Lecture 7. Kepler: Laws of Planetary Motion

Johannes Kepler (1571-1630)

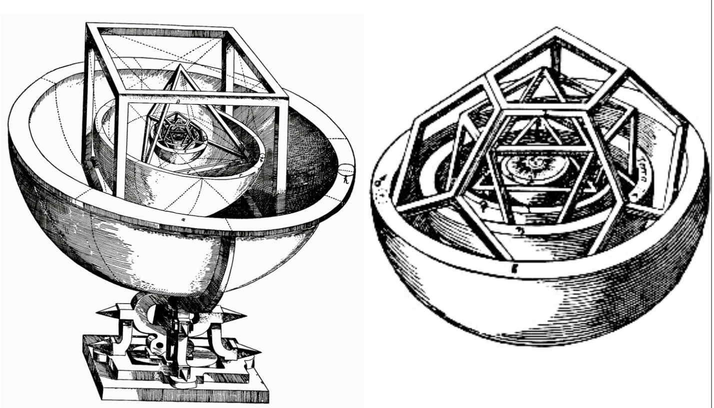
Kepler

- Johannes Kepler came from a poor Protestant family in Germany.
- He became aware of Copernicus' work at the University of Tübingen, where he completed a master's degree.
- He then began studying theology, but before finishing his degree he was offered a job teaching mathematics at a (Lutheran) school in Austria.
- Kepler was impressed with the Pythagorean idea that numbers underlie the mysteries of the cosmos.
- He was especially interested in why there are only 5 planets, and why they orbit at the distances they do from the Sun.

Mysterium Cosmographicum

- While teaching astrology, Kepler was struck with a great idea.
- There are 5 regular solids: the cube, the tetrahedron, the octahedron, the dodecahedron, and the icosahedron.
- If the orbit of each planet were circumscribed by one of these regular solids, and each of these was nested inside the next, he achieved a solar system with distances close to the distances calculated by Copernicus accurate to about 5%.
- This then accounted for the number of planets as well as their distances from the Sun.
- "I wanted to become a theologian, and for long I was uneasy, but now, see how through my efforts God is being celebrated in astronomy."

Kepler's solar system



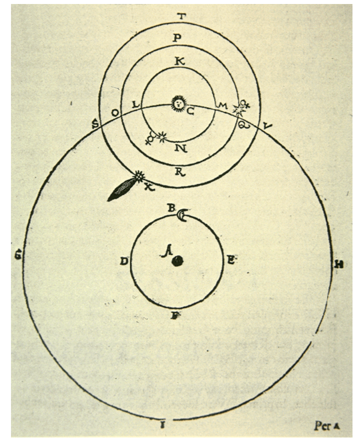
Kepler

- Published *Mysterium cosmographicum* in 1597.
- N.B. Kepler is doing something that marks the beginning of a more empirical approach to cosmology: he is testing his model against measurements.
- The book was read and added support to the Copernican movement.
- In 1598 Kepler was forced to leave Graz when he refused to convert to Catholicism.
- In 1600 he finally got a job with the great observational astronomer Tycho Brahe in Prague at the court of Rudolf II.
- His job was to use Tycho's data on Mars to fit with Tycho's cosmological model.

Tycho Brahe (1546-1601)

Tycho's cosmos

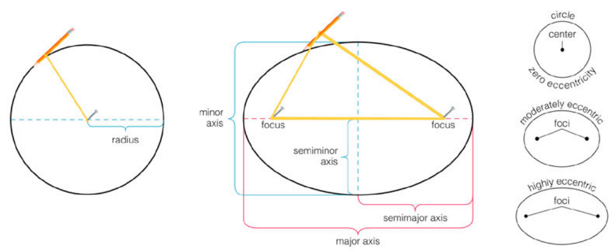
- Tycho had made the most accurate observations obtained at that time on the planets.
- Tycho's model of the heavens had the planets orbiting round the Sun and the Sun orbiting round the Earth at the center of the Universe.



Kepler's laws of planetary motion

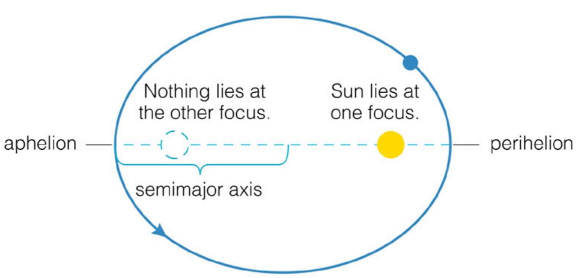
- Tycho died after two years and Kepler inherited his data and his title: Imperial Mathematician to the Emperor Rudolf II.
- Kepler dutifully attempted to reconcile the Mars data using models of Ptolemy, Copernicus, and Tycho.
- None were successful at representing Tycho's accurate data for Mars.
- After six years of work he gave up attempting to use circles for the planetary orbits.
- Kepler realized Mars moves in an ellipse around the Sun.
- In his *Astronomia Nova* (1609) he presented his first two laws of planetary motion.

Geometry of ellipses



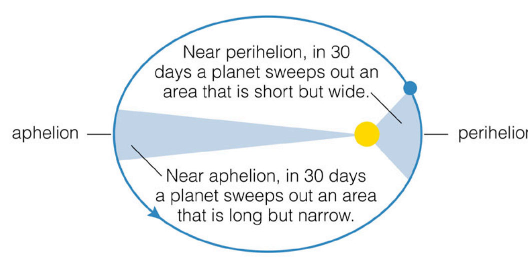
Kepler's First Law

First Law: Planets move in elliptical orbits with the Sun at one focus of the ellipse.



Kepler's Second Law

Second Law: A line from the Sun to the planet sweeps out equal areas in equal times, *i.e.* planets don't move at constant speed.



The areas swept out in 30-day periods are all equal.

Kepler's Third Law

- Kepler was a committed Pythagorean, and he searched for 10 more years to find a mathematical law to describe the motion of planets around the Sun.
- In *Harmony of the World* (1619) he enunciated his Third Law:
- (Period of orbit)^2 proportional to (semi-major axis of orbit)^3.
- In symbolic form: $P^2 \propto a^3$.
- If two quantities are proportional, we can insert a proportionality constant, k, which depends on the units adopted for P and a, and get an equation:
- $P^2 = ka^3$.

Using Kepler's 3rd Law

Kepler's third law: $P^2 \propto a^3$

Therefore $P^2 = ka^3$, where k is some constant number.

We can find k if P is expressed in (Earth's orbit) years and a is expressed in terms of the distance between Sun & Earth. This distance is called 1 Astronomical Unit, or 1 A.U.

Then substituting above values into $P^2 = ka^3$ we find:

$(1)^2 = k(1)^3$.

Solving for k, we find $k = 1$.

When these units for P and a are used:

$P^2 = a^3$ with P in years and a in A.U.

Example

An asteroid is found and its orbital semi-major axis around the Sun is measured to be 4 A.U. What is the period of its orbit round the Sun?

$P^2 = a^3$ with P in years and a in A.U.

Since $a = 4$ *A.U,*

$P^2 = a^3 = (4)^3 = 64$.

So $P = 8$ *years*.

Celestial dynamics?

- But what makes the planets move in this way?
- Kepler has now lost Aristotle's theory of motion, and there was nothing to replace it.
- He realized that planets move more slowly the further they were away from the Sun.
- Does this mean it is the Sun that is the source of their motion?
- Kepler guessed that the planets are moved in some way by the Sun, and the further away a planet is, the slower it orbits.
- He thought it might be some kind of magnetic force.

Kepler's intellectual legacy

- The three laws of planetary motion are an intellectual high water mark.
- The Sun was now firmly at the center of the Solar system.
- For the first time, Kepler introduced celestial motion that was not circular. (The Greek idea that motion in the heavens must be circular had lasted over 2,000 years.)
- Kepler was aware of the fact that his model was lacking in dynamics. He had discarded Aristotle's model, and with it his theory of motion, but there was now no theory to say what made the planets move in these ellipses.
- Kepler suggested there was some magnetic force emanating from the Sun.
- This became a major intellectual problem in the 17th C.