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ACCA-UK/BR Second International Workshop

Tiago Anselmo, Dept of Physics, UFPE, Brazil

"Schwarz-Christoffel accessory parameter for quadrilaterals via isomonodromy"

We exploit monodromy features associated to conformal mappings from the UHP to polycircular arc domains in order to determine, as a "zero curvature" limit, the accessory parameter of Schwarz-Christoffel mappings to four-sided polygons. The Painlevé VI tau function plays a prominent role in relating the accessory parameter to the geometry of the target domain. We show that this new method explicitly yields the well known result for the aspect ratio of rectangles as a function of the accessory parameter while the relevant tau function assumes a closed form in terms of special functions – the Picard solution. To demonstrate the precision of the method when there is no closed formula for the tau function available, we use asymptotic expansions to calculate the conformal modules of trapezoids and compare the results with tabulated values. [Joint work with R. Nelson, B. Carneiro da Cunha, e D. Crowdy]

Bruno Carneiro da Cunha, Dept of Physics, UFPE, Brazil

"The isomonodromic approach for constructing conformal maps"

We show that the isomonodromic tau function defined for isomonodromic flows can be used to find the accessory parameters of the Fuchsian differential equation satisfied by the uniformizing map of a wide class of domains in the complex plane. The conditions are stated in terms of the geometric information obtained from the graphical representation of the domain. Moreover, a systematic algorithm for construction of these maps can be implemented from the proposed expansion of the relevant tau function. Specializing to domains with four vertices linked by circular arcs, we find a novel application for the Painlev VI tau function, and solve for a family of vibration modes (quasi-normal modes) of black holes in five-dimensional anti-de Sitter space.

Giovani Vasconcelos, Dept of Physics, Federal University of Paraná, Brazil

"Exact solutions for the unsteady motion of multiple bubbles in a Hele-Shaw cell"

The unsteady motion of a finite assembly of bubbles in a Hele-Shaw channel is analyzed in the case when surface tension is neglected. A general exact solution is obtained in terms of a conformal map from a multiply connected circular domain to the fluid region exterior to the bubbles. The corresponding mapping function is given explicitly in terms of certain special transcendental functions known as the secondary Schottky-Klein prime functions. These solutions demonstrate that all bubbles reach an asymptotic steady velocity, U, which is precisely twice greater than the velocity, V, of the uniform background flow, i.e., U = 2V. The result does not depend on the number of bubbles. This confirms the prediction that the velocity selection does not require surface tension.

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Matt Turner, University of Surrey, UK

"Numerical simulation of fluid sloshing in rectangular vessels using conformal mappings"

This talk gives an overview of how time-dependent conformal mappings can help us to devise computationally efficient numerical methods for studying the sloshing of an inviscid, irrotational fluid in a rectangular vessel. An outline of the method is presented for the case of infinite-depth and finite-depth fluids in order to discuss the advantages of this method to other numerical approaches, and results presented. Finally, work is presented which utilizes maps to and from multiply-connected domains via the Schottky-Klein prime function to simulate fluid sloshing in a vessel with infinitely thin, horizontal baffles connected to the side walls. Details of this map are given for the case of a pre-described time-dependent free-surface and results of the map presented.

Nick Moore, Florida State University, USA

"Riemann-Hilbert problems to link flow-driven erosion, dissolution, and melting"

A variety of landscapes are formed by the action of flowing fluids, either air or water. In these settings, the development of morphology is a reciprocal process: as a structure is carved by a fluid, its changing shape alters the local flow. In the laboratory we can examine this coevolution of shape and flow by immersing erodible or soluble bodies in fast-flowing water. I will discuss a simplified Prandtl model that accounts for the vanishing rates and emergent shapes observed in the experiments. In particular, determining the terminal shape can be posed as a singular Riemann-Hilbert problem, and this analysis links the different processes of erosion, dissolution, and melting. Time permitting, I will also discuss new work on erosion of multiple bodies in the Stokes regime and possible porous-media applications.

Eric Keaveny, Imperial College London

"Linking individual and collective motion of swimming cells"

Many swimming cells and microorganisms propel themselves by periodically deforming flexible appendages. These deformations generate flows in the surrounding fluid that lead to interactions with neighbouring cells. In this talk, I will present results from simulations of collections of swimmers that resolve the time- and length-scales associated with cell deformation and I will discuss how they impact longer-time suspension dynamics. In particular, we find that the details of how the individual swimming bodies deform themselves can greatly impact suspension-scale dynamics leading to clustering or aggregation when the swimmers are nearly synchronized, or a turbulence-like state if the deformation frequencies differ substantially from one swimmer to another.

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Elisabeth Guazzelli, Aix Marseille Univ, CNRS, IUSTI, Marseille, France "Falling clouds of particles"

The dispersion of a collection of particles is relevant to many natural phenomena such as pyroclastic flows (i.e. fast-moving currents of hot gas and rock fragments produced by a volcanic eruption), turbidity currents (i.e. sediment-laden flows down a slope usually in lakes and oceans) and the mixing and spreading of pollutants in lakes and oceans. Here, we consider the motion of a cloud of particles settling because of gravity in an otherwise pure liquid and enquire about its following time evolution. A cloud composed of solid spherical particles falling under gravity in a quiescent fluid can be regarded as an effective medium of excess mass and the problem can be related to that of the sedimentation of a spherical drop of heavy fluid in an otherwise lighter fluid. However, the cloud is unstable even in the complete absence of inertia and without needing to perturb its initial shape. It has been observed first to remain roughly spherical with a leakage of particles in a vertical tail and then to evolve into a torus which breaks up into two droplets in a repeating cascade. The discrete nature of the particles is fundamental in the understanding of these instabilities. Simple simulations using a point-particle approach, which contains the minimal physics of the long-range interactions, capture this dynamics. Faster breakup is observed for clouds of anisotropic particles such as fibers, due to the self motion of the anisotropic particles. When inertia is finite, the cloud also deforms into a flat torus that eventually destabilizes and breaks up into a number of secondary droplets but particle leakage is much weaker. While this evolution resembles that observed in the Stokes regime, the physical mechanisms involved are qualitatively different. The cloud evolution can be strongly determined by the importance of wake-mediated interactions. If the cloud settles now in a flowing fluid instead of a quiescent fluid, there is a coupling between particle-particle and particlefluid interactions. This two-way coupling is evidenced for a cloud settling in a cellular flow field which is a simple model flow capturing key features of vortical effects on particles. This work has been done in collaboration with L. Bergougnoux, G. Bouchet, J. E. Butler, M. L. Ekiel-Jezewska, D. Lopez, B. Marchetti, B. Metzger, M. Nicolas, J. Park, F. Pignatel.]

Related references:

Ekiel-Jezewska M. L., Metzger B., and Guazzelli E. 2006 Spherical cloud of point particles falling in a viscous fluid, Phys. Fluids, 18, 038104.

Metzger B., Nicolas M., and Guazzelli E.2007 Falling clouds of particles in viscous fluids, J. Fluid Mech. 580, 283-301.

Park J., Metzger B., Guazzelli E., and Butler J.E. 2010 A cloud of rigid fibres sedimenting in a viscous fluid, J. Fluid Mech. 648, 351-362.

Pignatel F., Nicolas M, and Guazzelli E. 2011 A falling cloud of particles at a small but finite Reynolds-number, J. Fluid Mech. 671, 34-51.

Bergougnoux L., Bouchet G., Lopez D., and Guazzelli E. 2014 The motion of solid spherical particles falling in a cellular flow field at low Stokes number, Phys. Fluids 26, 093302.