Heterogeneous Effects

Linear Panel Event Studies

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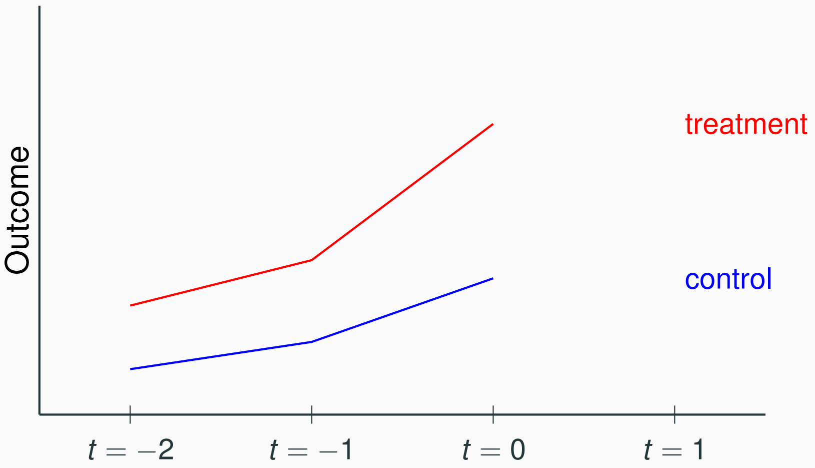
Motivation

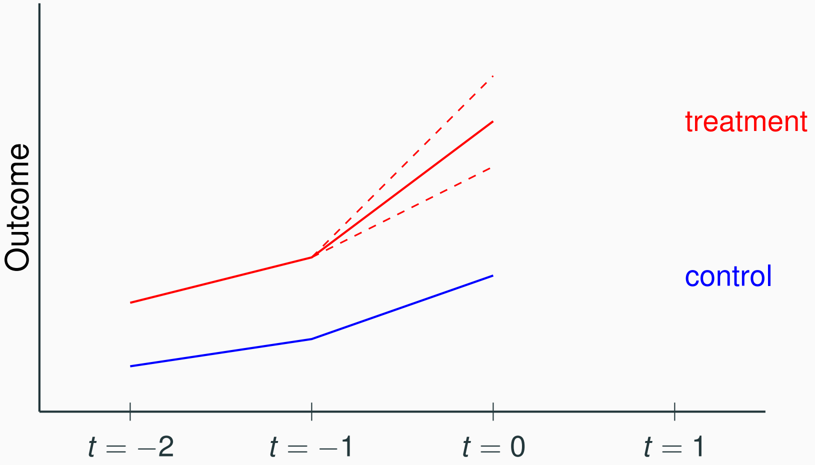
- Suppose that sufficient identifying assumptions hold
  - No anticipation
  - Parallel trends
- But policy of interest can affect different units differently
  - e.g., minimum wage has larger effects on employment at less productive firms
- Discuss implications for identification of average effects
- Show pitfalls with common estimators
- And discuss alternatives studied in the literature

Single Event

Single Event, Homogeneous Effects



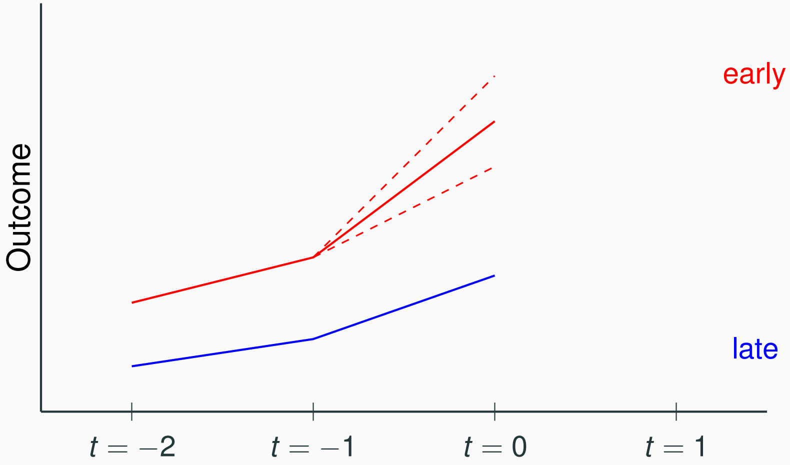
Single Event, Heterogeneous Effects



- Can recover an average treatment effect under no anticipation and parallel trends.

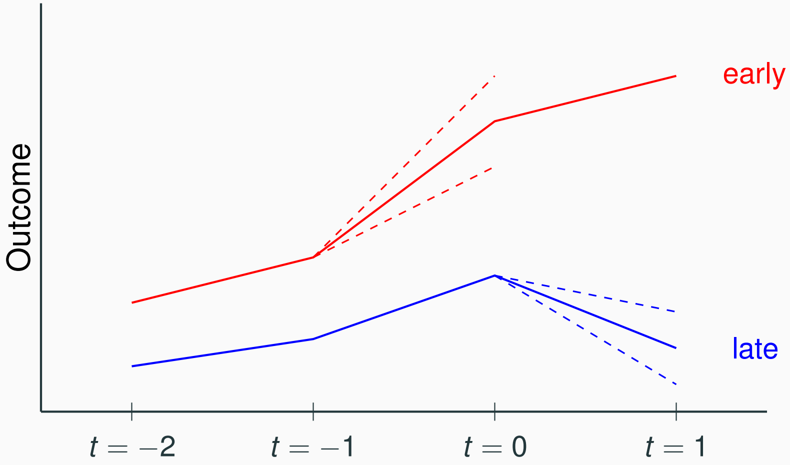
Heterogeneity Under Staggered Adoption

Staggered Adoption, Heterogeneous Static Effects



- Can still recover an average treatment effect under no anticipation and parallel trends.

Staggered Adoption, Heterogeneous Static Effects



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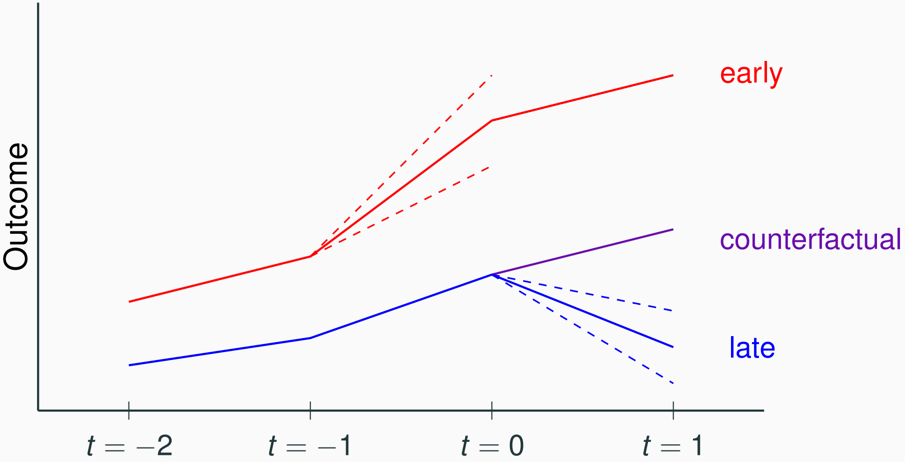
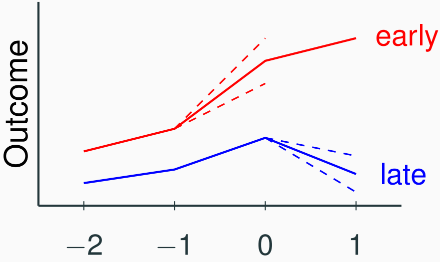
Staggered Adoption, Heterogeneous Static Effects



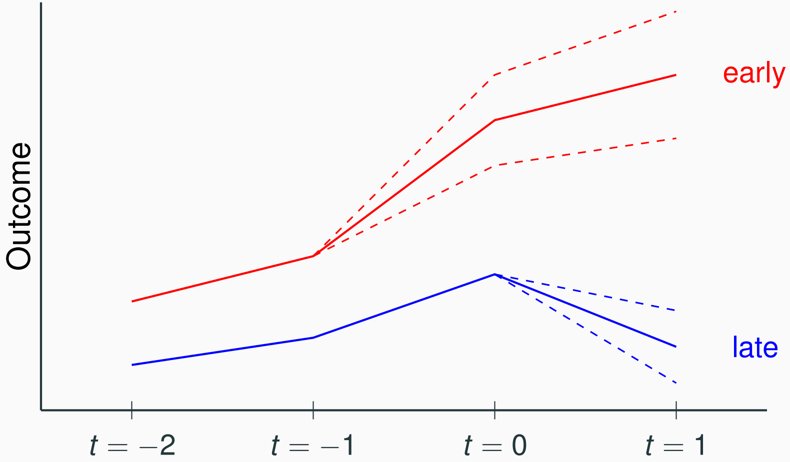
- Can still recover an average treatment effect under no anticipation and parallel trends.

Staggered Adoption, Heterogeneous Static Effects



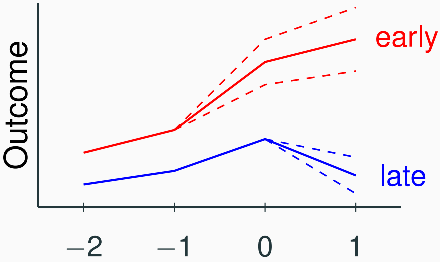
- For early adopters, late adopters are a valid control for the effect in the first period after adoption.
  - If trends diverge, it is because of effect of adoption on early adopters.
- For late adopters, early adopters are a valid control for the effect in the first period after adoption.
  - If trends diverge, it is because of effect of adoption on late adopters.

Staggered Adoption, Heterogeneous Dynamic Effects



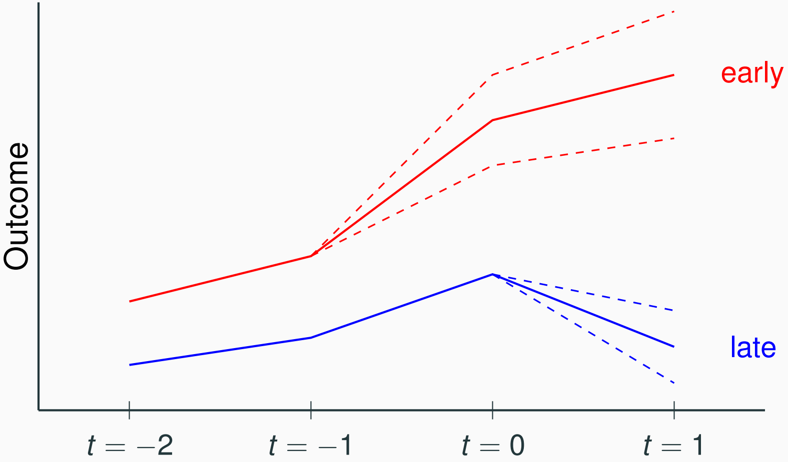
- Cannot recover an average dynamic treatment effect, even under no anticipation and parallel trends.

Staggered Adoption, Heterogeneous Dynamic Effects



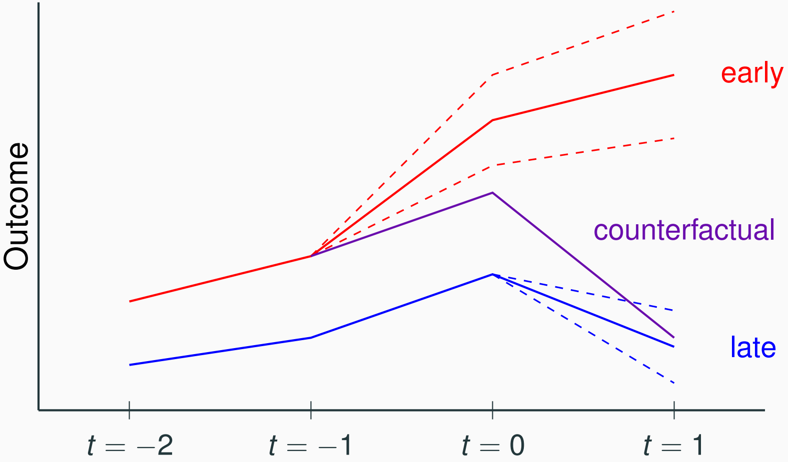
- For early adopters, late adopters are a valid control for the effect in the first period after adoption.
  - If trends diverge, it is because of effect of adoption on early adopters.
- For late adopters, early adopters are not a valid control for the effect in the first period after adoption.
  - If trends diverge, it could be...
    - ...because of static effect of adoption on late adopters, or...
    - ...because of dynamic effect of adoption on early adopters.

Staggered Adoption, Heterogeneous Dynamic Effects



- Notice that we'd be fine if we knew that the effect on late adopters is the same as the effect on early adopters....

Staggered Adoption, Semi-Homogeneous Dynamic Effects



- ...because then we could impute a counterfactual path for early adopters in the second period.

Lessons

- If we're not prepared to restrict
  - dynamics of treatment effects
  - heterogeneity of treatment effects
- Then for each average effect we want to recover we will need a control group that is
  - unaffected (or not yet affected) by treatment
  - measured simultaneously with the treated group
- *Approaches we consider require observing such a group*

Approaches Under Staggered Adoption

Reminder: Regression Representation

$$y_{it} = \alpha_i + \gamma_t + \sum_{-\infty}^{\infty} \delta_k \Delta z_{i,t-k} + \varepsilon_{it}$$

- Post-treatment indicator z_{it}
- Unit fixed effect α_i
- Time fixed effect γ_t
- Cumulative dynamic treatment effects $\{\delta_k\}_{-\infty}^{+\infty}$

Heterogeneous Dynamic Effects

$$y_{it} = \alpha_i + \gamma_t + \sum_{-\infty}^{\infty} \delta_{ik} \Delta z_{i,t-k} + \varepsilon_{it}$$

- Each unit i now has its own dynamic treatment effect $\{\delta_{ik}\}_{-\infty}^{+\infty}$
- Can't say much about $\{\delta_{ik}\}_{-\infty}^{+\infty}$ outside of special cases
- Staggered adoption is one case where we can
    - Recall that in staggered adoption, unit i adopts in period $g(i)$
    - Treatment timing relates to i only through $g(i)$

Staggered Adoption

$$y_{it} = \alpha_i + \gamma_t + \sum_{-\infty}^{\infty} \delta_{g(i)k} \Delta z_{i,t-k} + \varepsilon_{it}$$

- Relative to model with homogeneous effects, have added interactions with group $g(i)$
- With sufficient untreated / late-treated groups in sample, can estimate via interacted regression
- Can then aggregate the estimates $\{\delta_{gk}\}_{-\infty}^{+\infty}$, for example via a weighted average

Implementation

- Interaction regression
    - Stata: eventstudyinteract
    - R: fixest
- Using pre-treatment periods to estimate time effects
    - Stata: did_imputation
    - R: didimputation
- Averaging DiD estimators
    - Stata: did_multiplegt, csdid
    - R: DIDmultiplegt, did
- NB: List based on forthcoming survey articles by de Chaisemartin and D’Haultfoeuille (forthcoming) and Roth et al. (forthcoming).

Further Reading

de Chaisemartin, Clément and Xavier D'Haultfoeuille. 2020. Two-way fixed effects estimators with heterogeneous treatment effects. In *American Economic Review.*

Sun, Liyang and Sarah Abraham. 2021. Estimating dynamic treatment effects in event studies with heterogeneous treatment effects. In *Journal of Econometrics.*

Callaway, Brantly and Pedro H. C. Sant'Anna. 2021. Difference-in-differences with multiple time periods. In *Journal of Econometrics.*

Borusyak, Kirill, Xavier Jaravel, and Jan Spiess. 2023. Revisiting event study designs: Robust and efficient estimation. In *arxiv [econ].*

Pitfalls Under Staggered Adoption

What Can Go Wrong

- Suppose we estimate

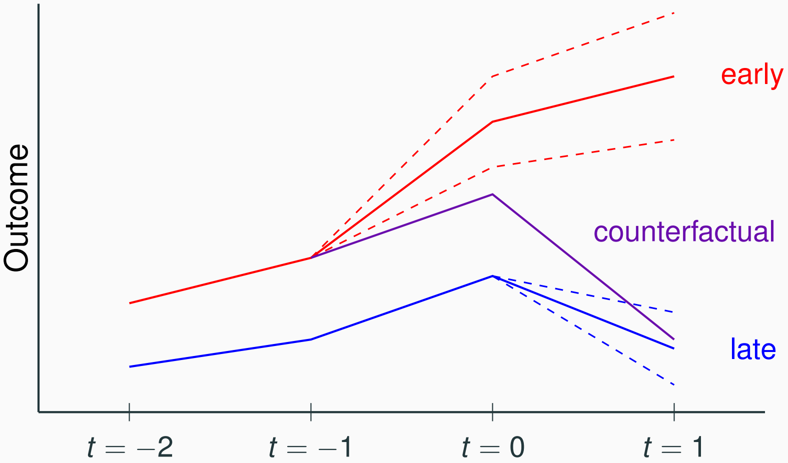
$$y_{it} = \alpha_i + \gamma_t + \sum_{-\infty}^{\infty} \delta_k \Delta z_{i,t-k} + \varepsilon_{it}$$

but the correct model is

$$y_{it} = \alpha_i + \gamma_t + \sum_{-\infty}^{\infty} \delta_{g(i)k} \Delta z_{i,t-k} + \varepsilon_{it}$$

- Recall our plot...

What Can Go Wrong



- Counterfactual path for late adopters is control group for second-period effect for early adopters
- Fine if assumptions correct; maybe not fine otherwise

What Can Go Wrong

- The path $\{\delta_k\}_{-\infty}^{+\infty}$ may not be even a weighted average of the paths $\left\{\delta_{gk}\right\}_{-\infty}^{+\infty}$
- Might estimate an effect larger or smaller than all true effects

Recommendations

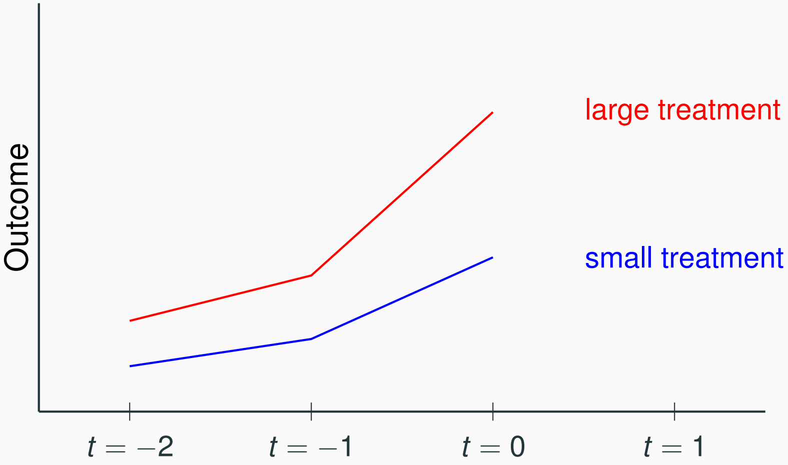
- Under staggered adoption
  - Restrict dynamics / heterogeneity of treatment effects, and/or
  - Use an estimator that leverages an untreated control

Heterogeneity Outside of Staggered Adoption

(Extra) Challenge

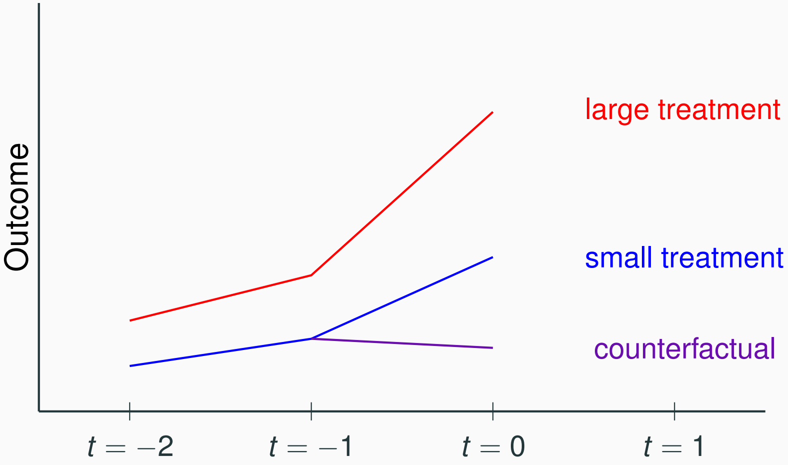
- Consider designs outside of staggered adoption
  - e.g., continuous treatment, multiple treatment
- Can be difficult to define a control group
  - Therefore difficult to estimate interesting objects without restricting treatment effects
- Example: Medicare (Finkelstein 2007)
  - Medicare increases insurance penetration in all states, some more than others

Single Event, Varying Treatment Intensity



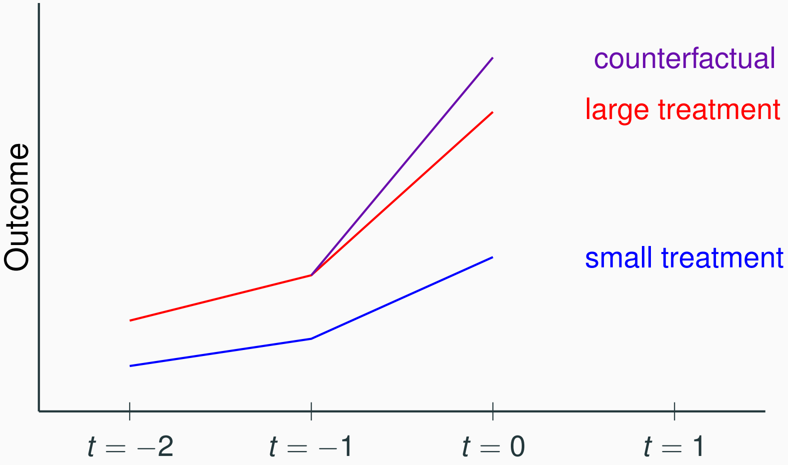
- Quiz: What is the treatment effect here?

Single Event, Varying Treatment Intensity



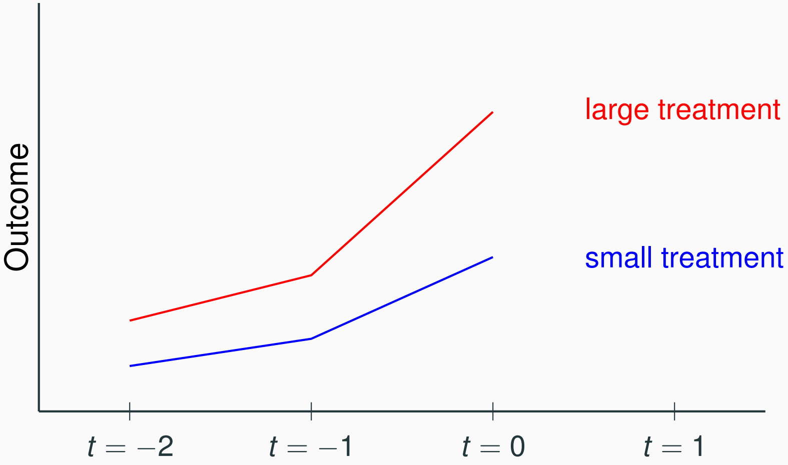
- Does treatment increase outcome, so more affected units increase faster than less affected?

Single Event, Varying Treatment Intensity



- Or does treatment decrease outcome, more so for less affected than for more affected units?

Single Event, Varying Treatment Intensity

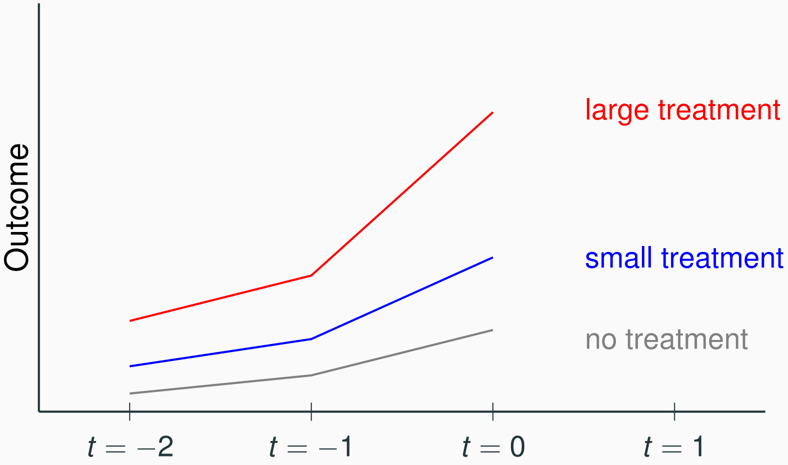


- Even under parallel trends, impossible to say anything about treatment effect without further restrictions.

Recommendations

- Outside staggered adoption
  - Restrict dynamics / heterogeneity / functional form of treatment effects, and/or
  - Use an estimator that leverages an untreated control

Single Event, Varying Treatment Intensity



Further Reading

de Chaisemartin, Clément and Xavier D'Haultfoeuille. 2017. Fuzzy difference-in-differences. In *The Review of Economic Studies.*

de Chaisemartin, Clément and Xavier D'Haultfoeuille. 2023. Difference-in-differences estimators of intertemporal treatment effects. In *SSRN.*

Sun, Liyang and Jesse M. Shapiro. 2022. A linear panel model with heterogeneous coefficients and variation in exposure. In *Journal of Economic Perspectives.*

Callaway, Brantly, Andrew Goodman-Bacon, and Pedro H. C. Sant'Anna. 2021. Difference-in-differences with a continuous treatment. In *arxiv [econ].*

Today

- ~~Overview (Jesse)~~
- ~~Basics of identification and estimation (Liyang)~~
- ~~Basics of plotting (Jesse)~~
- ~~Pitfalls and some solutions~~
  - ~~Confounds and pre-trend testing (Liyang)~~
  - ~~Heterogeneous effects (Jesse)~~
- Conclusions (Liyang)