Foundations of the international system of units (SI)

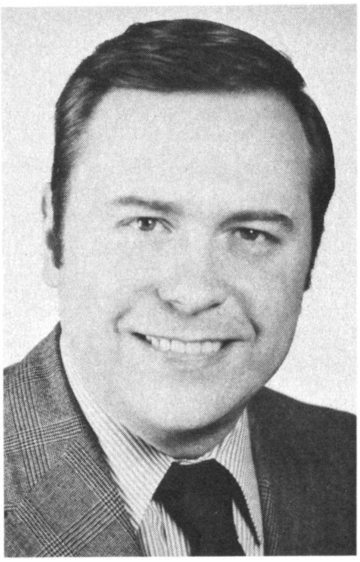
ROBERT A. NELSON

Lord Kelvin once said, "When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of *science*, whatever the matter may be."[1] If the touchstone for science is the measurement of physical quantities by numbers, then the touchstone for measurement itself must be the existence of a system of units.

A measurement is the result of a specified series of operations and calculations. Every measurement consists of two parts: a number and a unit. A *unit* specifies the character of a physical quantity and represents the measure of the reference chosen for comparison. It is an intrinsically abstract idealization. In contrast, a *standard* is an artifact or reproducible phenomenon which is the physical embodiment of the unit. The metric system of units, originally developed in France during the last decade of the eighteenth century, has been adopted by scientists because of its logic, precision, and simplicity.

The modern metric system is known as the International System of Units, *Le Système International d'Unités*, abbreviated SI in all languages. It consists of seven base units, two supplementary units, and derived units as shown in Table I. SI is administered by the International Bureau of Weights and Measures (BIPM), whose headquarters is located in Sèvres, France, across the Seine River from Paris. The fundamental scientific decisions are made by the International Committee for Weights and Measures (CIPM). It is assisted by the advice of eight Consultative Committees specializing in particular areas of metrology.* The decisions of the CIPM are submitted to the General Conference on Weights and Measures (CGPM) for ratification. The CGPM now meets every four years.

The research of the BIPM is reported in the journal *Metrologia*. Unit definitions and symbols and important decisions of the CGPM are summarized in *The International System of Units (SI)* [NBS Spec. Publ. 330 (U.S. Govt. Printing Office, Washington, 1977)], a translation of the official BIPM document. Recommended metric practice in the United States is outlined in *American National Standard for Metric Practice* [Z210.1-1976 (American National Standards Institute, New York, 1976)].† I have prepared a manual published by AAPT,** *SI: The International System of Units*, which is intended as a source for definitions, symbols, conventions, tables, and references for SI units. In this article I will trace the events that led to the creation of the BIPM, discuss how the units have been represented by their standards, and investigate how the original metric system evolved into SI.



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*These are electricity, photometry and radiometry, thermometry, the meter, the second, radioactivity, units, and mass.

†Also available from the American Society for Testing and Materials, Philadelphia [ASTM E 380-79], and the Institute of Electrical and Electronics Engineers, New York [IEEE Std 268-1976], the organizations which jointly prepared the document.

**AAPT Publications Dept., Graduate Physics Building, SUNY, Stony Brook, NY 11794 ($3.00 postpaid; $3.50 outside U.S.A.).

Table I.
Structure of SI units

Quantity	Name of unit	Symbol	Expression in terms of other SI units
Base Units			
length	meter	m	
mass	kilogram	kg	
time	second	s	
electric current	ampere	A	
temperature	kelvin	K	
luminous intensity	candela	cd	
amount of substance	mole	mol	
Supplementary Units			
plane angle	radian	rad	
solid angle	steradian	sr	
Examples of Derived Units			
velocity			m/s
acceleration			m/s^2
force	newton	N	kg•m/s^2
pressure	pascal	Pa	N/m^2
work, energy	joule	J	N•m or kg•m^2/s^2
power	watt	W	J/s
impulse, momentum			N•s or kg•m/s
frequency	hertz	Hz	s^{-1}
angular velocity			rad/s
angular acceleration			rad/s^2
charge	coulomb	C	A•s
potential difference, emf	volt	V	J/C
resistance	ohm	Ω	V/A
conductance	siemens	S	Ω^{-1}
inductance	henry	H	Wb/A
capacitance	farad	F	C/V
magnetic flux	weber	Wb	V•s
electric field (E)			V/m or N/C
magnetic field (B)	tesla	T	Wb/m^2 or N/(A•m)
luminous flux	lumen	lm	cd•sr
illuminance	lux	lx	lm/m^2
radioactivity	becquerel	Bq	s^{-1}

Origin of the metric system

Prior to the metric system there existed in France a multiplicity of names and methods of subdivision for the units of weights and measures. Over the centuries there evolved a diverse assortment of standards. Often units having the same name, such as the *aune* (ell) for cloth, varied in the measure they represented from one town to the next.

In Paris the unit of length was the *Pied de Roi* (0.325 m), or royal foot, which can be traced back to the reign of Charlemagne (742 or 743-814). A larger unit used in surveying was the *toise* (1.949 m), which represented six *pieds*. The *Toise du Chatelet* was introduced as the legal standard of length in 1668. It was represented by an iron bar attached to the outer wall of the Grand Chatelet, an ancient Paris fortress. This standard was replaced in 1766 by the *Toise du Perou* that had been used to measure the figure of the earth. The unit of mass was the *Livre poids de marc* (0.4895 kg), which represented two *marcs*. The *livre* was defined as 1/25 of the 15th-century standard known as the *Pile de Charlemagne*, consisting of 13 nested copper rings with a total mass of 50 *marcs* (12.2375 kg).[2]

All attempts by the Estates-General to impose the "Parisian" units on the whole country were fruitless. Although many proposals were made to bring order to the disparate system of weights and measures, they were opposed by the guilds and nobles who benefited from the confusion.

The advocates of reform sought to guarantee the permanence of the units by basing them on properties derived from nature. In 1670 Gabriel Mouton, vicar of St. Paul's church in Lyons, proposed a unit of length equal to one minute of arc on the earth's surface, which he called the *milliare*. He subdivided this unit by powers of ten to

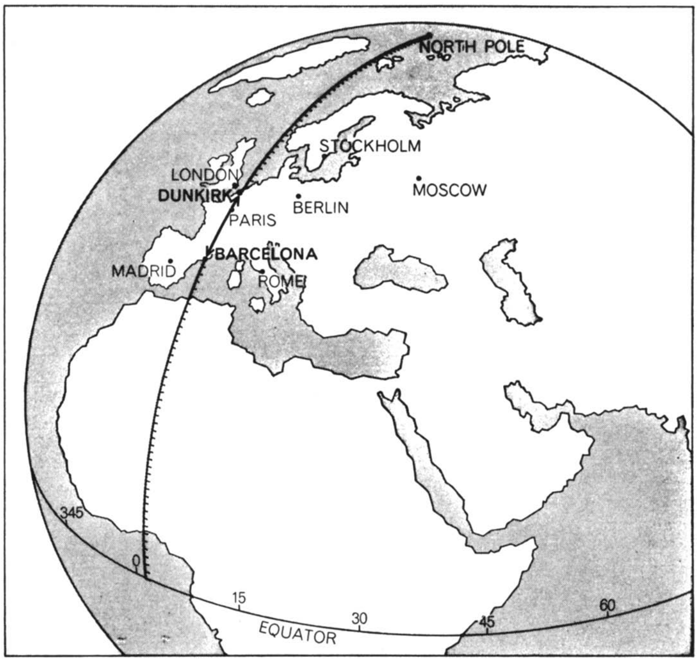

Fig. 1. Portion of quadrant of the earth surveyed between Dunkirk and Barcelona. From measurements of star positions the difference in latitude was determined, permitting extrapolation of the actual distance measured to the entire quadrant. The meter was defined as the length of one ten-millionth of the earth's quadrant. (From "Standards of Measurement" by Allen V. Astin. Copyright ©June, 1968 by *Scientific American*, Inc. All rights reserved.)

obtain smaller units. Mouton proposed that the length of a pendulum with a specified period be used as a standard to preserve the unit of length. The idea of the pendulum was repeated by the astronomer Jean Picard.

The occasion that made reform possible was the French Revolution of 1789. In 1790 the National Assembly, with the support of its president, Charles Maurice de Talleyrand, empowered a committee of the French Academy of Sciences to devise a new system of units. The committee recommended a decimal system for length, mass, and currency. A second committee (Borda, Lagrange, Laplace, Monge, and Condorcet) was appointed to choose the standard of length. The length of a pendulum with a particular period was rejected because the period depends on the local gravitational field strength. A fraction of a quadrant of the earth's equator was also rejected as being impractical. Instead the new unit, given the name *mètre* in May 1793 (after the Greek word *metron*, a measure), was defined as the length of one ten millionth of the quadrant of the earth along a meridian passing through Paris. The proposal was accepted by the National Assembly on March 26, 1791 and was enacted into law by King Louis XVI four days later.

The field work necessary to establish the length of the meter was begun in June, 1792. It was decided to survey the distance between Dunkirk, in France on the English Channel, and Barcelona, in Spain on the coast of the Mediterranean Sea (Fig. 1). Part of this route had been surveyed twice previously. The new measurement was conducted by the astronomers Jean Baptiste Delambre and P.F.A. Méchain (Fig. 2).

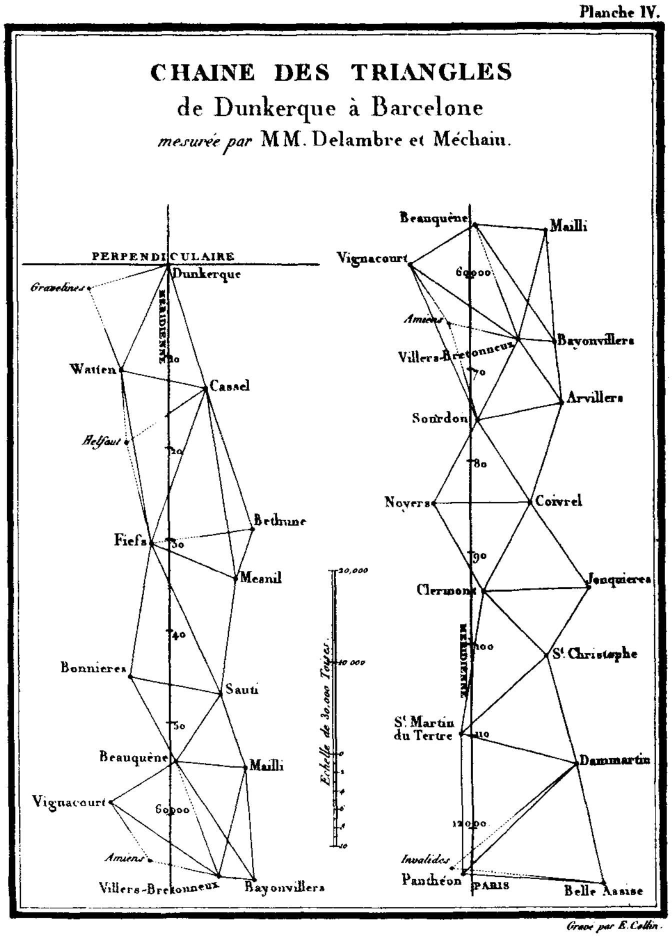

Fig. 2. Part of the system of triangles measured in survey of meridian from Dunkirk to Barcelona by Delambre and Méchain from 1792 to 1798. [From *Base du Système Métrique Décimal* by J. B. Delambre (Paris, 1806), Vol. 1, Plate IV; reproduced by permission of Science and Technology Research Center, New York Public Library, Astor, Lenox, and Tilden Foundations.]

Meanwhile the unit of volume, the *pinte*, was defined as the volume of a cube having a side equal to one tenth of a meter. The unit of mass, the *grave* (from the Latin *gravis*, heavy), was defined as the mass of one *pinte* of distilled water at the temperature of melting ice.

On August 1, 1793 the National Convention, which by then ruled France, issued a decree adopting the preliminary definitions and terms. The "methodical" nomenclature, specifying fractions and multiples of the units by Latin prefixes, was chosen in favor of the "common" nomenclature, involving separate names. A provisional value for the meter was calculated and a brass weight was derived to represent the *grave* (Fig. 3). The next month a "Temporary Commission for Republican Weights and Measures" was appointed to study the project further. The commission recommended that in addition to the metric reforms underway, the Réaumur temperature scale of 1732 (freezing and boiling points of water at 0°R and 80°R) should be changed to a *centésimal* or centigrade scale – now called the Celsius scale – with fixed points at 0°C and 100°C.

Among the many changes in society during the revolutionary period was the adoption of a new calendar consisting of twelve months of thirty days each, concluded

Fig. 3. The grave, a provisional standard of mass constructed in 1793. It was succeeded by the kilogram in 1795. (National Bureau of Standards photo.)

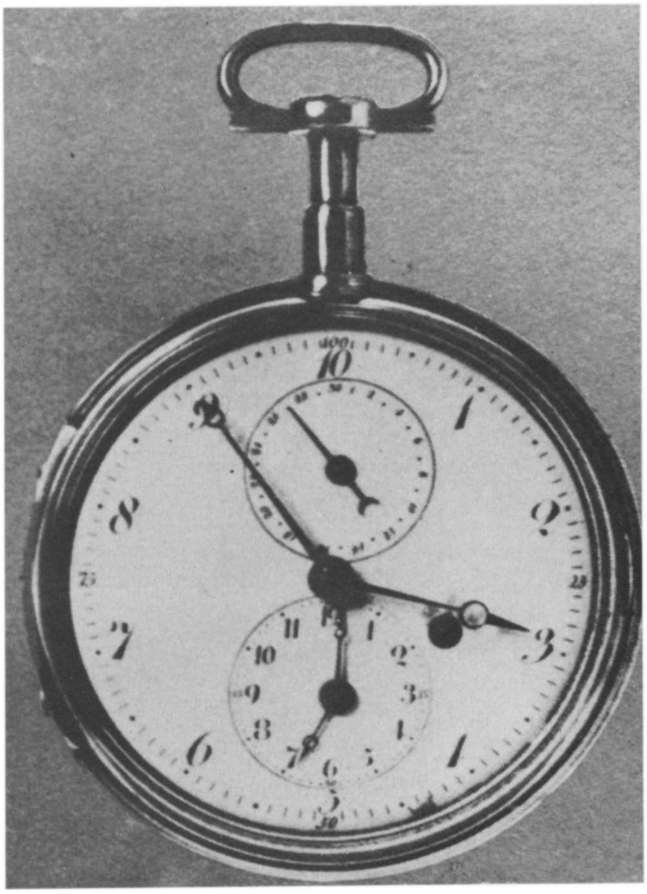

Fig. 4. Ten-hour watch, constructed in 1794 in anticipation of conversion to decimal time in France. The time reads "2.9 o'clock," which corresponds to the traditional time "7 o'clock a.m." as indicated by the small inner dial. (National Bureau of Standards photo.)

by a five or six day holiday. Each month was divided into three ten-day weeks or decades. Furthermore, the day itself was divided into ten hours, each hour was divided into 100 decimal minutes, and each minute was divided into 100 decimal seconds (Fig. 4). However, unlike the calendar reform, which remained in effect for twelve years, the new method of keeping the time of day was never truly adopted. The Danish astronomer Thomas Bugge reported that he saw only two decimal clocks when he visited Paris in 1798 as a delegate to the international commission for the metric system.[3]

The date marking the official inception of the metric system was April 7, 1795. On this day the revolutionary government issued a decree (*Loi du 18 germinal, an III*) formalizing the adoption of the metric units and the terms that are in use today. A brass bar constructed by Lenoir was presented to the Commission of Public Instruction to represent the provisional meter. The *pinte*, or *cadil*, was renamed the *litre* (from the Greek *litra*, an ancient unit of volume). The unit of mass was the *gramme* (from the Greek *gramma*, an ancient unit of mass), equal to the mass of a cubic centimeter of water at 0°C, and the *grave* was replaced by the *kilogramme*.

The survey to determine the meter was completed in 1798, having been carried on continuously during the "reign of terror" and the turmoil of revolution. From the measurements of Delambre and Méchain, Laplace obtained 5 130 740 *toises* for the length of the quadrant. The final value of the meter was thus 0.513 074 *toise* or 443.296 *lignes*. The provisional value had been 443.44 *lignes*.* We now know that the quadrant is 10 002 290 m instead of exactly 10 000 000 m as originally planned. The principal source of error was the assumed value of the earth's flattening used in correcting for oblateness. The provisional meter was actually closer to the intended value.[4]

The work to determine the unit of mass was begun by Lavoisier and Haüy and was completed by Lefèvre-Gineau and Fabbroni. They discovered that the temperature at which the density of water is maximum is 4°C and not 0°C as had been supposed. The French Academy therefore revised the definition of the kilogram to be the mass of 1000 cm^3 of water at the temperature of its greatest density. It was found that the kilogram represented 2.042 877 *livre* or 18 827.15 *grains* of the *Pile de Charlemagne*.† We now know that the intended mass was 0.999 972 kg, i.e., 1000.028 cm^3 for the volume of 1 kg of pure water at 4°C.[5]

Permanent standards made from platinum were con-

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*1 toise = 864 lignes. (1 toise = 6 pieds; 1 pied = 12 pouces; 1 pouce = 12 lignes; 1 ligne = 12 points.) The provisional meter was based on the survey that had been completed in 1740 by LaCaille.

†1 livre = 9216 grains. (1 livre = 2 marcs; 1 marc = 8 onces; 1 once = 8 gros; 1 gros = 3 deniers; 1 denier = 24 grains.)

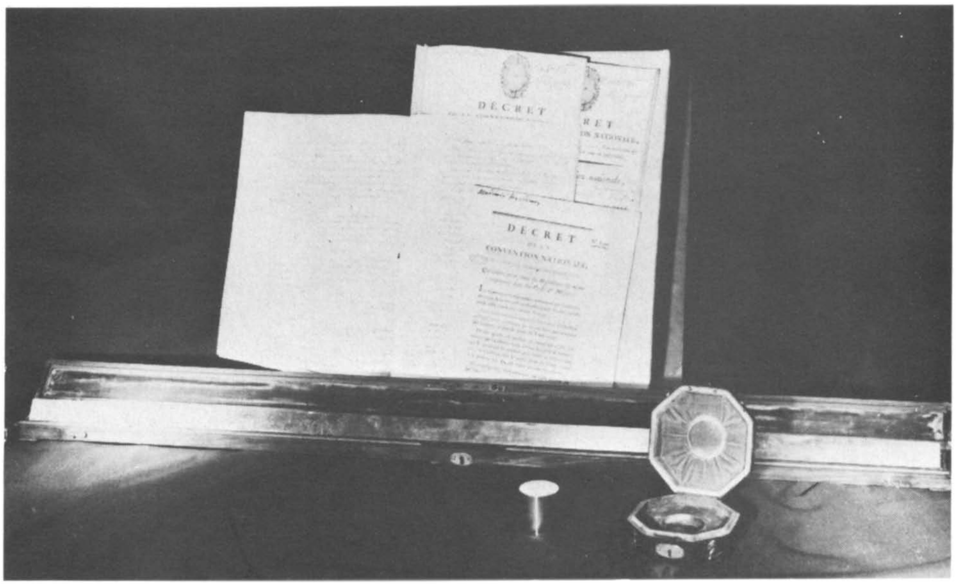

Fig. 5. *Mètre et Kilogramme des Archives* of 1799, a platinum bar and cylinder deposited in the *Archives de la République*. These standards served as the legal representations of the metric units. In the background are the decrees of 1793 and 1795 which established the metric system in France. (Photo Archives Nationales, Paris.)

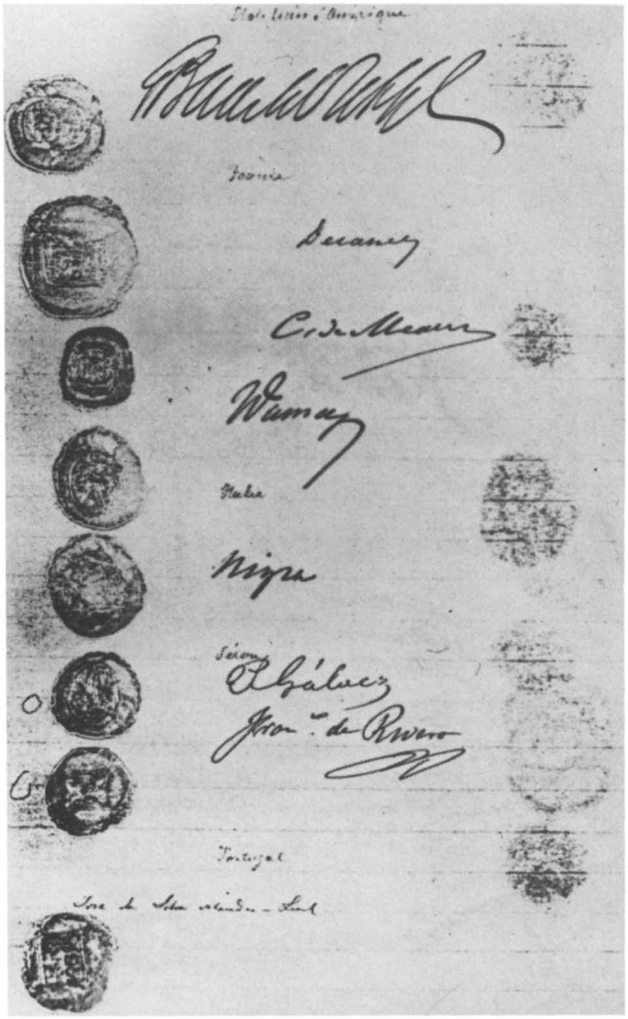

Fig. 6. Page of signatures from the Treaty of the Meter of 1875. The top signature is that of Elihu Benjamin Washburne, the American plenipotentiary. (International Bureau of Weights and Measures photo.)

structed to serve as the legal representations of the metric units. On June 22, 1799 they were deposited in the *Archives de la République* and hence were known as the *Mètre et Kilogramme des Archives* (Fig. 5). They became official by an act of December 10, 1799.

In spite of its auspicious beginning, the metric system was not quickly adopted in France. During the Napoleonic era several regressive acts were passed due to lack of funds to distribute secondary standards. Nevertheless the system was taught in the schools. Finally, in 1837 the state established a three-year transition period and made the metric system compulsory throughout France as of January 1, 1840.

The Treaty of the Meter

A series of international expositions in the middle of the 19th century enabled the French government to promote the metric system for world use. At the Paris Exposition in 1867 the superiority of the metric system was acknowledged by a "committee for weights, measures, and currencies." The International Geodetic Association also expressed interest but was concerned over the permanence of the *Mètre des Archives*. The full length of the bar defined the meter and there was a possibility that the ends had become worn through use.

In 1870 an international commission of scientists met in Paris to consider the design of new international metric standards. The meeting was suspended because of the Franco-Prussian War but was resumed in 1872. Formal diplomatic approval of the scientists' decisions was secured by the Diplomatic Conference on the Meter, convened in Paris on March 1, 1875. The resulting Treaty of the Meter was signed by representatives of 17 countries, including the United States, on May 20, 1875 (Fig. 6). The treaty established the International Bureau of Weights and Measures in Sèvres, France. It also provided for the creation of the International Committee for Weights and Measures to run the Bureau and the General Conference on Weights and Measures to ratify new proposals as the need arose. The French government offered the Pavillon de Breteuil, once a small royal palace, to serve as headquarters for the Bureau

Fig. 7. The Pavillon de Breteuil, headquarters of the International Bureau of Weights and Measures in Sevres, France. The Pavillon was originally constructed in 1743 but has undergone several restorations. On the left are laboratories built in 1878 and in 1929. (International Bureau of Weights and Measures photo.)

Fig. 8. Vault at the International Bureau of Weights and Measures. On the top shelf is the international prototype meter bar of 1889 in its protective case. On the bottom shelf is the international prototype kilogram, its six check standards, two thermometers (one of the maximum-minimum type), and a hygrometer. (International Bureau of Weights and Measures photo.)

(Fig. 7). The grounds of the estate form "a tiny international enclave within French territory"[2] much as the United Nations does within New York.

The new meter and kilogram standards were made from an alloy of platinum and iridium. *The Mètre et Kilogramme des Archives* in their existing states were taken as the points of departure. A total of 30 meter bars and 43 kilogram cylinders were manufactured by Johnson, Matthey, and Company of London from a single ingot of the alloy. The standards were intercompared at the International Bureau between 1886 and 1889. One meter and one kilogram were selected to represent the international prototypes (Fig. 8). The work was approved by the First General Conference on Weights and Measures in 1889 and the remaining standards were distributed by drawing lots. The United States drew meters 21 and 27 and kilograms 4 and 20. Thermometers which could be read to 0.001°C were also distributed to accompany the meter bars.

On January 2, 1890 the seals to the shipping cases for meter No. 27 and kilogram No. 20 were broken in an official ceremony at the White House. Among those invited by President Harrison to witness the occasion was the physicist E.W. Morley whose contribution to the definition of the meter will be described later. Three years later these standards became fundamental in the U.S. under a directive of Thomas C. Mendenhall, Superintendent of the U.S. Coast and Geodetic Survey and its Office of Standard Weights and Measures.

From its founding in 1901 until 1959 the National Bureau of Standards used the relations 1 yard = 3600/3937 m and 1 pound-mass = 0.453 592 427 7 kg. On July 1, 1959 the definitions were fixed by international agreement to be 1 yard = 0.9144 m and 1 pound-mass = 0.453 592 37 kg exactly.[6] The metric system had thus become the ultimate basis for all legal units of measure in the civilized world.

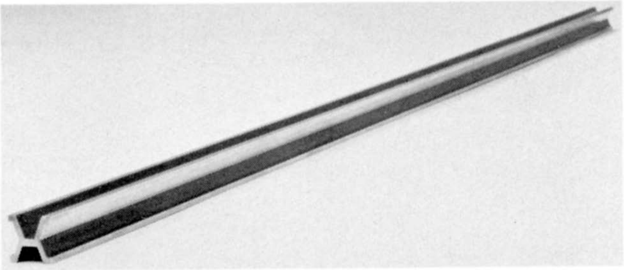

Fig. 9. International prototype meter bar of 1889. It is made of an alloy of 90% platinum and 10% iridium. The X-shaped Tresca cross section is inscribable in a square 20 mm on a side. The meter was defined as the distance between two lines ruled on a polished region of the bridge at each end of the bar when the bar was at 0°C, standard atmospheric pressure, and supported by two cylinders placed symmetrically 571 mm apart. In practice, standards being compared were immersed in a water bath at a known temperature and a correction was applied using the known coefficient of thermal expansion. (International Bureau of Weights and Measures photo.)

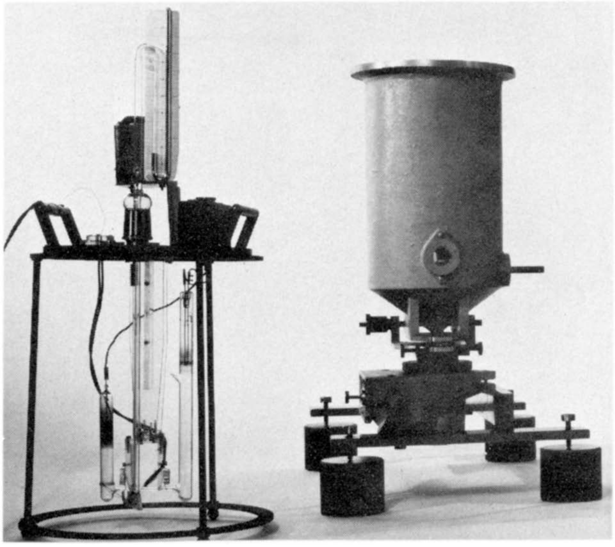

Fig. 10. Engelhard krypton lamp, removed from its cryostat and shown resting on a laboratory stand. The glass capillary at the bottom of the lamp is viewed end-on through the porthole in the cryostat. The standard radiation is produced by exciting a gas of ^{86}Kr atoms with an electrical current when the lamp is immersed in liquid nitrogen at the temperature of the triple point, 63.15 K. (Photo courtesy of the National Physical Laboratory, Teddington, England. Crown copyright reserved.)

The meter

The design of the new meter was based on four major considerations: (1) An alloy of 90% platinum and 10% iridium was selected because of its inalterability, hardness, luster, high coefficient of elasticity, and low coefficient of expansion. (2) A special X-shaped cross section designed by the French physicist Henri Tresca was adopted for maximum rigidity. (3) The meter was defined by the distance between two engraved lines on the neutral plane (top surface of the bridge) instead of the distance between the end faces (Fig. 9). (4) The meter was derived from the *Mètre des Archives* in its existing state and reference to the earth was abandoned. In 1927 the Seventh General Conference specified the manner in which the meter bar should be supported during a comparison.

The permanence of the international prototype was verified by comparison with three companion bars called "check standards." The only organized intercomparison of the national meters subsequent to the original one began in 1921 and lasted 15 years. It was found that the family of prototype meter bars preserved the unit to within 0.2 μm.[4]

In addition to periodic comparisons with the check standards there were nine measurements of the international prototype meter in terms of the red line of cadmium between 1892 and 1940. The first of these measurements was carried out by A. A. Michelson using the interferometer which he invented.

Michelson believed that a practical standard of length could be defined in terms of a specified wavelength of light. In 1887 he and E. W. Morley published a paper entitled "On a Method of Making the Wave Length of Sodium Light the Actual and Practical Standard of Length." It appeared in the same volume of the *American Journal of Science* as their paper on the famous Michelson-Morley experiment which yielded no evidence of an "ether" wind.

In searching for a suitable spectral line Michelson discovered the phenomena of fine structure and hyperfine structure that were to play important roles in the development of quantum mechanics and relativity. He investigated the spectra of sodium and mercury but settled on the red line of cadmium. At the International Bureau in Sèvres, Michelson and J. R. Benoît, the Bureau's director, determined with the Michelson interferometer that the meter was equivalent to 1 553 163.5 cadmium wavelengths (the value was affected slightly by the humidity of the air, which was not measured). For this work Michelson received the Nobel Prize in 1907. He was the first American to win the physics award.[7]

In 1905 Benoît, Fabry, and Perot found that the meter was equal to 1 553 164.13 cadmium wavelengths using the Fabry-Perot interferometer. This value was accepted by the Seventh General Conference on Weights and Measures in 1927 as an alternative definition of the meter. The conference, however, refrained from declaring it the absolute standard. Although the wavelength measurements demonstrated the feasibility of an optical standard, they did not offer a sufficient advance in precision to warrant replacing the older definition.

Significant improvements in the construction of monochromatic light sources occurred following World War II at the U.S. National Bureau of Standards and at the International Bureau. A krypton-86 lamp was judged to be the most satisfactory source, combining both the availability of a convenient emission line and excellent operation at low temperatures (Fig. 10). No spectral line is truly discrete. However, extraneous components can be eliminated by using the gas of a single isotope. By choosing an isotope with an even atomic number and an even mass number, the hyperfine structure is also eliminated because there is then no interaction between the spins of the electrons and the nucleus. The motion of the atoms themselves precludes a perfectly sharp line due to Doppler broadening. This property is minimized by using a gas of heavy atoms at a low temperature.[8]

The value of the krypton wavelength was obtained, not by a remeasurement of the international prototype

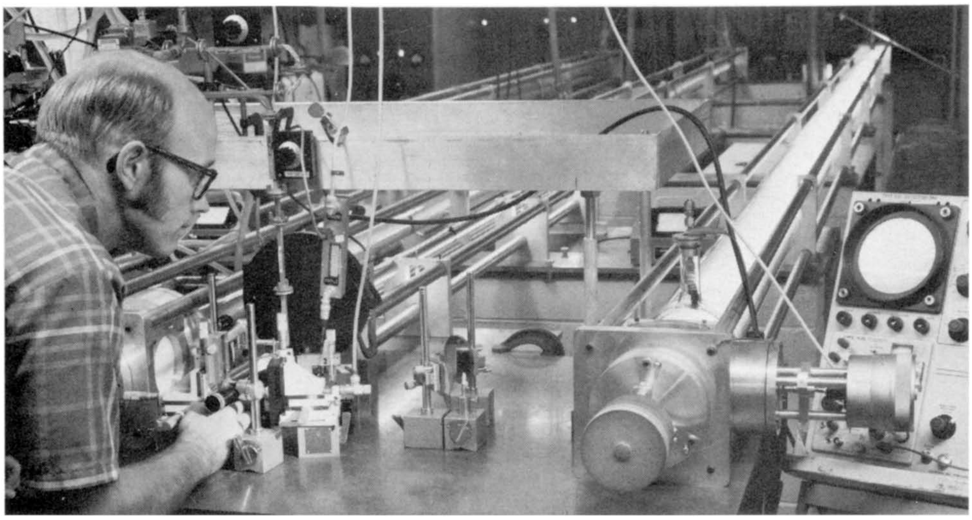

Fig. 11. Kenneth Evenson adjusting diode used in calibration of frequency of helium-neon laser at the Boulder, CO laboratories of the National Bureau of Standards. This measurement resulted in a new value for the speed of light, which eventually may be used as the basis for a new definition of the meter. (National Bureau of Standards photo.)

meter bar, but by direct comparison with the cadmium wavelength. The value of the cadmium wavelength in a vacuum was reduced from its value in air by applying a correction.[4] In 1960 the Eleventh General Conference on Weights and Measures defined the meter as "the length equal to 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom." This is the definition in use today. The standard red-orange radiation is emitted when the excited electrons fall from the $5d_5$ energy level to the $2p_{10}$ energy level. The spectral line from the krypton lamp is isolated by means of a monochromator or special interference filters.

A fundamental disadvantage of the present definition is that, since the light is incoherent, an interferometer measurement cannot be extended over a distance of about 40 cm in a single step. This difficulty can be removed by using a laser as the source of light.

At the Boulder, Colorado laboratories of the National Bureau of Standards a team headed by K. M. Evenson has succeeded in stabilizing a laser wavelength by passing the light through a gas, such as methane, which possesses a very narrow absorption line (Fig. 11). The wavelength was measured by direct comparison with the krypton-86 wavelength. Evenson's group was also able to measure the frequency in a series of measurements directly traceable to the cesium-133 atomic frequency standard. The procedure was analogous to the method of beats used by a piano tuner who starts with a tuning fork as a reference. By multiplying the independently determined wavelength and frequency a new value for the speed of light c was obtained. The principal source of error was the realization of the krypton wavelength itself. The spectral profile has a finite width and is slightly asymmetric. With respect to the "center of gravity" the value is $c = 299\,792\,456.2$ m/s. However, if the point of maximum intensity is used, $c = 299\,792\,458.7$ m/s. The uncertainty in both measurements is ± 1.1 m/s.

In June, 1975 the Fifteenth General Conference on Weights and Measures adopted the provisional value 299 792 458 m/s for the speed of light, representing the best compromise for the experimental values and now regarded as exact. It is possible to accept this value as a defined constant which would serve to define the meter. The Consultative Committee for the Definition of the Meter has recommended a new definition of the meter as "the distance traveled in a time interval of 1/299 792 458 of a second by plane electromagnetic waves in vacuum." Other, alternative wordings are being considered by the Consultative Committee for Units. It is expected that the final proposed definition will be presented for approval to the Seventeenth General Conference on Weights and Measures in the fall of 1983. The working standard for the meter would then become a series of stabilized lasers.[9]

The kilogram

In 1889 the First General Conference adopted a new standard of mass which was copied from the *Kilogramme des Archives*. It was assigned the value of one kilogram by definition. The definition has survived to the present.

The international prototype kilogram is made from the same platinum-iridium alloy as the meter bar of 1889. The material is not subject to oxidation, it is very hard, and has high density (almost twice that of lead). The kilogram is in the shape of a cylinder with a height equal to its diameter, 3.9 cm, with slightly rounded edges (Fig. 12). For a cylinder these dimensions present the smallest surface to volume ratio.

The standard is preserved at the International Bureau and is stored under triple bell jars in a vault 8 m below the ground (Fig. 8). There is an inevitable loss of material whenever it is used. A reduction of only 0.4 nm, or one layer of atoms, over the base area of 12 cm^2 would be sufficient to cause a loss of 0.01 mg. Therefore, the international prototype kilogram has been used on only three occasions: before 1889 (42 weighings), in 1939 (7 weighings), and in 1946 (14 weighings). Permanence of the unit is verified by comparison of the international prototype with its six check standards. There have been two periodic intercomparisons of the national prototypes with the international prototype. A third verification is being planned.[10]

The kilogram is the only unit still defined in terms of an arbitrary artifact instead of a natural phenomenon. This is because kilogram-size masses can be compared with one another with a precision 1000 times greater than the mass of an individual atom can be measured. Increased precision is being sought, therefore, through the design of improved balances. An atomic definition of the kilogram is not being contemplated at the present time.

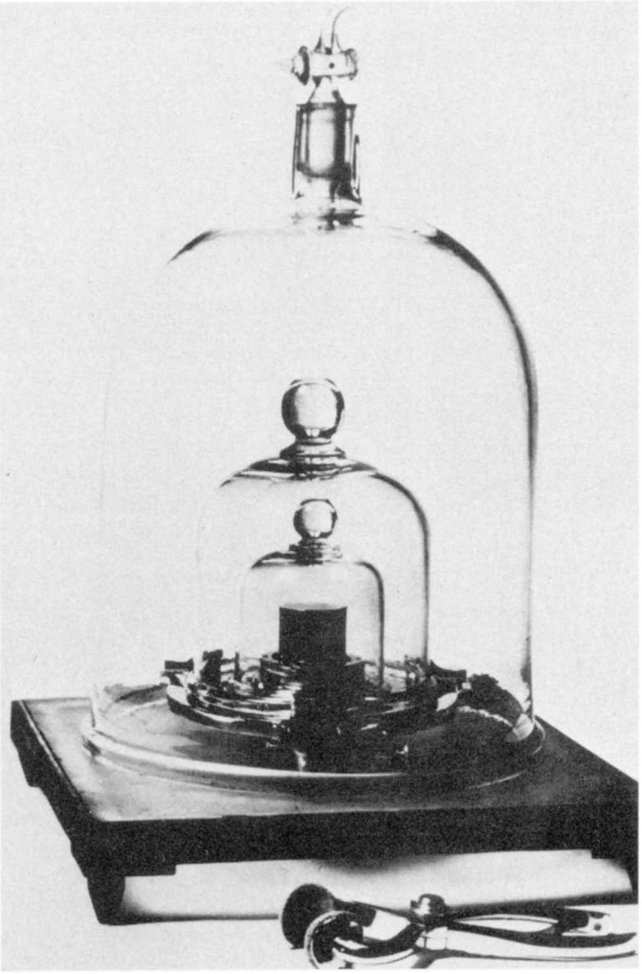

Fig. 12. International prototype kilogram, protected by triple bell jars. In the foreground are special calipers used in handling the kilogram. The standard has been used on only three occasions: before 1889, in 1939, and in 1946. (Photo courtesy of the French Embassy, Press and Information Division, New York.)

The second

Historically, time was defined in terms of the rotation of the earth. This measure of time is called mean solar time; when reckoned from midnight on the meridian of Greenwich it is called Universal Time (UT). The mean solar second is defined as 1/86 400 mean solar day. During the past century, however, three types of variation in the earth's rotational period have been discovered: a steady increase, periodic changes, and random fluctuations.

The steady (secular) increase in the length of the day is due to tidal friction. This phenomenon is caused by the moon's tide-raising force in the shallow seas. It is known that the length of the day increases at a rate of about 0.0016 second/day/century, or in terms of the rotation of the earth, 4.3 x 10^{-22} rad/s^2. This means that today is about 0.0016 s longer than a day just a century ago. Although the amount seems miniscule, the effect is cumulative. The accumulation over a century is 29.22 seconds of time. In the past 2000 years the earth acting as a clock has lost over three hours. The effect can be measured by comparing the predicted paths of ancient solar eclipses with those actually recorded (Fig. 13). There is also evidence in the form of fossils of coral which exhibit both daily and annual growth rings.[11]

The measurement of Universal Time is influenced by the variation in latitude. This effect is associated with the free precession of the earth and is called the Chandler wobble (not to be confused with the precession of the equinoxes, which is a forced precession). The North Pole wanders in a circle of radius 8 m with a period of 14 months as the figure of the earth precesses about the axis of rotation.

In 1956 the International Astronomical Union resolved to adopt special time scales that remove the regular inequalities of practical importance. UT0 is Universal Time at a local observatory. UT1 is UT0 corrected for migration of the earth's poles. It is the true astronomical measure of the earth's rotation and is used in navigation and surveying. UT2 is UT1 corrected for seasonal variation and is nearly uniform. Time signals were based on this scale until 1972.

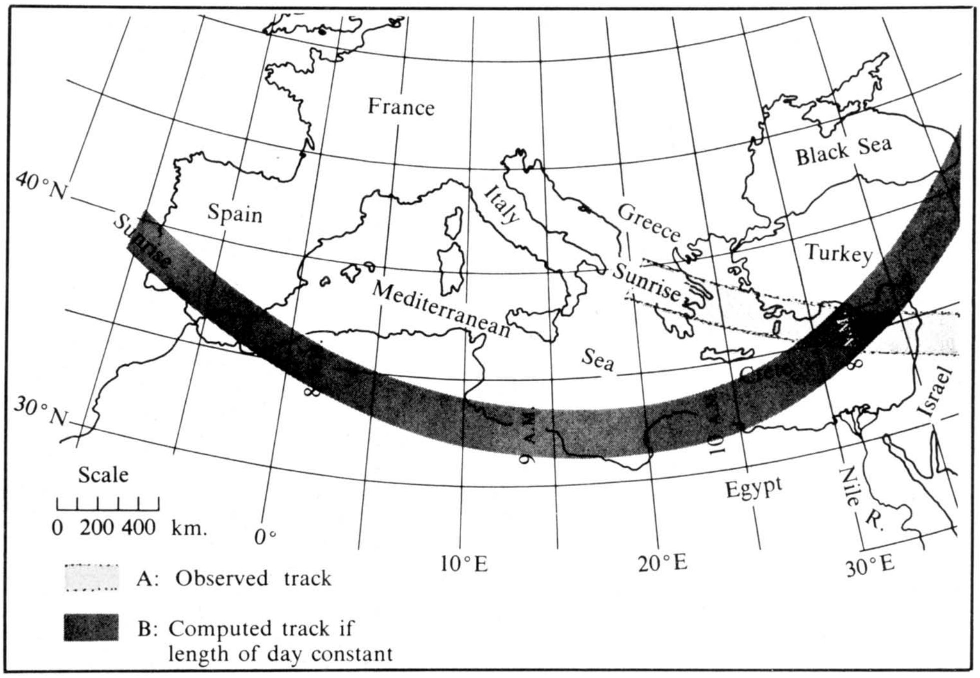

Fig. 13. Eclipse of January 14, 484 A.D. Difference in longitude of 30° between locations of observed eclipse and those computed assuming a uniform rate of rotation of the earth is evidence that the length of the day has been increasing by about 0.0016 s/day/century. [From *Gravitation* by Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler (W.H. Freeman, San Francisco, 1973)]

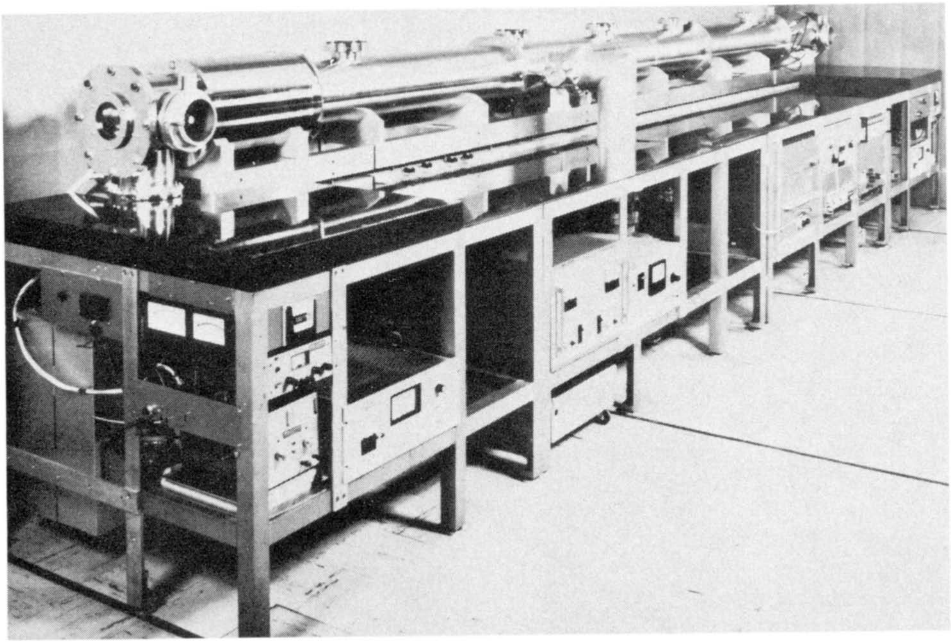

Fig. 14. Cesium-beam atomic clock, NBS-5, operated at the Boulder, CO laboratories of the National Bureau of Standards. The clock vacuum system is a stainless steel tube 6 m long. Each end has both a cesium oven and a detector so that the beam may be sent in either direction. At the center of the clock may be seen an oscillator circuit which sends microwaves through a waveguide into a 3.74-m long resonating cavity. A feedback circuit keeps the oscillator at the standard frequency. The cesium beam thus acts as the "balance wheel" of the atomic clock. (National Bureau of Standards photo.)

Because the variations in the rotation of the earth are complex and cannot be predicted precisely, the International Committee for Weights and Measures referred the study of a new definition of the second to the International Astronomical Union in 1948. At the suggestion of G. M. Clemence, the IAU decided that the new standard of time ought to be based on the period of revolution of the earth around the sun, as represented by the *Tables of the Sun* computed by the American astronomer Simon Newcomb in 1895. The measure of time defined in this way is called Ephemeris Time (ET). Accordingly, in 1956 the International Committee for Weights and Measures defined the ephemeris second to be 1/31 556 925.974 7 of the tropical year 1900 January 0^d 12^h ET. The operational significance of this definition was to adopt Newcomb's values for certain constants in the formula used to compute the sun's theoretical longitude.[12] Ephemeris Time is determined by comparison of the observed and predicted positions of the sun with respect to the fixed stars. (In practice this is done indirectly by observing the moon.)

It is now possible to measure time with atomic clocks. The precision of 1 part in 10^{13} far exceeds the precision attainable by any other kind of physical measurement. The "balance wheel" of an atomic clock is a beam of cesium atoms. The clock is run by an rf oscillator whose frequency can be adjusted precisely against the natural frequency of the cesium hyperfine transition via a feedback mechanism (Fig. 14). In 1967 the Thirteenth General Conference defined the second as "the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom." The definition was made to be in agreement with the ephemeris second within the limits of experimental uncertainty.

The Bureau International de l'Heure in Paris coordinates the timekeeping activities of the world. Using an adopted weighting procedure applied to the atomic time scales of various national laboratories, the BIH establishes the time scale known as International Atomic Time (TAI). In contrast, the time scale of civil clocks is Coordinated Universal Time (UTC), which is intended to approximate UT1. UTC corresponds in rate with TAI but differs from TAI by an integral number of seconds. UTC is disseminated in the United States by the NBS radio stations WWV and WWVH on carrier frequencies of 2.5, 5, 10, 15, and 20 MHz.

The second defined by the cesium hyperfine transition is essentially equal to the ephemeris second defined by Newcomb's *Tables of the Sun*. It is therefore roughly equal to the average value of the mean solar second during the 18th and 19th centuries, the time span from which Newcomb's data was obtained. The atomic or ephemeris second hypothetically would have been equal to the mean solar second in 1779.[13] Since then the length of the day has increased by 0.0032 s. Therefore, the length of the mean solar day is now about 86 400.0032 (fixed) seconds of TAI instead of exactly 86 400 (variable) seconds of UT1. The difference accumulates to about 1 s in the course of a year. Since 1972 the difference between TAI and UT1 has been accounted for by inserting "leap second" adjustments into TAI about once a year to obtain UTC. In this way UTC is kept within 0.9 s of UT1 at all times while still beating atomic seconds defined by TAI. (Due to unpredictable fluctuations, however, it is conceivable that more than one, none, or even negative leap seconds may be required in any given year.)

The electrical units

During the initial phase of electrical research in the late 18th and early 19th centuries investigators used units that were completely arbitrary and unrelated to other branches of physics. The possibility of defining the electrical units in terms of mechanical measurements was first pointed out by K. F. Gauss in 1833 and W. Weber in 1851. Also, the fundamental expression for electrical work was derived independently by H. von Helmholtz in 1847 and by Lord Kelvin (then W. Thomson) in 1848. At that time, and for at least the next 25 years, the "cardinal operation in electricity" was the measurement of electric resistance.[14]

In 1861 Kelvin persuaded the British Association for the Advancement of Science to create a committee for the adoption of internationally acceptable absolute electrical units and standards. The committee considered two distinct

Table II

Dimensions of the three-dimensional electrical units

As recommended by the B. A. committee.

Quantity	**Electrostatic subsystem**	**Electromagnetic subsystem**	**Practical System (x CGS em unit)**
		Base Units	
length	L	L	10^9
mass	M	M	10^{-11}
time	T	T	10^0
		Derived Units	
charge	$L^{3/2} M^{1/2} T^{-1}$	$L^{1/2} M^{1/2}$	10^{-1}
current	$L^{3/2} M^{1/2} T^{-2}$	$L^{1/2} M^{1/2} T^{-1}$	10^{-1}
potential difference, emf	$L^{1/2} M^{1/2} T^{-1}$	$L^{3/2} M^{1/2} T^{-2}$	10^8
resistance	$L^{-1} T$	$L T^{-1}$	10^9
inductance	$L^{-1} T^2$	L	10^9
capacitance	L	$L^{-1} T^2$	10^{-9}
energy, work	$L^2 M T^{-2}$	$L^2 M T^{-2}$	10^7
power	$L^2 M T^{-3}$	$L^2 M T^{-3}$	10^7

questions: recommendation of (1) the most convenient *unit* of resistance and (2) the best form and material for a *standard* to represent the unit.

A fundamental principle stressed by the B.A. Committee was that a system of units should be built up from a limited number of base units and that derived units should be expressed as products or quotients of other units within the system without requiring numerical factors. This property is called coherence. For example, the unit of resistance should be equal to the unit of potential difference divided by the unit of current.

The committee chose as base units the units of length, mass, and time. It recommended a three-dimensional meter-gram-second (MGS) system. Any electrical unit could be expressed in either electrostatic or electromagnetic measure, depending on whether the law of force for electric charges or magnetic poles was taken as fundamental. The dimensions of these electrical units are given in Table II.*

It was recognized that the MGS units would not be of convenient size for electrical engineers, who were concerned primarily with telegraphy. Therefore, the committee endorsed a plan to define a set of "practical" units derived from the MGS electromagnetic units by appropriate multiples of 10.

In 1863 the committee issued the B.A. standard of resistance, which was intended to represent 10^7 m/s in

Fig. 15. B.A. standard of resistance. This standard resistor, constructed in 1863 and 1864, was the first electrical standard to be distributed widely. It consisted of a coil of platinum-silver wire wrapped in silk, imbedded in paraffin and enclosed in a thin brass case. The heavy leads were made of copper. In operation the standard was used in a water bath. Soon after it was introduced the "B.A. unit" became known as the "ohm." (National Bureau of Standards photo.)

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*In the electrostatic subsystem the unit of charge was defined by Coulomb's law $F = qq'/r^2$ for the force F between two point charges q and q' separated by a distance r. Thus the es dimensions of charge were $[q] = [F]^{1/2} [r] = L^{3/2} M^{1/2} T^{-1}$ where L, M, and T are the dimensions of length, mass, and time. In the electromagnetic subsystem the unit of current was equivalent to that defined by the law $F = 2II'\ell/d$ for the force F between two long parallel wires of length ℓ and perpendicular separation d and carrying currents I and I'. Thus the em dimensions of current were $[I] = [F]^{1/2} = L^{1/2} M^{1/2} T^{-1}$. In either subsystem charge and current were related by $I = q/t$ where t is time. Hence in es units $[I] = L^{3/2} M^{1/2} T^{-2}$ while in em units $[q] = L^{1/2} M^{1/2}$.

MGS electromagnetic units (resistance had the dimensions of velocity).* The unit was preserved in terms of the resistance of a coil of wire of specified design (Fig. 15). It was the first electrical standard to be distributed widely.

Soon after it was introduced the "B.A. unit" became known as the "ohm." By 1875 the units of emf and capacitance were called the "volt" and the "farad." The concept of naming units after eminent scientists had been proposed by Sir Charles Bright and Latimer Clark in 1861.[15]

As an appendix to the 1863 report, James Clerk Maxwell (assisted by F. Jenkin) wrote a definitive treatise on the establishment of a system of electrical units. The ratio of the electrostatic to the electromagnetic unit of charge or current was a certain velocity. Maxwell observed that this "ratio of the units" was equal to the velocity of light according to the best experimental measurements. He was able to prove this crucial property in his paper "A Dynamical Theory of the Electromagnetic Field" – read before the Royal Society on December 8, 1864 – in which he predicted the existence of electromagnetic waves.

In their publications several committee members substituted the centimeter as the fundamental unit of length because, as emphasized by Kelvin, in the centimeter-gram-second system the density of water is substantially unity. The CGS system of electrical units was officially recommended by the British Association's committee "for the selection and nomenclature of dynamical and electrical units" in 1873. The names *dyne* and *erg* were introduced for the CGS units of force and energy.

The union of electricity and magnetism through Maxwell's theory of electrodynamics required a new, unified system of electrical units. The Gaussian system was created in the 1880s by Helmholtz and Hertz in order to combine the electrostatic and electromagnetic subsystems for theoretical work.[16] In addition, still another system was introduced by Heaviside and Lorentz to bring about a more satisfying form to the theoretical equations such that the factors 2π and 4π would occur where they logically belong on the basis of symmetry. Heaviside gave the name "rationalization" to this modification. The contributions of Hertz and Heaviside were a result of their work on the modern formulation of Maxwell's equations.†

In 1881 the First International Electrical Congress in Paris adopted the recommendations of the B.A. Committees and defined five "practical" electrical units as certain powers of 10 of the CGS electromagnetic units: the ohm, farad, volt, ampere, and coulomb. The Second Congress (Paris, 1889) added the joule, watt, and a unit of inductance, the quadrant. The quadrant was changed to the henry at the Fourth Congress (Chicago, 1893). The practical units formed a limited coherent system for electromagnetic quantities but were not coherent with respect to units of mechanical quantities.

In 1901 Giorgi demonstrated that the practical electrical units and the mechanical units could be incorporated into a single coherent system by (1) selecting the meter, kilogram, and second as the base units for mechanical quantities, and (2) expanding the number of base units to four, including one of an electrical nature (he recommended the ohm). However, at that time the CGS system was too firmly established for this proposal to be considered seriously.

The eventual adoption of the Giorgi proposal came about as part of efforts to correct a situation caused by the introduction of the so-called "international" electrical units by the London "International Conference on Electrical Units and Standards" in 1908. In this system the ohm and the ampere were defined in terms of independent, reproducible standards. However, the precision of these standards was not as great as had been hoped and their utility diminished with the growth of the national standards laboratories.

The Sixth General Conference on Weights and Measures amended the Treaty of the Meter in 1921 to cover electricity and photometry. Subsequently, in 1929, the International Committee resolved to return to an absolute system of electrical units, i.e., defined in terms of a mechanical experiment. The resolution was ratified by the General Conference in 1933. After prolonged deliberations by the International Electrotechnical Commission and the SUN Commission for Symbols, Units, and Nomenclature of the International Union of Pure and Applied Physics, the International Committee decided in 1935 to (1) adopt an absolute MKS system and (2) choose the ampere as the fourth fundamental unit. In 1938 the IEC (Torquay) adopted the name "newton" for the MKS unit of force and chose the permeability of free space μ_0 (rationalized or unrationalized) as "the connecting link between the electrical and mechanical units."** Finally, in 1946, by authority granted to it in 1933, the International Committee officially adopted the absolute system of electrical units to take effect January 1, 1948. The decision was approved retroactively by the Ninth General Conference the following October.[17] In 1950 the IEC adopted the rationalized form of the MKSA system, thus favoring development of the electrical units in terms of the ampere as the "fourth principal unit" and rationalization of the equations. This action served to assign the value $\mu_0 = 4\pi \times 10^{-7}$ N/A^2 without affecting the magnitudes of the units.

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*From mechanics the dimensions of work are known to be $L^2 M T^{-2}$. Since the general definition of voltage (due to Kelvin) is work done per unit charge, the em dimensions of voltage are $[V] = [W][q]^{-1} = L^{3/2} M^{1/2} T^{-2}$. Thus by Ohm's law the em dimensions of resistance are $[R] = [V][I]^{-1} = L T^{-1}$, a velocity.

†In the Gaussian system Coulomb's law is $F = qq'/r^2$ and the force between two long parallel wires is $F = 2II'\ell/c^2 d$ where c is the speed of light. In the Heaviside-Lorentz system the force laws are $F = qq'/4\pi r^2$ and $F = II'\ell/2\pi c^2 d$, respectively. Here the factor 4π occurs where there is spherical symmetry and the factor 2π appears where there is circular symmetry. [For a discussion of Maxwell's equations in the various systems of electrical units, see J.D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975) appendix, pp. 811-821.]

**In the rationalized MKSA system the force between two long parallel wires is given by $F = \mu_0 II'\ell/2\pi d$ where $\mu_0 = 4\pi \times 10^{-7}$ N/A^2. In the unrationalized MKSA system the force law is $F = 2\mu_0 II'\ell/d$ but $\mu_0 = 10^{-7}$N/A^2. In both systems the unit of current (the ampere) is the same and is defined as "that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length." Coulomb's law is $F = qq'/4\pi\epsilon_0 r^2$ (rationalized) or $F = qq'/\epsilon_0 r^2$ (unrationalized). The permittivity of free space ϵ_0 is related to μ_0 for either system by $\epsilon_0\mu_0 = 1/c^2$.

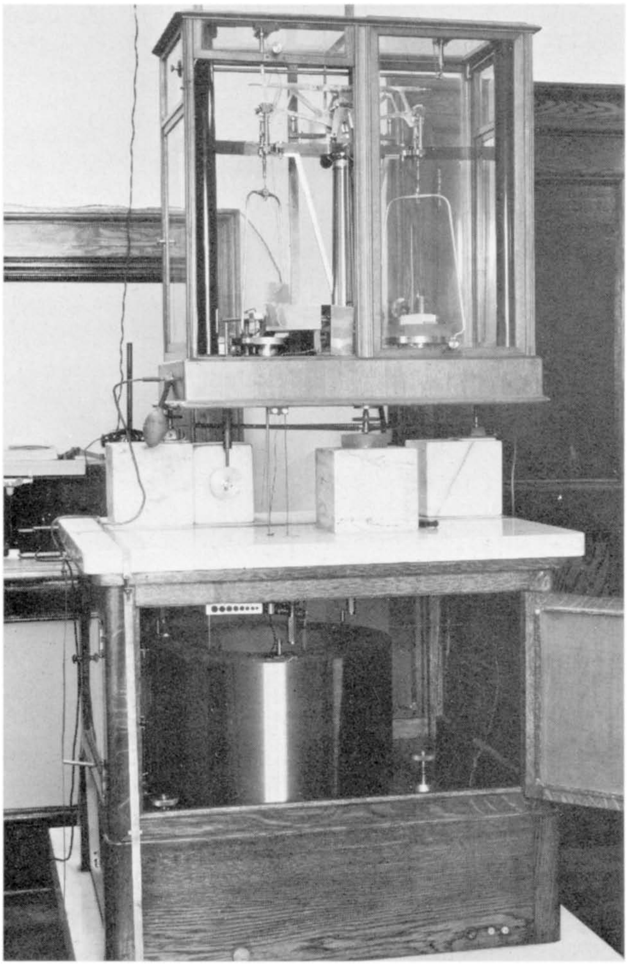

Fig. 16. Helical-coil current balance used in 1957 by R. L. Driscoll and R. D. Cutkosky to determine the relation between the NBS ampere, as maintained by standard resistors and standard cells, and the absolute ampere. Rear view is shown with operating room in background. The large fixed outer coil is composed of two separate coils with a center tap. A small movable coil is suspended within the two coils from an arm of the precision balance. All three coils carry the same current. When the current in the fixed coils is reversed the direction of the force on the movable coil changes. The value of the current in absolute amperes is calculated from the change in force and the dimensions of the coils. (National Bureau of Standards photo.)

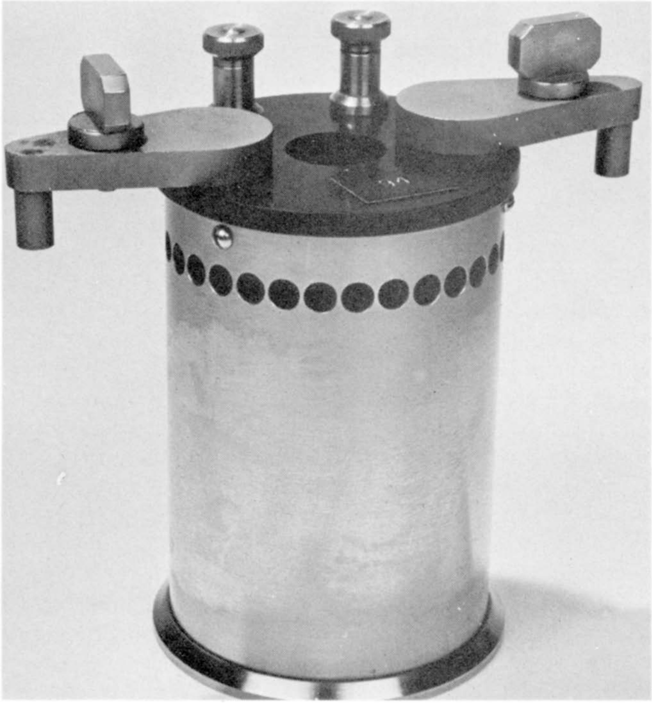

Fig. 17. Standard 1-Ω resistor. The outer terminals are for current and the inner terminals are for potential difference. With this arrangement the effect of the resistance of the leads can be eliminated. This resistor is of the double-walled or Thomas-type. A bifilar coil of manganin wire is wound on a tubular form hermetically sealed in dry air within the outer container. The series of holes near the top are above the sealed space and facilitate cooling by an oil bath. Since 1939 a bank of 10 similar resistors has been used to maintain the ohm at the National Bureau of Standards. Resistance was calibrated by comparison with the reactance at a known frequency of a large inductor whose self-inductance could be calculated from its dimensions and the assigned permeability of free space μ_0. However, resistance standards are now calibrated by comparison with the reactance of a calculable capacitor, a method made feasible in 1956 by the Thompson-Lampard theorem of electrostatics. Only a single measurement of length is required. (The permittivity of free space ϵ_0 must also be known but this is given by $\epsilon_0 = 1/\mu_0 c^2$.) According to a determination made in 1974 the NBS ohm was equivalent to 0.999 999 18 absolute ohm with an uncertainty of ± 0.06 part per million. (National Bureau of Standards photo.)

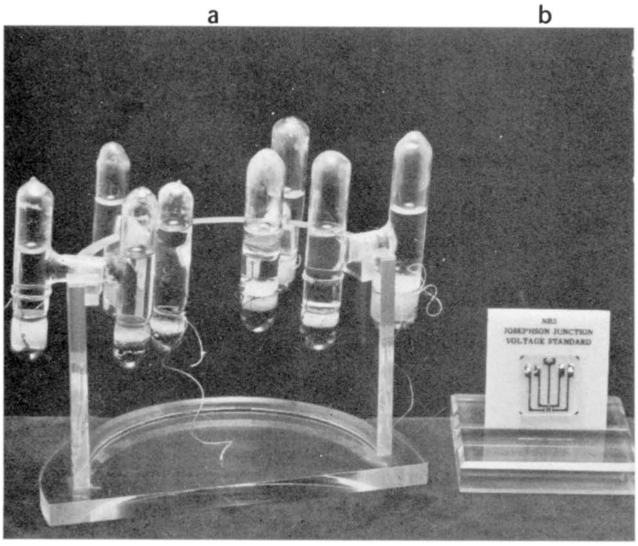

Fig. 18. Standards of voltage. (a) On the left are several Weston saturated cadmium sulfate standard cells. Between 1910 and 1972 the U.S. legal volt was maintained at the National Bureau of Standards by a large group of similar cells, called the National Reference Group (NRG), kept at constant temperature in an oil bath. In 1969, when the internationally accepted value for the Weston cell was fixed at 1.018 328 6 V at 20°C, the NRG consisted of 44 Weston cells. (b) On the right is a Josephson junction device on a 2.54-cm square glass substrate. The middle and outer strips are an evaporated film of lead covered by a layer of lead oxide approximately 1 nm thick. The strips in between are a second film of lead. Four Pb-PbO-Pb Josephson junctions are formed at the bottom where the strips cross. In operation the device is mounted near one end of a waveguide and the apparatus is submerged in liquid helium at a temperature of about 1.2 K in a liquid nitrogen/liquid helium cryostat. At this temperature the lead films assume the superconducting state. When the Josephson junctions are irradiated by microwaves an ac current of electron pairs is induced to tunnel through the lead oxide insulating layer with a frequency equal to the frequency f of the radiation. The current-voltage characteristic is a step function in which the dc bias current increases discontinuously at discrete voltage intervals given by $V = (2e/h)^{-1} f$, where $2e$ is the charge of an electron pair and h is Planck's constant. By comparison with the mean emf of the National Reference Group of standard cells, the value of $2e/h$ was found to be 483.593 420 THz per NBS volt. This value was adopted as the basis for the U.S. legal volt at the NBS on July 1, 1972. (National Bureau of Standards photo.)

The ampere is defined in terms of the force of attraction experienced between two parallel, infinitely long current-bearing wires. In practice, however, a primary calibration experiment involves the measurement of the force between two coils of a current balance (Fig. 16). Although the base unit for electricity is the ampere, electrical units are realized through the volt and the ohm. A standard resistance coil can be calibrated by comparison with the reactance of an inductor or a capacitor, whose values are computed from their dimensions, at a known frequency (Fig. 17). The customary method of maintaining the electrical units has thus been through banks of standard cells and standard resistors (Fig. 18a).

During the past ten years the ac Josephson effect has been employed as a standard for voltage. A Josephson junction reduces the measurement of voltage to a measurement of frequency through a precisely known proportionality constant (Fig. 18b). It is likely that eventually a new, indirect definition of the ampere will be adopted based on this technique. (Some metrologists believe that the volt might even replace the ampere as the fourth base unit.)

The International System of Units

By 1948 the International Bureau of Weights and Measures and its organs were responsible for the units and standards of length, mass, electricity, photometry, temperature (Fig. 19), and radioactivity. At this time the International Union of Pure and Applied Physics and various other scientific organizations requested the General Conference on Weights and Measures to develop a comprehensive MKS system of units for international use. The plan called for the definition of appropriate base units and derived units. Derived units had not been considered previously because they do not require separate standards.

The Tenth General Conference selected a meter-kilogram-second-ampere-kelvin-candela system of units in 1954, considering the advice of the IAU for the second and the IEC for the ampere. In 1960 the Eleventh General Conference completed the task and introduced the name "International System of Units (SI)." During this period the metric unit definitions, symbols, and terminology were extensively revised. In addition, the radian and the steradian were placed in a special class of supplementary units. The mole was added as a seventh base unit in 1971. SI has rapidly become the one system of units advocated for universal use.

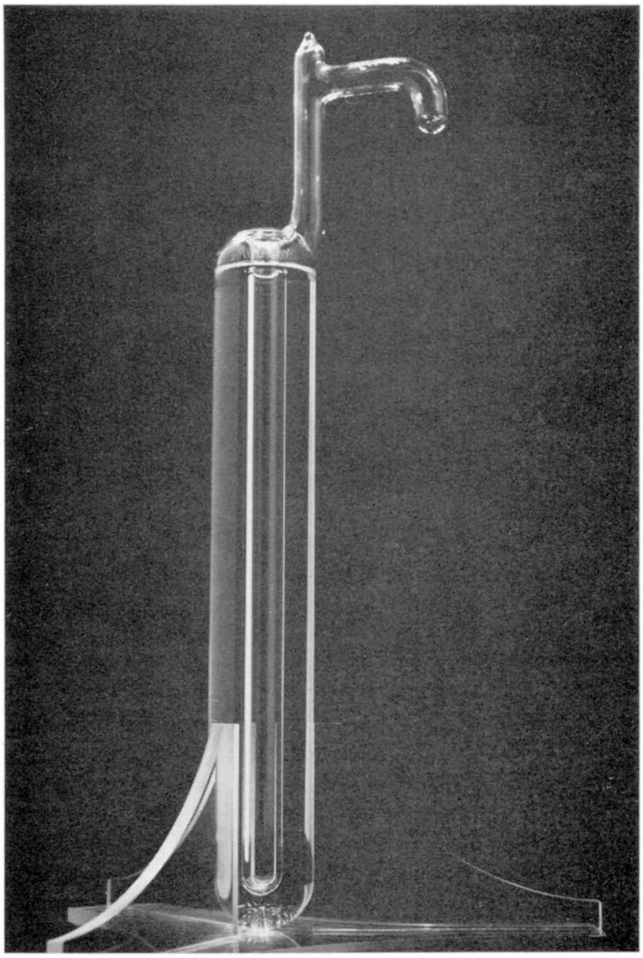

Fig. 19. Triple-point cell. This apparatus is a sealed vessel containing pure water and no air. It is immersed in a water-ice bath and is used to establish the triple point of water, whose temperature is defined as 273.16 K (0.01°C). At this point ice, water, and water vapor coexist (the vapor pressure is only 610.6 Pa, or 4.58 mmHg). The central well may hold a thermometer or thermocouple to be calibrated. (National Bureau of Standards photo.)

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