $t = 10$ sec.

Table 1: Quadrotor Parameters.

<i>Symbol</i>	<i>Value</i>	<i>Symbol</i>	<i>Value</i>
m	0.486 (Kg)	I_z	7.65e-3 (KgM ²)
l	0.25 (m)	g_{ft}	9.81 (Ms ²)
k_d	3.23e-7 (NMs ²)	$k_{f_{ix}}, k_{f_{iy}}$	5.56e-4 (Ns/M)
k_p	2.98e-5 (Ns ²)	$k_{f_{tz}}$	6.35e-4 (Ns/M)
J_r	2.83e-5 (KgM ²)	k_{f_r}, k_{f_r}	5.56e-4 (Ns/r)
I_x	3.82e-3 (KgM ²)	k_{f_r}	6.35e-4 (Ns/r)
I_y	3.82e-3 (KgM ²)		

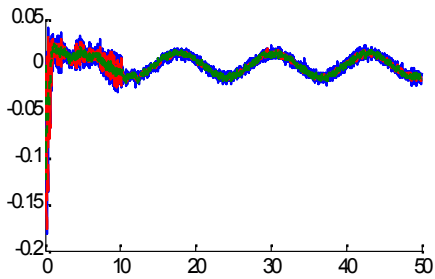
Table 2: Controller and Observer Parameters.

<i>Controller</i>		<i>Proposed Observer</i>	
<i>Symbol</i>	<i>Value</i>	<i>Symbol</i>	<i>Value</i>
	$diag(10, 10, 10)$	$\mu_{x,y,z,f}$	2
	$diag(1, 1, 10)$	$\mu_{, , f}$	2
K	$diag(1.5, 1.5, 20)$	$x,y,z,1$	10
K	$diag(0.2, 0.2, 0.2)$	$, , ,1$	5
x, y	0.01	$x,y,2$	0.1
z	0.01	$, ,z,2$	0.2
$,$	0.01	$,2$	0.3
$,$	0.01	$x,y, ,$	20
$F_{W_{x,y,z}}$	$3 I_{3,3}$	z	30
$F_{W, , ,}$	$I_{3,3}$		40
$F_{V_{x,y,z}}$	$5 I_{5,5}$	$x,y,z,1$	10
$F_{V, , ,}$	$I_{5,5}$	$x,y,z,2$	40
x, y	0.1	$, , ,1$	5
z	0.1	$, ,2$	20
$,$	0.1	$,2$	30
$,$	0.1	x,y,z	0.5
		$, ,$	0.8
		$\bar{\epsilon}_{x,y,z,\phi,\theta}$	0.01
		$\bar{\epsilon}_{\psi}$	0.05

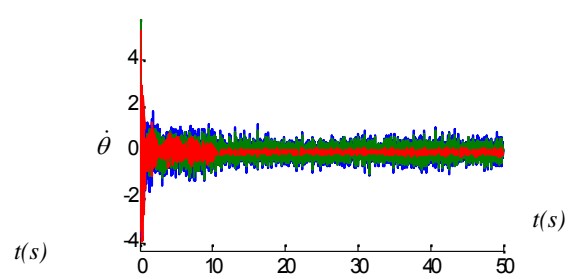
The conventional high gain observer produces large measurement noise amplification has created degradations in system's pursuit and by consequence an important control signal as is presented as in Figure.3. a which can cause instability if the measurement noise is so important.

Unlike the results achieved by the proposed observer where a significant minimization of unfavorable effect was obtained for the system output and control signals as is shown in the Figure. 3. b where control signal amplitude is ten times less than the one gotten by using ordinary observer.

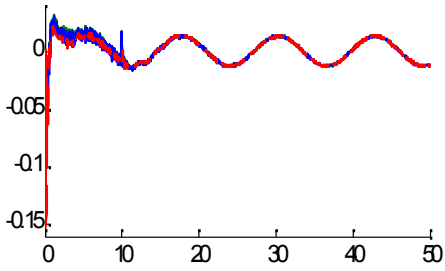
— desired output ■■■■■ Real output — estimated output — Signal control



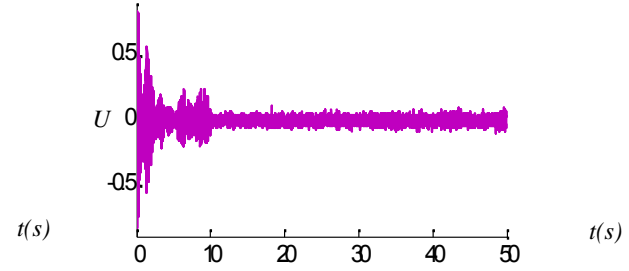
(3. a. 1)



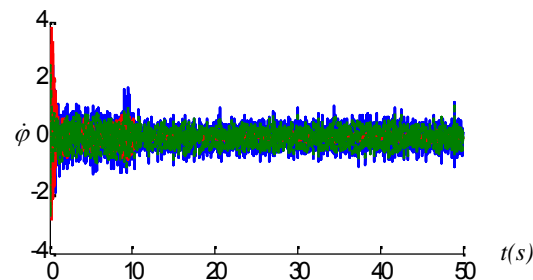
(3. a. 5)



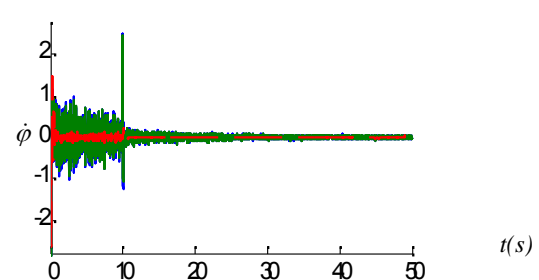
(3. b. 1)



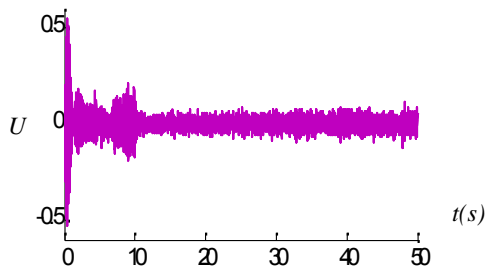
(3. a. 6)



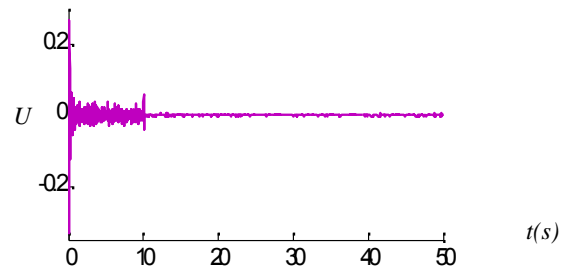
(3. a. 2)



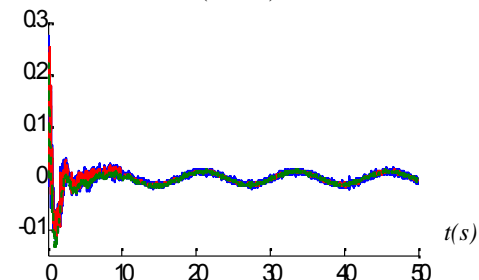
(3. b. 2)



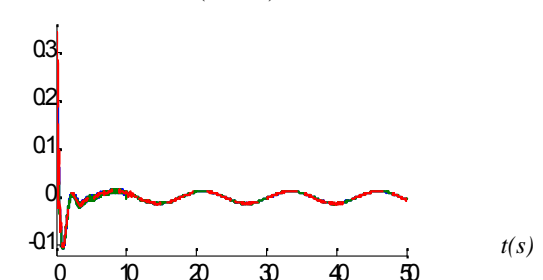
(3. a. 3)



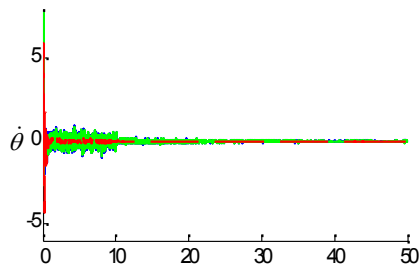
(3. b. 3)



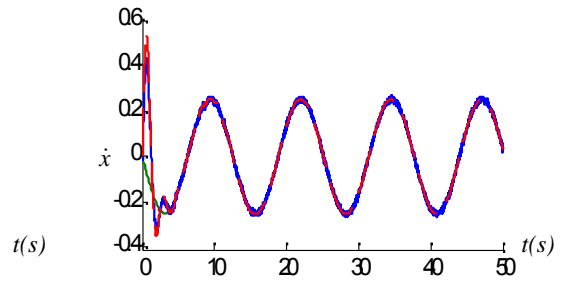
(3. a. 4)



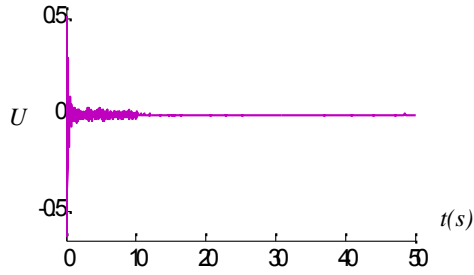
(3. b. 4)



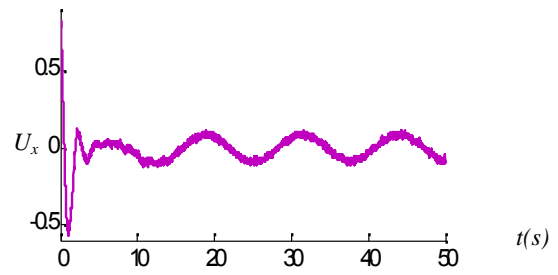
(3. b. 5)



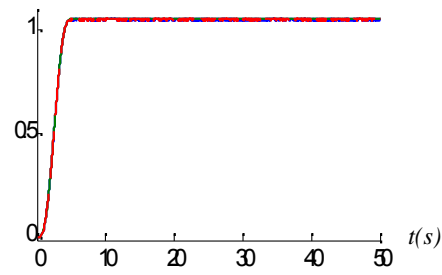
(3. a. 10)



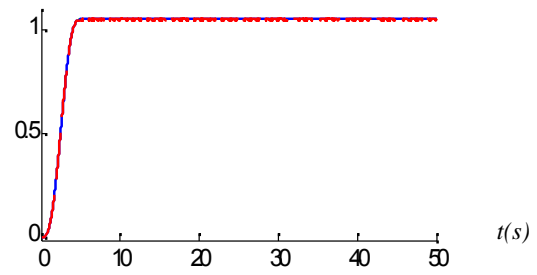
(3. b. 6)



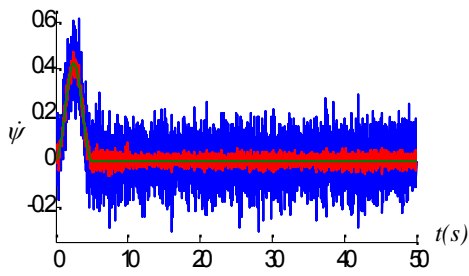
(3. a. 11)



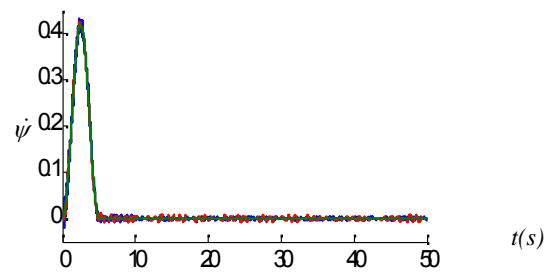
(3. a. 7)



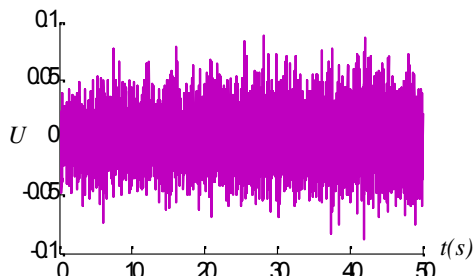
(3. b. 7)



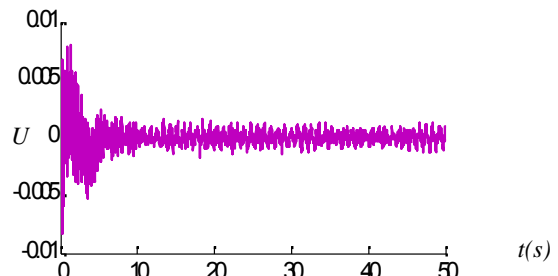
(3. a. 8)



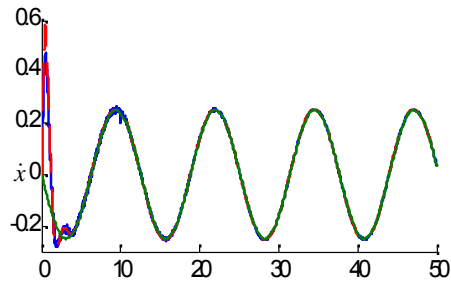
(3. b. 8)



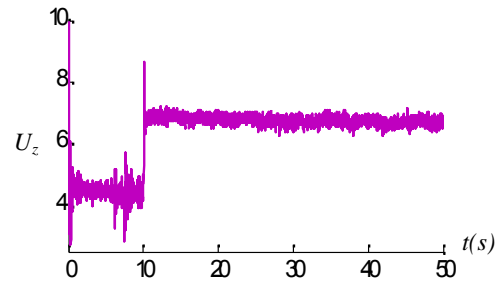
(3. a. 9)



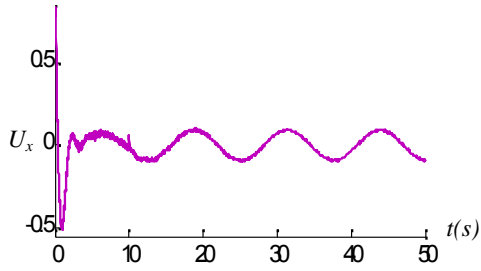
(3. b. 9)



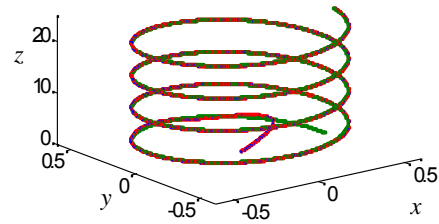
(3. b. 10)



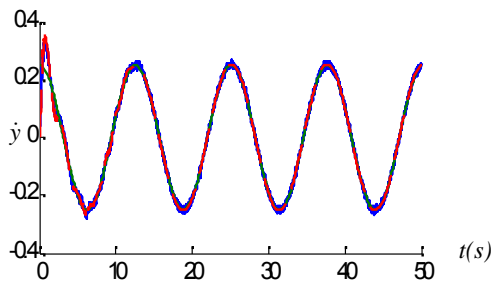
(3. a. 15)



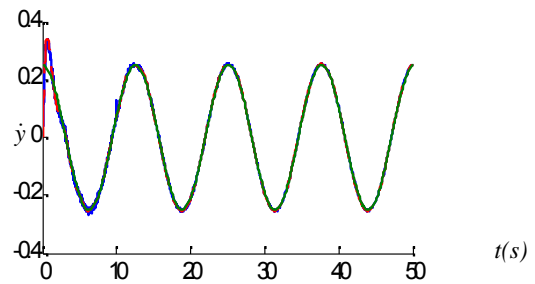
(3. b. 11)



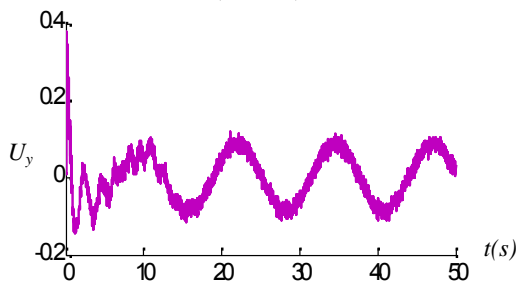
(3. a. 16)



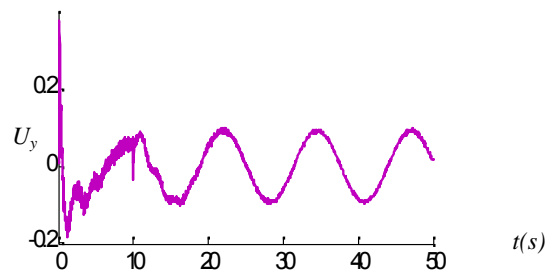
(3. a. 12)



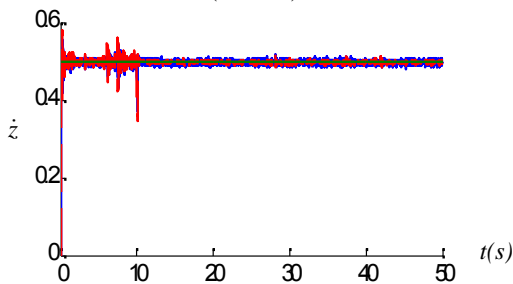
(3. b. 12)



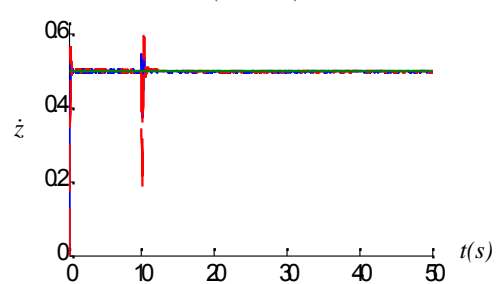
(3. a. 13)



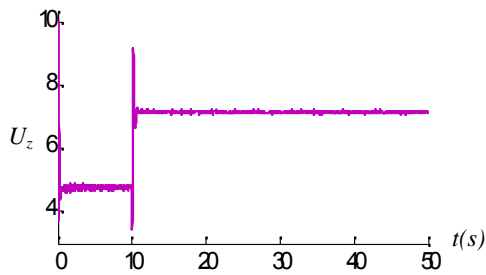
(3. b. 13)



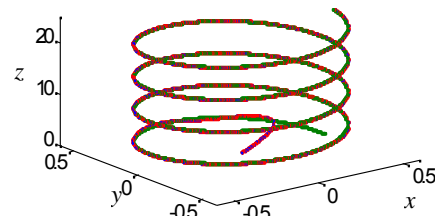
(3. a. 14)



(3. b. 14)



(3. b. 15)



(3. b. 16)

Figure. 3: Simulation results

7. CONCLUSION

A new adaptive observer was developed for an under actuated Quadrotor UAV. The control scheme is based on the use of SHL neural networks for each Quadrotor subsystem in order to estimate on line the unknown nonlinear dynamical modal in Quadrotor helicopter. The proposed observer is based on conventional high gain observer structure where new parameters are added to the last one in order to reduce the amplification of measurement noise generated by ordinary observer. The proposed observer have proved its capacity where a very important minimization in the amplitude and the variance of the noisy signal is gotten without having degradations in the performances as it was shown by the numeric simulation results. This method does not require any prior knowledge about dynamic model and states variables. The uniformly ultimately boundedness of the tracking error and all signals in the overall closed-loop system is proved using Lyapunov's direct method. The effectiveness of the proposed approach was validated by simulation of a Quadrotor control at noise measurement presence.

REFERENCES

- [1] P. Adigbli : Nonlinear Attitude and Position Control of a Micro Quadrotor using Sliding Mode and Backstepping Techniques. 3rd US-European Competition, MAV07 & EMAV07, 17-21, France, 2007
- [2] S.Bouabdallah and al : Backstepping and sliding mode Techniques Applied to an Indoor Micro Quadrotor. In Proc 2005 IEEE ICRA, 2005
- [3] T. Madani and A.Benallegue: Backstepping Control with Exact 2-Sliding Mode Estimation for a Quadrotor Unmanned Aerial Vehicle. Proc of IEEE/RSJ International CIRS, 2007.
- [4] A.Mokhtari, A.Benallegue and B.Daachi: Robust Feedback Linearization and GH^∞ Controller for a Quadrotor Unmanned aerial vehicle. Journal of Electrical Engineering, vol.57, No.1, pp 20.27, 2006.
- [5] S.Bouaddallah and al : PID vs. LQ control Techniques Applied to an Indoor Micro Quadrotor. In Proc IEEE ICIRS, Japan, 2004.
- [6] A.Das, F.Lewis and K.Subbarao : Backstepping Approach for Controlling a Quadrotor Using Lagrange Form Dynamics. J Intell Robot syst, pp 127-151, 2009.
- [7] C. Nicol, & al: Robust Neural Network of a Quadrotor Helicopter, in proc IEEE Fuzzy information PSC, pp. 454-458, 2008.
- [8] J.C.Raimundez & al : Adaptive Tracking control for a Quad-rotor. ENOC-2008, saint Petersburg, Russia July 2008.
- [9] O.Bouhali and H.Boudjedir : Neural Network control with neuro-sliding mode observer applied to Quadrotor Helicopter. In Proc INISTA'11, pp 24-28, Istanbul, Turkey 2011.
- [10] T.Dieks and S.Jagannathan: Neural Networks Out Feedback of a Quadrotor UAV. Proc IEEE, 47th CDC, pp 3633-3639, Mexi, Dec 2008.
- [11] H.Khalil : High-Gain Observers in Nonlinear Feedback Control. Proc IEEE, IC on Control, Automation and Systems, 2008

- [12] M.Farza and al : Dynamic High Gain Observer Design. Proc IEEE, 6th International Multi-Conference on systems, signals and Device, 2009.
- [13] E.Bullinger and F.Allgower : An Adaptive High-Gain Observer for Nonlinear Systems. Proc IEEE, the 36th CDC USA, 1997.
- [14] J.J.Slotine : Applied nonlinear control. Prentice Hall, 1991.
- [15] S.Jagannathan : Neural networks control of nonlinear Discrete-time systems. CRC press, Taylor & Francic Group, 2006.